

# Power extraction in regular and random waves from an OWC in hybrid wind-wave energy systems

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## Abstract

A mathematical model is developed to analyse the hydrodynamics of a novel oscillating water column (OWC) in a hybrid wind-wave energy system. The OWC has a coaxial cylindrical structure in which the internal cylinder represents the mono-pile of an offshore wind turbine while the external cylinder has a skirt whose scope is to guide the wave energy flux inside the chamber. This layout is not casual, but consistent with the current approach to harnessing wave energy through hybrid systems. The device shape is rather complex and the boundary value problem is solved by applying the matching-method of eigenfunctions. Within the framework of a linearised theory, we model the turbine damping effects by assuming the airflow to be proportional to the air chamber pressure. Consequently, the velocity potential can be decomposed into radiation and diffraction problems. We study the effects of both skirt and internal radius dimensions on the power extraction efficiency for monochromatic and random waves. We show that the skirt has strong effects on the global behaviour, while the internal cylinder affects the values of the sloshing eigenfrequencies. Finally, we validate the analytical model with laboratory data and show a good agreement between analytical and

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experimental results.

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## 1. Introduction

The oscillating water column (OWC) is one of the most studied devices to extract energy from water waves. For an extensive review concerning related theories and experiments we refer to the works of McCormick (1981), Falnes (2002), Babarit et al. (2012) and Babarit (2018). Substantially, the OWC is a partially-immersed structure open at its bottom that confines air above the internal fluid free-surface. Incident waves induce oscillations inside the chamber, thus the airflow is forced to pass through a turbine usually located at the top of the OWC. The turbine is coupled to a generator to produce electricity. In this paper we perform a novel analysis of a coaxial cylindrical OWC in a hybrid wind-wave energy system.

Analytical theories concerning immersed floating structures open at one end have been applied in several contexts. One of the main contributions is due to Garrett (1970), who examined cylindrical bottomless harbours. Other authors such as Mavrakos (1985) analysed the effects of the wall thickness of a floating cylindrical body on the diffracted wave field. Extension of hydrodynamical theories to OWC devices includes the analysis of both bi-dimensional (Evans, 1982; Sarmiento and Falcão, 1985) and three-dimensional configurations. For example, Evans and Porter (1997) were the first to solve the case of a partially immersed cylindrical OWC in open sea by applying the Galerkin method to integral equations.

Several recent studies consider the OWC to be installed in fixed structures for coastal protection. This is mainly due to economical reasons and difficulties in developing wave energy absorbers on their own. Concerning analytical models for OWCs combined with external structures, Martin-Rivas and Mei (2009a) and Martin-Rivas and Mei (2009b) solved the linearised problem of an OWC at the tip of a breakwater and the case of an OWC installed on a straight coast. More recently, Lovas et al. (2010) extended the theory of Martin-Rivas and Mei (2009b) to examine the hydrodynamic wave field when the OWC is installed in correspondence of convex or concave corners, Deng et al. (2013) and Deng et al. (2014) took into account the presence of a coaxial supporting structure to examine possible benefits on the energy

33 conversion efficiency, while Zhou et al. (2018) solved the case of a concentric  
34 axisymmetric OWC including an internal mono-pile. Wave-structure inter-  
35 action in hybrid wave farms with OWC devices was investigated by Zheng  
36 et al. (2018), while recently, breakwater-integrated OWCs were studied by  
37 means of a semi-analytical model by Zheng et al. (2019).

38 Despite the large number of theories developed so far, the wave energy  
39 sector is suffering from slow technological progress combined with difficulties  
40 in attracting funds (Magagna and Uihlein, 2015). This is mainly due to  
41 the large levelised cost of energy (LCOE) (Astariz et al., 2015, 2016). One  
42 way to attract funds and confidence in industry is to combine both wind  
43 and wave energy technologies. This is a recent research effort that aims to  
44 develop a more sustainable and affordable device to extract clean energy  
45 against fossil fuels. We refer to the work of Perez-Collazo et al. (2015) for an  
46 extensive review of alternatives that combine wave and offshore wind energy  
47 technologies.

48 Motivated by this recent technology concept, Perez-Collazo et al. (2018a)  
49 and Perez-Collazo et al. (2018b) tested a 1:50 novel hybrid wind-wave system  
50 that integrates a skirt, a cylindrical OWC and an offshore wind turbine on  
51 a jacket frame structure. The Authors investigated experimentally the hy-  
52 drodynamic response of the device to monochromatic and random waves for  
53 different orifice diameters simulating different air turbines. Promising results  
54 were obtained; however further analytical work is needed to understand the  
55 influence of the device parameters on the global hydrodynamic behaviour of  
56 the system.

57 For all these reasons, in this paper we develop a mathematical model for  
58 the OWC designed by Perez-Collazo et al. (2018b) by adding a concentric  
59 cylinder that represents a wind turbine installed on a mono-pile. This system  
60 appears similar to the theoretical models already solved by the Authors pre-  
61 viously mentioned. However, the skirt connected to the OWC removes any  
62 axial symmetry and the boundary value problem increases in complexity.

63 Here we apply an eigenfunction expansion method and solve the corre-  
64 sponding velocity potentials in terms of Bessel functions and modified Bessel  
65 functions of the first and second kind, respectively (Linton and McIver, 2001;  
66 Mei et al., 2005). First, we analyse the case of monochromatic waves and  
67 derive several integral relations based on the Green's theorem which can be  
68 useful to check the numerical computations of the radiation and diffraction  
69 velocity potentials. Then we show that both the skirt and the internal cylin-  
70 der play an important role in the power extraction and the sloshing dynamics

71 inside the air chamber.

72 Next, we extend the theory to the case of random waves described by the  
73 JONSWAP spectrum (Goda, 2000) and characterise the power extraction  
74 efficiency by applying the superposition principle to the different incident  
75 wave frequency components. We find that the resonant peaks related to  
76 the Helmholtz and sloshing modes decrease in intensity with respect to the  
77 monochromatic case, and that the the random waves have a broadening effect  
78 on the capture factor curve. Interestingly, similar results were also found  
79 in the context of oscillating wave surge converters (Michele et al., 2016a,b;  
80 Sarkar et al., 2014).

81 Laboratory experimental models usually simulate the damping effects of  
82 a turbine by means of an orifice of a certain diameter connecting the air  
83 chamber with the atmosphere (Perez-Collazo et al., 2018b). In this case,  
84 the air turbine is of the impulse type; therefore, a quadratic relation exists  
85 between the airflow through the orifice and the pressure head between the  
86 air chamber and the atmosphere, thus the boundary condition on the free  
87 surface becomes nonlinear (Pereiras et al., 2015; López et al., 2016, 2014).  
88 We then non-dimensionalise the corresponding equation by using adequate  
89 scales (Michele et al., 2018, 2019a; Michele and Renzi, 2019; Sammarco et  
90 al., 1997a,b) and apply a perturbation technique to the velocity potential.  
91 We show that if the ratio between the orifice and OWC diameter is not very  
92 small, the air pressure inside the chamber and the corresponding airflow  
93 through the orifice are governed by the diffraction potential at the leading  
94 order. Finally, we compare our analytical model with the 1:50 scale model  
95 in Perez-Collazo et al. (2018b) and show good matching of the theoretical  
96 results with those evaluated experimentally.

## 97 **2. Governing Equations**

98 With reference to figure 1, consider an OWC device embedded in a hybrid  
99 wave-wind energy extraction system. Let us define a Cartesian reference  
100 system with the  $x$  and  $y$ -axes coincident with the undisturbed free-surface  
101 level and the  $z$ -axis pointing vertically upward. The concentric cylindrical  
102 structure of the OWC has inner radius equal to  $R_i$ , while the external radius  
103 corresponds to  $R_e$ . The internal cylinder spans the entire water depth and  
104 is fixed with the horizontal bottom at  $z = -h$ . The external cylinder does  
105 not have constant draught in water but includes a skirt of height  $h_s$  and  
106 corresponding arc-length  $R_e(\theta_2 - \theta_1)$  with  $\theta_2 > \theta_1$ . The remaining part of

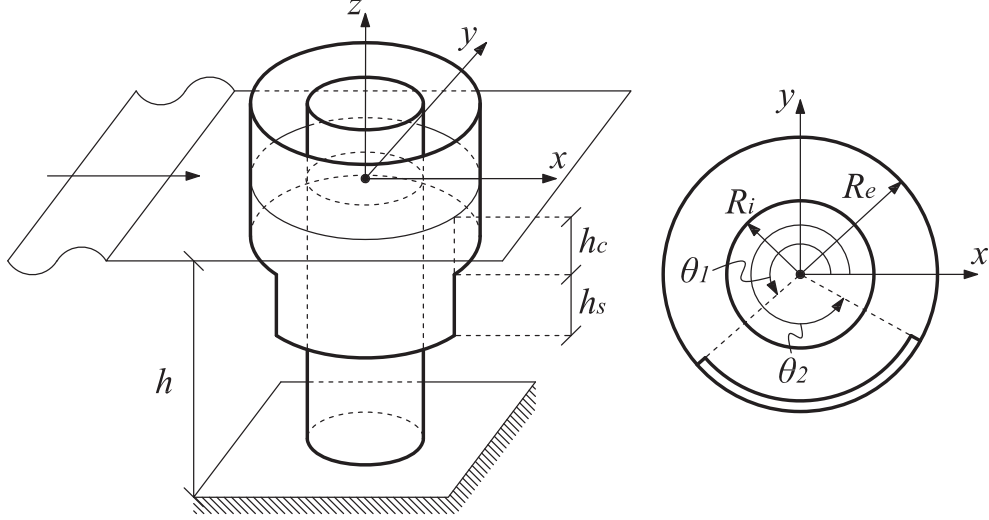


Figure 1: Three-dimensional representation and horizontal cross-section of the hybrid wind-wave energy converter.

107 the external structure has draught equal to  $h_c$ . The chamber is open at  
 108 its base, while at the top an energy conversion system transforms the airflow  
 109 through it in electricity (Falnes, 2002). Let us define the solid wetted surfaces  
 110 of the OWC

$$S_{R_i} = \{r = R_i, \theta \in [0, 2\pi), z \in [-h, 0]\}, \quad (1)$$

$$S_{R_e} = \{r = R_e, \theta \in [0, 2\pi), z \in [-h_c, 0]\} \\ \cup \{r = R_e, \theta \in [\theta_1, \theta_2], z \in [-h_c - h_s, -h_c]\}, \quad (2)$$

111 and the fluid surface  $S_f$  representing the gap under the OWC,

$$S_f = \{r = R_e, \theta \in [0, 2\pi), z \in [-h, 0]\} \setminus S_{R_e}, \quad (3)$$

112 where  $r = \sqrt{x^2 + y^2}$  represents the radial coordinate and  $\theta$  is the angu-  
 113 lar coordinate positive anticlockwise. Moreover, let us define the following  
 114 surfaces

$$S_i = \{r \in [R_i, R_e], \theta \in [0, 2\pi), z = 0\}, \quad (4)$$

$$115 \quad S_e = \{r \in [R_e, \infty), \theta \in [0, 2\pi), z = 0\}, \quad (5)$$

116 where  $S_i$  denotes the free surface inside the chamber, while  $S_e$  is the external  
 117 free surface in contact with air at constant atmospheric pressure. The fluid

118 is inviscid and incompressible, while the fluid flow can be assumed to be irro-  
 119 tational. Then the governing equation for the velocity potential  $\Phi(x, y, z, t)$   
 120 satisfies the Laplace equation in the fluid domain  $\Omega(x, y, z)$ . On the free  
 121 surfaces  $S_e$  and  $S_i$  we have the linearised kinematic condition

$$\zeta_t = \Phi_z, \quad z = 0, \quad (6)$$

122 and the linearised mixed boundary conditions (Mei et al., 2005)

$$\Phi_{tt} + g\Phi_z = 0, \quad \text{on } S_e, \quad (7)$$

$$\Phi_{tt} + g\Phi_z = -\frac{P_{at}}{\rho}, \quad \text{on } S_i, \quad (8)$$

123 where  $\zeta$  represents the free-surface elevation,  $g$  is the acceleration due to  
 124 gravity,  $P_a$  denotes the oscillating pressure of the air inside the chamber  
 125 depending on time  $t$  and  $\rho$  is the water density. We further require tangential  
 126 fluid velocity at the bottom and on the solid surfaces, hence

$$\Phi_n = 0, \quad \text{on solid boundaries}, \quad (9)$$

127 where  $n$  denotes the normal derivative to the relevant surface. The problem is  
 128 forced by monochromatic incident waves of frequency  $\omega$ , hence let us assume  
 129 harmonic motion

$$\{\Phi, \zeta, P_a\} = \text{Re} \{(\phi, \eta, p_a) e^{-i\omega t}\}, \quad (10)$$

130 with  $i$  being the imaginary unit. We shall now write the Laplace equation  
 131 and the boundary conditions (6)-(9) in terms of the spatial variables  $(\phi, \eta, p_a)$   
 132 only. Thus we get

$$\nabla^2 \phi = 0, \quad \text{in } \Omega, \quad (11)$$

$$-i\omega\eta = \phi_z, \quad z = 0, \quad (12)$$

$$-\omega^2 \phi + g\phi_z = 0, \quad \text{on } S_e, \quad (13)$$

$$-\omega^2 \phi + g\phi_z = i\omega \frac{p_a}{\rho}, \quad \text{on } S_i, \quad (14)$$

$$\phi_n = 0, \quad \text{on solid boundaries}. \quad (15)$$

133 Finally we require that the velocity potential  $\phi$  be outgoing for  $r \rightarrow \infty$ .

134 Following the method of Evans and Porter (1997) we decompose the ve-  
 135 locity potential in two parts, i.e.

$$\phi = \phi^D + \phi^R, \quad (16)$$

136 where  $\phi^D$  is the diffraction potential satisfying the boundary conditions (12)-  
 137 (15) with  $p_a = 0$  and  $\phi^R$  is the radiation potential that satisfies the same  
 138 conditions with the unknown forcing pressure  $p_a \neq 0$ . Let us decompose  
 139 the fluid domain  $\Omega$  by defining  $\Omega_i$  and  $\Omega_e$ , respectively, as the internal and  
 140 external fluid subdomains:

$$\Omega_i = \{r \in (R_i, R_e), \theta \in [0, 2\pi), z \in (-h, 0)\}, \quad (17)$$

141

$$\Omega_e = \{r \in (R_e, \infty), \theta \in [0, 2\pi), z \in (-h, 0)\}, \quad (18)$$

142 and let  $\phi_i^D$  ( $\phi_i^R$ ) be the diffraction (radiation) potential in  $(r, \theta, z) \in \Omega_i$  and  
 143  $\phi_e^D$  ( $\phi_e^R$ ) the diffraction (radiation) potentials in  $(r, \theta, z) \in \Omega_e$ .

144 The boundary value problem for the external velocity potentials  $\phi_e^{D,R}$  is

$$\nabla^2 \phi_e^{D,R} = 0, \quad \text{in } \Omega_e, \quad (19)$$

$$-\omega^2 \phi_e^{D,R} + g \phi_{e_z}^{D,R} = 0, \quad \text{on } S_e, \quad (20)$$

$$\phi_{e_r}^{D,R} = 0, \quad \text{on } S_{R_e}, \quad (21)$$

$$\phi_{e_z}^{D,R} = 0, \quad z = -h \quad (22)$$

$$\phi_e^{D,R} = \phi_i^{D,R}, \quad \text{on } S_f, \quad (23)$$

$$\phi_{e_r}^{D,R} = \phi_{i_r}^{D,R}, \quad r = R_e, \quad (24)$$

145 where the conditions (23)-(24) represent respectively continuity of the po-  
 146 tential (pressure) and of the velocity field between the external velocity po-  
 147 tentials  $\phi_e^{D,R}$  and the internal velocity potentials  $\phi_i^{D,R}$ . The boundary value  
 148 problem for the internal velocity potentials  $\phi_i^{D,R}$  is governed by

$$\nabla^2 \phi_i^{D,R} = 0, \quad \text{in } \Omega_i, \quad (25)$$

$$-\omega^2 \phi_i^D + g \phi_{i_z}^D = 0, \quad \text{on } S_i, \quad (26)$$

$$-\omega^2 \phi_i^R + g \phi_{i_z}^R = i\omega \frac{p_a}{\rho}, \quad \text{on } S_i, \quad (27)$$

$$\phi_{i_r}^{D,R} = 0, \quad \text{on } S_{R_i}, \quad (28)$$

$$\phi_{i_r}^{D,R} = 0, \quad \text{on } S_{R_e}, \quad (29)$$

$$\phi_{i_z}^{D,R} = 0, \quad z = -h, \quad (30)$$

149 and the coupling matching conditions (23)-(24).

150 In the following sections we solve the diffraction and radiation potential in  
 151  $\Omega_e$  and  $\Omega_i$  by integrating the matching conditions on the common boundaries.

### 152 2.1. Diffraction potential solution

153 Let us assume for simplicity incident waves with direction parallel to the  
 154  $x$ -axis and amplitude  $A$ . The generalized angles  $\theta_1$  and  $\theta_2$  can be properly  
 155 modified in order to investigate the effects of oblique incident waves. For the  
 156 sake of example, a skirt described by the angles  $\bar{\theta}_1 = \theta_1 + \alpha$  and  $\bar{\theta}_2 = \theta_2 + \alpha$   
 157 simulates the effects of incoming waves with angle of incidence  $\pi - \alpha$  on the  
 158 same OWC.

159 Use of cylindrical coordinates yields the following general solution for the  
 160 diffraction potential in  $(r, \theta, z) \in \Omega_e$

$$\begin{aligned}
 \phi_e^D = & -\frac{iAg}{\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0 (h+z)}{\cosh k_0 h} \left\{ \mathcal{B}_{n0}^D \frac{\sin n\theta H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)} \Big|_{r=R_e} \right. \right. \\
 & \left. \left. + \cos n\theta \left[ \epsilon_n i^n J_n(k_0 r) + \mathcal{A}_{n0}^D \frac{H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)} \Big|_{r=R_e} \right] \right\} + \\
 & \left. \sum_{l=1}^{\infty} \frac{(\mathcal{A}_{nl}^D \cos n\theta + \mathcal{B}_{nl}^D \sin n\theta) K_n(\bar{k}_l r) \cosh k_l (h+z)}{K_{n_r}(\bar{k}_l r) \Big|_{r=R_e} \cosh k_l h} \right\}, \quad (31)
 \end{aligned}$$

161 while the diffraction potential in  $(r, \theta, z) \in \Omega_i$  can be written as

$$\begin{aligned}
 \phi_i^D = & -\frac{iAg}{\omega} \\
 & \times \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0 (h+z)}{\cosh k_0 h} \left\{ \cos n\theta \left[ \mathcal{C}_{n0}^D \frac{J_n(k_0 r)}{J_{n_r}(k_0 r)} \Big|_{r=R_i} + \mathcal{D}_{n0}^D \frac{Y_n(k_0 r)}{Y_{n_r}(k_0 r)} \Big|_{r=R_i} \right] \right. \\
 & \left. + \sin n\theta \left[ \mathcal{E}_{n0}^D \frac{J_n(k_0 r)}{J_{n_r}(k_0 r)} \Big|_{r=R_i} + \mathcal{F}_{n0}^D \frac{Y_n(k_0 r)}{Y_{n_r}(k_0 r)} \Big|_{r=R_i} \right] \right\} \\
 & + \sum_{l=1}^{\infty} \frac{\cosh k_l (h+z)}{\cosh k_l h} \left\{ \cos n\theta \left[ \mathcal{C}_{nl}^D \frac{I_n(\bar{k}_l r)}{I_{n_r}(\bar{k}_l r)} \Big|_{r=R_i} + \mathcal{D}_{nl}^D \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)} \Big|_{r=R_i} \right] \right. \\
 & \left. + \sin n\theta \left[ \mathcal{E}_{nl}^D \frac{I_n(\bar{k}_l r)}{I_{n_r}(\bar{k}_l r)} \Big|_{r=R_i} + \mathcal{F}_{nl}^D \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)} \Big|_{r=R_i} \right] \right\}. \quad (32)
 \end{aligned}$$



162 In the latter expressions,  $A$  denotes the amplitude of the incident waves,  $\epsilon_n$   
 163 is the Jacobi symbol defined as

$$\epsilon_0 = 1, \quad \epsilon_n = 2 \quad n = 1, \dots, \infty, \quad (33)$$

164 the terms  $k_l$ 's are the roots of the dispersion relation (Mei et al., 2005)

$$\left. \begin{aligned} \omega^2 &= gk_0 \tanh k_0 h, \\ \omega^2 &= -g\bar{k}_l \tan \bar{k}_l h, \quad k_l = i\bar{k}_l, \quad l = 1, \dots, \infty \end{aligned} \right\}, \quad (34)$$

165  $J_n$  and  $Y_n$  are the Bessel functions of order  $n$ ,  $H_n^{(1)}$  is the Hankel function  
 166 of the first kind and order  $n$ ,  $I_n$  and  $K_n$  are the modified Bessel functions  
 167 of order  $n$  and finally  $\mathcal{A}_{nl}^D, \mathcal{B}_{nl}^D, \mathcal{C}_{nl}^D, \mathcal{D}_{nl}^D, \mathcal{E}_{nl}^D, \mathcal{F}_{nl}^D$  are complex constants yet  
 168 unknown. The no-flux condition (28) yields

$$\mathcal{C}_{nl}^D = -\mathcal{D}_{nl}^D, \quad \mathcal{E}_{nl}^D = -\mathcal{F}_{nl}^D, \quad (35)$$

169 thus expression (32) reads now

$$\begin{aligned} \phi_i^D &= -\frac{iAg}{\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0 (h+z)}{\cosh k_0 h} (\cos n\theta \mathcal{D}_{n0}^D + \sin n\theta \mathcal{F}_{n0}^D) \mathcal{T}_n \right. \\ &\quad \left. + \sum_{l=1}^{\infty} \frac{\cosh k_l (h+z)}{\cosh k_l h} (\cos n\theta \mathcal{D}_{nl}^D + \sin n\theta \mathcal{F}_{nl}^D) \mathcal{U}_{nl} \right\}, \end{aligned} \quad (36)$$

170 where

$$\mathcal{T}_n = \frac{Y_n(k_0 r)}{Y_{n_r}(k_0 r)|_{r=R_i}} - \frac{J_n^{(1)}(k_0 r)}{J_{n_r}(k_0 r)|_{r=R_i}}, \quad \mathcal{U}_{nl} = \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)|_{r=R_i}} - \frac{I_n(\bar{k}_l r)}{I_{n_r}(\bar{k}_l r)|_{r=R_i}}. \quad (37)$$

171 Substituting expressions (36) and (31) in the matching condition (24) and  
 172 integrating over  $S_f \cup S_{R_e}$ , gives

$$\epsilon_n i^n J_{n_r}(k_0 r)|_{r=R_e} + \mathcal{A}_{n0}^D = \mathcal{D}_{n0}^D \mathcal{T}_{n_r}|_{r=R_e}, \quad \mathcal{B}_{n0}^D = \mathcal{F}_{n0}^D \mathcal{T}_{n_r}|_{r=R_e}, \quad (38)$$

173

$$\mathcal{A}_{nl}^D = \mathcal{D}_{nl}^D \mathcal{U}_{nl_r}|_{r=R_e}, \quad \mathcal{B}_{nl}^D = \mathcal{F}_{nl}^D \mathcal{U}_{nl_r}|_{r=R_e}. \quad (39)$$

174 The external diffraction potential can be written in terms of the coefficients  
 175  $\mathcal{D}^D$  and  $\mathcal{F}^D$

$$\begin{aligned}
 \phi_e^D = & -\frac{iAg}{\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0 (h+z)}{\cosh k_0 h} \left\{ \cos n\theta \left[ \epsilon_n i^n J_n(k_0 r) + \frac{H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)} \Big|_{r=R_e} \right. \right. \right. \\
 & \times \left. \left. \left. \left( \mathcal{D}_{n0}^D \mathcal{T}_{n_r} |_{r=R_e} - \epsilon_n i^n J_n(k_0 r) |_{r=R_e} \right) \right] + \mathcal{F}_{n0}^D \mathcal{T}_{n_r} |_{r=R_e} \frac{\sin n\theta H_n^{(1)}(k_0 r)}{H_{n_r}^{(1)}(k_0 r)} \Big|_{r=R_e} \right\} \\
 & + \left. \sum_{l=1}^{\infty} \frac{\cosh k_l (h+z)}{\cosh k_l h} \frac{K_n(\bar{k}_l r)}{K_{n_r}(\bar{k}_l r)} \Big|_{r=R_e} \mathcal{U}_{nl_r} |_{r=R_e} \left( \mathcal{D}_{nl}^D \cos n\theta + \mathcal{F}_{nl}^D \sin n\theta \right) \right\}. \tag{40}
 \end{aligned}$$

176 For the sake of brevity, we introduce the following integrals for the vertical  
 177 eigenfunctions

$$\mathbb{I}_{sl}^{(1)} = \int_{-h}^{-hc-hs} \frac{\cosh k_l (h+z) \cosh k_s (h+z)}{\cosh k_l h \cosh k_s h} dz, \tag{41}$$

178

$$\mathbb{I}_{sl}^{(2)} = \int_{-hc-hs}^{-hc} \frac{\cosh k_l (h+z) \cosh k_s (h+z)}{\cosh k_l h \cosh k_s h} dz, \tag{42}$$

179

$$\mathbb{I}_{sl}^{(3)} = \int_{-hc}^0 \frac{\cosh k_l (h+z) \cosh k_s (h+z)}{\cosh k_l h \cosh k_s h} dz, \tag{43}$$

180 and the following integrals involving the angular eigenfunctions

$$cc_{pn} = \int_{\theta_2}^{\theta_1} \cos n\theta \cos p\theta d\theta, \quad ss_{pn} = \int_{\theta_2}^{\theta_1} \sin n\theta \sin p\theta d\theta, \tag{44}$$

181

$$sc_{pn} = \int_{\theta_2}^{\theta_1} \sin n\theta \cos p\theta d\theta, \quad cs_{pn} = \int_{\theta_2}^{\theta_1} \cos n\theta \sin p\theta d\theta, \tag{45}$$

182 whose values can be found straightforwardly.

183 Multiplying the condition (23) by  $\cosh k_s (h + z) \cos p\theta / \cosh k_s h$  and in-  
 184 tegrating over the fluid surface  $S_f$ , yields

$$\begin{aligned}
 & \frac{4\mathbb{I}_{s0}^{(1)} \epsilon_n i^{n+1} \delta_{pn}}{\epsilon_p R_e H_{n_r}^{(1)}(k_0 r) \Big|_{r=R_e}} + \sum_{m=0}^{\infty} \frac{2\text{cc}_{pm} \mathbb{I}_{s0}^{(2)} \epsilon_m i^{m+1}}{R_e \pi H_{m_r}^{(1)}(k_0 r) \Big|_{r=R_e}} = \frac{2\mathcal{D}_{n0}^D \mathbb{I}_{s0}^{(1)} \delta_{pn} \pi \tau_n}{\epsilon_p} \\
 & + \sum_{m=0}^{\infty} \mathbb{I}_{s0}^{(2)} \tau_m (\mathcal{D}_{m0}^D \text{cc}_{mn} + \mathcal{F}_{m0}^D \text{sc}_{pm}) + \sum_{l=1}^{\infty} \frac{2\mathcal{D}_{nl}^D \mathbb{I}_{sl}^{(1)} \delta_{pn} \pi \gamma_{nl}}{\epsilon_p} \\
 & + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathbb{I}_{s0}^{(2)} \gamma_{ml} (\mathcal{D}_{ml}^D \text{cc}_{pm} + \mathcal{F}_{ml}^D \text{sc}_{pm}), \quad s = 0, 1, \dots, p = 0, 1, \dots, \quad (46)
 \end{aligned}$$

185 where  $\delta_{pn}$  is the Kronecker delta, while  $\tau_m$  and  $\gamma_{ml}$  have the following expres-  
 186 sions

$$\tau_m = \mathcal{T}_m \Big|_{r=R_e} - \frac{H_m^{(1)}(k_0 r) \Big|_{r=R_e}}{H_{m_r}^{(1)}(k_0 r) \Big|_{r=R_e}} \mathcal{T}_{m_r} \Big|_{r=R_e}, \quad (47)$$

187

$$\gamma_{ml} = \mathcal{U}_{ml} \Big|_{r=R_e} - \frac{K_m(\bar{k}_l r) \Big|_{r=R_e}}{K_{m_r}(\bar{k}_l r) \Big|_{r=R_e}} \mathcal{U}_{m_l r} \Big|_{r=R_e}. \quad (48)$$

188 Multiplying again the condition (23) by  $\cosh k_s (h + z) \sin p\theta / \cosh k_s h$  and  
 189 integrating over the fluid surface  $S_f$  gives now

$$\begin{aligned}
 & \pi \mathcal{F}_{n0}^D \mathbb{I}_{s0}^{(1)} \delta_{pn} \tau_n + \sum_{m=0}^{\infty} \mathbb{I}_{s0}^{(2)} \tau_m (\mathcal{D}_{m0}^D \text{cs}_{pm} + \mathcal{F}_{m0}^D \text{ss}_{pn}) \\
 & + \sum_{l=1}^{\infty} \pi \mathcal{F}_{nl}^D \mathbb{I}_{sl}^{(1)} \delta_{pn} \gamma_{nl} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathbb{I}_{sl}^{(2)} \gamma_{ml} (\mathcal{D}_{ml}^D \text{cs}_{pm} + \mathcal{F}_{ml}^D \text{ss}_{pm}) = 0. \quad (49)
 \end{aligned}$$

190 An additional condition for the internal diffraction potential can be obtained  
 191 by multiplying  $\phi_{i,r}^D = 0$  respectively by  $\cosh k_s (h + z) \cos p\theta / \cosh k_s h$  and  
 192  $\cosh k_s (h + z) \sin p\theta / \cosh k_s h$ . Integrating over the relevant domain  $S_{R_e}$   
 193 gives us

$$\begin{aligned}
 & \sum_{l=1}^{\infty} \frac{2\pi \mathcal{D}_{nl}^D \mathbb{I}_{sl}^{(3)} \delta_{pn} \mathcal{U}_{nl_r} \Big|_{r=R_e}}{\epsilon_p} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathbb{I}_{sl}^{(2)} \mathcal{U}_{m_l r} \Big|_{r=R_e} (\mathcal{D}_{ml}^D \overline{\text{cc}}_{pm} + \mathcal{F}_{ml}^D \overline{\text{sc}}_{pm}) \\
 & + \frac{2\pi \delta_{pn} \mathbb{I}_{s0}^{(3)} \mathcal{D}_{n0}^D \mathcal{T}_{n_r} \Big|_{r=R_e}}{\epsilon_p} + \sum_{m=0}^{\infty} \mathbb{I}_{s0}^{(2)} \mathcal{T}_{m_r} \Big|_{r=R_e} (\mathcal{D}_{m0}^D \overline{\text{cc}}_{pm} + \mathcal{F}_{m0}^D \overline{\text{sc}}_{pm}) = 0, \quad (50)
 \end{aligned}$$

$$\begin{aligned}
& \pi \delta_{pn} I_{s0}^{(3)} \mathcal{F}_{n0}^D \mathcal{T}_{nr}|_{r=R_e} + \sum_{m=0}^{\infty} I_{s0}^{(2)} \mathcal{T}_{mr}|_{r=R_e} (\mathcal{D}_{m0}^D \overline{\mathcal{C}}_{spm} + \mathcal{F}_{m0}^D \overline{\mathcal{S}}_{spm}) \\
& + \sum_{l=1}^{\infty} \pi \mathcal{F}_{nl}^D I_{sl}^{(3)} \delta_{pn} \mathcal{U}_{nlr}|_{r=R_e} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} I_{sl}^{(2)} \mathcal{U}_{mlr}|_{r=R_e} (\mathcal{D}_{ml}^D \overline{\mathcal{C}}_{spm} + \mathcal{F}_{ml}^D \overline{\mathcal{S}}_{spm}) = 0,
\end{aligned} \tag{51}$$

194 where  $\overline{\mathcal{C}}_c, \overline{\mathcal{S}}_s, \overline{\mathcal{C}}_s, \overline{\mathcal{S}}_c$  are defined as follows

$$\overline{\mathcal{C}}_{pn} = \int_{\theta_1}^{\theta_2} \cos n\theta \cos p\theta \, d\theta, \quad \overline{\mathcal{S}}_{pn} = \int_{\theta_1}^{\theta_2} \sin n\theta \sin p\theta \, d\theta, \tag{52}$$

195

$$\overline{\mathcal{S}}_{pn} = \int_{\theta_1}^{\theta_2} \sin n\theta \cos p\theta \, d\theta, \quad \overline{\mathcal{C}}_{pn} = \int_{\theta_1}^{\theta_2} \cos n\theta \sin p\theta \, d\theta. \tag{53}$$

196 Summation of (46) and (50) plus summation of (49) and (51) gives an  
197 inhomogeneous linear system in the unknown coefficients  $\mathcal{D}^D$  and  $\mathcal{F}^D$ . Once  
198 they are known, the coefficients  $\mathcal{A}^D$  and  $\mathcal{B}^D$  can be obtained through (38)-  
199 (39), while  $\mathcal{C}^D$  and  $\mathcal{E}^D$  are given by (35).

## 200 2.2. Radiation potential solution

201 The problem is linear, hence the radiation velocity potential outside and  
202 inside the OWC can be assumed proportional to the pressure acting on the  
203 free surface  $S_i$ . The general solutions are similar to (31)-(32):

$$\begin{aligned}
\phi_e^R = & -\frac{ip_a}{\rho\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0(h+z)}{\cosh k_0 h} \frac{H_n^{(1)}(k_0 r)}{H_{nr}^{(1)}(k_0 r)|_{r=R_e}} (\mathcal{A}_{n0}^R \cos n\theta + \mathcal{B}_{n0}^R \sin n\theta) \right. \\
& \left. + \sum_{l=1}^{\infty} \frac{\cosh k_l(h+z)}{\cosh k_l h} \frac{K_n(\bar{k}_l r)}{K_{nr}(\bar{k}_l r)|_{r=R_e}} (\mathcal{A}_{nl}^R \cos n\theta + \mathcal{B}_{nl}^R \sin n\theta) \right\}, \tag{54}
\end{aligned}$$

$$\begin{aligned}
\phi_i^R = & -\frac{ip_a}{\rho\omega} \sum_{n=0}^{\infty} \left\{ \frac{\cosh k_0(h+z)}{\cosh k_0 h} \mathcal{T}_n (\mathcal{D}_{n0}^R \cos n\theta + \mathcal{F}_{n0}^R \sin n\theta) \right. \\
& \left. + \sum_{l=1}^{\infty} \frac{\cosh k_l(h+z)}{\cosh k_l h} \mathcal{U}_{nl} (\mathcal{D}_{nl}^R \cos n\theta + \mathcal{F}_{nl}^R \sin n\theta) \right\} - \frac{ip_a}{\rho\omega}, \tag{55}
\end{aligned}$$

204 except for the forcing term that takes into account for the pressure  $p_a$  on  $S_i$ .

205 Continuity of the fluid velocity across the cylindrical surface  $r = R_e$  yields

$$\mathcal{A}_{n0}^R = \mathcal{D}_{n0}^R \mathcal{T}_{n_r}|_{r=R_e}, \quad \mathcal{B}_{n0}^R = \mathcal{F}_{n0}^R \mathcal{T}_{n_r}|_{r=R_e}, \quad (56)$$

206

$$\mathcal{A}_{nl}^R = \mathcal{D}_{nl}^R \mathcal{U}_{nl_r}|_{r=R_e}, \quad \mathcal{B}_{nl}^R = \mathcal{F}_{nl}^R \mathcal{U}_{nl_r}|_{r=R_e}, \quad (57)$$

207 while multiplying the condition (23) by  $\cosh k_s (h + z) \cos p\theta / \cosh k_s h$  and

208 integrating over  $S_f$  gives

$$\begin{aligned} & -2\pi\delta_{p0} \frac{\sinh k_s (h - h_c - h_s)}{k_s \cosh k_s h} - \text{cc}_{p0} \frac{\sinh k_s (h - h_c) - \sinh k_s (h - h_c - h_s)}{k_s \cosh k_s h} = \\ & \frac{2\mathcal{D}_{n0}^R \mathcal{I}_{s0}^{(1)} \delta_{pn} \pi \tau_n}{\epsilon_p} + \sum_{m=0}^{\infty} \mathcal{I}_{s0}^{(2)} \tau_m (\mathcal{D}_{m0}^R \text{cc}_{pm} + \mathcal{F}_{m0}^R \text{sc}_{pm}) + \sum_{l=1}^{\infty} \frac{2\mathcal{D}_{nl}^R \mathcal{I}_{sl}^{(1)} \delta_{pn} \pi \gamma_{nl}}{\epsilon_p} \\ & + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathcal{I}_{s0}^{(2)} \gamma_{ml} (\mathcal{D}_{ml}^R \text{cc}_{pm} + \mathcal{F}_{ml}^R \text{sc}_{pm}). \end{aligned} \quad (58)$$

209 Similarly, multiplying continuity condition (23) by  $\cosh k_s (h + z) \sin p\theta / \cosh k_s h$

210 and integrating over the surface  $S_f$  yields

$$\begin{aligned} & \pi \mathcal{F}_{n0}^R \mathcal{I}_{s0}^{(1)} \delta_{pn} \tau_n + \sum_{m=0}^{\infty} \mathcal{I}_{s0}^{(2)} \tau_m (\mathcal{D}_{m0}^R \text{cs}_{pm} + \mathcal{F}_{m0}^R \text{ss}_{pm}) \\ & + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathcal{I}_{sl}^{(2)} \gamma_{ml} (\mathcal{D}_{ml}^R \text{cs}_{pm} + \mathcal{F}_{ml}^R \text{ss}_{pm}) + \sum_{l=1}^{\infty} \pi \mathcal{F}_{nl}^R \mathcal{I}_{sl}^{(1)} \delta_{pn} \gamma_{nl} \\ & = \frac{(\cos p\theta_2 - \cos p\theta_1) [\sinh k_s (h - h_c) - \sinh k_s (h - h_c - h_s)]}{pk_s \cosh k_s h}. \end{aligned} \quad (59)$$

211 Finally the condition  $\phi_{i_r}^R = 0$  on  $S_{R_e}$  allows to obtain

$$\begin{aligned} & \frac{2\pi\delta_{pn} \mathcal{I}_{s0}^{(3)} \mathcal{D}_{n0}^R \mathcal{T}_{n_r}|_{r=R_e}}{\epsilon_p} + \sum_{m=0}^{\infty} \mathcal{I}_{s0}^{(2)} \mathcal{T}_{m_r}|_{r=R_e} (\mathcal{D}_{m0}^R \overline{\text{cc}}_{pm} + \mathcal{F}_{m0}^R \overline{\text{sc}}_{pm}) \\ & + \sum_{l=1}^{\infty} \frac{2\pi \mathcal{D}_{nl}^R \mathcal{I}_{sl}^{(3)} \delta_{pn} \mathcal{U}_{nl_r}|_{r=R_e}}{\epsilon_p} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \mathcal{I}_{sl}^{(2)} \mathcal{U}_{ml_r}|_{r=R_e} (\mathcal{D}_{ml}^R \overline{\text{cc}}_{pm} + \mathcal{F}_{ml}^R \overline{\text{sc}}_{pm}) = 0, \end{aligned} \quad (60)$$

$$\begin{aligned}
& \pi \delta_{pn} I_{s0}^{(3)} \mathcal{F}_{n0}^R \mathcal{T}_{n_r}|_{r=R_e} + \sum_{m=0}^{\infty} I_{s0}^{(2)} \mathcal{T}_{m_r}|_{r=R_e} (\mathcal{D}_{m0}^R \overline{\mathcal{C}}_{\mathcal{S}pm} + \mathcal{F}_{m0}^R \overline{\mathcal{S}}_{\mathcal{S}pm}) \\
& + \sum_{l=1}^{\infty} \pi \mathcal{F}_{nl}^R I_{sl}^{(3)} \delta_{pn} \mathcal{U}_{nl_r}|_{r=R_e} + \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} I_{sl}^{(2)} \mathcal{U}_{ml_r}|_{r=R_e} (\mathcal{D}_{ml}^R \overline{\mathcal{C}}_{\mathcal{S}pm} + \mathcal{F}_{ml}^R \overline{\mathcal{S}}_{\mathcal{S}pm}) = 0.
\end{aligned} \tag{61}$$

213 As in the case of the diffraction potential, summation of equation (58) and  
214 (60) and summation of (59) and (61) yield an inhomogeneous linear system  
215 in  $\mathcal{D}^R$  and  $\mathcal{F}^R$ . The remaining constants for the external velocity potential  
216  $\mathcal{A}^R, \mathcal{B}^R$  can be evaluated by applying (56)-(57).

### 217 3. Power extraction efficiency in regular waves and integral rela- 218 tions

219 Within the framework of a linear theory, the relation between the flux  
220 rate  $Q$  through the turbine and the air pressure inside the chamber  $P_a$  can  
221 be defined as follows (Martin-Rivas and Mei, 2009b)

$$q = \left( \frac{KD}{N\rho_a} - \frac{i\omega V_0}{c_a^2 \rho_a} \right) p_a, \tag{62}$$

222 where  $K$  is an empirical coefficient depending on the turbine characteristics,  
223  $D$  is the outer diameter of the turbine rotor,  $\rho_a$  the air density,  $V_0$  the volume  
224 of air in the chamber when  $\eta = 0$ ,  $c_a$  the speed of sound in air, while  $q$   
225 represents the complex part of  $Q$  independent on time, i.e.

$$Q = \text{Re} \{ q e^{-i\omega t} \} = \text{Re} \left\{ e^{-i\omega t} \int_{S_i} \frac{\partial \Phi}{\partial z} dS_i \right\}. \tag{63}$$

226 The flux  $q$  can be further decomposed into radiation and diffraction compo-  
227 nents

$$q = q^D + q^R, \quad q^D = \int_{S_i} \frac{\partial \phi_i^D}{\partial z} dS_i = \Gamma A, \quad q^R = \int_{S_i} \frac{\partial \phi_i^R}{\partial z} dS_i = -(\mathcal{B} - i\mathcal{C}) p_a, \tag{64}$$

228 with  $\Gamma$  being the complex exciting force, while the real quantities  $\mathcal{B}$  and  $\mathcal{C}$   
 229 represent respectively the radiation damping and the added mass due to the  
 230 radiation wave field. The averaged power output over a wave period is

$$\bar{P}_{out} = \frac{KD}{2N\rho_a} |p_a|^2 = \frac{A^2 |\Gamma|^2 KD}{2N\rho_a \left[ \left( \frac{KD}{N\rho_a} + \mathcal{B} \right)^2 + \left( \frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2 \right]}, \quad (65)$$

231 hence the corresponding capture factor can be defined as the ratio between  
 232 the generated power (65) and the energy influx of incident waves with am-  
 233 plitude  $A$  per OWC width  $2R_e$  (Michele et al., 2016b)

$$C_F = \frac{|\Gamma|^2 KD}{2\rho g R_e C_g N\rho_a \left[ \left( \frac{KD}{N\rho_a} + \mathcal{B} \right)^2 + \left( \frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2 \right]}, \quad (66)$$

234 where  $C_g$  is the group velocity (Mei et al., 2005). Maximum efficiency of  
 235 the capture factor (66) can be achieved if both the resonance condition, i.e.  
 236  $(\mathcal{C} - \omega V_0/c_a^2 \rho_a) = 0$ , and the identity  $\mathcal{B} = KD/N\rho_a$  are satisfied. However,  
 237 resonance is not always possible, mainly because of the difficulty in changing  
 238 the structural parameter  $V_0/c_a^2 \rho_a$ . In any case, if  $(\mathcal{C} - \omega V_0/c_a^2 \rho_a) \neq 0$ , the  
 239 optimal damping force exerted by the turbine can be chosen such that the  
 240 derivative of  $C_F$  (66) with respect to  $KD/N\rho_a$  is zero. This condition holds  
 241 if

$$\frac{KD}{N\rho_a} = \sqrt{\mathcal{B}^2 + \left( \frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2}. \quad (67)$$

242 Substitution of the latter expression in (66) yields the optimized capture  
 243 factor  $C_{Fopt}$

$$C_{Fopt} = \frac{|\Gamma|^2 \sqrt{\mathcal{B}^2 + \left( \frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2}}{2\rho g R_e C_g \left[ \left( \sqrt{\mathcal{B}^2 + \left( \frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2} + \mathcal{B} \right)^2 + \left( \frac{\omega V_0}{c_a^2 \rho_a} + \mathcal{C} \right)^2 \right]}. \quad (68)$$

244 If also resonance occurs, expression (68) yields the the maximum value of the  
 245 capture factor  $C_{Fmax}$

$$C_{Fmax} = \frac{|\Gamma|^2}{8\rho g R_e C_g \mathcal{B}}. \quad (69)$$

246 Note that in the case of axisymmetric bodies, i.e. without the skirt, the  
 247 latter relation yields after some algebra

$$C_{Fmax} = \frac{1}{2k_0 R_e}, \quad (70)$$

248 i.e. two times smaller than the maximum that can be reached by an oscil-  
 249 lating flap-type wave surge converter in open sea having width equal to  $2R_e$   
 250 (Michele et al., 2016b). Now we derive several integral relations to perform a  
 251 numerical check of the hydrodynamic quantities  $\Gamma$  and  $\mathcal{B}$ . Applying Green's  
 252 theorem to  $\phi^R$  and its complex conjugate over the entire fluid domain  $\Omega$   
 253 yields

$$\frac{1}{\rho\omega} \text{Re} \left\{ \int_{S_i} p^* \phi_i^R dS_i \right\} = \text{Re} \left\{ \int_{S_\infty} -i\phi_e^R \phi_e^{R*} dS_\infty \right\}, \quad (71)$$

254 where  $(\cdot)^*$  denotes the complex conjugate of  $(\cdot)$  and  $S_\infty$  is a vertical cylinder  
 255 of large radius  $r \rightarrow \infty$  and height  $h$ . The radiation potential  $\phi_e^R$  in the far  
 256 field can be approximated by

$$\phi_e^R \sim \frac{\mathcal{A}(\theta)}{\sqrt{k_0 r}} e^{ik_0 r} \cosh k_0 (h + z), \quad (72)$$

257 in which  $\mathcal{A}(\theta)$  represents the angular variation of the radiated waves at large  
 258 distances. From (54) we get

$$\mathcal{A}(\theta) = -\frac{ip_a}{\rho\omega \cosh k_0 h} \sqrt{\frac{2}{\pi}} \sum_{n=0}^{\infty} \frac{e^{-i(\pi/4+n\pi/2)} (\mathcal{A}_{n0}^R \cos n\theta + \mathcal{B}_{n0}^R \sin n\theta)}{H_{n_r}^{(1)}(k_0 r) \Big|_{r=R_e}}. \quad (73)$$

259 Substitution of (64), (72) and (73) into the integrals (71) yields after some  
 260 algebra

$$\begin{aligned} \mathcal{B} &= \frac{\rho\omega C_0}{|p_a|^2} \int_0^{2\pi} |\mathcal{A}(\theta)|^2 d\theta \\ &= \frac{2D_0}{\rho\omega \cosh^2 k_0 h} \left( \frac{2|\mathcal{A}_{00}^R|^2}{H_{0_r}^{(1)}(k_0 r) \Big|_{r=R_e}} + \sum_{n=1}^{\infty} \frac{|\mathcal{A}_{n0}^R|^2 + |\mathcal{B}_{n0}^R|^2}{H_{n_r}^{(1)}(k_0 r) \Big|_{r=R_e}} \right), \end{aligned} \quad (74)$$

261 where

$$C_0 = \int_{-h}^0 \cosh^2 k_0 (h + z) dz = \frac{2k_0 h + \sinh 2k_0 h}{4k_0}. \quad (75)$$



262 Expression (74) relates the radiation damping and the amplitude of the radi-  
 263 ated waves at large distances and can be used for numerical check purposes.  
 264 Similarly, applying Green's theorem to the radiation and diffraction velocity  
 265 potentials gives

$$\frac{ip}{\rho\omega} \int_{S_i} \phi_{iz}^D dS_i = - \int_{S_\infty} (\phi^I \phi_{er}^R - \phi_e \phi_r^I) dS_\infty, \quad (76)$$

266 where  $\phi^I$  is the velocity potential of the incident waves with amplitude  $A$   
 267 and frequency  $\omega$  directed along the  $x$ -axis

$$\phi^I = -\frac{iAg \cosh k_0(h+z)}{\omega \cosh k_0 h} e^{ik_0 r \cos \theta}. \quad (77)$$

268 By the method of the stationary phase, after a long but straightforward  
 269 algebra we obtain

$$\Gamma = \frac{4gC_0}{\omega \cosh^2 k_0 h} \sum_{n=0}^{\infty} \frac{\mathcal{A}_{n0}^R e^{-in\pi/2} \cos n\pi}{H_{nr}^{(1)}(k_0 r) \Big|_{r=R_e}}. \quad (78)$$

270 This is the Haskind-Hanaoka formula for the OWC in open sea and relates the  
 271 exciting force  $\Gamma$  with the amplitude of the radiated waves in the direction  
 272 opposite to the incoming waves  $\theta = \pi$ . The latter relation is used in the  
 273 next section to check numerical evaluations for the radiation and diffraction  
 274 velocity potentials.

### 275 3.1. Results and discussion

276 In this section we examine the effects of the OWC geometry and tur-  
 277 bine characteristics on the hydrodynamic behaviour and energy extraction  
 278 efficiency. For the sake of example, let us consider the following fixed param-  
 279 eters:  $A = 1$  m,  $h = 10$  m,  $\rho = 1000$  kg m<sup>-3</sup>,  $\rho_a = 1$  kg m<sup>-3</sup> and  $c_a = 340$  m  
 280 s<sup>-1</sup>. Since in the expressions for the velocity potentials (31)-(32)-(54)-(55)  
 281 there are infinite terms, we need to truncate the summations up to a limiting  
 282 value  $n = N$  and  $l = L$  for practical computations. In this work we use  
 283  $N = L = 100$  to achieve a precision of 2 decimal places (Deng et al., 2013).

#### 284 3.1.1. Effects of the skirt height

285 Here we examine the effects of the skirt height  $h_s$  on the exciting force  $\Gamma$ ,  
 286 radiation damping  $\mathcal{B}$  and added mass  $\mathcal{C}$ . Let us fix the following parameters

287  $R_e = h/2$ ,  $\theta_1 = 5\pi/4$  rad,  $\theta_2 = 3\pi/4$  rad and  $h_c = 0.2 \times h$ . The latter  
 288 numerical values are the same adopted in Deng et al. (2013), thus we can  
 289 perform several validations of our numerical results. As in Deng et al. (2013),  
 290 let us define the non-dimensional hydrodynamic quantities

$$\tilde{\Gamma} = \Gamma \frac{\sqrt{g/h}}{gh}, \quad \tilde{\mathcal{B}} = \mathcal{B} \frac{\rho \sqrt{g/h}}{g}, \quad \tilde{\mathcal{C}} = \mathcal{C} \frac{\rho \sqrt{g/h}}{g}, \quad (79)$$

291 and take as a first case  $R_i = 0$ . Figures 2(a)-2(c) shows the behaviour of  $\tilde{\Gamma}$ ,  
 292  $\tilde{\mathcal{B}}$  and  $\tilde{\mathcal{C}}$  versus the non-dimensional frequency of the incident waves  $\omega^2 h/g$   
 293 for five different configurations. Each configuration has a specific value of  
 294 the skirt height, that varies from  $h - h_c$  to zero. The limiting value in which  
 295  $h_s = h - h_c$ , corresponds to the case of an OWC supported by a coaxial  
 296 tube-sector-shaped structure analysed by Deng et al. (2013), while the case  
 297  $h_s = 0$  corresponds to the model developed by Evans and Porter (1997).  
 298 Excellent agreement with the numerical results of Deng et al. (2013) (Fig. 4  
 299 and Fig. 5, case  $d/h = 0.2$ ) is obtained. This test validates the effectiveness  
 300 of the method of solution adopted in this work for the novel device.

301 As in Garrett (1970), Evans and Porter (1997) and Deng et al. (2013),  
 302 resonant interactions of the heave and sloshing modes inside the chamber  
 303 occur. This is the reason why each peak for each hydrodynamic parameter  
 304 is localized at the same frequency. Since  $R_i = 0$ , the resonances appear ap-  
 305 proximately at zeros of the Bessel function  $J'_n(k_0 R_e)$  satisfying the boundary  
 306 condition for the sloshing modes inside a vertical cylinder of radius  $R_e$  and  
 307 height  $h$ .

308 Note that the OWC without the skirt does not excite the sloshing modes  
 309 proportional to  $\cos \theta$  or  $\sin \theta$ . In this case, the first sloshing resonance satisfies  
 310 the second zero of  $J'_0(k_0 R_e)$  which occurs for  $k_0 R_e \simeq 3.831$ . Similar results  
 311 are obtained in Evans and Porter (1997). On the contrary, if  $h_s \neq 0$ , we  
 312 obtain two additional peaks between the Helmholtz mode and the frequency  
 313 corresponding to  $k_0 R_e \simeq 3.831$ . These peaks are related to the firsts roots of  
 314  $J'_1(k_0 R_e)$  and  $J'_2(k_0 R_e)$ .

315 As the height  $h_s$  increases, the first resonant peak of  $\tilde{\Gamma}$ ,  $\tilde{\mathcal{B}}$  and  $\tilde{\mathcal{C}}$  related  
 316 to the Helmholtz mode moves towards small frequencies. In particular, in  
 317 the case of  $\tilde{\Gamma}$  shown in figure 2(a), the first peak tends to become higher and  
 318 sharper as well. On the other hand, the peaks related to the sloshing modes  
 319 seem to be unaffected by  $h_s$ .

320 The optimal capture factor  $C_{F_{opt}}$  (68) is shown in figure 2(d). Within  
 321 this range of  $\omega$ , except for the case without the skirt, four modes are excited

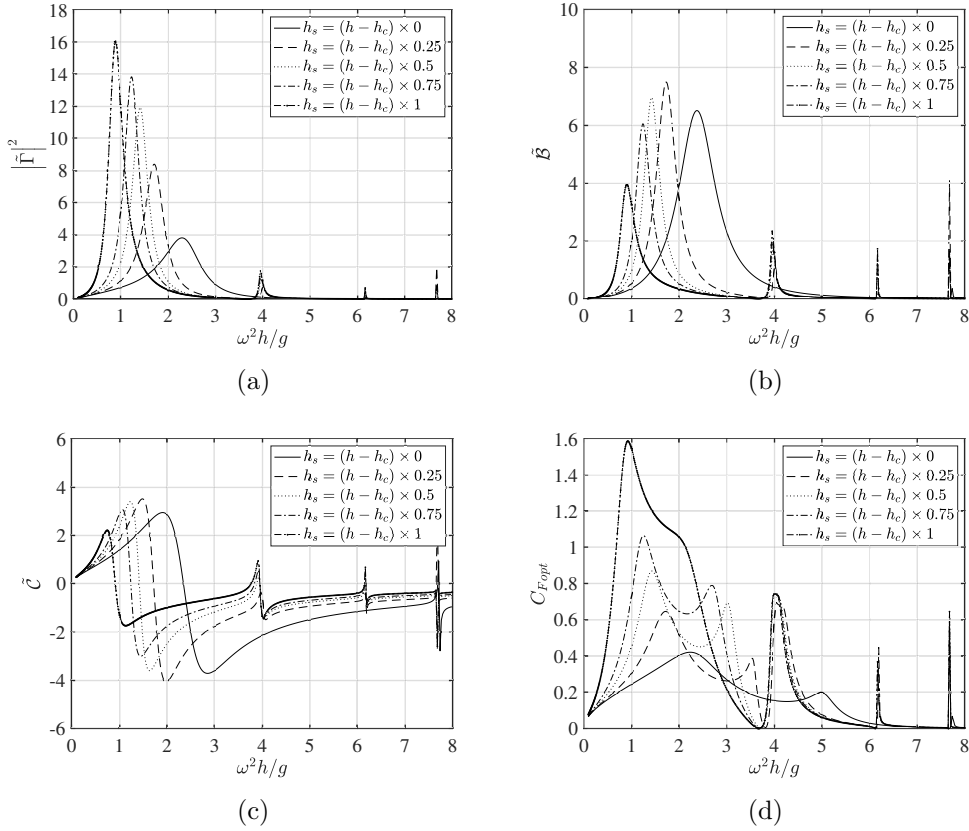


Figure 2: The effects of the skirt height  $h_s$  on the hydrodynamic behaviour. 2(a) non-dimensional exciting force  $|\tilde{\Gamma}|^2$ , 2(b) non-dimensional radiation damping  $\tilde{B}$ , 2(c) non-dimensional added mass  $\tilde{C}$  and 2(d) optimal capture factor  $C_{Fopt}$  of each configuration versus non-dimensional incident wave frequency  $\omega^2 h/g$ .

322 hence four maxima occur for  $C_{Fopt}$ . The same figure shows that the first  
323 resonant peak is the widest and sometimes, depending on  $h_s$ , the largest.  
324 Values of  $C_{Fopt}$  can be larger than 1, i.e. larger than the maximum of a  
325 bi-dimensional absorber in a channel flume (Mei et al., 2005). Note that the  
326 most efficient configuration corresponds to the case of a skirt extending to  
327 the bottom. Next, let the internal radius be  $R_i = 0.75 \times R_e$  and evaluate  
328  $\tilde{\Gamma}$ ,  $\tilde{\mathcal{B}}$ ,  $\tilde{\mathcal{C}}$  and  $C_{Fopt}$  for the same configurations analysed before. Now the  
329 internal radius differs from zero, thus the resonant peaks are associated with  
330 the sloshing modes of an isolated annular cylinder with fluid occupying the  
331 volume  $\Omega_i$ . Since the general solution of the velocity potential includes both  
332  $J_n$  and  $Y_n$  the corresponding wave-number  $k_0$  for each sloshing mode must  
333 satisfy the following eigenvalue condition

$$J_n(k_0 R_i) Y_n(k_0 R_e) - J_n(k_0 R_e) Y_n(k_0 R_i) = 0. \quad (80)$$

334 Figure 3 shows that four peaks are present in the computed range of fre-  
335 quencies. Maxima of all the resonant peaks are almost unaffected, while the  
336 peaks corresponding to the sloshing modes tend to move towards smaller fre-  
337 quencies. Note also that the second and third peaks for  $C_{Fopt}$  increase their  
338 width, hence in this case the presence of an internal radius has benefits in  
339 terms of power extraction efficiency.

### 340 3.1.2. Effects of the skirt opening

341 Now we analyse the effects of the skirt opening  $\theta_2 - \theta_1$  on the same  
342 hydrodynamic parameters analysed in the previous section  $\Gamma$ ,  $\mathcal{B}$  and  $\mathcal{C}$ . Let  
343 the external radius be  $R_e = h/2$  and fix both skirt height  $h_s = 0.5 \times (h - h_c)$   
344 and internal radius  $R_i = 0$ . Five skirt opening angles have been analysed,  
345 respectively described by  $\theta_1 = \theta_2 = \pi$ ,  $\theta_1 = 5\pi/4$  and  $\theta_2 = 3\pi/4$ ,  $\theta_1 = 3\pi/2$   
346 and  $\theta_2 = \pi/2$ ,  $\theta_1 = 7\pi/4$  and  $\theta_2 = \pi/4$ ,  $\theta_1 = 2\pi$  and  $\theta_2 = 0$  rad.

347 Figure 4 shows the effects of the opening angle for different incident wave  
348 non-dimensional frequencies on the hydrodynamic parameters  $\Gamma$ ,  $\mathcal{B}$  and  $\mathcal{C}$   
349 and the optimal capture factor  $C_{Fopt}$ . As shown by figures 4(a)-4(c), when  
350 the opening increases, the Helmholtz mode resonant peaks decrease while  
351 the corresponding resonant frequencies increase. This is less visible for the  
352 sloshing modes whose position is almost unvaried. We shall point out that  
353 similar results are obtained by Deng et al. (2013) for a skirt extending from  
354 the OWC to the sea bottom.

355 Figure 4(d) shows the behaviour of the optimal capture factor  $C_{Fopt}$ . The  
356 best configuration with larger and wider peaks corresponds to the symmetric

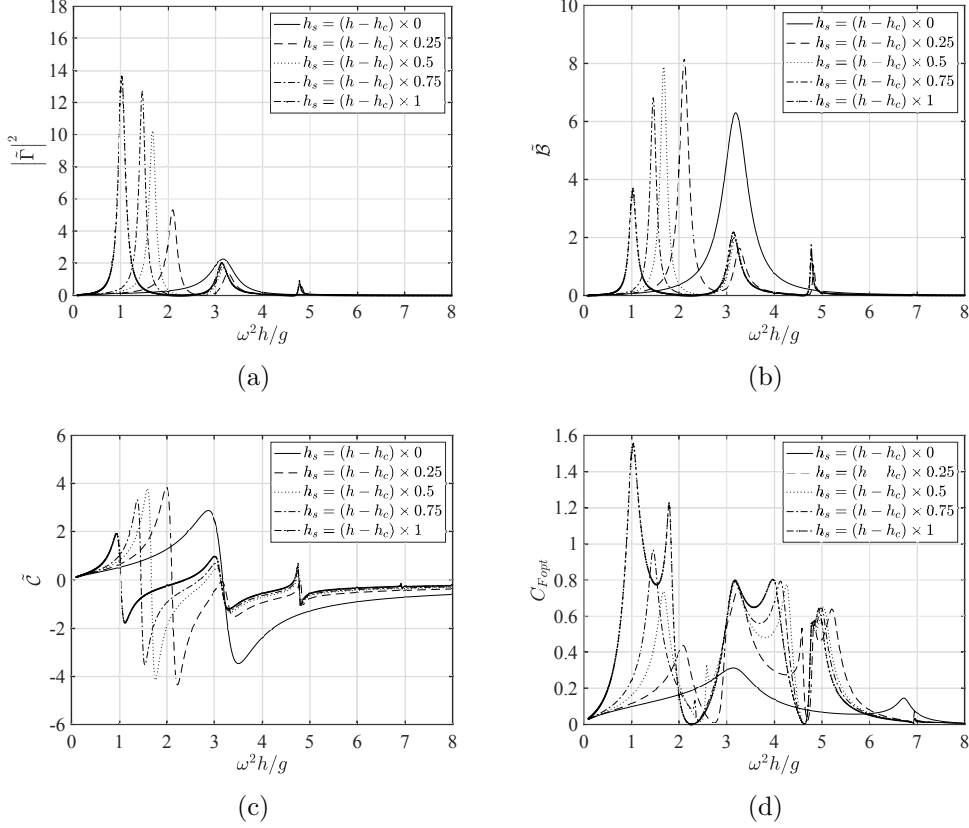


Figure 3: The effects of skirt height  $h_s$  and internal radius  $R_i$  on the hydrodynamic behaviour. 3(a) non-dimensional exciting force  $|\tilde{\Gamma}|^2$ , 3(b) non-dimensional radiation damping  $\tilde{B}$ , 3(c) non-dimensional added mass  $\tilde{C}$  and 3(d) capture factor  $C_{Fopt}$  of each configuration versus non-dimensional incident wave frequency  $\omega^2 h/g$ . The value of the internal radius corresponds to  $R_i = 0.75 \times R_e$ .

357 case  $\theta_1 = 3\pi/2$  and  $\theta_2 = \pi/2$  with opening angle equal to  $\pi$  rad. This  
 358 result suggests that the skirt plays an important role on the power extraction  
 359 efficiency, however one should take care of its effects on the OWC structural  
 360 resistance that could penalise the overall behaviour and durability in real  
 361 seas. Now we change the internal radius to  $R_i = 0.75 \times R_e$ . Figures 5(a)-  
 362 5(d) show  $\tilde{\Gamma}$ ,  $\tilde{\mathcal{B}}$ ,  $\tilde{\mathcal{C}}$  and  $C_{Fopt}$  versus  $\omega^2 h/g$ . The same considerations of  
 363 the previous section can be extended here, i.e. the maximum values of the  
 364 resonant peaks almost preserve their values, while the peaks of the sloshing  
 365 modes become wider.

#### 366 4. Power extraction efficiency in random waves

367 In this section we investigate the effects of random waves on the generated  
 368 power. Without loss of generality, we can adopt the JONSWAP spectrum  
 369  $S_\zeta$  to describe the incident wave field (Goda, 2000)

$$S_\zeta(\omega) = \frac{\alpha H_s^2}{\omega} \left(\frac{\omega_p}{\omega}\right)^4 \exp\left[-1, 25 \left(\frac{\omega_p}{\omega}\right)^4\right] \gamma^{\exp[-(\omega/\omega_p-1)^2/(2\sigma)]}, \quad (81)$$

370 in which  $H_s$  is the significant wave height,  $\omega_p$  denotes the peak frequency  
 371 and

$$\alpha = \frac{0.0624(1.094 - 0.01915 \ln \gamma)}{0.23 + 0.0336\gamma - 0.185(1, 9 + \gamma)^{-1}}, \quad \sigma = \begin{cases} 0.07 & : \omega \leq \omega_p \\ 0.09 & : \omega > \omega_p \end{cases}, \quad \gamma = 3.3. \quad (82)$$

372 Because of linearity, the pressure oscillation inside the OWC can be written  
 373 as

$$P_a(t, \omega_p) = \sum_{n=1}^{\infty} \sqrt{2S_\zeta(\omega_n) \Delta\omega \text{RAO}(\omega_n)} \cos(\omega_n t + \delta_n), \quad (83)$$

374 where  $\omega_n$  is the  $n$ th component of the discretised spectrum,  $\Delta\omega$  is the fre-  
 375 quency step,  $\delta_n$  is a random phase related to  $\omega_n$  while the term RAO is the  
 376 response amplitude operator for the air pressure  $p_a$ , i.e.

$$\text{RAO}(\omega_n) = \left| \frac{\Gamma(\omega_n)}{\left(\frac{KD}{N\rho_a} + \mathcal{B}(\omega_n)\right) - i\left(\frac{\omega_n V_0}{c_a^2 \rho_a} + \mathcal{C}(\omega_n)\right)} \right|. \quad (84)$$

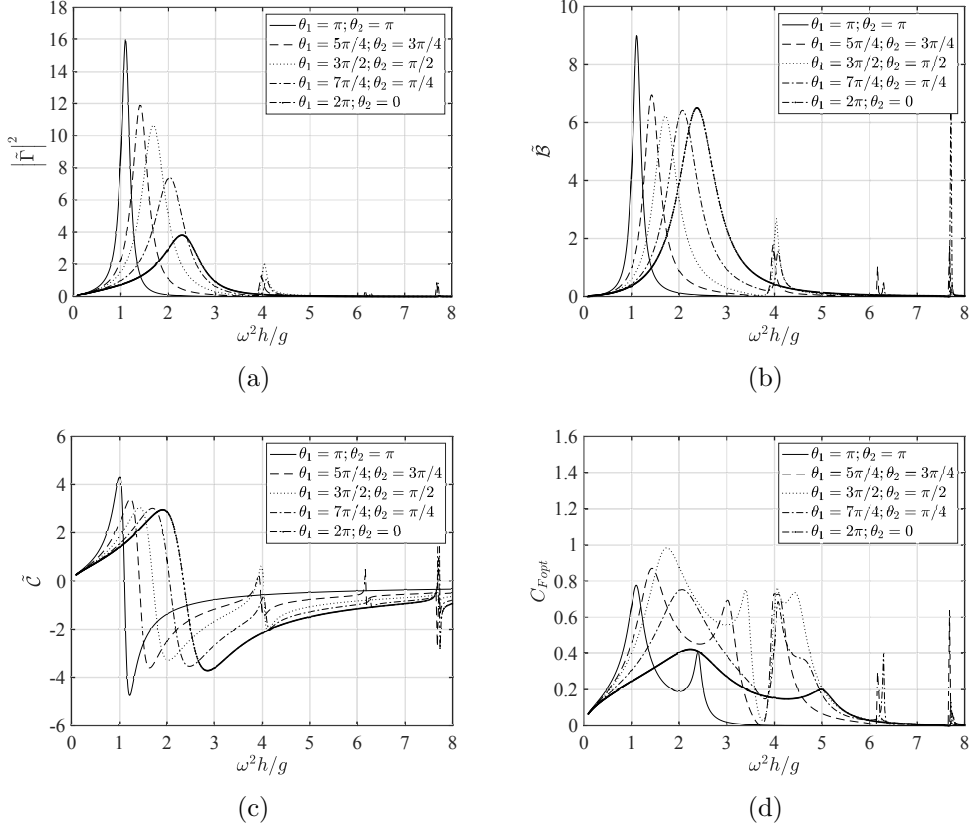


Figure 4: The effects of the skirt opening  $\theta_2 - \theta_1$ . 4(a) non-dimensional exciting force  $|\tilde{\Gamma}|^2$ , 4(b) non-dimensional radiation damping  $\tilde{B}$ , 4(c) non-dimensional added mass  $\tilde{C}$  and 4(d) optimized capture factor  $C_{Fopt}$  of each configuration versus non-dimensional incident wave frequency  $\omega^2 h/g$ .

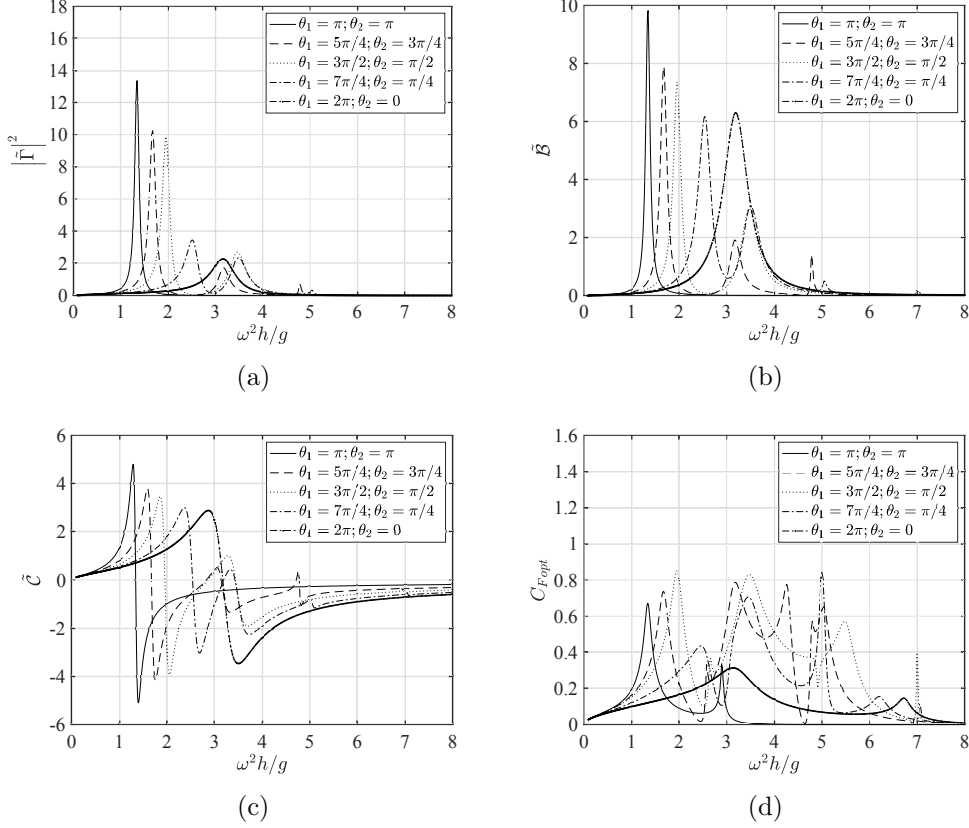


Figure 5: The effects of the skirt opening  $\theta_2 - \theta_1$  and internal radius  $R_i$  on the hydrodynamic behaviour. 5(a) non-dimensional exciting force  $|\tilde{\Gamma}|^2$ , 5(b) non-dimensional radiation damping  $\tilde{B}$ , 5(c) non-dimensional added mass  $\tilde{C}$  and 5(d) optimized capture factor  $C_{Fopt}$  of each configuration versus non-dimensional incident wave frequency  $\omega^2 h/g$ . The value of the internal radius corresponds to  $R_i = 0.75 \times R_e$ .



377 Then, the instantaneous generated power is

$$\begin{aligned}
P_s(t, \omega_p) &= \frac{KD}{N} \left[ \sum_{n=1}^{\infty} \sqrt{2S_{\zeta}(\omega_n) \Delta\omega \text{RAO}(\omega_n)} \cos(\omega_n t + \delta_n) \right]^2 \\
&\quad - \frac{V}{c_a^2} \sum_{n=1}^{\infty} \sqrt{2S_{\zeta}(\omega_n) \Delta\omega \text{RAO}(\omega_n)} \cos(\omega_n t + \delta_n) \\
&\quad \times \sum_{n=1}^{\infty} \sqrt{2S_{\zeta}(\omega_n) \Delta\omega \text{RAO}(\omega_n)} \omega_n \sin(\omega_n t + \delta_n). \tag{85}
\end{aligned}$$

378 From the foregoing expression we obtain the averaged generated power (Michele  
379 et al., 2016b)

$$\bar{P}_s(\omega_p) = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^{\tau} P_s dt = \frac{KD}{N} \sum_{n=1}^{\infty} S_{\zeta}(\omega_n) \Delta\omega \text{RAO}^2(\omega_n), \tag{86}$$

380 whose expression in the limit  $\Delta\omega \rightarrow 0$  becomes

$$\bar{P}_s(\omega_p) = \frac{KD}{N} \int_0^{\infty} S_{\zeta}(\omega) \text{RAO}^2(\omega) d\omega. \tag{87}$$

381 Defining  $P_{\zeta}$  as the total incident wave power per unit crest width

$$P_{\zeta}(\omega_p) = \int_0^{\infty} \rho g C_g(\omega) S_{\zeta}(\omega) d\omega, \tag{88}$$

382 the capture width ratio in random seas  $C_{F\zeta}$  can then be written as

$$C_{F\zeta}(\omega_p) = \frac{\bar{P}_s}{2R_e P_{\zeta}}. \tag{89}$$

383 Let us compare a fixed configuration when excited by random and monochro-  
384 matic waves. Here we assume  $A = 1$  m,  $h = 10$  m,  $\rho = 1000$  kg m<sup>-3</sup>,  $\rho_a = 1$   
385 kg m<sup>-3</sup>,  $c_a = 340$  m s<sup>-1</sup>, the external radius  $R_e = h/2$  and two values of  
386 the internal radius,  $R_i = 0$  and  $R_i = 0.75 \times R_e$ , respectively. Let us fix  
387 the optimal value of  $KD/N\rho_a$  that maximizes power extraction for the fixed  
388 frequency  $\omega = 1$  rad s<sup>-1</sup> and assume the symmetric configuration  $\theta_1 = 3\pi/2$   
389 and  $\theta_2 = \pi/2$  rad maximizing power extraction efficiency. In other words, we  
390 have fixed both OWC geometry and turbine characteristics and optimized

391 them for a frequency representing the wave climate of a particular area.  
 392 This situation can be of practical interest because of the difficulty in tuning  
 393 the turbine speed/geometry with a wide range of incident wave frequencies  
 394 (López et al., 2014).

395 Figure 6(a) shows the behaviour of  $C_{F\zeta}$  and  $C_F$  for the case with null  
 396 internal radius  $R_i = 0$ . The abscissa for  $C_{F\zeta}$  refers to the peak frequency  $\omega_p$   
 397 of the JONSWAP spectrum, while the abscissa for  $C_F$  refers to the frequency  
 398 of the monochromatic incident waves. In both cases the maxima of  $C_{F\zeta}$   
 399 are smaller than the resonant peaks of  $C_F$ , while the system becomes more  
 400 efficient outside the resonant frequencies. Furthermore, the narrow peak  
 401 related to the resonance of the first sloshing mode decreases significantly  
 402 and reduces to a small hump. This fact is consistent with the bad coupling  
 403 between the incident wave spectrum and the natural modes characterized  
 404 by small radiation damping. Similar results are obtained in the context of  
 405 flap-type oscillating wave surge converters by Michele et al. (2016a), Michele  
 406 et al. (2016b) and Sarkar et al. (2014).

407 Figure 6(b) shows  $C_{F\zeta}$  and  $C_F$  respectively versus  $\omega_p$  and  $\omega$  for the second  
 408 configuration with  $R_i = 0.75 \times R_e$ . As before, we optimize  $KD/N\rho_a$  for the  
 409 fixed frequency  $\omega = 1 \text{ rad s}^{-1}$ . Also in this case the maxima of  $C_{F\zeta}$  are  
 410 smaller than those of  $C_F$  and the spreading effect of the spectrum is evident.  
 411 Differently, in the case shown in figure 6(a) the sloshing mode has a significant  
 412 contribution because of the small sharpness of the resonant peak in  $C_F$ .

## 413 5. Theoretical and experimental comparisons

414 The damping force exerted by the turbine is usually modelled by an orifice  
 415 above the OWC (Perez-Collazo et al., 2018b). In this case, in which an  
 416 impulse turbine is used, the relation between the airflow  $Q$  through the orifice  
 417 and the air pressure  $P$  inside the OWC chamber is quadratic (López et al.,  
 418 2016), hence the linear relation used to model Wells turbines (62) fails and  
 419 cannot be used here. Applying Bernoulli's theorem in correspondence of the  
 420 orifice cross section we obtain

$$P_a = \rho_a C_q^2 \frac{Q|Q|}{2\Omega_o}, \quad (90)$$

421 where  $C_q \simeq 0.6$  is the dimensionless coefficient of discharge depending on the  
 422 orifice geometry and  $\Omega_o$  is the area of the orifice. Substitution of the latter

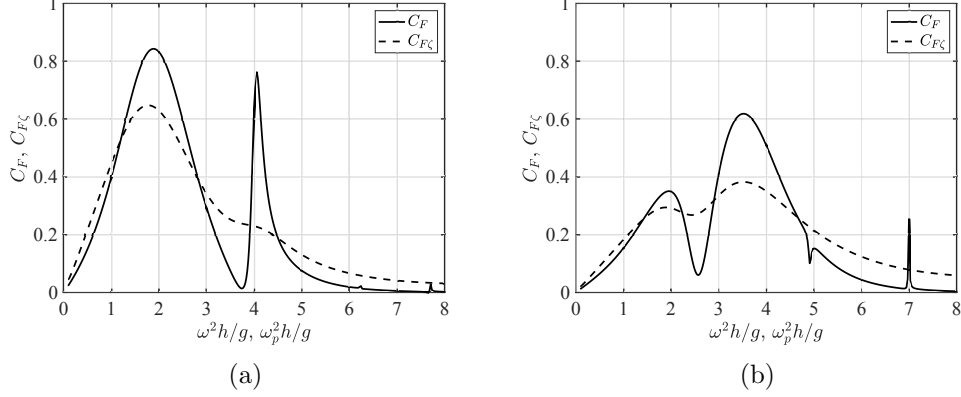


Figure 6: Comparison between the capture factor in random waves  $C_{F\zeta}$  and the capture factor for monochromatic incident waves  $C_F$  respectively versus peak spectral frequency  $\omega_p$  and incident frequency  $\omega$ . Figure 6(a) refers to the configuration with null internal radius  $R_i$  while figure 6(b) is related to the case with  $R_i = 0.75 \times R_i$ . The turbine characteristics are optimized for the frequency  $\omega = 1 \text{ rad s}^{-1}$ .

423 expression in the nonlinear mixed boundary condition on the free surface  
 424 yields (Mei et al., 2005):

$$\Phi_{tt} + g\Phi_z + |\nabla\Phi|_t^2 + \frac{1}{2}\nabla\Phi \cdot \nabla|\nabla\Phi|^2 = -\frac{\rho_a C_q^2}{2\rho\Omega_o^2} \left[ \int_{S_i} \Phi_z \, dS_i \left| \int_{S_i} \Phi_z \, dS_i \right| \right]_t, \text{ on } S_i. \quad (91)$$

425 Now, by introducing the following non-dimensional quantities denoted by  
 426 primes (Michele et al., 2018, 2019a; Michele and Renzi, 2019; Sammarco et  
 427 al., 1997a,b):

$$\begin{aligned} (x', y', z') &= (x, y, z) / \lambda, & \Phi' &= \Phi / (A\omega\lambda), & t' &= t\omega, \\ G &= g / (\omega^2\lambda), & \epsilon &= A/\lambda, \end{aligned} \quad (92)$$

428 expression (91) becomes

$$\begin{aligned} \Phi'_{t't'} + G\Phi'_{z'} &= \epsilon |\nabla'\Phi'|_{t'}^2 + \epsilon^2 \frac{1}{2} \nabla'\Phi' \cdot \nabla' |\nabla'\Phi'|^2 \\ &- \epsilon \frac{\rho_a C_q^2}{2\rho\Omega_o^2} \left[ \int_{S_i} \Phi'_{z'} \, dS_i \left| \int_{S_i} \Phi'_{z'} \, dS_i \right| \right]_{t'}, \text{ on } S_i, \end{aligned} \quad (93)$$

429 thus, if the wave steepness is small, i.e.  $\epsilon \ll 1$ , and the ratio between the area  
 430  $S_i$  and the area of the orifice  $\Omega_o$  is of order  $O(1/\epsilon^2)$ , the nonlinear terms on

431 the right hand side of (93) become small and weak if compared to the linear  
 432 part on the left hand side. Applying the standard perturbation expansion  
 433 technique to the velocity potential

$$\Phi' = \Phi'_1 + \epsilon\Phi'_2 + O(\epsilon^2), \quad (94)$$

434 gives the condition (93) homogeneous and unforced at the leading order  $O(1)$ :

$$\Phi'_{1_{z'}} + G\Phi'_{1_{z'}} = 0, \quad \text{on } S_i. \quad (95)$$

435 If we now return in physical variables and assume both harmonic motion and  
 436 incident waves at  $O(1)$ , equation (95) becomes identical to the boundary  
 437 condition on  $S_i$  for  $\phi_i^D$  (26), hence the solution of the velocity potential  $\Phi_1$   
 438 corresponds to the diffraction velocity potential already found in Section 2.1.  
 439 As a consequence, the air pressure inside the chamber at the leading order  
 440 can be approximated by the following expression

$$P_a = \rho_a C_q^2 \frac{\text{Re}\{q^D e^{-i\omega t}\} |\text{Re}\{q^D e^{-i\omega t}\}|}{2\Omega_o^2}. \quad (96)$$

441 The latter expression yields the averaged rate of work done by the air pressure  
 442 inside the chamber

$$\bar{P}_{out} = 2\rho_a C_q^2 \frac{|q^D|^3}{3\pi\Omega_o^2}, \quad (97)$$

443 and the corresponding capture factor

$$C_{Fexp} = \frac{2\rho_a C_q^2 |q^D|^3}{3\pi\Omega_o^2 R_e A^2 \rho g C_g}. \quad (98)$$

444 In order to validate the theory, comparisons are made with the experimental  
 445 results of Perez-Collazo et al. (2018b). Channel flume and OWC character-  
 446 istics are fixed and listed in Table 1.

### 447 5.1. Monochromatic waves

448 Figure 7 shows the values of the capture factor  $C_{Fexp}$  versus the wave  
 449 period  $T$  in prototype values for both the analytical (expression (98)) and  
 450 experimental model (see figure 10 in Perez-Collazo et al. (2018b)). In par-  
 451 ticular, figure 7(a) and figure 7(b) refer to the different orifice diameters  
 452  $d_0 = 0.015$  m and  $d_0 = 0.019$  m, respectively. The amplitude of the incident

Table 1: Channel and OWC characteristics

Parameters	Symbol	Dimensions
Depth	$h$	1 m
External radius	$R_e$	0.08 m
Internal radius	$R_i$	0 m
OWC draft	$h_c$	0.076 m
Skirt height	$h_s$	0.04 m
Skirt angle 1	$\theta_1$	$3\pi/2$ rad
Skirt angle 2	$\theta_2$	$\pi/2$ rad

regular waves is  $A = 1$  m. The agreement between both models is good at large periods, however, for the case shown in figure 7(a) the theoretical capture factor is clearly overestimated when  $T \in [7, 8]$  s. This is a consequence of the Helmholtz-mode resonance around  $T = 5.5$  s. In this range of periods, nonlinearities, viscous dissipation and effects due to vortex shedding at the lower edges (Xu et al., 2016; Xu and Huang, 2019) are not weak anymore and become important. Moreover, the smaller the value of  $d_0$ , the greater the differences between the models. This is because the ratio  $S_i/\Omega_o$  increases and strengthens the order of magnitude of the last term on the right-hand side of (93).

### 5.2. Random waves

In this section we analyse the amplitude response of the free surface elevation inside the air chamber in irregular wave conditions. Within the framework of a linearised theory we can write the spectrum of the averaged amplitude response as (Michele et al., 2016a):

$$S_{\bar{\eta}} = \sqrt{2 |\bar{\eta}|^2 S_{\zeta} \Delta\omega}, \quad (99)$$

where  $\bar{\eta}$  represents the averaged free-surface amplitude response inside the OWC chamber in monochromatic waves with  $A = 1$  m.

For the sake of example, let us consider the configuration with orifice diameter  $d_0 = 0.015$  m, significant wave height  $H_s = 3.5$  m and peak period  $T_p = 13.3$  s in prototype values (Series C07 in Perez-Collazo et al. (2018b)). Figure 8 shows the theoretical and experimental spectra of the averaged amplitude response  $S_{\bar{\eta}}$  versus the period  $T_n$  of each  $n$ th wave component. The

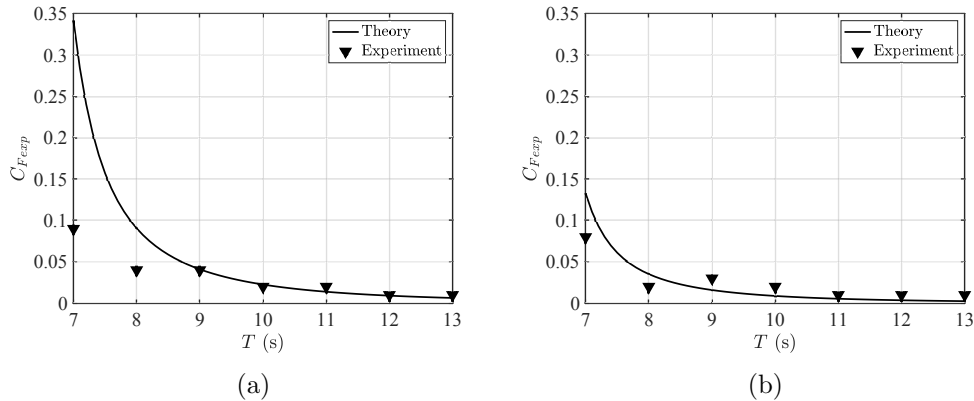


Figure 7: Behaviour of the capture factor  $C_{Fexp}$  versus incident wave period  $T$  in prototype values for two orifice diameters. 7(a)  $d_0 = 0.015$  m, 7(b)  $d_0 = 0.019$  m. The solid line indicates the analytical results given by expression (98), while the triangular markers correspond to the experimental results of Perez-Collazo et al. (2018b).

475 theory predicts one peak around  $T_n = 5.5$  s, while the experimental response  
 476 spectrum tends to decay towards small periods. As in the case of regular  
 477 waves, this discrepancy is due to the linearised theory that tends to overes-  
 478 timate the amplitude response in resonance conditions. Indeed, the peak is  
 479 located in correspondence of the Helmholtz pumping mode eigenfrequency.  
 480 Beyond  $T_n = 5.5$  s we stay in the range of validity of the scales (92) and good  
 481 matching between theory and experiment is obtained.

## 482 6. Conclusions

483 We developed a linearised theory for a cylindrical OWC installed in hybrid  
 484 wind-wave energy systems. The novel OWC model presented here has a skirt  
 485 structure integral with the OWC whose task is to increase power extraction  
 486 efficiency.

487 We evaluated the dependence of the hydrodynamic quantities such as  
 488 added inertia, radiation damping and exciting force on the incident wave fre-  
 489 quency. Our results show that large resonant peaks occur in correspondence  
 490 of the frequencies very close to the eigenfrequencies of a cylindrical tank hav-  
 491 ing depth equal to  $h$ . Furthermore, we performed a numerical check of the  
 492 latter quantities and therefore of the accuracy of the results by deriving some  
 493 useful integral identities based on Green's theorem.

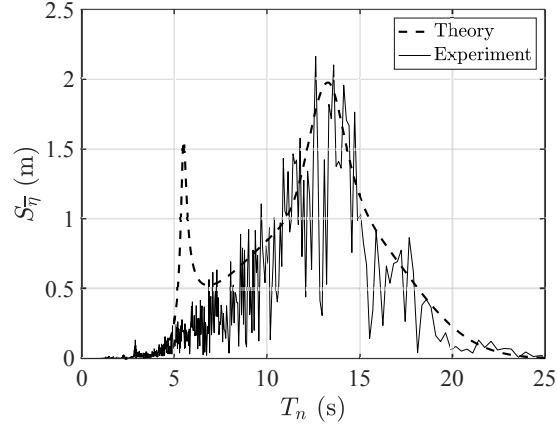


Figure 8: Behaviour of the response amplitude spectrum  $S_{\bar{\eta}}$  versus the  $n$ th wave component period  $T_n$  in prototype values for  $H_s = 3.5$  m and peak period  $T_p = 13.3$  s. The dashed line represents the analytical results given by (99), while the continuous line corresponds to the spectrum of the time series obtained by Perez-Collazo et al. (2018b).

494 Then we investigated the effects of the skirt height and opening angle on  
 495 the hydrodynamic behaviour and efficiency. We found that the greater the  
 496 skirt height, the greater the efficiency when the Helmholtz pumping resonates  
 497 while the narrow sloshing resonant peaks are almost unaffected and maintain  
 498 their shape. This means that the sloshing dynamics depend mainly on the  
 499 internal and external OWC radius. Indeed, we showed that when an internal  
 500 cylinder is present, wide peaks on the capture factor behaviour can be ob-  
 501 tained at large frequencies. Concerning the skirt opening angle, we obtained  
 502 that the optimal configuration maximizing power extraction corresponds to  
 503 the symmetric case  $\theta_1 = 2\pi/3; \theta_2 = \pi/2$  rad.

504 We also investigated the OWC response to random incident waves de-  
 505 scribed by the JONSWAP spectrum. We showed that the presence of a  
 506 broad range of wave frequencies does not couple well with the narrow res-  
 507 onant peaks of some sloshing modes. This is less true for the broad band  
 508 Helmholtz-mode at low frequencies. In this case we have large radiation  
 509 damping and the resonant peak almost keeps its shape. Outside resonance  
 510 the efficiency is larger or comparable to that for the monochromatic case and  
 511 the benefits of random waves are evident. Similar results are already well  
 512 known for flap-type OWSCs in open sea.

513 Subsequently, we validated the analytical model with the experimental

514 set-up developed by Perez-Collazo et al. (2018b). First, we derived the non-  
515 linear boundary condition on the free surface inside the air chamber. This  
516 condition is completely generalised and therefore valid for any OWC labo-  
517 ratory model that uses orifices to simulate the presence of a turbine. We  
518 solved the problem by applying the perturbation expansion to the velocity  
519 potential and showed that the air pressure and the corresponding airflow  
520 through the orifice depend mainly on the diffraction potential at the leading  
521 order. We evaluated the corresponding theoretical capture factor and com-  
522 pared it with that obtained experimentally by Perez-Collazo et al. (2018b).  
523 Good agreement between both models was found especially for large incident  
524 wave periods and large orifice diameters. Finally, we compared theory and  
525 experiments by analysing the response spectra of the free-surface amplitude  
526 inside the OWC chamber in irregular waves. Good matching was obtained  
527 for frequencies not close to the resonant Helmholtz pumping mode.

528

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## 532 **References**

## 533 **References**

534 Astariz, S., Vazquez, A., Iglesias, G., 2015. Evaluation and comparison of the  
535 levelized cost of tidal, wave, and offshore wind energy. *J. Renew. Sustain.*  
536 *Ener.* 7, 1–11. (doi:10.1063/1.4932154)

537 Astariz, S., Vazquez, A., Iglesias, G., 2016. Wave energy vs. other energy  
538 sources: a reassessment of the economics. *Int. J. Green Energy* 13, 744–  
539 755. (doi:10.1080/15435075.2014.963587)

540 Babarit, A., Hals, J., Muliawan, M., Kurniawan, A., Moan, T., Krokstad,  
541 J., 2012. Numerical benchmarking study of a selection of wave energy  
542 converters. *Renew. Energy* 41, 44–63. (doi:10.1016/j.renene.2011.10.002)

543 Babarit A., 2018. *Ocean wave energy conversion*. Elsevier.



- 544 Deng, Z., Huang, Z., Law, A.W.K., 2013. Wave power extraction  
545 by an axisymmetric oscillating-water-column converter supported by a  
546 coaxial tube-sector-shaped structure. *Appl. Ocean. Res.* 42, 114–123.  
547 (doi:10.1016/j.apor.2013.05.006)
- 548 Deng, Z., Huang, Z., Law, A.W.K., 2014. Wave power extraction from a  
549 bottom-mounted oscillating water column converter with a v-shaped chan-  
550 nel. *Proc. R. Soc. Lond. A* 470, 20140074. (doi:10.1098/rspa.2014.0074)
- 551 Evans, D.V., 1982. Wave power absorption by systems of oscillat-  
552 ing surface pressure distributions. *J. Fluid Mech.* 114, 481–499.  
553 (doi:10.1017/S0022112082000263)
- 554 Evans, D.V., Porter, R., 1997. Efficient calculation of hydrodynamic proper-  
555 ties of owc-type devices. *J. Offshore Mech. Arctic Engng.* 119, 210–218.
- 556 Falnes, J., 2002. *Ocean waves and oscillating systems*. Cambridge University  
557 Press.
- 558 Garrett, J.C.R., 1982. Bottomless harbours. *J. Fluid Mech.* 43, 433–449.  
559 (doi:10.1017/S0022112070002495)
- 560 Goda, Y., 2000. *Random Seas and Design of Maritime Structures*. World  
561 Scientific, Singapore.
- 562 Linton, C.M., McIver, P., 2001. *Mathematical techniques for wave/structure*  
563 *interactions*. Chapman & Hall/CRC .
- 564 López, I., Pereiras, B., Castro, F., Iglesias, G., 2016. Holistic performance  
565 analysis and turbine-induced damping for an OWC wave energy converter.  
566 *Renew. Energ.* 85, 1155–1163. (doi:10.1016/j.renene.2015.07.075)
- 567 López, I., Pereiras, B., Castro, F., Iglesias, G., 2014. Optimisa-  
568 tion of turbine-induced damping for an OWC wave energy converter  
569 using a RANS-VOF numerical model. *Appl. Energ.* 127, 105–114.  
570 (doi:10.1016/j.apenergy.2014.04.020)
- 571 Lovas, S., Mei, C.C., Liu, Y., 2010. Oscillating water column at a  
572 coastal corner for wave power extraction. *Appl. Ocean. Res.* 32, 267–283.  
573 (doi:10.1016/j.apor.2010.06.004)

- 574 Magagna, D., Uihlein, A., 2015. Ocean energy development in Europe:  
575 Current status and future perspectives. *Int. J. Mar. Energy.* 11, 84–104.  
576 (doi:10.1016/j.ijome.2015.05.001)
- 577 Martin-Rivas, H., Mei, C.C., 2009a. Wave power extraction from an os-  
578 cillating water column along a straight coast. *Ocean Eng.* 36, 426–433.  
579 (doi:10.1016/j.oceaneng.2009.01.009)
- 580 Martin-Rivas, H., Mei, C.C., 2009b. Wave power extraction from an oscillat-  
581 ing water column at the tip of a breakwater. *J. Fluid Mech.* 629, 394–414.  
582 (doi:10.1017/S0022112009005990)
- 583 Mavrakos, S.A., 1985. Wave loads on stationary floating bottomless cylin-  
584 drical body with finite wall thickness. *Appl. Ocean. Res.* 7, 213–224.  
585 (doi:10.1016/0141-1187(85)90028-8)
- 586 McCormick, M.E., 1981. *Ocean wave energy conversion.* Wiley Interscience.
- 587 Mei, C.C., Stiassnie, M., , D. K.-P., 2005. *Theory and applications of ocean*  
588 *surface waves.* World Scientific, Singapore.
- 589 Michele, S., Sammarco, P., d’Errico, M., Renzi, E., Abdolali, A., Bellotti, G.,  
590 Dias, F., 2015. Flap gate farm: from Venice lagoon defense to resonating  
591 wave energy production. Part2: Synchronous response to incident waves  
592 in open sea. *Appl. Ocean Res.* 52, 43–61. (doi:10.1016/j.apor.2015.05.002)
- 593 Michele, S., Sammarco, P., d’Errico, M., 2016a. The optimal design of a flap  
594 gate array in front of a straight vertical wall: Resonance of the natural  
595 modes and enhancement of the exciting torque. *Ocean Eng.* 118, 152–164.  
596 (doi:10.1016/j.oceaneng.2016.04.002)
- 597 Michele, S., Sammarco, P., d’Errico, M., 2016b. Theory of the synchronous  
598 motion of an array of floating flap gates oscillating wave surge converter.  
599 *Proc. R. Soc. Lond. A* 472, 20160174. (doi:10.1098/rspa.2016.0174)
- 600 Michele, S., Sammarco, P., d’Errico, M., 2018. Weakly nonlinear theory for  
601 oscillating wave surge converters in a channel. *J. Fluid Mech.* 834, 55–91.  
602 (doi:10.1017/jfm.2017.724)
- 603 Michele, S., Renzi, E., Sammarco, P., 2019. Weakly nonlinear theory  
604 for a gate-type curved array in waves. *J. Fluid Mech.* 869, 238–263.  
605 (doi:10.1017/jfm.2019.223)

- 606 Michele, S., Renzi, E., 2019. A second-order theory for an array of  
607 curved wave energy converters in open sea *J. Fluid Struct.* 88, 315–330.  
608 (doi:10.1016/j.jfluidstructs.2019.05.007)
- 609 Pereiras, B., López, I., Castro, F., Iglesias, G., 2015. Non-dimensional  
610 analysis for matching an impulse turbine to an owc (oscillating wa-  
611 ter column) with an optimum energy transfer. *Energy* 87, 481–489.  
612 (doi:10.1016/j.energy.2015.05.018)
- 613 Perez-Collazo, C., Greaves, D., Iglesias, G., 2015. A review of combined  
614 wave and offshore wind energy. *Renew. Sust. Eenerg. Rev.* 42, 141–153.  
615 (doi:10.1016/j.rser.2014.09.032)
- 616 Perez-Collazo, C., Greaves, D., Iglesias, G., 2018a. Hydrodynamic response  
617 of the wec sub-system of a novel hybrid wind-wave energy converter. *Energ.*  
618 *Convers. Manage.* 171, 307–325. (doi:10.1016/j.enconman.2018.05.090)
- 619 Perez-Collazo, C., Greaves, D., Iglesias, G., 2018b. A novel hybrid wind-  
620 wave energy converter for jacket frame substructures. *Energies* 11, 637.  
621 (doi:10.3390/en11030637)
- 622 Sammarco, P., Michele, S., d’Errico, M., 2013. Flap gate farm: from Venice  
623 lagoon defense to resonating wave energy production. Part1: Natural  
624 modes. *Appl. Ocean Res.* 43, 206–213. (doi:10.1016/j.apor.2013.10.001)
- 625 Sammarco, P., Tran, H.H., Mei, C.C., 1997a. Subharmonic resonance of  
626 Venice gates in waves. Part 1. Evolution equation and uniform incident  
627 waves. *J. Fluid Mech.* 349, 295–325. (doi:10.1017/S0022112097006848)
- 628 Sammarco, P., Tran, H.H., Gottlieb, O., Mei, C.C., 1997b. Subharmonic  
629 resonance of Venice gates in waves. Part 2. Sinusoidally modulated incident  
630 waves. *J. Fluid Mech.* 349, 295–325. (doi: 10.1017/S0022112097006836)
- 631 Sarkar, D., Renzi, E., Dias, F., 2014. Wave farm modelling of oscil-  
632 lating wave surge converters. *Proc. R. Soc. Lond. A* 470, 20140118.  
633 (doi:10.1098/rspa.2014.0118)
- 634 Sarmiento, A.J.N.A., Falcão, A. F. de O., 1985. Wave generation by an oscil-  
635 lating surface-pressure and its applications in wave-energy extraction. *J.*  
636 *Fluid Mech.* 150, 467–485. (doi:10.1017/S0022112085000234)

- 637 Xu, C., Huang, Z., Deng, Z., 2016. Experimental and theoretical study of a  
638 cylindrical oscillating water column device with a quadratic power take-off  
639 model. *Appl. Ocean. Res.* 57, 19–29. (doi.org/10.1016/j.apor.2016.02.003)
- 640 Xu, C., Huang, Z., 2019. Three-dimensional CFD simulation of a circular  
641 OWC with a nonlinear power-takeoff: Model validation and a discussion  
642 on resonant sloshing inside the pneumatic chamber. *Ocean Eng.* 176, 184–  
643 198. (doi.org/10.1016/j.oceaneng.2019.02.010)
- 644 Zheng, S., Zhang, Y., Iglesias, G., 2018. Wave-structure inter-  
645 action in hybrid wave farms. *J. Fluid Struct.* 83, 386–412.  
646 (doi:10.1016/j.jfluidstructs.2018.09.012)
- 647 Zheng, S., Zhang, Y., Iglesias, G., 2019. Coast/breakwater-  
648 integrated owc: A theoretical model. *Mar. Struct.* 66, 121–135.  
649 (doi:10.1016/j.marstruc.2019.04.001)
- 650 Zhou, Y., Zhang, C., Ning, D., 2018. Hydrodynamic investigation of  
651 a concentric cylindrical owc wave energy converter. *Energies* 11, 95.  
652 (doi:10.3390/en11040985)