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Essays On Labor Markets With Search And Information Frictions

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Essays On Labor Markets With Search And Information Frictions

Abstract

This dissertation attempts to analyze the wage determination and matching probability in labor markets with search and information frictions.

In the first chapter, I provide a theoretical framework to analyze the role of job-to-job transition in a worker's lifetime wage growth.

I use the first-price auction models to analyze the wage determination and job-to-job transition process, and formulate the inference problem about a worker's quality revealed through the worker's past job history.

By doing so, I characterize the mechanism through which frequent job transitions convey a negative signal about the worker's quality.

The second chapter, joint work with Hanna Wang, focuses on the market for entry-level jobs where job seekers send out costly applications in order to match with a potential employer.

With the presence of coordination friction, we characterize the matching probability in a market with heterogeneous workers and show how restricting the search effort can improve social welfare.

The third chapter reviews the literature on search and information frictions and identifies the contribution of the two earlier papers.

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ESSAYS ON LABOR MARKETS WITH SEARCH AND INFORMATION FRICTIONS

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ABSTRACT

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Joonbae Lee

Benjamin Lester

This dissertation attempts to analyze the wage determination and matching probability in labor markets with search and information frictions. In the first chapter, I provide a theoretical framework to analyze the role of job-to-job transition in a worker's lifetime wage growth. I use the first-price auction models to analyze the wage determination and job-to-job transition process, and formulate the inference problem about a worker's quality revealed through the worker's past job history. By doing so, I characterize the mechanism through which frequent job transitions convey a negative signal about the worker's quality. The second chapter, joint work with Hanna Wang, focuses on the market for entry-level jobs where job seekers send out costly applications in order to match with a potential employer. With the presence of coordination friction, we characterize the matching probability in a market with heterogeneous workers and show how restricting the search effort can improve social welfare. The third chapter reviews the literature on search and information frictions and identifies the contribution of the two earlier papers.

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CHAPTER 1 : Wage Dynamics with Developing Asymmetric Information

1.1. Introduction

In this paper, I propose a theory of workers' lifetime wage dynamics that incorporates asymmetry of information –the current employer (the incumbent) knows more about the worker than an outsider (the poacher)– in the Employment-to-Employment transition. By doing so, I can identify the channel through which on-the-job search affects both the individual wage profile and equilibrium inference on worker's quality. This is important because on-the-job worker search alone cannot account for qualitative differences in job transition –a transition may occur as a result of poor performance or upon receipt of a better outside offer regardless of job performance. Indeed, data show that a worker's wage is negatively correlated with his transition frequency. This contradicts the theoretical result which indicates that worker search causes wage growth. [1]

Job-to-job transition is an important element in understanding a worker's wage growth. Many researchers have modeled it as a worker's ability to engage in on-the-job search and analyzed its implications to wages, most notably, Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002). Indeed, in many industries, bringing in a competitor and negotiating wage is widely accepted as a norm for driving up a worker's wage. An anecdote from the consulting industry suggests that, on average, a consultant triggers wage renegotiation following on-the-job search once every three years.[2] This observation is more generic, as Moscarini and Postel-Vinay (2017) shows, the macro-level Employment-to-Employment (henceforth EE) transition rate co-moves strongly with wage growth. [3]

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[1] Light and McGarry (1998) consider two different models of job transition, the "search goods" model and the "experience goods" model. They show that workers who are more mobile have lower wage paths, which is consistent with the mechanics of 'experience goods' model, in which low current match expectations cause turnover.

[2] I thank Jay Park, who is working in the industry, for this comment.

[3] The authors conclude that this possibly reflects workers' ability to extract rents following an outside offer. In another paper, Moscarini and Postel-Vinay (2016) consider an extension of the Burdett and Mortensen (1998) model. The model incorporates business cycle fluctuations, and its results fit well with the dynamics of the EE transition rate and wage growth.

In the model, information about worker quality is revealed gradually over time, and the learning is shared only among the matched party, the worker and his/her current employer. The information is modeled with a Poisson news process that generates arrival over time with rate α. Outside firms other than the matched party do not observe this news process. However, I assume that each worker's job separation and hiring are public information, as these events are recorded in a credible and verifiable CV that the worker carries. This is the public information that is available to outside firms.

An employee working for this firm (incumbent) brings about a meeting with a new firm (poacher) at Poisson rate λ, which denotes the worker's on-the-job search intensity.[4] When an outside firm (poacher) contacts this worker, the firm competes against the current firm (incumbent) for the future service of the worker by playing a first-price auction game like that in Postel-Vinay and Robin (2002). This is a two-player auction where the bidders are asymmetrically informed.

In this setting, I first focus on the informational content of the worker's employment history. When the current employer knows more about the quality of the worker, the new firm faces the problem of adverse selection – low-quality workers that current employers have low willingness to pay are the workers most likely to be attracted by the poaching firm. Therefore, low-quality workers shift jobs more often, while high-quality workers selectively stay longer in the match. In turn, the average quality of workers increases with tenure.

Next, starting from initial uncertainty about a worker's type, I analyze how the worker's history affects his wage determination. A high-quality worker makes fewer transitions, and the worker's wage gradually increases over time. The employer of the worker earns information rent until the poachers' estimate about the worker's quality is accurate. Due to the asymmetry of information, a low-quality worker can exploit new employers by taking advantage of their inaccurate belief of his type. However, in the long run, their wage

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[4]I follow the notations in Burdett and Mortensen (1998), Postel-Vinay and Robin (2002). I do not distinguish between search by workers who are employed and workers who are unemployed.

decreases with job transition, which is consistent with data.

The model has implications for legislation recently introduced in some states, which bars employers from asking job applicants about wage history. In auctions with asymmetrically-informed bidders, it is known that the incumbent firm's information rent decreases as additional dimensions of worker information (e.g., wage history) become public. In my setting, this implies that, without the wage information, the wage dispersion between high- and low-quality workers increases. The low-quality workers gain from the policy, while high-quality workers' wage growth is hurt by adverse selection.

The paper is organized as follows. In the next subsection, I review the literature that is related to this paper. In Section 2.3, I present the model, and in Section 1.3, I define the equilibrium of the model. In Section , I present two cases that yield closed-form solutions and analyze their properties. In Section 1.5, I discuss the implications of the model and the role of underlying assumptions. Section 1.6 concludes.

1.1.1. Literature Review

This paper contributes to the literature on the labor markets with search friction by adding in the elements of adverse selection and asymmetric information.

Burdett and Mortensen (1998) first showed that a worker's ability to engage in on-the-job search can support continuum of posted wages in equilibrium. For their results, it was crucial that firms commit to posted wage contracts, that they could not respond to outside offers. Postel-Vinay and Robin (2002) and Moscarini (2005) relaxed the assumption and solved for the wage-bargaining problem after the arrival of a poacher. In this paper I adopt their premise that wage is determined by an auction. [5]

---

[5]There are some papers that explicitly solves for the alternating-offers bargaining, such as Cahuc et al. (2006). However, their bargaining equilibrium is qualitatively similar to to that of the Bertrand auction equilibrium. Modeling the wage-negotiation process as an auction is a commonly accepted practice in the macro-labor context, although some authors attempt to endogenize the choice of bargaining protocol as in Doniger (2015) and Flinn et al. (2017). Contrary to the "experience goods" models described in the next paragraph, these papers focus on aggregate equilibrium.

Alongside the "search goods" model of the job mobility, many researchers focus on "experience goods" nature of the jobs, incorporating gradual learning of match quality (Jovanovic (1979) and Jovanovic (1984).) Moscarini (2005) tried to synthesize both approaches by nesting the "experience goods" job search model into a general equilibrium. My model focuses on learning about worker quality, rather than idiosyncratic match quality, but replicates the turnover pattern whereby low-quality matches dissolve quickly. This effect seems to be prevalent in the data, as shown by Light and McGarry (1998).

My innovation is in incorporating the component of asymmetric-information into these models. I use the first-price auction model with asymmetrically informed bidders studied extensively by Engelbrecht-Wiggans et al. (1983), and Milgrom and Weber (1982). In a static setting, the auction game has a well-defined solution, while my technical contribution is solution of the game in a dynamic environment. [6] I solve for the dynamic equilibrium where the value of an object sold in the market is endogenous, as it takes into account the future outcome of the auction. This is also true in Postel-Vinay and Robin (2002) and Moscarini (2005), but in my model, the values are non-stationary because the information asymmetry also evolves over time.

The information asymmetry in my model is generated by the arrival of private information to an incumbent firm. In that regard, this paper is also closely related to the literature on dynamic adverse selection, notably, Kim (2017), Hwang (2018) and Camargo and Lester (2014). These papers analyze trading dynamics in markets for goods and financial assets when a buyer's inference on the quality is influenced both by the public information (calendar time), and the correct anticipation of the seller's equilibrium behavior. It is reasonable to believe that these effects are also present in labor markets where current employers know worker characteristics better than others.

---

[6]Wolinsky (1988) solved for seller valuation in a sequential auction game in which a random number of bidders are attracted over time. In my setting, because I am interested in wage negotiation initiated by on-the-job search, the auction always has two bidders. Furthermore, Wolinsky (1988) focuses on stationary equilibrium, while my model exhibits non-stationarity due to the evolution of an observable component of a worker's history.

Surprisingly little attention has been paid to information frictions in wage determination, with the exception of Carrillo-Tudela and Kaas (2015), which introduces adverse selection and screening considerations to the framework used in Burdett and Mortensen (1998). We both consider an environment in which a worker's quality is initially unknown to firms. However, in their setting, workers know their quality, and in return, firms offer screening contracts that separate good types from bad types. They also assume a particular class of contract that promotes/demotes based on the realization of a perfectly revealing signal.

In this paper, I focus instead on information asymmetry between firms. I think this is a natural assumption for many occupations, unless all past worker performance is public. Although I focus on interaction between firms in job transitions, I generate a similar dynamic to that of Carrillo-Tudela and Kaas (2015), because the good worker and the bad worker are treated differently by the incumbent in a wage auction. We both show that unobserved quality (worker type) can account for the correlation between high job mobility and low wages.

My work also contributes to the understanding of wage-tenure profile as in Burdett and Coles (2003) and Stevens (2004), which show that firms optimally choose to backload wages in order to retain workers. Contrary to their findings, my model generates reverse causality, in which tenure in a firm increases a worker's bargaining power. I also provide a learning channel through which workers drive up their wages, contrary to other explanations such as human capital accumulation.

1.2. Model

1.2.1. Basic Setting

Model Setup

- *Time*: Time is continuous. Calendar time is indexed by $t \in [0, \infty)$.

- *Firms*: The economy is populated with measure 1 of identical, risk neutral firms. A firm can hire multiple workers.

- *Workers*: Workers are either of type H and L (with notation: $\{H, L\}$) standing for *High* and *Low* productivity. Both risk neutral. Assume that worker types are sole input into the production.

- *Types (Productivity)*: Type L workers produce observable flow output normalized to 0. In a small interval of time, type H workers might generate a lump-sum output Y (*breakthrough*), at Poisson rate α. Otherwise, they produce 0.

- *Payoffs*: For a firm with discount rate $r > 0$, the expected continuation value of a type H worker's output is $\frac{\alpha Y}{r}$. The continuation value is 0 for a type L worker's output. Accordingly, a firm hiring a worker who is H with probability p, at flow wage w, accrues expected flow profit of

$$p\alpha Y - w.$$

  Flow payoff of the worker is wage w.

- *Learning*: Firms learn from the output their workers generate. If an employee first produces a positive output (Y), then the employer immediately knows that the employee is an H. The employer is uncertain about an employee that has produced nothing to date. Nevertheless, the longer an employee of a firm produces nothing, the more likely his/her employer thinks the worker is unproductive.

- *On-the-job Search*: Outside employment opportunities for a worker arrive at Poisson rate λ, in ther form of a competing wage offer from an outside *poaching* firms.

- *Asymmetry of Information*: I assume that the output is observed only by the current employer (*incumbent*). Outsiders, or a *poacher* only observes a worker's employment history.

Discussion Since the focus of this paper is on the adverse selection, I assume that worker types are the sole input into the production and abstract away from idiosyncratic match productivity. Also, I will adopt the setting of Burdett and Mortensen (1998), in which a firm can hire multiple workers, which is not true in a matching model. I also analyze in the level of an individual worker following the worker's employment history. A worker's history effectively starts with the first job and is not affected by unemployment, which I do not include in my model.

According to the payoff structure, a worker's type is learned through a *good news* process that generates news with rate α for H workers but generates no news for L workers. It is reasonable to think of this arrival of this process as a private '*breakthrough*'. There are several ways to think about the private breakthrough process. First of all, it may be the output of a worker which cannot be transferred out of the firm because it is the firm's property, or because it is confidential. Examples include research output, coded program, or sales performance. On the other hand, we can think of the information as a subjective performance measure, which is accurate and correlated across firms. Examples would be a senior professor's evaluation of assistant professor, beyond the public output, such as publication. Lastly, the model might apply to a worker's teamwork ability, and leadership which has the feature of an experience good the poachers have less accurate knowledge over.

Even though the perfect good news is assumed for tractability, the mechanics of the model goes through as long as the incumbent is better informed than a poacher.

1.2.2. Histories and Beliefs

In this section, I define two equilibrium objects that evolve as a function of a worker's observable history. I assume that the worker's employment history is a public information, as carried around in the form of a resume, or a CV.

Definition 1 (history). *A worker's (employment) history, $h(t)$, is a chronological list of all*

firms and tenures up until age t:

$$h(t) = (\tau_1, \tau_2, \ldots, \tau_n), \quad t = \sum_{i=1}^{n} \tau_i$$

where τ_i is the tenure at the i-th firm this worker was employed at.

The firms $1, 2, \ldots, n$ are chronologically ordered so that the n-th firm is the last firm to employ the worker: the current employer, which we call an *incumbent* firm. Note that τ_n is defined to be $t - \sum_{i=1}^{t-1} \tau_i$. In particular, in contrast to the previous tenures (τ_i, with $i \neq n$) that terminated by a transition, the worker need not be ending his tenure at the current firm (n) at time t. For later reference, it is useful to distinguish between histories continuing with tenure τ_n and histories with switch at time t.

Definition 2. Define by $h(t)$ the history continuing with tenure τ_n at time t, and $\tilde{h}(t)$ the history with tenure τ_n ending at t.

I define the two equilibrium objects.

Definition 3 (beliefs). $p(h(t))$ is the *incumbent's belief* that the worker is H, if the incumbent did not observe any good news output (breakthrough) after hiring him/her (for τ_n duration).

$x(h(t))$ is the *poacher's belief* about the incumbent's knowledge that the worker is H.

Discussion on Histories The space of histories consists of partitions of age t, (work history of length t) into a vector of past tenures in the firms that the worker was employed at. Formally, we denote the full set of public employment histories of a worker of age t by $\mathcal{H}(t)$, where

$$\mathcal{H}(t) = \{(\tau_1, \tau_2, \ldots, \tau_n) \mid \sum_{i=1}^{n} \tau_i = t\},$$

while a particular employment history is an element $h(t) \in \mathcal{H}(t)$. A worker is never unemployed in my model, and the set of tenures add up to the age t. Firm n is the incumbent firm that currently hires the worker. Note that τ_n is the continuation of current tenure.

Unlike other tenures τ_i, $(i < n)$, τ_n might or might not terminate at t.

Since I assume a continuum of firms being drawn randomly in a meeting process, any two firms in a worker's employment history are distinct firms. Furthermore, concerning the meeting process (happening at a Poisson rate λ) outlined above, note that the history reflects only the meetings that resulted in transition. It may be that a worker generated a meeting, but did not transit, as these events are not reflected in the history.

Potentially, there may be other elements of a worker's history that are also observable and informative, such as wages, or the identity of the firm. I abstract away from the possibilities for now, but later discuss what happens when the wage is also public.

Discussion on Beliefs Poachers (outside firms) do not observe the output process and form belief about the information in the current match, conditioning only on the public information, the employment history.

To elaborate on the belief p, note first that the incumbent's belief that the worker's quality is high, P, is a mapping

$$P : \mathcal{H}(t) \times \mathbf{1}(\tau_n) \to [0,1],$$

where $\mathbf{1}(\tau_n)$ is a indicator random variable of whether the incumbent firm has observed the H output for the duration of τ_n.

Given an employment history $h(t)$, the mapping is given by

$$\begin{cases} P(h(t), 1) = 1 \\ P(h(t), 0) = p(h(t)) := \frac{p_0 e^{-\alpha \tau_n}}{p_0 e^{-\alpha \tau_n} + (1 - p_0)} \end{cases}$$

where p_0 is the initial expectation of the worker's quality at the point of hire. I relegate to the later sections the details on p_0. In essence, it is pinned down by the history of the worker up until $t - \tau_n$, $(\tilde{h}(t - \tau_n))$, and the bid the incumbent firm made at the point of attracting the worker.

Since the arrival of a good output follows a Poisson news process of arrival rate α, the probability that a H type worker generates no arrival for the duration of τ_n ($\tau_n > 0$) is $e^{-\alpha \tau_n}$. The belief $p(h(t))$ is obtained using Bayes' rule, or by solving the ODE for the Poisson good-news drift starting from an initial belief p_0:[7]

$$p'(\tau_n) = -\alpha p(\tau_n)(1 - p(\tau_n)), \quad p(0) = p_0.$$

The drift equation reflects the fact that the incumbent firm becomes more pessimistic about the worker quality, as time elapses without observing a good output. It is sufficient to track only the pessimistic belief $p(h(t))$, since the belief jumps up to 1 with the arrival of a good output.

Now, we focus on the belief of the *poacher* (outside firm), $x(h(t))$. Define $x(h(t))$ as the poacher's Bayes rational belief over the incumbent's observation of output:

$$x(h(t)) := Pr(\{P(h(t), \mathbf{1}(\tau_n)) = 1\}).$$

The belief is affected by several elements. First, it is affected by the incumbent's additional information about the worker, belief about the initial expectation (p_0) and the good output realization within τ_n. In order to avoid complication, assume the incumbent and the poacher agrees on the initial expectation p_0.

Assumption 1. Assume that the incumbent and the poacher agrees on the initial expectation about the worker's quality p_0, given the public employment history $h(t - \tau_n)$.

A particular example of an implementation of this assumption is presented in Section 1.4.1, in the form of Assumption 5.

If both firms agree on the initial quality, p_0, then the only divergence in information is whether the incumbent has observed a good ouput for the last τ_n duration. From this fact,

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[7]Derivation is contained in Appendix 1.7.1 for the readers who are not familiar with continuous time belief process with Poisson news.

	Incumbent	Poacher
Tenure in the current firm	O	O
Past jobs and transitions	O	O
Good output (Y)	O	X
Failed poaching attempts	O	X
Belief	$p(h(t))$ or 1	$x(h(t))$

Table 1: Summary of Information Structure

it might be tempting to say that $x(h(t))$ is given by

$$x(h(t)) = p_0\left(1 - e^{-\alpha\tau_n}\right).$$

However, this is not the whole story because the poacher does not know if there was any other failed poaching attempts that are not reflected in the public employment history. The Bayes-rational outsider knows the following: the poaching attempts arrive at the Poisson rate λ, and among them, only those that resulted in a job transition are shown in the employment history. The outsider's estimate is biased without taking into account the failed poaching attempts that are not observed. I summarize the elements of information structure in Table 1.

1.2.3. Wage Auction

While the worker is hired, he/she generates meeting with a new firm (poacher) at Poisson rate λ. This is an exogenous process at which the worker meets another firm, and is the only opportunity for a worker to shift to a new job. If the worker does not shift to a new job, the worker continues in the current job.

Assume that, once the poacher contacts the worker, the two firms (incumbent and poacher) compete for the future service of the worker through a *common value first-price sealed-bid auction*. Furthermore, we assume that, by initiating this auction, the worker fully commits to accept the result of the auction, by shifting to whichever firm that offers higher bid.

From the assumption, strategic players of an auction are the informed incumbent and the

uninformed poacher. Engelbrecht-Wiggans et al. (1983) have already studied the first price auction game between informed and uninformed bidders, in a static and symmetric payoff setting. For reference, we summarize the main results of the paper in the Appendix 1.7.2 and proceed to think about how the result modifies in our setting.

In our setting, the informed bidder receives a fully revealing signal about the value of an object (the employee): the firm's value takes on one of two possible numbers, depending on whether the bidder received information (arrival of α) or not. The Bayes rational belief about the firms' information x, corresponds to the distribution of the firm's signal. Formally:

Definition 4. The *symmetric-payoff, common-value first-price auction game with asymmetric information* at time t, when the beliefs are $x = x(h(t))$ and $p = p(h(t))$, consists of

- Two bidders: Informed I (*Incumbent*), and Uninformed P (*Poacher*)

- Informed I can be of two types: $\{Ih, I0\}$ for observing/not observing the news.

  Uninformed P is of only one type: $P\emptyset$ since he does not know about the news arrival.

- Two ex-post valuations: $0 \leq \Pi_{I0} < \Pi_{Ih}$, for I, while the expected value of the worker

$$\Pi_{P\emptyset} = x\Pi_{I0} + (1 - x)\Pi_{Ih}$$

- Signals: I observes binary signal that informs

$$\begin{cases} \Pi_{Ih} & \text{with probability } x \\ \Pi_{I0} & \text{with probability } 1 - x \end{cases}$$

Applying the result from Engelbrecht-Wiggans et al. (1983):

Proposition 1. The game has a unique Bayesian Nash equilibrium where:

1. The support of the bids is $[\Pi_{I0}, \tilde{\Pi}]$, where $\tilde{\Pi} = x\Pi_{Ih} + (1-x)\Pi_{I0}$.

2. Both players submit mixed bids according to the distribution

$$G(b) = \frac{(1-x)(\Pi_{Ih} - \Pi_{I0})}{\Pi_{Ih} - b}, \quad b > \Pi_{I0}$$

where, mixed strategy for player $Ih, I0$ and $P\emptyset$ satisfy:

$$G_{P\emptyset}(b) = G(b),$$

$$xG_{Ih}(b) + (1-x)G_{I0}(b) = G(b),$$

and $G_{I0}(b) = 1$ for all $b \geq \Pi_{I0}$ and 0 otherwise.

Figure 1 depicts a particular pair of distributions. This result is derived using indifference conditions as in Appendix 1.7.3. We note that the equilibrium strategies imply that with positive probability there are ex-post instances where bidder $P\emptyset$ regrets winning. This is the well-known "winner's curse" in auctions with common values and asymmetric information. However, in order for the bidder $P\emptyset$ to participate in the auction, and to bid non-trivial bids, there has to be some instances in which bidder $P\emptyset$ wins positive profit, which happens when Ih loses the auction.

Note that, in the equilibrium of the auction game, there is non-zero probability of tie at Π_{I0}. For later sections, in order to make probability calculations easy, I assume that in case of a tie, $P\emptyset$ wins. Later, I show that this corresponds to an efficient tie breaking rule.

Assumption 2. Tie is resolved in favor of the poacher $P\emptyset$.

Discussion I am effectively ruling out the worker's decision in the wage determination. It is restrictive, because as we later show in the equilibrium section, that a worker might have profitable deviation of reneging on his/her commitment to the auction outcome, and wait for the next wage auction where the poachers have more favorable belief about the
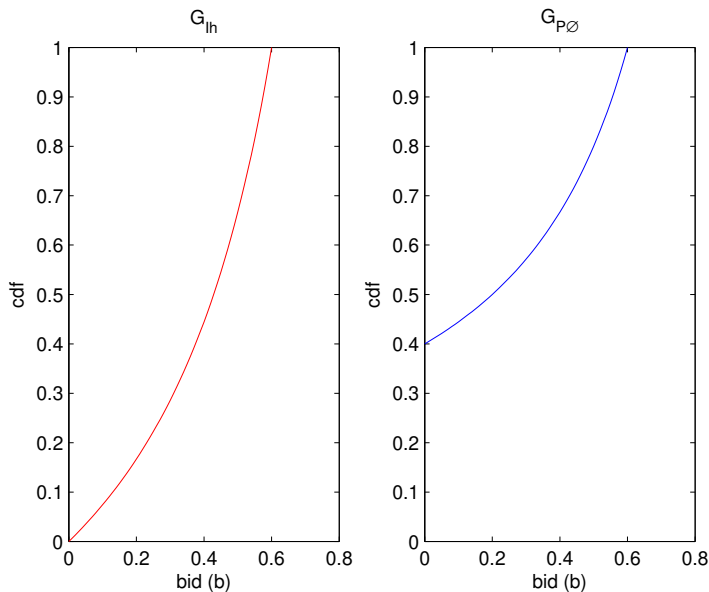
Figure 1: Distribution of Bids G_{Ih} and $G_{P\emptyset}$, with $x = 0.6$, $\Pi_{Ih} = 1$, $\Pi_{I0} = 0$

worker. However, although continual switching is not optimal in the early phase of the career, the result will go through for later phase of the worker's career, because the worker has to ultimately cash-in the benefits by initiating an auction. Furthermore, a worker's incentive to renege on the auction outcome will be less if the worker anticipates that by reneging, the incumbent will exploit the worker until the next auction. Hence, we highlight our main assumption as:

Assumption 3. The players in the wage auction game are two firms (incumbent and poacher). The worker abides by the auction rule and chooses a firm that offers better wage.

1.3. Equilibrium

I solve for equilibrium the in a dynamic environment. In essence, the value of the firm from winning the auction is the annuity value of the worker's output net the future bids the firm has to make in order to retain the worker. The auction equilibrium with the given values exhibits differential rate at which H and L workers are leaving the current match. Hence, the Bayes rational beliefs take into account the win probabilities of the auction. The path of beliefs in turn affects the evolution of values, which affects the present discounted value

of the future profits. The equilibrium is a fixed point in which the objects are consistent with each other.

Formally, the equilibrium objects are given by

Proposition 2 (Equilibrium)**.** The equilibrium consists of paths of beliefs $x, p : \mathcal{H}(t) \rightarrow [0, 1]$, paths of values $\Pi_{Ih}, \Pi_{I0} : \mathcal{H}(t) \rightarrow \mathbb{R}$, and the mixed bidding strategies $G_{Ih}, G_{I0}, G_{P\emptyset}$ that maps from $\mathcal{H}(t)$ such that:

- During the continuation of tenure, $x(t) = x(h(t))$ evolves according to the ODE:

$$x'(t) = \left(\int G_{P\emptyset}(v|h(t)) \, dG_{Ih}(v|h(t)) \right) \lambda x(t)(1 - x(t)) + \alpha(1 - x(t))p(h(t))$$

- During the continuation of tenure, $p(t) = p(h(t))$ evolves according to the ODE:

$$p'(t) = -\alpha p(t)(1 - p(t)),$$

- During the continuation of tenure, $\Pi_{Ih}(t) = \Pi_{Ih}(h(t))$ and $\Pi_{I0}(t) = \Pi_{I0}(h(t))$ solves the ODE's:

$$-\Pi'_{Ih}(t) + (r + \lambda)\Pi_{Ih}(t) = \alpha Y + \lambda \int (\Pi_{Ih}(t) - v)G_{P\emptyset}(v|h(t)) \, dG_{Ih}(v|h(t)) \quad \text{(Ih)}$$

$$-\Pi'_{I0}(t) + (r + \lambda + \alpha p(t))\Pi_{I0}(t) = \alpha p(t)\left(Y + \Pi_{Ih}(t)\right) \quad \text{(I0)}$$

- $\{G_{Ih}, G_{I0}, G_{P\emptyset}\}$ are equilibrium mixed strategies of an auction with values $\Pi_{Ih}(h(t))$, $\Pi_{I0}(h(t))$ for an incumbent where $h(t)$ is a history with continuation of tenure at time t, and values $\Pi_{Ih}(\tilde{h}(t))$, $\Pi_{I0}(\tilde{h}(t))$ for a poacher, where $\tilde{h}(t)$ is a history with job shift at time t.

The equilibrium solves for the fixed points of the beliefs, values and the auction equilibrium. The readers who are interested in the characterizations of the equilibrium can skip to the

next Section 1.4. For detailed derivation of the conditions, readers can refer to the Appendix 1.7.4 where we invoke a limit argument from a discrete model as $dt \to 0$.

Discussion on Beliefs   The drift of belief p is coming from the Bayes rule, taking into account the incumbent's growing pessimism with no arrival of good output. The initial condition is what the incumbent firm, and the outside firms, believe about the quality of the worker when the incumbent poached the worker from the previous firm. Note that, with maximum bid on the support, the firm expects to win for sure, and the expected quality of the worker is at most $x + (1 - x)p < 1$ because of the adverse selection.

Lastly, the drift equation for the belief x is the addition of two components:

$$x'(t) = \underbrace{\left(\int G_{P\emptyset}(v|t) \, dG_{Ih}(v|t) \right) \lambda x(t)(1 - x(t))}_{(1)} + \underbrace{\alpha(1 - x(t))p(h(t))}_{(2)}$$

The first component (1) is the bad news drift coming from the differential rate at which the good and bad workers are leaving the firm. Due to the tie-breaking rule assumption, if any other poacher was to arrive (at rate λ) before a poacher's arrival at τ, the earlier poacher must have taken the no news worker for sure, while the good workers have non-zero probability of being retained by the incumbent. Therefore, job-to-job transition in a worker's history is an imperfect bad news about the worker's type. (1) shows that there is growing optimism about the worker's type as the worker's tenure in the firm increases. Part (2) is additional flow of learning (αp) from the pool of workers with no news so far.

Discussion on (Ih), $(I0)$ These are all local conditions, or annuity equations for the expected future value of the firms. Although most of the results follow directly from the definition of the auction equilibrium and annuity equations, it is worth mentioning a few intuitions from the expressions. Note that the last condition implies, from no knowledge about the worker's type except for the initial expectation p, that the poacher's value is Π_{I0} with the starting beliefs. Integrating the annuity equation $(I0)$ yields the following

expression for the value:

$$\Pi_{I0}(\tilde{h}(t)) = \alpha p(\tilde{h}(t)) \int_0^\infty e^{-(\alpha+\lambda+r)s} \Big(Y + \Pi_{Ih}(\tilde{h}(t+s)) \Big) \, ds \qquad \text{(I0')}$$

where $\tilde{h}(t+s)$ is the history with continuation in this firm for duration of s. The derivation of the result is relegated to Appendix 1.7.5.

The equation is intuitive because it is the discounted value of transition to the learned state Ih at any future point $t + s$, with discount rate is the sum of three components, (1) the rate of transition to state Ih, α, (2) the rate of losing the worker before any transition arrival, λ, (3) and the discount rate r. Since the poacher does not observe any output, $x(\tilde{h}(t)) = 0$. The match starts from initial belief $p(\tilde{h}(t))$. Note, however, the Π_{I0} is not stationary because Π_{Ih} is changing over time.

Now we move on to the value of learning the good type at the history of continuation at time t, $\Pi_{Ih}(h(t))$. The discounted value of the future auctions matters both in terms of continuation probability and the expected bid payment. However, since all the bids on the support of the auction equilibrium induces same expected payoff, we can use the indifference condition to write down the value equation as if the incumbent firm wins all future auctions with probability 1:

$$-\Pi'_{Ih}(t) + (r + \lambda)\Pi_{Ih}(t) = \alpha Y + \lambda \Big(\Pi_{Ih}(t) - \bar{V}(t) \Big) \qquad \text{(Ih')}$$

where $\bar{V}(t)$ is the maximum bid over the support of $G_{P\emptyset}(v|h(t))$. The flow profit is given by

$$\alpha Y - \lambda \bar{V}(t)$$

flow expected produce minus the expected cost of retainment. Integrating over time yields:

$$\Pi_{Ih}(t) = \frac{\alpha Y}{r} - \frac{\lambda}{r} \int_t^\infty e^{-r(s-t)} \tilde{V}(s) \, ds$$

which has a clear interpretation of expected future value of the worker net the expected future (maximum) bids, in order to retain the worker.

In particular, when the cost is assumed to be held constant forever at $\bar{V}$,

$$\Pi_{Ih}(t) = \frac{\alpha Y}{r} - \lambda \int_t^\infty e^{-r(s-t)} \bar{V} \, ds$$
$$= \frac{\alpha Y}{r} - \frac{\lambda}{r} \bar{V}.$$

A natural reason for $\bar{V}$ to be fixed is because the worker quality is known to be H and the belief does not evolve. In case, $\bar{V}$ is the value of starting a new employment from belief 1, which is given by:

$$\Pi_{I0} = \alpha \int_0^\infty e^{-(\alpha+\lambda+r)s} [Y + \Pi_{Ih}] \, ds.$$

Substituting in the Π_{Ih} allows us to solve for Π_{I0}:

$$\Pi_{I0} = \frac{\alpha}{\alpha+\lambda+r} \left(Y + \frac{\alpha Y}{r} - \frac{\lambda}{r} \Pi_{I0} \right)$$

Algebra yields:[8]

$$\Pi_{I0} = \Pi_{Ih} = \frac{\alpha Y}{r+\lambda},$$

a stationary expression for both Ih and $I0$. This is intuitive because when the worker's type is known to be *High*, everytime the worker meets another firm, Bertrand auction drives down the value of the firm to 0. Furthermore, both Ih and $I0$ know the worker's type and is anticipated to receive the same expected profit.

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[8]Expanding,

$$\left(1 + \frac{\alpha}{\alpha+\lambda+r} \frac{\lambda}{r} \right) \Pi_{10} = \frac{\alpha}{\alpha+\lambda+r} \frac{\alpha+r}{r} Y$$
$$\frac{(r+\alpha)(r+\lambda)}{(\alpha+\lambda+r)r} \Pi_{10} = \frac{\alpha}{\alpha+\lambda+r} \frac{\alpha+r}{r} Y$$

and

$$\Pi_{1h} = \frac{\alpha Y}{r} - \frac{\lambda}{r} \frac{\alpha Y}{r+\lambda} = \frac{\alpha Y}{r+\lambda}.$$

Discussion on Auction The auction equilibrium characterized in Section 1.2.3 mostly applies to the equilibrium. However, the auction is equilibrium is more general because the values of winning the auction is different for the two bidders. The discrepancy comes from the history of the worker. If the incumbent firm wins, the worker's history is a continuation history $h(t)$, but if the poacher wins, the transition is made public in the new history $\tilde{h}(t)$. In general, the firm's values as calculated by $(I0)$ and (Ih) need not coincide for the two histories. In the next section, I focus on the cases in which I can characterize the equilibrium despite the complications.

1.4. Characterization of the Equilibrium

In order to solve for the equilibrium, it is necessary to keep track of the evolution of beliefs and the values at the same time. Below, I provide two examples where I can solve for the equilibrium objects in the closed form. For the first case, I assume short-lived firms whose values are stationary. From the first case, I derive an implication for the inference on worker quality from the observed employment history. For the second case, I impose an additional assumption on the auction rule. By doing so, I characterize the evolution of a firm's value, and its implication to the lifetime wage profile.

1.4.1. Solvable Case 1: Stationarity by Replacement of Firms

In this section I impose an assumption that makes firm values stationary. From there, I can solve for the Markov equilibrium in which the aforementioned two beliefs are state variables.

Note that the complications arise because the expected future profit of the firm has to take into account the future auction outcomes. In order to get around the issue, I assume that the firms are 'replaced' by a new firm every time a poacher arrives at rate λ. This can have interpretation of a new department within the same firm competing for the worker, or that there is a new manager introduced every new round of an auction. Another way to justify it is to say that the lifetime of a manager is short enough compared to the poaching attempt, which arrives only occassionally. The auction game is still played between the informed

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bidder (successor firm/manager of the 'incumbent') and an uninformed bidder ('poacher').

Assumption 4. The manager is replaced with the arrival of a poacher (at rate λ), and his/her expected lifetime is $\frac{1}{\lambda}$.

Values Assuming so, the future auction outcomes are effectively ruled out from the value calculation, and I take into account only the use value of this worker until the next auction. The values are stationary due to the memoryless property of a Poisson process. The expected duration of the match is always $\frac{1}{\lambda}$.

Proposition 3 (Values $(I0)$, (Ih)). Given a history $h(t)$ and beliefs $x = x(h(t))$ and $p = p(h(t))$, the values of the incumbent are $\Pi_{Ih}(h(t)) = \frac{\alpha Y}{r+\lambda}$, and $\Pi_{I0}(h(t)) = p\frac{\alpha Y}{r+\lambda}$.

The value of a poacher is $\Pi_{I0}(\tilde{h}(t)) = \tilde{p}\frac{\alpha Y}{r+\lambda}$, where $\tilde{p}$ is the initial belief on worker quality, having won the auction. $\tilde{p}$ satisfies $p \leq \tilde{p} \leq x + (1-x)p$.

In this environment, the value of a worker is linear in its belief about quality p. With the observation of a good output, the value jumps to $\frac{\alpha Y}{r+\lambda}$.

Auction Equilibrium Since the values are stationary, it suffices to know only the point beliefs $x = x(h(t))$ and $p = p(h(t))$ to characterize the values at the time of an auction.

Although information variables evolve over time, the auction game itself is a repeated static auction: after a transition shock (λ), there is a new auction game between two bidders; one informed and the other uninformed.

The informed bidder's value for the worker takes two points: $\frac{\alpha Y}{r+\lambda}$ with probability x, and $\alpha p\frac{Y}{r+\lambda}$ ($p = p(h(t))$) with probability $1 - x$. Since the winning poacher does not know whether the worker is H at the time of poaching, the game is different from the static auction, in which the value of the object is revealed instantly after winning. However, the two auctions are very similar in the sense that, in the static auction, the uncertainty is taken into account at the bidding stage, while in this environment, the uncertainty matters after winning the auction. Indeed, the auction equilibrium is very similar to the static case

as shown below:

Proposition 4. The equilibrium bids of the auction game with belief $p = p(h(t))$ and $x = x(h(t))$ come from the support

$$\left[p\frac{\alpha Y}{r+\lambda}, \ (x + (1-x)p)\frac{\alpha Y}{r+\lambda}\right].$$

Bidder $I0$ bids $p\frac{\alpha Y}{r+\lambda}$, and the equilibrium expected profit of the uninformed poacher is 0.

The distribution of bidder Ih bids, G_{Ih}, solves the indifference condition for bidder $P\emptyset$:

$$\left(\frac{xG_{Ih}(v) + (1-x)p}{xG_{Ih}(v) + (1-x)}\right)\frac{\alpha Y}{r+\lambda} - v = 0,$$

for all $v \in \left[p\frac{\alpha Y}{r+\lambda}, \ (x + (1-x)p)\frac{\alpha Y}{r+\lambda}\right]$.

The distribution of bidder $P\emptyset$ bids, $G_{P\emptyset}$, solves the indifference condition for bidder Ih:

$$G_{P\emptyset}(v)\left(\frac{\alpha Y}{r+\lambda} - v\right) = \frac{\alpha Y}{r+\lambda} - (x + (1-x)p)\frac{\alpha Y}{r+\lambda},$$

for all $v \in \left[p\frac{\alpha Y}{r+\lambda}, \ (x + (1-x)p)\frac{\alpha Y}{r+\lambda}\right]$.

From the assumption, in case of a tie, which happens with positive probability at $p\frac{\alpha Y}{r+\lambda}$, the poacher wins the worker.

Beliefs Given the auction equilibrium, I can calculate the winning probabilities. The incumbent $I0$ loses the worker for sure, while the incumbent Ih wins with probability $1 - \frac{1}{2}x$. The probability can be calculated explicitly from the bidding strategies, as in Appendix 1.7.6. Using this information, the drift of beliefs with the continuation in tenure forms an autonomous system:

Proposition 5. The pair of beliefs (x, p) as function of tenure τ in a firm solves the following ODE's:

$$x'(\tau) = \lambda\left(1 - \frac{1}{2}x(\tau)\right)x(\tau)(1 - x(\tau)) + \alpha(1 - x(\tau))p(\tau)$$

$$p'(\tau) = -\alpha p(\tau)(1 - p(\tau))$$

from starting belief $p(0)$ and $x(0) = 0$.

The evolution of beliefs reflect that the workers with good output is retained in the firm with probability $1 - \frac{1}{2}x$.

Earlier, in Section 1.2.2, I made an assumption (Assumption 1) that the incumbent and the poacher agree on the starting belief p_0. In the current setting, this can be done by making a technical assumption that the poaching wage is publicly observed.

Assumption 5. A worker's employment history shows the winning bid a poacher has made at the time of the worker's transition.

The assumption is made purely for a technical reason and does not have a substantive element. Under this assumption, the poacher's expected profit from winning an auction is 0, and the starting belief from a winning bid v is given by:

$$\frac{xG_{Ih}(v) + (1 - x)p}{xG_{Ih}(v) + (1 - x)},$$

where $x = x(h(t))$, $p = p(h(t))$, and the bid distribution, G_{Ih}, is from the auction equilibrium calculated above, with beliefs x and p. In essence, given the equilibrium bidding strategies, it is the expected quality of a worker derived from Bayes rule.

To summarize, the divergence of histories $h(t)$ and $\tilde{h}(t)$ at time t is reflected in the beliefs (x, p) given by $\big(x(h(t)), p(h(t))\big)$, and

$$x(\tilde{h}(t)) = 0, \quad p(\tilde{h}(t)) = \frac{xG_{Ih}(v) + (1 - x)p}{xG_{Ih}(v) + (1 - x)}.$$

Simulated Wage Paths   Since the system of beliefs is autonomous, an individual's career path can be readily simulated given a prior belief, transition shocks, and the outcomes of auctions, the bids from which governing the starting belief in a new tenure.

Figures 2, 3 depict a simulated path of beliefs and wages for two types of workers, using the identical transition shocks that arrived at times (0.61 1.22 2.73 3.84 6.42 7.06 11.63). The arrival of transition shocks are denoted with vertical dotted red lines.

Note that the L type worker shifted every time there was a transition shock because incumbent did not want to bid for the worker. These episodes are shown by the vertical dashed red lines exhibiting sharp drop in beliefs. The last episode at 11.63 pushed the belief down close to 0 because the poacher could win the worker with a very small bid.

Notice also that the belief path of H type workers is more smooth because some of the transition shocks did not result in transition. For this example, poaching attempts at time 0.61, 1.22, 3.84 and 11.63 were deterred by the incumbent; note the wage jumps at these points despite the smooth evolution of beliefs. These are instances of the incumbent matching the worker's outside offer. Even when the worker transitted to a new firm, (points 2.73, 6.42, and 7.06), the drop in beliefs were milder compared to the L workers because the poacher had to bid high enough to win against the incumbent. But also note that two close-by transition shocks at 6.42 and 7.06 resulted in sharp drop in the belief at point 7.06. Probably what happened is that the worker could not generate news during the short duration of tenure, and the poacher at 7.06 could win the worker with a relatively small bid.

In the end, the beliefs converge to the correct level. The histories diverge for the two workers because high type workers on average has longer tenure in a firm; with the arrival of a good output, the H workers are bid by two firms, while the L workers are always bid only by the poacher. In the limit, for a long enough history, public information would be enough to distinguish the two types. However, note that the belief about H worker's type jumps down whenever there is a shift to a new firm, since L workers are more likely to leave the match (this is an imperfect bad news); while the L workers also exhibit upward belief drift between any two transition shocks.

Figure 2: Simulated Belief and Wage Paths for Case 1, L type worker

Figure 3: Simulated Belief and Wage Paths for Case 1, H type worker

1.4.2. Solvable Case 2: Restriction on Auction Rule

For this case, I make the following assumption which is convenient, while retaining the main mechanism of the auction game:

Assumption 6. *Recall option*: The incumbent can buy the worker back from the poacher by bribing the poacher in case the poacher won the H worker. The incumbent pays the future value of the worker to the poaching firm.

The incumbent is indifferent between paying out the future value of the worker and losing the worker. The poacher still has incentive to bid in the auction because there is positive probability that he might win over the high type worker and is bribed, or reimbursed in cash by the incumbent. This assumption has the benefit of fixing the transition rule ex-ante for different types of workers (H worker always stays, while L worker always shifts), and making the auction game that of *symmetric* values.

Beliefs   Under this new auction rule, when the poacher wins the auction and realizes that the incumbent does not bribe, the firm immediately knows that the worker has not generated a news in the previous firm. On top of that, since the worker transfers if and only if the worker did not generate any news in any of the previous employers, the belief p is effectively the function of starting belief p_0 and elapsed time in the market t only:

$$p(h(t)) = p(t),$$

where

$$p(t) = \frac{p_0 e^{-\alpha t}}{p_0 e^{-\alpha t} + (1 - p_0)}$$

from intial belief p_0.

The belief x reverts back to 0 everytime a worker makes transition, and the drift of belief

at tenure τ, calendar time t is:

$$x'(t) = \lambda x(t)(1 - x(t)) + \alpha(1 - x(t))p(t)$$

starting from $x(t - \tau) = 0$. This equation can be solved for x in closed form, as attached in Appendix 1.7.7.

Auction Equilibrium The time variable t is calendar time, with knowledge of the point of the last job transition, $\sum_{i=1}^{n-1} \tau_i$. $\Pi_{Ih}(h(t))$ and $\Pi_{I0}(h(t))$ are the values of the incumbent. It is clear that the candidate lower bound of the bids is $\Pi_{I0}(h(t))$ since $I0$ bidder never bids above it.

The poacher's value at time t is evaluated over a different history, $\tilde{h}(t)$. However, from the bribing assumption, the poacher only obtains a no news worker. That is, when the poacher indeed wins a worker, the expected value of the worker $\Pi_{I0}(\tilde{h}(t))$ starts from initial belief $p(t)$ and $x(t) = 0$:

$$\Pi_{I0}(\tilde{h}(t)) = \alpha p(t) \int_0^\infty e^{-(\alpha+\lambda+r)s} \Pi_{Ih}(\tilde{h}(t+s)) \, ds$$

In case the poacher is bribed, the poacher receives transfer

$$\Pi_{Ih}(h(t))$$

from the incumbent.

Overall, the poacher expects at most

$$x(t)\Pi_{Ih}(h(t)) + (1 - x(t))\Pi_{I0}(\tilde{h}(t)),$$

from bidding the upper bound of the support, which is

$$x(t)\Pi_{Ih}(h(t)) + (1 - x(t))\Pi_{I0}(h(t)).$$

Therefore, the poacher's expected profit from the auction is:

$$(1 - x(t))(\Pi_{I0}(\tilde{h}(t)) - \Pi_{I0}(h(t)))$$

where $\tilde{h}(t)$ is continuation from starting belief $p(t)$ and $x(t) = 0$, in comparison to $h(t)$ from $p(t)$ and $x(t) \geq 0$. I later verify that the expression is positive in equilibrium. Intuitively, the two paths agree on the quality of the worker $p(t)$, but differs only in what others believe about the worker, $x(t)$. $h(t)$ is a history that is more costly to fight off a poacher.

In this case, given $x = x(t)$, the indifference condition for the poacher that defines distribution G_{Ih} is given by:

$$xG_{Ih}(v|t)(\Pi_{Ih}(h(t)) - v) + (1 - x)(\Pi_{I0}(\tilde{h}(t)) - v) = (1 - x)(\Pi_{I0}(\tilde{h}(t)) - \Pi_{I0}(h(t))).$$

In turn, the support of the bids is

$$\left[\Pi_{I0}(h(t)), \ x(t)\Pi_{Ih}(h(t)) + (1 - x(t))\Pi_{I0}(h(t))\right].$$

Since the upper bound of the support contains information about how much the incumbent has to pay in order to keep the match with probability 1, this feature allows us to write the incumbent's value autonomously in terms of $\Pi_{Ih}(h(t))$ and $\Pi_{I0}(h(t))$ only, without knowing about the worker's value in a new match.

Values   Denote by $\Pi_{Ih}(t)$ and $\Pi_{I0}(t)$ the continuation values for the incumbent, following history $h(t)$. From the set of beliefs, the values solve the system of equations:

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$$-\Pi'_{I0}(t) + (r + \lambda + \alpha p(t))\Pi_{I0}(t) = \alpha p(t)[Y + \Pi_{Ih}(t)] \tag{I0}$$

-

$$-\Pi'_{Ih}(t) + (r + \lambda)\Pi_{Ih}(t) = \alpha Y + \lambda \int (\Pi_{Ih}(t) - v)G_{P\emptyset}(v|t)\, dG_{Ih}(v|t) \tag{Ih}$$

Proposition 6. Using the fact that $\bar{v} = x(t)\Pi_{Ih}(t) + (1 - x(t))\Pi_{I0}(t)$, the second ODE can be rewritten:

$$-\Pi'_{Ih}(t) + (r + \lambda)\Pi_{Ih}(t) = \alpha Y + \lambda(1 - x(t))(\Pi_{Ih}(t) - \Pi_{I0}(t)). \tag{Ih'}$$

Note that this is a non-homogeneous system of ODE's ($x(t)$ and $p(t)$ are changing over time) with two variables Π_{Ih} and Π_{I0}. Still, it is an autonomous system of two variables, in which I can characterize some properties of the solution:

Proposition 7. Fix time t, and the beliefs $x(t)$, and $p(t)$. The value at time t that the incumbent firm has to pay in order to retain the worker for sure (i.e., upper bound of the bidding support) is given by

$$\alpha Y \int_t^\infty e^{-(r+\lambda)(z-t)}\Big(x(z) + (1 - x(z))p(z)\Big)\, dz,$$

which is the integral over future path of the quality of the workers in the firm $x(z) + (1 - x(z))p(z)$, for $z \in [t, \infty)$.

The expression is intuitive in the sense that it integrates over the path of future beliefs regarding the quality of the worker staying in the firm. The firm's profit is shown to be decreasing over time, as the cost term increases with time. The proof involves solving the system of equations using substitution, and the readers can refer to Appendix 1.7.8 for full proof. The full path of firm profits Π_{Ih}, Π_{I0} can be also found accordingly, although the expressions are much messier. Note that in the limit, $x(z) + (1 - x(z))p(z)$ approaches 1,

and the cost term goes to
$$\frac{\alpha Y}{r + \lambda}.$$

This verifies that a firm's information rent decreases with time.

A few basic properties of the value function is outlined here:

Proposition 8. For any $h(t)$, $\Pi_{Ih}(h(t))$ is decreasing in α and decreasing in λ. A poacher's value $\Pi_{I0}(\tilde{h}(t))$ is increasing in α and increasing in $p_0 = p(\tilde{h}(t))$.

Proof. Since the closed form of x and p are known, the closed form for the $\tilde{x}(t) = x(t) + (1 - x(t))p(t)$ is:

$$\tilde{x}(t) = \frac{p_0 \frac{\alpha}{\alpha + \lambda}\left(1 - e^{-(\alpha + \lambda)t}\right) + p_0 e^{-(\alpha + \lambda)t}}{p_0 \frac{\alpha}{\alpha + \lambda}\left(1 - e^{-(\alpha + \lambda)t}\right) + e^{-\lambda t}\left(p_0 e^{-\alpha t} + (1 - p_0)\right)}.$$

The result is just a substitution of this expression into the Proposition 7. Details of the procedure is contained in Appendix 1.7.9 □

Simulated Path of Wages Below is a simulated path of beliefs x and p for a particular realization of Poisson arrivals, when both λ and α are set at $\frac{1}{3}$. The blue line stands for the poacher's belief x about the news arrival in the incumbent; while the green line stands for the incumbent's belief that the worker's type is H, p. The green line is gradually decreasing from p_0 according to a good news drift. Note that with the arrival of output at point 4.2, the incumbent's belief (green line, on the right) jumps up to 1 and stays there after.

Note also that in the left graph of Figure 4, the slope of the poacher's belief gets smaller as the time goes on. This is because the growth of x is affected by learning, which happens at rate $\alpha p(t)$. Everytime the worker transits, the belief starts to grow again from 0, and the initial growth rate is affected by the initial level of optimism about the worker's type. Therefore, even for the same H type of workers, if one happens to draw the news at a later point in career, the wage growth rate thereafter will be slower than when the worker
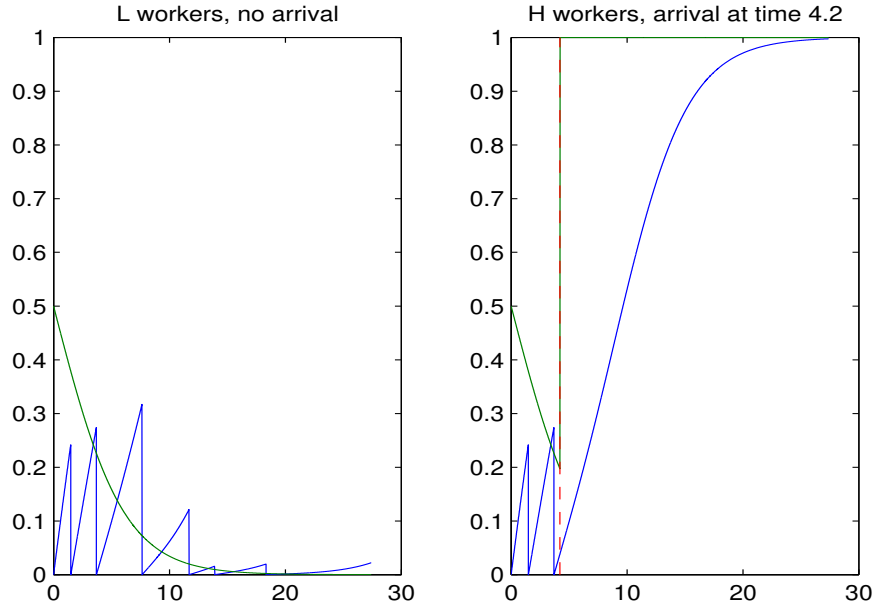
Figure 4: Simulated Path of Beliefs

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happened to draw the news at an early point in career.

For these two paths simulated for the length of 30, we see that they start to diverge after the arrival of the good news. Before then, the transition shocks (rate λ) affects the two workers in the same way by driving the outside firm's belief back into 0. For L type workers, everytime the worker meets another firm, the worker transits to a new firm, which drives down the poacher's belief, x, back to 0. H type workers are not explicitly affected since there is no transition. However, the competitors infer postive news about the worker's type as the worker's tenure increases (smooth increasing part in the right figure). Ultimately, the belief reaches the correct one, $x = 1$ for sufficiently long tenure.

I now investigate how the belief path translates to wages paid to the worker. I use the

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[9]x in blue and p in green. Initial belief set at $p_0 = 0.5$. Arrival of transition shocks, which arrives at rate λ, shown by the kinks at the left Figure. For H workers, learning shock occured at time 4.2, indicated by red dotted line.
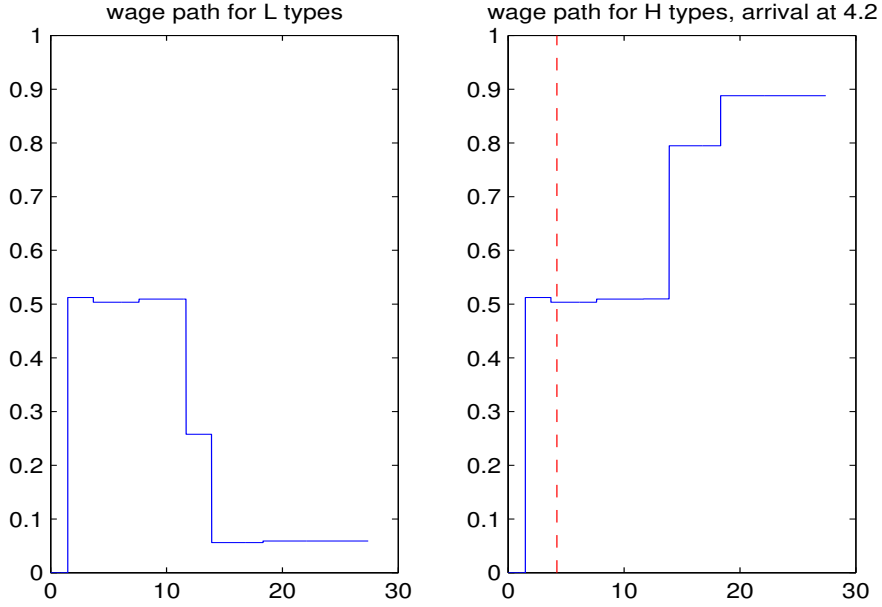
Figure 5: Sample Wage Paths for the Two Types of Workers

proxy, the retaining cost term, which we wrote as

$$\alpha Y \int_t^\infty e^{-(r+\lambda)(s-t)} \Big(x(s) + (1 - x(s))p(s) \Big) \, ds$$

These are calculated from the last observed transition, and is a function of tenure. We solve for this integral and normalize for the termination rate of the match $r + \lambda$ to be the proxy for the constant wage paid to the worker. Note that this is an upper bound of the actual wage paid since actual wage is determined by the outcome of the auction. We graph a corresponding sample wage paths below:

The wage paths start to diverge after the arrival of news: for the H type workers, longer duration of stay in the match signals outside firms that the worker is of high quality, and the incumbent firm has to pay more in order to fight off the poachers who are bidding more aggressively for this worker. In the end, the wage is driven up to the productivity of the match, αY. For the L types, although they might initially draw high wage by luck, the wage converges to 0 as the work history drives belief down close to 0.

I expect the wage path of L workers to exhibit a hump pattern. Although the wage growth in the middle age is driven by the fact that they start from wage 0, it is also magnified by firm's information rent.

1.5. Discussions

1.5.1. Role of Job Transition in Wage Growth and Long-Run Convergence

For both the case 1 and case 2, the long-run convergence is obvious. For case 2, a worker with a good news is kept with probability 1, and the belief about the worker's quality gradually converges to 1. In case 1, whenever a transition shock arrives, L workers shift with probability 1, while H workers shift with probability less than 1. After observing employment history for long enough, the firms can distinguish the two types.

Proposition 9. For case 1, the average tenure of a L worker is $\frac{1}{\lambda}$, while the average tenure of a H worker is greater than $\frac{1}{\lambda}$. For long enough job history, the firms correctly learn the worker's type.

Since both workers start from a pool with initial belief p_0, the proposition implies that H workers, in the long run, raise their wage to 1, while for L workers, wage goes down to 0.
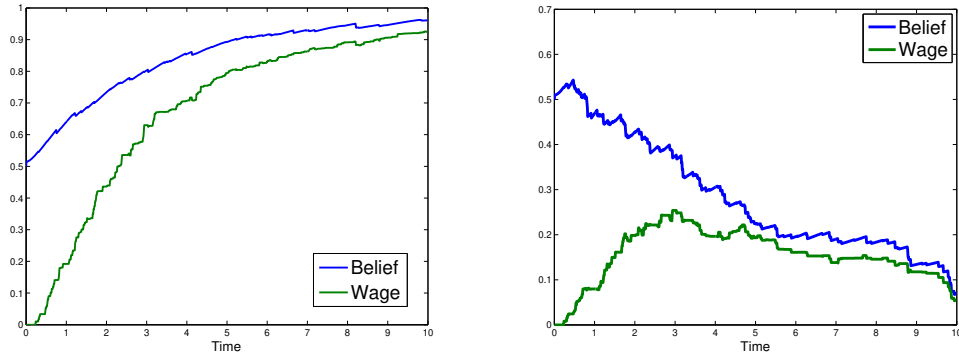

Figure 6: Simulated Average Belief and Wage Path for H Types (Left) and L Types (Right)

Figure 6 shows an average path of belief and wage for the two types respectively, an average path of 50 different simulated wage paths, following case 1 environment. Note that the path still exhibits many kinks due to a large heterogeneity of path realizations. However, there

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is noticeable trend of the belief and the wage converging to the correct level.

The model implies that, on average, workers with frequent job transitions are likely to be low-quality workers, and the number of transitions is negatively correlated with wage. Light and McGarry (1998) shows that unobserved heterogeneity can account for negative correlation between number of job transitions and low wages. This is in contrast to 'search goods' explanation of job transition, in which job transition indicates that a worker moving to a better paying job, and hence, more transitions increase a worker's wage. The authors conclude that the data is more consistent with 'experience goods' model, in which the transition is initiated by dissolution of a match that is doing poorly.

Carrillo-Tudela and Kaas (2015) has also pointed out that perfect information model of job transition cannot account for the unobserved heterogeneity. Their modelling method differs from this paper, in that they focus on firms' screening and worker sorting. Both the theory and data shows that it is important to take into account unobservable heterogeneity of the workers, and a natural modelling choice is to incorporate asymmetry of information. This paper presents one way of modelling asymmetry of information, in a natrual environment in which some employers know more about the worker than the others.

In a different line, there are papers showing that lifetime earning paths diverge significantly for workers in different percentiles of income. A report by Guvenen et al. (2015) analyses the lifetime earnings of more than 4 million workers using their W2 record. What they show is a large heterogeneity in lifetime earnings growth. Bottom 20 percentile of workers exhibit lifetime earnings decrease, while a median worker experiences 38% increase in lifetime earnings. Even after controlling for extreme outliers, top percentile workers' gain in lifetime earnings is about 1500%.

There are many ways to account for the heterogeneity, and learning is one possible explanation. Farber (2005) used survey data of displaced workers (due to slack work, plant closing, or position abolishment) and showed that about 10% of the displaced workers permanently

leave the labor force after displacement. Even for those who return, 13% are hired at part-time jobs after losing a full-time job, and the re-employment income is on average 13% less than the previous job. There are many factors underlying this phenomenon, including the age effect (the authors focus on life-cycle earnings), and firm specific human capital that is lost at displacement. However, it is also documented that there is noticeable 'stigma' effect to workers who lose jobs, and in order to account for the effect, it is necessary to incorporate learning in the model. This paper suggests one step into the direction, and future work will involve identifying the effect of learning within a comparable sample of small cohort with similar characteristics (same college, MBA program, etc.) and checking if the job transition affect these workers differently.

1.5.2. *Implications to the Information Policy*
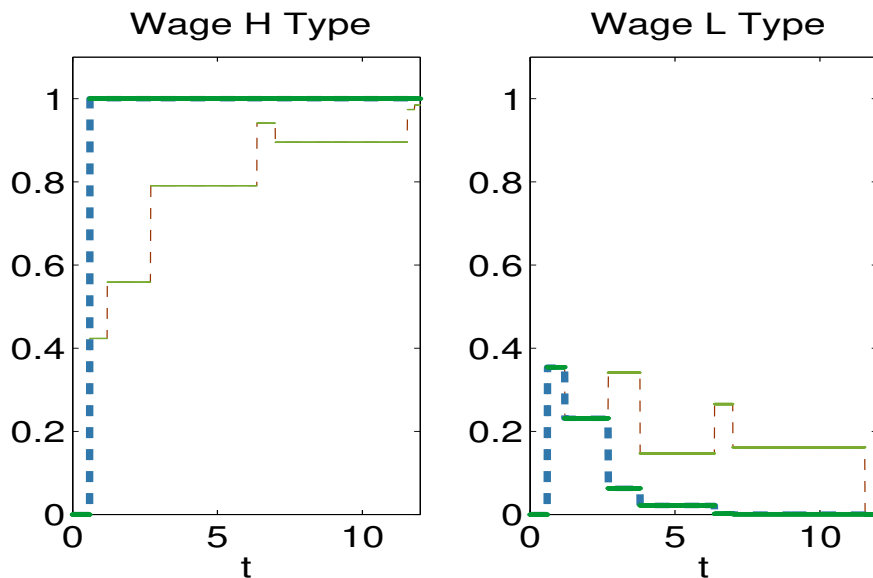
Figure 7: Simulated Wage Profile for Full-Information

10

It is worthwhile comparing with the benchmark case where the learning is a public news. The benchmark case is a very simple example of Postel-Vinay and Robin (2002) model

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[10] As in Postel-Vinay and Robin (2002). The thicker lines (full-information wage profile) allow for comparison with the case with the thin lines (wage profile under asymmetric information, from the previous section). Note that H type workers boost up the wage to 1 in early career, while the wage of L type workers gradually decline along the belief path $p(t)$.

where there is no firm heterogeneity and workers can take at most two types. Since our first price auction limits to the Bertrand competition when there is no asymmetry of information, every time there is an auction, the wage gets driven up to the belief about the worker at the point of auction.

Due to the Bertrand competition, the value of a H worker to a firm is discounted by rate λ, arrival of a poacher. The profit the firm can obtain by keeping the worker is

$$\frac{\alpha Y}{r + \lambda}.$$

Therefore, starting from the initial belief p about the worker's type, the value of the worker is

$$\alpha p \int_0^\infty e^{-(\alpha + \lambda + r)t} [Y + \frac{\alpha Y}{r + \lambda}] \, dt$$

which can be decomposed into the use value of the worker when the worker actually generates a breakthrough before a poacher arrives (y term) and the continuation value starting from the breakthrough, $\frac{\alpha Y}{r+\lambda}$, appropriately discounted taking into account transition and poaching rate. The value can be extended to be

$$\alpha p \Big(\frac{Y}{\alpha + \lambda + r} + \frac{1}{\alpha + \lambda + r} \frac{\alpha Y}{r + \lambda} \Big) = \frac{\alpha p Y}{r + \lambda}$$

Therefore, we expect that, everytime a worker with no breakthrough shifts a job, to receive the future value $p \frac{\alpha Y}{r+\lambda}$ at that time, and the wage to drive up to the marginal productivity $\frac{\alpha Y}{r+\lambda}$ with the arrival of a breakthrough.

From this example, it is worth noting that the worker who has generated a news does benefit from making the news public. While it has negative consequences for bad workers because their wage deterministically drifts down along with the belief. In our model example of two worker types, the wage is almost perfectly informative about the worker's type because any wage increase by retention is a perfect signal that the worker is a High type.

This sheds light onto the recent California legislature which bans firms from inquiring about the worker's past wage history. The legislature was introduced on the grounds that the worker quickly loses bargaining power once the wage information is revealed, especially for the workers who were receiving wage that are below average/expectation to the new employer. According to our model, the high type workers who have generated the good news would still want to credibly convey the wage information to the new firm, while the low type workers would want to hide the information. Along with consideration for pooling/what the firms can infer from declined report, the model tells us what the effect would be if the ban is to be strictly enforced to preclude all wage information. Furthermore, we expect our main mechanism to go through if there is small chance that high type workers would also like to hide their information about their wage history, because if the current wage is not a favorable signal to the worker's type, the worker benefits by hiding it rather than disclosing it.

It should be noted, however, that this exercise is only a benchmark. A limitation is that I assumed short-lived firms. If firms internalize the informational content of the wage in future auctions, the element would alter the firms' bidding strategy. However, restricting to a perfect news benchmark is not a large deviation from the policy exercise. In the perfect good news output, as I describe here, a poacher would immediately tell that a worker is H type if the worker received a promotion in the current firm. If the poacher sees that a worker did not receive a promotion, then the poacher is more pessimistic about the worker's quality.

1.5.3. Role of the Firm's Commitment

In this section, I explore the role of firm's commitment in driving the main mechanism of the model. This can serve both as a robustness check for the main results, or serve as the gauge for the genericity of the model in terms of capturing different real world dynamics. For instance, if we consider an object that is open for appraisal, such as real-estate property, it is more natural to think that the good assets trade faster, while the less attractive ones

stay in the market. In the relevant paper, Kaya and Kim (2018) shows how the trading dynamics would flip if we take into account these 'appraisal' possibility.

We construct a realistic example with drastically different results. Assume that the firms instead learn by 'bad news'. That is, instead of assuming that α accompanies the lump-sum produce of Y, think of it as accompanying a lump-sum deficit $-Y$, which makes it inefficient for the firm to retain the worker afterwards. Assume that a worker with no news generates flow profit of $b > 0$ to the firm. Expected flow profit of the H worker is $b > 0$, while for L worker, it is $b - \alpha Y \leq 0$.

The match starts with prior belief p_0 and drifts according to the equation

$$p'(t) = \alpha p(t)(1 - p(t)), \quad p(t) = \frac{p_0 e^{\alpha t}}{p_0 e^{\alpha t} + (1 - p_0)}.$$

Since the bad news worker is not kept in this firm, both the poacher and the incumbent agree that a worker with tenure t is likely to be H type with probability $p(t)$. Opportunity for the poacher to 'appraise' the good, or to 'interview' a worker will make the poacher better informed than the incumbent.

Definition 5. With an 'interview', the poacher privately observes the news generated by the worker for next T duration of time.

We assume that the news process the interviewer observes is the same as the bad news breakthrough process that they see as an employer. The result still goes through for other types of news process, such as perfect good news process, or any other imperfect news. Assuming bad news process, from the incumbent's point of view, the poacher's belief about the worker takes at most two points:

$$\begin{cases} \tilde{p} = \frac{p(t)}{p(t) + (1 - p(t))e^{-\alpha T}} > p(t) & \text{if no bad news, probability } x(t) \\ 0 & \text{if bad news, probability } 1 - x(t) \end{cases}$$

where, according to the definition of interview: $x(t) = p(t) + (1 - p(t))e^{-\alpha T}$ and $1 - x(t) =$

$(1 - p(t))(1 - e^{-\alpha T})$. This is exactly the mirror case of the main model, where now we let $x(t)$ be the probability that the *incumbent* attaches to the event that there were no bad news observed in the interview. Effectively, the incumbent now has to worry about overpaying for the worker who the poacher identified as unproductive.

In the replacement case (Example 2), the support of the bids is the interval $[0, p(t)]$, where the poacher with no bad news $(P0)$ wins with probability $1 - \frac{1}{2}x(t) = 1 - \frac{1}{2}\left(p(t) + (1 - p(t))e^{-\alpha T}\right)$ and bids to make incumbent firm $(I\emptyset)$ satisfy zero-profit, indifference condition: (assume that the incumbent firm's bid is disclosed)

$$0 = \frac{x\tilde{p}G_{P0}(v) + (1 - x) \cdot 0}{xG_{P0}(v) + (1 - x)} \frac{\alpha Y}{r + \lambda} - v$$

Since the probability of winning is 1 at the upper bound of the support, $\bar{v}$, we pin down the upper bound:

$$\bar{v} = (x\tilde{p} + (1 - x) \cdot 0)\frac{\alpha Y}{r + \lambda} = p(t)\frac{\alpha Y}{r + \lambda} = \Pi(p(t)).$$

Poacher with bad news (PL) wins with zero probability, while the incumbent firm $(I\emptyset)$ bids in order to make $P0$ bidder indifferent:

$$G_{I\emptyset}(v) = \frac{\Pi(\tilde{p}) - \Pi(p(t))}{\Pi(\tilde{p}) - v} = \frac{(\tilde{p} - p(t))\frac{\alpha Y}{r + \lambda}}{\tilde{p}\frac{\alpha Y}{r + \lambda} - v} \quad \text{over } v \in \left[0, p(t)\frac{\alpha Y}{r + \lambda}\right].$$

Since only a no bad news poacher, $P0$, makes non-trivial bid, successful poaching drives up belief to $\tilde{p}$ from $p(t)$. If the incumbent succeeded in retaining the worker, due to the interview, the poacher might have been type PL who saw bad news about the worker. Every time the worker is retained with bid v, belief jumps down taking into account the adverse selection:

$$p(h(t)) = \frac{x\tilde{p}G_{P0}(v) + (1 - x) \cdot 0}{xG_{P0}(v) + (1 - x)} < p(t).$$

1.6. Conclusion

This paper focuses on the relationship between the wage profile and the job-to-job transition of a worker, by exploring the interaction between the micro-auction game and the macro-labor model. I incorporate learning and information friction into the traditional model of on-the-job search, and show that asymmetry of information can generate high job transition rate for low type workers, and low job transition for high type workers. This finding is consistent with data which shows that escape from a bad match is a more likely cause of job transition than worker search alone. Using the results from the literature on auctions, I characterize the auction equilibrium in the dynamic setting, and analyse the effect of information policy. I show that hiding a worker's wage history helps low type workers, while it might have a negative consequence of hurting high type workers. At the same time, a worker may on average be worse off when the law switches off learning from wage history. The model shows that the interaction between a well-informed incumbent and less-informed poacher results in positive signal on worker quality as a worker's tenure grows. Depending on the industry, this might not be a case. If a worker's performance is public, or if the firm lacks commitment to keep a bad worker, it is possible that a longer tenure convey negative signal about quality. I hope this discussion can shed light to different connotations to job transitions attached in various industries.

1.7. Appendix to Chapter 1

1.7.1. Good News Drift

Starting from initial probability $p(t) \in [0,1]$, the probability that no news arrives in dt interval is given by:

$$P(H)Pr(\text{no news}|H) + P(L)Pr(\text{no news}|L) = p(t)(1 - \alpha dt) + (1 - p(t)).$$

Applying Bayes rule, $p(t + dt)$ is given by:

$$p(t + dt) = \frac{p(t)(1 - \alpha dt)}{p(t)(1 - \alpha dt) + (1 - p(t))}.$$

Subtracting $p(t)$ from both sides:

$$p(t + dt) - p(t) = \frac{p(t)(1 - p(t))(1 - \alpha dt) - p(t)(1 - p(t))}{p(t)(1 - \alpha dt) + (1 - p(t))}$$
$$= \frac{p(t)(1 - p(t))(-\alpha dt)}{p(t)(1 - \alpha dt) + (1 - p(t))}.$$

Dividing both sides by dt and taking limit $dt \to 0$,

$$p'(t) = -\alpha p(t)(1 - p(t)).$$

In general, this is true even if match dissolves with some positive rate, as long as the rate for H and L types are identical. Starting from any $p_0 = p(t)$, and using the Bayes' rule, the belief after dt is given by

$$p(t + dt) = \frac{p(t)(1 - rdt - \alpha dt - \lambda dt)}{p(t)(1 - rdt - \alpha dt - \lambda dt) + (1 - p(t))(1 - rdt - \lambda dt)}$$

$$p(t + dt) - p(t) = \frac{p(t)(1 - p(t))(1 - rdt - \alpha dt - \lambda dt) - p(t)(1 - p(t))(1 - \lambda dt)}{p(t)(1 - rdt - \alpha dt - \lambda dt) + (1 - p(t))(1 - rdt - \lambda dt)}$$

$$p'(t) = -\alpha p(t)(1 - p(t)), \quad p(0) = p_0$$

This is true as long as both the high and low types leave at the same rate λ.

1.7.2. Results from Engelbrecht-Wiggans et al. (1983)

Definition 6. Z is the value of the object, X is the private signal of an informed bidder, U is an independent uniform random variable on $[0, 1]$. Let $H = E[Z|X]$.

Definition 7. Define the informed bidder's *distributional type* be $T = T(H, U)$, uniformly

distributed on $[0, 1]$, where

$$T(h, u) = Pr(\{H < h, \text{ or } H = h \text{ and } U < u\}).$$

Let

$$H(t) = \inf\{h | P(H \le h) > t\}$$

to have $H = H(T)$ almost surely.

We solve for the equilibrium strategies $\beta : [0, 1] \to \mathbb{R}_+$ for the informed bidder, and the bid distribution G for uninformed bidder.

Proposition 10 (Engelbrecht-Wiggans et al. (1983))**.** The equilibrium bid distribution β of the informed bidder is,

$$\beta(t) = E[H(T) | T \le t] = \frac{1}{t} \int_0^t H(s) ds$$

with $\beta(0) = H(0)$, and $\beta(1) = E[H]$.

The equilibrium bid distribution G of the uninformed bidder is

$$G(b) = Pr(\beta(T) \le b).$$

Proof. Suppose the informed bidder type is $T = t$. If he bids $\beta(\tau)$, then he wins with probability τ, yielding an expected payoff

$$[H(t) - \beta(\tau)]\tau = \int_0^\tau (H(t) - H(s)) \, ds$$

which is maximized at $\tau = t$.

For any uninformed bidder, any bid below $H(0)$ yields zero payoff, while bid greater than

$E[H]$ generates negative payoff. Consider a bid $b = \beta(t)$. Its expected payoff is

$$E[Z - \beta(t) | T \leq t] t$$

However, $E[Z - \beta(t) | T \leq t] = E[H(T) | T \leq t] - \beta(t) = 0$. □

Applying the results to our environment, we get the distribution

$$H(t) = \begin{cases} \Pi_{I0} & t \leq 1 - x \\ \Pi_{Ih} & t > 1 - x \end{cases}$$

which implies

$$\beta(t) = \frac{1}{t} \int_0^t H(s)\, ds = \begin{cases} \Pi_{I0}, & t \leq 1 - x \\ \frac{1}{t}\Big((1 - x)\Pi_{I0} + (t - (1 - x))\Pi_{Ih}\Big), & t > 1 - x \end{cases}$$

1.7.3. Common Value Auction: A Constructive Proof

In this Appendix, we show that in equilibrium:

- $I0$ submits degenerate bid Π_{I0}. Ih and $P\emptyset$ submit randomized bids over support $[\Pi_{I0}, \tilde{\Pi}]$, where $\tilde{\Pi} = x\Pi_{Ih} + (1 - x)\Pi_{I0}$.

- Bidding distributions for the three players are

$$G_{Ih}(b) = \frac{1 - x}{x} \frac{b - \Pi_{I0}}{\tilde{V} - b}$$

$$G_{P\emptyset}(b) = \frac{(1 - x)(\Pi_{Ih} - \Pi_{I0})}{(\Pi_{Ih} - b)}$$

G_{I0} is degenerate at Π_{I0}.

Claim 1. There does not exist a pure strategy NE of this auction game.

Proof. The value of the object at sale is at least Π_{I0}.

Any bid $b < \Pi_{I0}$ is not played in equilibrium because any opponent player can make profitable deviation to b' with $b < b' < \Pi_{I0}$.

Suppose bidder $P\emptyset$ always bids $b = \Pi_{I0}$. Then bidder $1H$ has profitable deviation for any bid $b' > \Pi_{I0}$, from which he can bid slightly less, $b' > b'' > \Pi_{I0}$.

Suppose bidder $P\emptyset$ bids $\Pi_{I0} < b \leq x\Pi_{Ih} + (1-x)\Pi_{I0}$. Bidder Ih's best response is to bid slightly above b and win for sure. In which case, the bidder 2 expects to gain negative profit $\Pi_{I0} - b < 0$, and would rather get 0 payoff by bidding Π_{I0}.

The bidder $P\emptyset$ never bids above $x\Pi_{Ih} + (1-x)\Pi_{I0}$ in equilbrium. Since bidder $I0$'s best response to $b > \Pi_{I0}$ of bidder $P\emptyset$ is to bid below b, bidder $P\emptyset$ always wins over $I0$ in equilibrium. Hence his maximum willingness to pay for the object is $x\Pi_{Ih} + (1-x)\Pi_{I0}$. $\square$

Therefore, if there is an equilibrium, it has to be in mixed strategies, the support is given by the following claim.

Claim 2. In equilibrium, bidder $P\emptyset$'s bid distribution $G_{P\emptyset}$ is continuous and strictly increasing over $(\Pi_{I0}, x\Pi_{Ih} + (1-x)\Pi_{I0}]$.

Proof. First, note that given the candidate bidder $P\emptyset$'s strategy, bidder $I0$ never puts positive probability on bids above Π_{I0}. Therefore, the strategy is best response to bidder Ih's. Furthermore, Ih and $P\emptyset$ shares the same support. We can also rule out any atom in the interior of the support, otherwise there is profitable deviation. We can also rule out $G_{P\emptyset}$ having atom at $\tilde{\Pi}$ because Ih would deviate. $\square$

In equilibrium, Ih and $P\emptyset$ mixes over bids to make the opponent indifferent over all bids on the support. That is, G_{Ih} solves:

$$xG_{Ih}(b)(\Pi_{Ih} - b) + (1-x)(\Pi_{I0} - b) = 0$$

and $G_{P\emptyset}$ solves:

$$G_{P\emptyset}(b)(\Pi_{Ih} - b) = (\Pi_{Ih} - \tilde{\Pi}) = (1 - x)(\Pi_{Ih} - \Pi_{I0}).$$

When ties are resolved with the toss of a fair coin, the winning probabilities are:

$$\begin{cases} \frac{1}{2}x + (1 - x) & \text{for } Ih \\[2mm] \frac{1}{2}(1 - x) & \text{for } I0 \\[2mm] \frac{1}{2} & \text{for } P\emptyset \end{cases}$$

Conditional on type realizations, Ih or $I0$, probability that bidder $P\emptyset$ wins is

$$Pr(P\emptyset \text{ wins}|Ih) = \frac{1}{2}x, \quad Pr(P\emptyset \text{ wins}|I0) = x + \frac{1}{2}(1 - x)$$

That is, bidder $P\emptyset$'s expected payment is

$$\frac{1}{2}x^2\Pi_{Ih} + (x(1 - x) + \frac{1}{2}(1 - x)^2)\Pi_{I0}$$

Bidder Ih expects to pay

$$\frac{1}{2}x\Pi_{Ih} + (1 - x)\Pi_{I0}$$

Bidder $I0$: $\frac{1}{2}(1 - x)\Pi_{I0}$.

1.7.4. Proposition 2: Derivation of Equilibrium Conditions

Beliefs Time t stands for tenure in the firm. The differential equation is obtained using Bayes rule:

$$x(t + dt) = \frac{x(t)\big(1 - \lambda dt + \lambda dt Pr(Ih \text{ wins}|t)\big) + (1 - x(t))p(t)\alpha dt}{x(t)\big(1 - \lambda dt + \lambda dt Pr(Ih \text{ wins}|t)\big) + (1 - x(t))(1 - \lambda dt + \lambda dt Pr(I0 \text{ wins}|t))}$$

where, given the bid distributions at time t,

$$Pr(Ih \text{ wins}|t) = \int G_{P\emptyset}(v|t) \, dG_{Ih}(v|t).$$

We assume that in case of a tie, the worker shifts to $P\emptyset$. In this case, $Pr(I0 \text{ wins}|t)$ is 0, and

$$x(t+dt)-x(t) = \frac{x(t)(1-x(t))\big(1-\lambda dt + \lambda dt Pr(Ih \text{ wins}|t)\big) + (1-x(t))(p(t)\alpha dt - x(t)(1-\lambda dt))}{x(t)\big(1-\lambda dt + \lambda dt Pr(Ih \text{ wins}|t)\big) + (1-x(t))(1-\lambda dt)}$$

Dividing by dt and taking $dt \to 0$:

$$x'(t) = \underbrace{\lambda Pr(Ih \text{ wins}|t)x(t)(1-x(t))}_{(1)} + \underbrace{\alpha(1-x(t))p(t)}_{(2)}$$

Note that the differential rate at which I wins affects the drift of the belief x. Intuitively, when the $I0$ workers are leaving with probability 1, staying at this firm is a partial good news about the worker's type, which is reflected in the drift component in (1). The second component, (2), is additional breakthrough flowing from $1 - x(t)$ to $x(t)$ pool.

Values It is informative to look at the recursion of the value equations in the discrete time:

$$\Pi_{Ih}(t) = Y \alpha dt + (1-rdt-\lambda dt)\Pi_{Ih}(t+dt) + \lambda dt \int (\Pi_{Ih}(t+dt)-v)G_{P\emptyset}(v|t+dt)dG_{Ih}(v|t+dt)$$

where the recursive expression $\Pi_{Ih}(t+dt)$ takes into account the the location of the future beliefs, which is taken as exogenous by the firm at the time of auction.

Since the highest bid $\bar{V}(t)$ wins the auction game for sure, substituting the indifference condition:

$$\Pi_{Ih}(t) = Y \alpha dt + (1 - rdt - \lambda dt)\Pi_{Ih}(t + dt) + \lambda dt(\Pi_{Ih}(t + dt) - \bar{V}(t))$$

Subtracting both sides by $\Pi_{Ih}(t)$ and dividing by dt:

$$0 = \alpha Y + \Pi'_{Ih}(t) - (r + \lambda)\Pi_{Ih}(t) + \lambda(\Pi_{Ih}(t) - \bar{V}(t)). \tag{1h'}$$

Since this holds for any t, integrating it for $[s, \infty)$ after mutiplying by $e^{-(r+\lambda)t}$ yields

$$0 = \frac{\alpha Y}{r + \lambda}e^{-(r+\lambda)s} - e^{-(r+\lambda)t}\Pi_{Ih}(t) + \int_t^\infty e^{-(r+\lambda)s}(\Pi_{Ih}(s) - \bar{V}(s))\,ds.$$

Similarly, for Π_{I0}, the value it derives are from expected value of transition to state Π_{Ih},

$$\Pi_{I0}(t) = (1 - r\,dt - \lambda dt - \alpha p(t)dt)\Pi_{I0}(t + dt) + \alpha p(t)\,dt\Pi_{Ih}(t + dt),$$

$$0 = \Pi'_{I0}(t) - (r + \lambda + \alpha p(t))\Pi_{I0}(t) + \alpha p(t)\Pi_{Ih}(t) \tag{I0}$$

1.7.5. Section 1.3: Expected Flow Value I0

Normalize all the expressions so that the starting period t is 0. First note that Π_{I0} starting from initial belief $\tilde{p}(0)$, by a direct integration of $I0$, is given by:

$$\Pi_{I0}(0) = \int_0^\infty e^{-\int_0^s (r+\lambda+\alpha\tilde{p}(z))dz}\alpha\tilde{p}(s)\Big(Y + \Pi_{Ih}(s)\Big)\,ds,$$

where $\Pi_{Ih}(s)$ is the value after learning the type of the worker, evaluated over the Bayes rational belief path starting from $\tilde{p}(0)$. We note that Π_{I0} is the expected future value from transition to the learned state Π_{Ih} and the lump-sum payoff Y that arrives at rate α in case the worker is indeed $High$ quality.

Due to our good news learning assumption, the rate of transition at tenure τ, $\alpha\tilde{p}(\tau)$, and

the evolution of $\tilde{p}(\tau)$ exactly cancels out in the integral:

$$\alpha\tilde{p}(s)e^{-\int_0^s \alpha\tilde{p}(z)\,dz} = \alpha\tilde{p}(s)e^{-\alpha s + \int_0^s \alpha(1-\tilde{p}(z))\,dz}$$

$$= \alpha\tilde{p}(s)e^{-\alpha s - \int_0^s \frac{\tilde{p}'(z)}{\tilde{p}(z)}\,dz}$$

$$= \alpha\tilde{p}(s)e^{-\alpha s}\frac{\tilde{p}(0)}{\tilde{p}(s)} = \alpha\tilde{p}(0)e^{-\alpha s}$$

Therefore, the expression can be simplified to:

$$\Pi_{I0}(0) = \alpha\tilde{p}(0)\int_0^\infty e^{-(r+\lambda+\alpha)s}\Big(Y + \Pi_{Ih}(s)\Big)\,ds.$$

Intuitively, Π_0 is the expected value of transition to Π_1 at rate α, with effective discount rate $\lambda + r$, coming from the dissolution of the match at rate λ. The good news drift allows us to replace the effect from change in transition rate over time, $\alpha\tilde{p}(s)$, with the initial probability $\tilde{p}(0)$ and constant rate of transition α.

1.7.6. Section 1.4.1: Case 1 Equilibrium Auction Outcomes

Using the indifference condition, $G_{P\emptyset}$ is given by:

$$G_{P\emptyset}(v) = \frac{\Pi_1 - \Pi_{I0}(x + (1-x)p)}{\Pi_1 - v} = \frac{(1-x)(1-p)\frac{\alpha Y}{r+\lambda}}{\frac{\alpha Y}{r+\lambda} - v}$$

with atom $1 - x$ at the lower bound, $p\frac{\alpha Y}{r+\lambda}$.

Again, using the indifference condition, G_{Ih} is given by:

$$G_{Ih}(v) = \frac{1-x}{x}\left(\frac{1-p}{1 - v\frac{r+\lambda}{\alpha Y}} - 1\right)$$

Since the value is a linear transformation of a belief, it is convenient to convert the v

variables into corresponding beliefs $\tilde{p} = \frac{r+\lambda}{\alpha Y} v$. Define the bid distributions $\tilde{G}_{Ih}$ and $\tilde{G}_{P\emptyset}$ as

$$\tilde{G}_{Ih}(\tilde{p}) = G_{Ih}(\tilde{p}\frac{\alpha Y}{r+\lambda}) = \tilde{G}_{Ih}(\tilde{p}) = \frac{1-x}{x}\left(\frac{1-p}{1-\tilde{p}} - 1\right),$$

$$\tilde{G}_{P\emptyset}(\tilde{p}) = G_{P\emptyset}(\tilde{p}\frac{\alpha Y}{r+\lambda}) = \frac{(1-x)(1-p)}{(1-\tilde{p})}.$$

for $\tilde{p} \in [p, x + (1-x)p]$.

The probability that the Ih type wins over $P\emptyset$ is given by:

$$\int_p^{x+(1-x)p} \tilde{G}_{P\emptyset}(\tilde{p})\, d\tilde{G}_{Ih}(\tilde{p}) = \int_p^{x+(1-x)p} \frac{(1-x)(1-p)}{1-\tilde{p}} \frac{1-x}{x} \frac{(1-p)}{(1-\tilde{p})^2}\, d\tilde{p}$$

The antiderivative of $\frac{1}{(1-\tilde{p})^3}$ being $\frac{1}{2(1-\tilde{p})^2}$, the definite integral that is to be multiplied by $\frac{(1-x)^2(1-p)^2}{x}$ is

$$\frac{1}{2}\frac{1}{(1-\tilde{p})^2}\Big|_p^{x+(1-x)p} = \frac{1}{2}\left(\frac{1}{(1-x)^2(1-p)^2} - \frac{1}{(1-p)^2}\right) = \frac{1}{2}\left(\frac{1-(1-x)^2}{(1-x)^2(1-p)^2}\right) = \frac{1}{2}\frac{x(2-x)}{(1-x)^2(1-p)^2}.$$

In the end, the probability is $\frac{1}{2}(2-x) = 1 - \frac{1}{2}x$.

1.7.7. Solution for x in Case 2

Using

$$\int_0^t e^{-\lambda s}\alpha e^{-\alpha s}\, ds = \frac{\alpha}{\alpha+\lambda}\int_0^t (\alpha+\lambda)e^{-(\alpha+\lambda)s}\, ds$$

$$= \frac{\alpha}{\alpha+\lambda}\left(1 - e^{-(\alpha+\lambda)t}\right),$$

Starting from initial belief p_0 and $x(0) = 0$, x is given by

$$x(t) = \frac{p_0 \int_0^t e^{-(r+\lambda)s} d(1-e^{-\alpha s})}{p_0 \int_0^t e^{-(r+\lambda)s} d(1-e^{-\alpha s}) + p_0 e^{-(r+\alpha+\lambda)t} + (1-p_0)e^{-(r+\lambda)t}}$$

$$= \frac{p_0 \frac{\alpha}{\alpha+\lambda}(1-e^{-(\alpha+\lambda)t})}{p_0 \frac{\alpha}{\alpha+\lambda}(1-e^{-(\alpha+\lambda)t}) + e^{-\lambda t}(p_0 e^{-\alpha t} + (1-p_0))}$$

Intuitively, only the subset of good workers who received good news (with probability $1 - e^{-\alpha s}$ within duration s) before the first poaching attempt, are the revealed good workers in this firm. It is easily seen in the expression $\frac{\alpha}{\alpha + \lambda}$ that the order of arrival of two independent Poisson processes, rates α and λ, matter for the numerator. The denominator represents the probability of survival in this firm for the duration of t, either by the arrival of news, or no poaching attempt $(e^{-\lambda t})$.

To verify that the solution to the ODE is indeed as above, we check our algebra by plugging in the expression for x into the differential equation:

$$x'(t) = \frac{p_0 \alpha e^{-(\alpha+\lambda)t}}{A(t)} - \frac{p_0 \frac{\alpha}{\alpha+\lambda}(1 - e^{-(\alpha+\lambda)t})A'(t)}{A(t)^2},$$

where

$$A(t) = p_0 \frac{\alpha}{\alpha + \lambda}(1 - e^{-(\alpha+\lambda)t}) + e^{-\lambda t}(p_0 e^{-\alpha t} + (1 - p_0)),$$

and

$$A'(t) = p_0 \alpha e^{-(\alpha+\lambda)t} - p_0(\alpha + \lambda)e^{-(\alpha+\lambda)t} - \lambda(1 - p_0)e^{-\lambda t}$$

$$= -\lambda p_0 e^{-(\alpha+\lambda)t} - \lambda(1 - p_0)e^{-\lambda t}.$$

Meanwhile,

$$1 - x(t) = \frac{p_0 e^{-(\alpha+\lambda)t} + (1 - p_0)e^{-\lambda t}}{A(t)},$$

hence,

$$\frac{x'(t)}{1 - x(t)} = \frac{p_0 \alpha e^{-(\alpha+\lambda)t}}{p_0 e^{-(\alpha+\lambda)t} + (1 - p_0)e^{-\lambda t}} - \frac{p_0 \frac{\alpha}{\alpha+\lambda}(1 - e^{-(\alpha+\lambda)t})(-\lambda)}{A(t)}$$

$$= \alpha p(t) + \lambda x(t)$$

1.7.8. Proof of Proposition 7

Define

$$D(t) = \Pi_{Ih}(t) - \Pi_{I0}(t).$$

Subtract the equations in the system to yield the following ODE in terms of D only:

$$-D'(t) + (r + \lambda)D(t) = \alpha(1 - p(t))Y + (\lambda(1 - x(t)) - \alpha p(t))D(t)$$

$$-D'(t) + (r + \lambda x(t) + \alpha p(t))D(t) = \alpha(1 - p(t))Y$$

Use the drift equation to substitute $\lambda x(t) + \alpha p(t)$ with $\frac{x'(t)}{1-x(t)}$:

$$-D'(t) + (r + \frac{x'(t)}{1 - x(t)})D(t) = \alpha(1 - p(t))Y$$

Since $\frac{x'(t)}{1-x(t)} = \frac{d}{dt}(-\log(1 - x(t)))$, we have

$$\exp(-rt + \log(1 - x(t))) = \exp(-rt)(1 - x(t))$$

multiplying by this number,

$$-e^{-rt}(1 - x(t))D'(t) + (re^{-rt}(1 - x(t)) + e^{-rt}x'(t))D(t) = \alpha e^{-rt}(1 - x(t))(1 - p(t))Y$$

which can be rearranged by

$$-\frac{d}{dt}e^{-rt}(1 - x(t))D(t) = e^{-rt}(1 - x(t))(1 - p(t))\alpha Y$$

Integrating from t to infinity, noting that D and x are bounded:

$$e^{-rt}(1 - x(t))D(t) = \int_t^\infty e^{-rs}(1 - x(s))(1 - p(s))\alpha Y \, ds$$

Plugging this information into (Ih'):

$$-\Pi'_{Ih}(t) + (r+\lambda)\Pi_{Ih}(t) = \alpha Y + \lambda(1-x(t))(\Pi_{Ih}(t) - \Pi_{I0}(t)) \tag{Ih'}$$

$$-e^{-(r+\lambda)t}\Pi'_{Ih}(t) + (r+\lambda)e^{-(r+\lambda)t}\Pi_{Ih}(t) = \alpha Y e^{-(r+\lambda)t} + \lambda e^{-(r+\lambda)t}(1-x(t))(\Pi_{Ih}(t) - \Pi_{I0}(t))$$

$$\frac{d}{dt} - e^{-(r+\lambda)t}\Pi_{Ih}(t) = \alpha Y e^{-(r+\lambda)t} + \lambda e^{-\lambda t}e^{-rt}(1-x(t))D(t)$$

$$= \alpha Y e^{-(r+\lambda)t} + \lambda e^{-\lambda t}\int_t^\infty e^{-rs}(1-x(s))(1-p(s))\alpha Y\, ds$$

Integrating over $[t,\infty)$:

$$e^{-(r+\lambda)t}\Pi_{Ih}(t) = \frac{\alpha Y}{r+\lambda}e^{-(r+\lambda)t} + \lambda\int_t^\infty e^{-\lambda s}\int_s^\infty e^{-rz}(1-x(z))(1-p(z))\alpha Y\, dz\, ds$$

Changing the order of integration

$$e^{-(r+\lambda)t}\Pi_{Ih}(t) = \frac{\alpha Y}{r+\lambda}e^{-(r+\lambda)t} + \int_t^\infty \left(e^{-\lambda t} - e^{-\lambda z}\right)e^{-rz}(1-x(z))(1-p(z))\alpha Y\, dz$$

Multiplying by $e^{(r+\lambda)t}$

$$\Pi_{Ih}(t) = \frac{\alpha Y}{r+\lambda} + \int_t^\infty \left(e^{-r(z-t)} - e^{-(r+\lambda)(z-t)}\right)(1-x(z))(1-p(z))\alpha Y\, dz$$

Use this to back out the $x(t)\Pi_{Ih}(t) + (1-x(t))\Pi_{I0}(t) = \Pi_{Ih}(t) - (1-x(t))D(t)$:

$$\Pi_{Ih}(t) - (1-x(t))D(t)$$
$$= \frac{\alpha Y}{r+\lambda} + \int_t^\infty \left(1 - e^{-\lambda(z-t)}\right)e^{-r(z-t)}(1-x(z))(1-p(z))\alpha Y\, dz$$
$$\quad - \int_t^\infty e^{-r(s-t)}(1-x(s))(1-p(s))\alpha Y\, ds$$
$$= \frac{\alpha Y}{r+\lambda} - \int_t^\infty e^{-(r+\lambda)(z-t)}(1-x(z))(1-p(z))\alpha Y\, dz$$
$$= \alpha Y\int_t^\infty e^{-(r+\lambda)(z-t)}\Big(x(z) + (1-x(z))p(z)\Big)\, dz$$

1.7.9. Proposition 8: Properties of Value Functions

Expression for $\tilde{x}$ Denote by $C(t)$, the maximum cost in order for the incumbent to retain the worker:

$$C(t) = \alpha Y \int_t^\infty e^{-(r+\lambda)(s-t)} \big(x(s) + (1 - x(s))p(s)\big)\, ds$$

Closed form for the $x(s) + (1 - x(s))p(s) := \tilde{x}(s)$:

$$\tilde{x}(t) = \frac{p_0 \frac{\alpha}{\alpha+\lambda}\left(1 - e^{-(\alpha+\lambda)t}\right) + p_0 e^{-(\alpha+\lambda)t}}{p_0 \frac{\alpha}{\alpha+\lambda}\left(1 - e^{-(\alpha+\lambda)t}\right) + e^{-\lambda t}\left(p_0 e^{-\alpha t} + (1 - p_0)\right)}$$

normalizing by p_0:

$$\tilde{x}(t) = \frac{\frac{\alpha}{\alpha+\lambda}\left(1 - e^{-(\alpha+\lambda)t}\right) + e^{-(\alpha+\lambda)t}}{\frac{\alpha}{\alpha+\lambda}\left(1 - e^{-(\alpha+\lambda)t}\right) + e^{-\lambda t}\left(e^{-\alpha t} + \frac{1-p_0}{p_0}\right)}$$

$$= \frac{\frac{\alpha}{\alpha+\lambda} + \frac{\lambda}{\alpha+\lambda}e^{-(\alpha+\lambda)t}}{\frac{\alpha}{\alpha+\lambda} + \frac{\lambda}{\alpha+\lambda}e^{-(\alpha+\lambda)t} + \frac{1-p_0}{p_0}e^{-\lambda t}}$$

Lemma 1.
$$\frac{\partial \tilde{x}}{\partial \lambda} > 0.$$

Proof. The result follows from the property of x. Suppose there are two paths of x's, which I denote as x_1 and x_2 for parameters $\lambda_1 < \lambda_2$, starting from the same p_0. For t close to 0, $x_2(t) > x_1(t)$ and the inequality should not flip because if so, there is an intersection and at the intersection, $x = x_2 = x_1$, we have $x_2' > x_1'$.

Therefore, an increase in λ increases x and $\tilde{x} = x + (1 - x)p$. $\qquad\square$

Lemma 2.
$$\frac{\partial \tilde{x}}{\partial \alpha} > 0.$$

Proof. From the last expression for $\tilde{x}$, focus on the term

$$\frac{\alpha}{\alpha + \lambda} + \frac{\lambda}{\alpha + \lambda} e^{-(\alpha + \lambda)t} := A$$

which I denote A. Its derivative with respect to α is given by:

$$(-1)\frac{\lambda e^{-(\alpha + \lambda)t}\left((\alpha + \lambda)t - e^{(\alpha + \lambda)t} + 1\right)}{(\alpha + \lambda)^2}$$

It is immediately shown that $1 + (\alpha + \lambda)t - e^{-(\alpha + \lambda)t} < 0$ and the result follows. $\qquad\square$

Expression for Π_{Ih}   From

$$\Pi_{Ih}(t) = \frac{\alpha Y}{r} - \lambda \int_t^\infty e^{-r(z-t)} C(z)\, dz,$$

we have

$$
\begin{aligned}
\Pi_{Ih}(t) &= \frac{\alpha Y}{r} - \lambda(\alpha Y) \int_t^\infty e^{-r(z-t)} \int_z^\infty e^{-r(s-z)} \tilde{x}(s)\, ds\, dz \\
&= \frac{\alpha Y}{r} - \lambda(\alpha Y) \int_t^\infty \int_t^s e^{-r(z-t)} e^{-r(s-z)} e^{-\lambda(s-z)} \tilde{x}(s)\, dz\, ds \\
&= \frac{\alpha Y}{r} - \lambda(\alpha Y) \int_t^\infty e^{-r(s-t)} \tilde{x}(s) \int_t^s e^{-\lambda(s-z)}\, dz\, ds \\
&= \frac{\alpha Y}{r} - \lambda(\alpha Y) \int_t^\infty e^{-r(s-t)} \tilde{x}(s)\left(\frac{1}{\lambda}\left(1 - e^{-\lambda(s-t)}\right)\right) ds \\
&= \alpha Y \int_t^\infty e^{-r(s-t)}\left(1 - \tilde{x}(s)(1 - e^{-\lambda(s-t)})\right) ds
\end{aligned}
$$

Last line by changing the order of integration. From the expression, it is immediate that

Proposition 11. Suppose αY is a constant. Then, an increase in α decreases $\Pi_{Ih}(t)$.

Proposition 12. An increase in λ increases $\tilde{x}$ and $(1 - e^{-\lambda s})$. Overall, decreases the term.

Expression for Π_{I0}   Let's turn to Π_{I0}. Starting from p_0, the value is

$$\alpha p_0 \int_0^\infty e^{-(\alpha + \lambda + r)t}\left[Y + \Pi_{Ih}(t)\right] dt$$

Expanding

$$\underbrace{\alpha p_0 \frac{Y}{\alpha + \lambda + r}}_{(1)} + \underbrace{\alpha p_0 \int_0^\infty e^{-(\alpha+\lambda+r)t} \int_t^\infty e^{-r(s-t)} \left(1 - \tilde{x}(s)(1 - e^{-\lambda(s-t)})\right) ds\, dt\, (\alpha Y)}_{(2)}$$

Let $r = 0$ and $\alpha Y = 1$ for simplicity, and focus on the second term, (2):

$$\alpha p_0 \int_0^\infty e^{-(\alpha+\lambda)t} \int_t^\infty \left(1 - \tilde{x}(s)(1 - e^{-\lambda(s-t)})\right) ds\, dt$$

Changing the order of integration:

$$\alpha p_0 \int_0^\infty \int_0^s e^{-(\alpha+\lambda)t} \left(1 - \tilde{x}(s)(1 - e^{-\lambda(s-t)})\right) dt\, ds$$
$$= \alpha p_0 \int_0^\infty \int_0^s e^{-(\alpha+\lambda)t}(1 - \tilde{x}(s))\, dt\, dt + \alpha p_0 \int_0^\infty \int_0^s e^{-\alpha t}\tilde{x}(s)e^{-\lambda s}\, dt\, ds$$

The first part:

$$\alpha p_0 \int_0^\infty \frac{1}{\alpha + \lambda}\left(1 - e^{-(\alpha+\lambda)s}\right)(1 - \tilde{x}(s))\, ds$$

The second part:

$$\alpha p_0 \int_0^\infty \int_0^s e^{-\alpha t}\tilde{x}(s)e^{-\lambda s}\, dt\, ds = \alpha p_0 \int_0^\infty \frac{1}{\alpha}(1 - e^{-\alpha s})\tilde{x}(s)e^{-\lambda s}\, ds$$

Overall,

$$\underbrace{\frac{p_0}{\alpha + \lambda}}_{(1)} + \underbrace{p_0 \int_0^\infty \frac{\alpha}{\alpha + \lambda}\left(1 - e^{-(\alpha+\lambda)s}\right)(1 - \tilde{x}(s))\, ds + p_0 \int_0^\infty (1 - e^{-\alpha s})\tilde{x}(s)e^{-\lambda s}\, ds}_{(2)}$$

Expanding (2):

$$p_0 \int_0^\infty \frac{\alpha}{\alpha + \lambda}(1 - e^{-(\alpha+\lambda)s}) - \tilde{x}(s)\frac{\alpha}{\alpha + \lambda}(1 - e^{-(\alpha+\lambda)s}) - \tilde{x}(s)e^{-(\alpha+\lambda)s} + \tilde{x}(s)e^{-\lambda s}\, ds$$

$$p_0 \int_0^\infty \frac{\alpha}{\alpha + \lambda}(1 - e^{-(\alpha+\lambda)s}) - \tilde{x}(s)\frac{\alpha}{\alpha + \lambda} - \frac{\lambda}{\alpha + \lambda}\tilde{x}(s)e^{-(\alpha+\lambda)s} + \tilde{x}(s)e^{-\lambda s}\, ds$$

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$$p_0 \int_0^\infty \frac{\alpha}{\alpha + \lambda}(1 - \tilde{x}(s)) + \frac{\lambda}{\alpha + \lambda}e^{-(\alpha+\lambda)s}(1 - \tilde{x}(s)) - \underbrace{e^{-(\alpha+\lambda)s}}_{(*)} + \tilde{x}(s)e^{-\lambda s}\, ds$$

The underbraced term $(*)$ cancels out with (1). Therefore, the expression is:

$$p_0 \int_0^\infty \left(\frac{\alpha}{\alpha + \lambda} + \frac{\lambda}{\alpha + \lambda}e^{-(\alpha+\lambda)s}\right)(1 - \tilde{x}(s)) + \tilde{x}(s)e^{-\lambda s}\, ds$$

To see how this increases with α, let

$$A(\alpha) = \frac{\alpha}{\alpha + \lambda} + \frac{\lambda}{\alpha + \lambda}e^{-(\alpha+\lambda)s}$$

and let

$$B = \frac{1 - p_0}{p_0}e^{-\lambda s}.$$

It follows immediately that

$$\tilde{x}(s) = \frac{A}{A + B}.$$

Rewrite the above expression in terms of A and B:

$$p_0 \int_0^\infty A\left(\frac{B}{A + B}\right) + \frac{A}{A + B}\frac{p_0}{1 - p_0}B\, ds$$

$$p_0 \int_0^\infty \frac{A}{A + B}\left(\frac{1}{1 - p_0}\right)B\, ds$$

Overall,

$$\int_0^\infty \frac{A}{A + B}e^{-\lambda s}\, ds$$

Note that p_0 affects the expression only through B. Increase in p_0 decreases B, and increases the expression. Therefore, I show increment in α and p_0.

CHAPTER 2 : Ranking and Search Effort in Matching (joint with Hanna Wang)

2.1. Introduction

Mismatch in markets with simultaneous applications is a widely-studied phenomenon. The macroeconomic search literature, following the Diamond-Mortensen-Pissarides framework, typically represents matching friction using a matching function which maps market tightness into a number of matches. The number of matches is always smaller than the number of vacancies or unemployed workers - the idea is simple: without coordination many workers apply to the same firms while other firms do not receive any applications, resulting in unemployment and unfilled vacancies.

The extent of this miscoordination affects the success probability of applications and workers' incentives to search for jobs. Dating back to Stigler (1962), many papers have noted the importance of endogenous search effort for labor market outcomes, namely it allows sampling better wage offers and increases the probability of finding any job. This paper considers the latter effect for workers commonly ranked by employers and choose a number of applications to send out with the goal of securing a job.

A range of papers have examined this problem for identical agents, e.g. Albrecht et al. (2003) and Kircher (2009), but to our best knowledge this is the first paper to study endogenous simultaneous search effort of heterogeneous agents. Workers within the same applicant pool might be favored differently by firms and face different incentives in the application process. A natural question is how this differentiation translates into differences in search intensity and matching outcome.

The source of heterogeneity in our model is workers' endowment of a characteristic over which employers have common preferences but cannot contract. Each employer offers the same wage to all workers and is not able to discriminate between workers by posting type-specific wage schedules but only by selecting a preferable applicant. This assumption follows

Peters (2010) and has the effect that differences in worker incentives are driven only by rank competition instead of employers' wage-posting.

Due to fixed wages workers cannot distinguish among vacancies evade others by directing applications to specific firms, intensifying competition across types. While this assumption is restrictive, it is true for some labor markets and potentially applies to many others to a certain extent. One example which we will use throughout the main part of this paper is the market for new public school teachers. Salaries and work conditions are negotiated by unions at district level and as noted by Boyd et al. (2013), who estimate an empirical matching model for the teaching market, the majority of teachers find employment in close proximity to where they grew up. Because salaries are only conditioned on years of experience and education, the variation for entry level positions conditional on education is minimal. This warrants the assumption of fixed wages within the local labor market. The authors find that schools have strong preferences over the selectivity of teachers' colleges and performance in a basic knowledge test. It is reasonable to assume the distribution of these qualifications are public knowledge, such that a public common ranking of candidates is justified.

One caveat are that teachers might have common preferences over schools' non-wage attributes, too. We argue, however, that these should be much more random. Boyd et al. (2013) find that all teachers prefer schools with a smaller fraction of poor students, but they also show teachers have strong preferences for schools that are close to the location where they grew up and preferences for racial composition of the student body depends on own race. Furthermore, it is conceivable teacher might prefer schools based on other privately known factors such as proximity to spouses' families, whether they attended a school themselves, friends or family members working at the school etc. This should substantially limit the ability of newly graduated teachers to predict the popularity of schools and the skill level of respective competing applicants.

This examples extends readily to other parts of the public sector or markets with unionized wages. Another way to interpret our setting is that the worker characteristic cannot be

priced because the value is difficult to quantify (e.g. for soft skills or reference letters) or that it is not legal to discriminate based on it (e.g. race, sex, motherhood status or physical attractiveness). One other possibility are individuals with similar qualifications applying to a tier of employers/institutions such that payoffs are uniform e.g. consulting job for college graduates from top universities with grades within a certain range or college applications to Ivy League Universities. We discuss a few applications in more details in Section 6.

The effect of rank on the number of applications is not obvious. Two opposing forces come to mind when we compare a low-rank to a high-rank worker's incentives. On the one hand, because success probability per application decreases with rank (which follows immediately from our model) and applications are costly, the low-rank worker might be discouraged from applying as often as the high-rank worker. On the other hand, the low-rank worker might want to send out additional applications as a form of insurance, because applications are more likely to fail.

Using our model we show that the relationship between rank and search effort is hump-shaped. That is it decreases with rank for workers above a certain threshold, while the reverse is true for workers below the threshold. As Shimer (2004) pointed out, traditional search models (e.g. Pissarides (2000)) generate the result that equilibrium search effort has to increase in the baseline arrival rate[1]. This would suggest that higher-rank workers search more given that they are favored over other workers. In contrast, we find that this is only true for workers which rank below the threshold.

Our result implies that above the threshold rank, lower-rank workers can gain on, if not overtake, higher-rank workers in terms of employment probability, because they counteract a lower per-application success rate with sending more applications. However, below the threshold, employment chance rapidly decreases for lower-rank workers because they both face an even lower per-application success rate and send out fewer applications than higher-

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[1]The arrival rate denotes the stochastic probability with which a worker receives a job offer in a given period. Search effort is often modeled as choosing a scaling parameter that is multiplied by the baseline rate.

ranked competitors.

In past literature, heterogeneous searchers have mostly been studied with fixed search effort and directed search, such as in Peters (2010), Shimer (2005), Shi (2002), Burdett and Coles (1997). One exception is Lentz (2010) who finds a monotone relationship between sequential search effort and worker type in his search model, confirming Shimer's (2004) prediction discussed earlier. In contrast to our goal of examining worker heterogeneity, past papers which model endogenous simultaneous search effort consider heterogeneity on the firm side, studying the role of wage dispersion and directed search for efficiency (e.g Albrecht et al. (2003), Galenianos and Kircher (2009), Kircher (2009)).

By relaxing the assumption of fixed wages and allowing employers to set different wages workers might tend to sort and more skilled workers exert more effort, possibly leading to a different application and matching pattern than our result predicts. Nevertheless, our model still offers insights on search effort whenever unequal searchers are in somewhat direct competition with one another and incur frictions due to the lack of ability to exactly predict the applicant skill level at each location.

Simultaneous endogenous search lets workers choose a number of applications to send out at the same time. Modelling this is a challenging task because firms' and workers' actions are interdependent and characterizing employment probabilities involves cumbersome combinatorics. In particular, a worker only gets hired by a particular firm if that firm does not hire a different worker, which in turn depends on whether that worker gets hired by another firm he applied to and so on. To limit the number of strategic interactions arising from many rounds of offers and acceptances, the majority of papers restrict firms to making a single offer (e.g. Shimer (2004), Albrecht et al. (2003), Galenianos and Kircher (2009)). We rely on the assumption that the equilibrium matching is stable to obtain well-defined expressions for employment probability. This concept was first introduced by Kircher (2009) in the search and matching in context, and has since been adapted by a handful of papers (Gautier and Holzner (2013, 2016), Wolthoff (2009)). A matching is stable if no firm or worker can

be strictly better off by matching with a different partner. Contrary to the one-shot offer settings in which some firms are left with open vacancies, even if they have unemployed applicants, stable matching effectively implies that workers and firms communicate back and forth until there are no more mutually beneficial matches left to be made.

In our model, a stable matching requires workers to only be unemployed when all firms they applied to hire higher-ranked workers. This lets us express the employment probability as a function of the mass of successfully employed higher-rank workers and derive a differential equation that characterizes how the latter changes with worker rank.

The paper proceeds as follows. In the next section we illustrate the basic idea using a simple example. Section 2.3 develops the full theoretical model and presents the main result. In Section 2.4 we provide an analysis of how market tightness and application costs affect application behavior and employment outcomes. Lastly, we discuss efficiency and the social planner solution in Section 2.5.

2.2. A Simple Discrete Example

Consider a labor market with M vacancies, each at a different firm, and three workers, $\{A, B, C\}$, ranked first, second, and third respectively by all firms. The workers decide how many applications to send to the firms. They apply to each firm at most once and can take at most one job.

Here, as well as for the full model, we make anonymity assumptions analogous to Kircher (2009). We assume workers do not know the identity of firms and applications are equivalent to random sampling of jobs without replacement. So, with m applications, a worker has $\binom{M}{m}$ ways of sampling firms. Workers also do not know where others apply, and might apply to the same job as higher-ranked competitors. When offered multiple employment opportunities, they randomly choose one without consideration for other applicants. This rules out coordination among workers to direct applications or purposeful selection of firm offers, e.g. a higher-rank worker chooses to take a specific job over another to leave the

(B,C)	1	2
1	$(\frac{1}{2}, \frac{1}{2}\frac{1}{2} + \frac{1}{2} \cdot 0)$	$(\frac{1}{2}, \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0)$
2	$(1,0)$	$(1,0)$

Table 2: Match Probabilities, Two Slots Case

MB	$1-0$	$2-1$
A	1	0
B	$\frac{1}{2}$	$\frac{1}{2}$
C (1)	$\frac{1}{4}$	$\frac{1}{4}$
C (2)	0	0

Table 3: Marginal Benefits of an Application for Two Slots Case

latter job to a lower-rank worker.

Note that worker A sends out at most one application, this application lands a job with certainty. Worker B however, risks unemployment if he only sends out one application, since he might apply to the same firm as worker A. By sending two applications he gets a job with certainty since at least one firm he applies to cannot hire worker A.

For $M = 2$, worker B applies to the same job as worker A with probability $\frac{1}{2}$ if he sends one application. In this case worker C only gets a job if worker B applies to the same firm as worker A and if he sends an application to the firm to which worker A and B do not apply. We can see that each worker only needs information on higher-rank workers in order to evaluate his own decision. A worker always has priority over lower-rank workers and applying to the same firms as these workers is not a concern. Fixing worker A's number of applications to be one, the following table shows the employment probability of each worker as a function of the number of applications.

The next table presents the marginal benefit of each additional applications for each worker. The third row and fourth row show worker C's marginal benefit when worker B sends out one or two applications respectively.

As mentioned above, the marginal benefit for worker C is a function of worker B's applications but not vice versa.

We denote the flat cost of applications as c. The worker's expected utility is the probability of getting a job multiplied by the wage minus the cost of application. So the workers increase then number of their applications as long as the marginal benefit of the application exceeds the cost. Setting the wage to one for all jobs, we derive that for application cost less than $\frac{1}{2}$ worker A applies once, worker B applies twice, while worker C does not apply at all. The pattern of application illustrates our main result: the number of applications is hump-shaped in rank. Workers of mid-range rank might apply more to insure against losing against higher-rank competitors while low-rank workers might apply less because too many jobs are taken by higher-rank workers (due to their priority in rank and high search effort).

2.3. The Model

Setting We now turn to the full model. We consider a bipartite matching problem of two sets of continuum of agents: workers $\mathcal{W}$, and firms $\mathcal{F}$. Let $\mathcal{W}$ be a unit mass of workers, indexed by $i \in [0, 1]$. Each worker is endowed with type $x \in [0, 1]$, and we denote workers' type distribution by F with support $[0, 1]$ and a pdf f. Define $X : [0, 1] \to [0, 1]$ which maps each worker to her type. Then F is induced by X, and Lebesgue measure λ over $\mathcal{W}$: $F(x) := \lambda(\{i : X(i) \leq x\})$. Let $\mathcal{F}$ be a positive mass $M(> 0)$ of firms indexed by $j \in [0, M]$.

As in our simple example, firms are identical with one vacancy each and we adopt the same anonymity assumptions as before. We now have large markets and add that firms cannot distinguish between workers of the same type, however the probability of receiving two applications from the same type is zero given the continuous type space. We define a strategy as a function k that maps from X into a discrete number of applications, which in turn are i.i.d. uniform draws from the firm pool.

We normalize each worker's value from matching with a firm to one and set the value of remaining unemployed to zero. A worker incurs a flat cost c for each application he sends.

We assume a firm's payoff is equal to the matched worker's type, or $-\varepsilon(< 0)$ if it remains unmatched. Hence, this implies firms strictly prefer to be matched with any worker than to remain unmatched. As we show later, worker types exactly correspond to their position in firm preference rankings which is the only object needed for equilibrium analysis.

Matching Process The application and hiring process begins with all workers simultaneously choosing $k \in \mathbb{Z}_+$. Then applications are realized, forming a network with 'links' from the worker to the firm pool. Given the network a stable matching, which define in the following part, is formed.

We define a *matching* as a function $\mu\colon \mathcal{W} \cup \mathcal{F} \to \mathcal{W} \cup \mathcal{F}$ such that

1. $\mu(i) = j \in \mathcal{F}$ if and only if $\mu(j) = i \in \mathcal{W}$,

2. If $\mu(i) \notin \mathcal{F}$, then $\mu(i) = i$,

3. If $\mu(j) \notin \mathcal{W}$, then $\mu(j) = j$.

For each $i \in \mathcal{W}$, and corresponding $k(i)$ applications i sent out in the first stage, define the set of firms receiving an application from i as $B(i) = \{j_1, j_2, \ldots, j_{k(i)}\}$. A matching μ in the second stage is *feasible* if $\mu(i) \in \{i\} \cup B(i)$ for all i. We require the second stage matching μ to be feasible, and *stable* in the following sense.

Definition 8. A feasible matching μ is stable if there is no pair (i, j) with $j \in B(i)$ such that $\mu(i) = i$ and either (i) $\mu(j) = j$, or (ii) $\mu(j) \neq j$ and $X(i) > X(\mu(j))$.

This is the notion of no blocking pair with both parties strictly better off. For any blocking pair (i, j) with $j \in B(i)$, since workers are indifferent over all firms, it must be $\mu(i) = i$. Furthermore, j strictly prefers i to his current match $\mu(j)$, himself or a worker with type lower than i.

While technically there can be multiple stable matchings, these matchings all exhibit the same aggregate measures. This property follows from the continuum agent assumption. In

particular, the mass of matched workers with ranks within any interval is constant across these matchings.

Employment Probability   We first characterize employment probability by type for a given number of applications made by each worker. There are $1 - F(x)$ workers of higher rank for a type x worker. We define the mass of successful applicants of types higher than x as $A(x)$, here on referred to as the accumulation function. $A(x)$ takes a deterministic value at any x due to the continuous type space, i.e. the mass of employed mass of workers above a certain rank is determined with certainty for any application behavior. Since $A(x)$ is also the mass of of jobs occupied by higher-rank workers, a worker will only be successful if he does not apply to any of these jobs. His success probability per application is thus $1 - \frac{A(x)}{M}$. Note that when workers make more than one application, $A(x)$ is not equal to the mass of firms receiving applications from workers of ranks higher than x. Not all firms with applications can hire higher-rank workers, since those workers will choose only one of the available firms they applied to.

Rank x worker's employment probability given k applications is given by $1 - \left(\frac{A(x)}{M}\right)^k$, the probability that he applies to at least one firm that does not hire a higher-rank worker. Due to the large-market properties, the probabilities that each of the applications succeed are independent.

Assuming the derivative of f exists everywhere, and f and f' are bounded, we establish the following differential equation:

Proposition 13. The change of the accumulation function in a small neighborhood of x if all workers within this neighborhood apply k times is given by:

$$-A'(x) = f(x) \left(1 - \left(\frac{A(x)}{M}\right)^k\right)$$

The equation simply states that the change in A at x is equal to the expected mass of type

x workers that succeeds in finding a job. The right-hand side is positive for $k > 0$ and $A(x) < M$; in this case the accumulation is strictly decreasing. For a better understanding of this equation the interested reader might turn to Appendix 2.7.1 in which we provide an alternative derivation using the per-application success rate $\varphi(x) = 1 - \frac{A(x)}{M}$. We express it as a function of the effective queue length and success rate of higher-ranked competitors applying to the same job.

The differential equation does not apply for cutoff types $\bar{x}$ that have types in any arbitrarily small neighborhood sending out different numbers of applications. At these types, $A(x)$ is not differentiable. However, note that $A(x)$ must be continuous, since $f(x)$ is continuous. This implies that the function $A(x)$ has a kink at $\bar{x}$, and the value $A(\bar{x})$ is the same as the left and the right limit of $A(x)$ as x goes to $\bar{x}$.

Worker Problem   We now turn to the worker's utility maximization problem for a given level of accumulation A. His utility is his employment probability minus the cost of application. We restrict our attention to nontrivial cost $c \in (0,1)$ so that at least some workers apply and the number of applications is finite. The worker solves:

$$\max_{K \in \mathbb{Z}_+} \left(1 - \left(\frac{A}{M}\right)^K\right) - cK.$$

To analyze this problem, it is useful to define the gross marginal benefit of the $k+1$th application MB_{k+1}. It is a function of $\frac{A}{M} \in [0,1]$:

$$MB_{k+1}\left(\tfrac{A}{M}\right) = 1 - \left(\frac{A}{M}\right)^{k+1} - \left(1 - \left(\frac{A}{M}\right)^k\right) = \left(\frac{A}{M}\right)^k \left(1 - \frac{A}{M}\right).$$

The marginal benefit is equal to the increase in employment probability due to the $k+1$th application. In other words it is the probability that the first k applications are not successful and the $k+1$th application is. We graph the marginal benefit of the first three applications for $\frac{A}{M} \in [0,1]$ in Figure 8 together with a cost of 0.07.
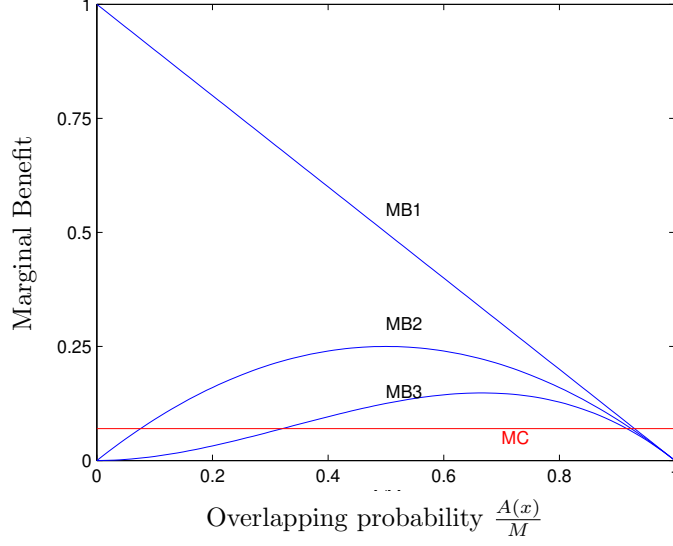
Figure 8: Marginal benefits of applications 1-3

Note that, for a fixed $\frac{A}{M}$, the marginal benefit always decreases in k. With constant marginal cost c, the solution to the worker's problem is to increase the number of applications as long as the marginal benefit exceeds cost. Denote by $K(\frac{A}{M})$ the set of solutions to the worker's problem, who faces probability $\frac{A}{M}$ of his application not being successful. $K(\frac{A}{M})$ satisfies the following properties:

1. If $A = 0$, then $K(\frac{A}{M}) = 1$. This is because employment probability is one for any K and additional applications only increase cost.

2. For all $A \in (0, M(1 - c))$:

   - If $MB_{k+1}(\frac{A}{M}) = (\frac{A}{M})^k(1 - \frac{A}{M}) \neq c$ for any $k \in \mathbb{Z}_+$, then $K(\frac{A}{M}) = \hat{k} + 1$ such that $(\frac{A}{M})^{\hat{k}}(1 - \frac{A}{M}) > c$ and $(\frac{A}{M})^{\hat{k}-1}(1 - \frac{A}{M}) < c$.

   - If $MB_{\tilde{k}+1}(\frac{A}{M}) = (\frac{A}{M})^{\tilde{k}}(1 - \frac{A}{M}) = c$, the worker is indifferent between $\tilde{k}$ and $\tilde{k} + 1$ applications, i.e. $K(\frac{A}{M})$ is then the set $\{\tilde{k}, \tilde{k} + 1\}$.

3. If $A > M(1 - c)$, then $MB_1(\frac{A}{M}) < c$ and $K(\frac{A}{M}) = 0$. If $A = M(1 - c)$, then $MB_1(\frac{A}{M}) = c$ and $K(\frac{A}{M}) = \{0, 1\}$.

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After examining the accumulation function and the worker's solution separately, we describe their consistency in equilibrium.

Equilibrium   Given cost c, firm mass M, and the distribution of worker types F, an equilibrium are functions (A, k) such that

- Given $A : [0,1] \to [0, M]$, the number of applications sent out by each worker $k : [0,1] \to \mathbb{Z}_+$ maximizes his respective utility. That is, for all x:

$$k(x) = \arg\max_{k \in \mathbb{Z}_+} 1 - \left(\frac{A(x)}{M}\right)^k - ck$$

- Given k, a continuous function A is a solution to the differential equation:

$$-A'(x) = f(x)\left(1 - \left(\frac{A(x)}{M}\right)^{k(x)}\right), \tag{1}$$

wherever applicable[2], with initial condition $A(1) = 0$.

An equilibrium exists, since each worker's solution only depends on the decisions of higher-ranked workers through $A(x)$ and we can solve for the equilibrium starting from the highest-ranked worker. In practice this involves identifying cutoff types and respective cutoff accumulations for which there is a discontinuity in $k(x)$ and kink point in $A(x)$. In the intervals between these cutoff types all worker types send out the same number of applications, so $A(x)$ follows the differential equation.

As we mentioned before it is possible for workers with a cutoff type to be indifferent between two adjacent numbers of applications. However, in equilibrium, varying the tie-breaking rule for these types can only affect a zero measure of workers[3]. For simplicity we restrict the

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[2]'Applicable' here refers to x within non-zero measure intervals of types sending the same number of application, for which the derivative exists.

[3]To elaborate, there are two different categories of cases where there is a tie. First of all, for types where $\frac{A(x)}{M} < 1 - c$, we have $k(x) > 0$ and $A(x)$ strictly decreasing. Worker types indifferent between k and $k+1$ are of measure zero since $MB_{k+1}(\frac{A(x)}{M}) = (\frac{A(x)}{M})^k(1 - \frac{A(x)}{M}) = c$ is only satisfied for maximal two points in the type space.

analysis to left-continuous k for the rest of this paper. In summary:

Proposition 14. The equilibrium exists and is unique up to a measure zero set of workers.

As we can see from Figure 8, the marginal benefit of the first application $(MB_1(\frac{A}{M}))$ is a monotone decreasing function of $\frac{A}{M}$, while the other marginal benefits are single peaked in $\frac{A}{M}$. Furthermore, as noted before, the marginal benefit decreases in k, so the curves are nested within each other, with higher numbers of applications being further on the inside. These properties of the marginal benefits and from the continuity and monotonicity of function $A : [0, 1] \to \mathbb{R}_+$, give rise to the main result.

Theorem 1. The equilibrium number of applications $k(x)$ is single-peaked in x.

We provide a detailed proof in Appendix 2.7.2.

This result implies that for workers of types to the right of the peak, the number of applications decreases with rank. These workers have a lower per-application success rate $(1 - \frac{A}{M})$ than higher type competitors but tend to make up for it by sending out more applications. To the left of the peak, the number of applications increases with type and lower type workers apply less because they are discouraged by the low per-application success rate. Therefore, the decline in employment probability is exacerbated for these workers, with the lowest type worker incurring the lowest probability.

Furthermore from the previous discussion on the workers' problem we know $k(0) = 0$ implies $A(0) < M(1 - c)$ and $k(0) > 0$ implies $A(0) < M(1 - c)$, so:

Corollary 1. If the lowest type workers send out zero applications then total employment $A(0)$ is given by $M(1 - c)$. If the lowest type workers send out a strictly positive number of applications, total employment is strictly less than $M(1 - c)$.

Secondly, there may be types who are indifferent between $k = 0$ and $k = 1$. Let x_0 be a type with $MB_1(\frac{A(x_0)}{M}) = 1 - \frac{A(x_0)}{M} = c$. In equilibrium there cannot be a non-zero measure of types $x < x_0$ sending out a strictly positive number of applications, so all lower types must have marginal benefit equal to $MB_1(\frac{A(x_0)}{M})$. The proof is by contradiction: Assume there is an interval of worker types sending out a strictly positive number of applications, then $A(x)$ is strictly increasing over this interval. Take a type x' in the interior of the interval, then $MB_1(\frac{A(x')}{M}) < c$ since $A(x') > A(x_0)$. Hence a strictly positive number of applications cannot be optimal for this type.

Normalization A property of our model is that the employment probability depends only on rank and not on the cardinality of types. Consequently, the equilibrium is invariant to the worker type distribution and we can express all objects of interests in terms of the worker's percentile ranking, i.e. the cdf $F(x)$, as transformed type.

Formally, for an equilibrium (A, K) with distribution F there exists an equivalent representation of an equilibrium $(\tilde{A}, \tilde{K})$ defined over types $\tilde{x} = F(x)$ satisfying the following:

$$-\tilde{A}'(\tilde{x}) = 1 - \left(\frac{\tilde{A}(\tilde{x})}{M}\right)^{\tilde{K}(\tilde{x})}$$

$$\tilde{K}(\tilde{x}) = \arg\max_k 1 - \left(\frac{\tilde{A}(\tilde{x})}{M}\right)^k - ck$$

This can be obtained by a straightforward change of variables as illustrated in Appendix 2.7.3. This allows us to state the following results:

Corollary 2. Denote by (A^F, K^F) and (A^G, K^G) equilibria from two distributions F and G, respectively. For any pair $(x, x') \in [0, 1]^2$ such that $F(x) = G(x')$, we have $A^F(x) = A^G(x')$.

Corollary 3. Total employment in the economy does not depend on F.

Without loss of generality, we restrict attention to a uniform distribution hereafter, by letting the type $\tilde{x}$ as x, and the equilibrium objects $(\tilde{A}, \tilde{K})$ as (A, K).

2.3.1. Example

Here we demonstrate how to find the equilibrium for a set of parameter values. Let $c = \frac{4}{27}$ and $M = 0.7$. For this cost at most two applications are sent out in equilibrium.

For $k = 1, 2$, there is an analytic solution for $A(x)$ for all $x < \hat{x}$ with initial value $A(\hat{x}) = A$:

$$A(x) = M(1 - (1 - \frac{A}{M})e^{-\frac{\hat{x}-x}{M}}) \qquad (k = 1)$$

$$A(x) = M \tanh(\tanh^{-1}(\frac{A}{M}) + \frac{\hat{x}-x}{M}) \qquad (k = 2)$$

where tanh stands for hyperbolic tangent function. We can solve for the three cutoff types
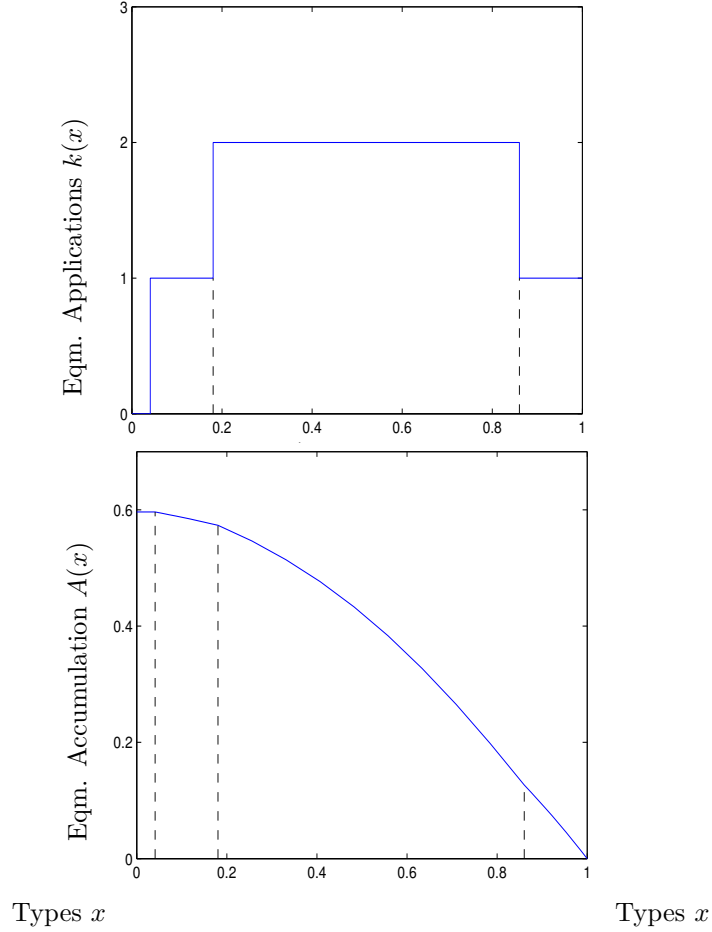
Figure 9: $k(x)$ and $A(x)$. Cutoff types indicated by dashed lines

which satisfy $\frac{A(x)}{M}(1 - \frac{A(x)}{M}) = c$, and $1 - \frac{A(x)}{M} = c$. Figure 9 shows the equilibrium schedule of applications and $A(x)$. Note $A(x)$ is steeper for types that apply twice and has a maximum value of $M(1 - c) \approx 0.6$.

As Figure 10 shows, the equilibrium employment probability jumps when the number of applications changes. Furthermore worker types in $[0.65, 0.87]$ have a similar probability to types in $(0.87, 0.97]$, while types below 0.65 are consistently less likely to find a job the lower their type.

Figure 10: Equilibrium Employment Probability

(B,C)	1	2	3
1	$(\frac{2}{3}, \frac{2}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3})$	$(\frac{2}{3}, \frac{2}{3}\frac{2}{3} + \frac{1}{3} \cdot 1)$	$(\frac{2}{3}, 1)$
2	$(1, \frac{1}{3})$	$(1, \frac{1}{3} \cdot 0 + \frac{2}{3})$	$(1, 1)$
3	$(1, \frac{1}{3})$	$(1, \frac{1}{3} \cdot 0 + \frac{2}{3})$	$(1, 1)$

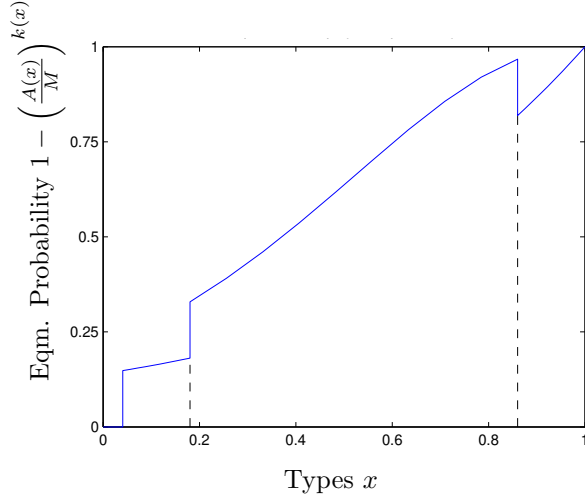Table 4: Match Probabilities, Three Slots Case

2.4. Comparative Statics

In this section we examine how equilibrium outcomes relate to the market tightness M and application cost c.

Simple Example Revisited We illustrate our results with a modified version of our simple example. Assume there are now $M = 3$ jobs. Employment probability is: Compared to the case with $M = 2$, the payoffs uniformly increased for both workers, which is intuitive. The marginal benefits are:

MB	$1 - 0$	$2 - 1$	$3 - 2$
A	1	0	0
B	$\frac{2}{3}$	$\frac{1}{3}$	0
C (1)	$\frac{4}{9}$	$\frac{3}{9}$	$\frac{2}{9}$
C (2)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Table 5: Marginal Benefits of an Application for Three Slots Case

Observe that a higher M has an ambiguous effect on the marginal benefit of applications. Worker B's first application is more valuable now, while the reverse is true for the second. For instance, for c between $\frac{1}{3}$ and $\frac{4}{9}$, worker B applies twice if there are two jobs and only once when there are three. Worker C, however, does not apply with two jobs and applies once with three. This suggest, that with higher M, workers ranked in the mid-range decrease their search effort, due to diminished insurance incentives, while low-rank workers increase their search effort, encouraged by greater availability of jobs.

We can further note that decreasing the cost of application might hurt worker C in equilibrium. For the above range of c the worker C has expected utility $\frac{2}{3} - c < \frac{1}{3}$, and worker B has $\frac{2}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3} - c = \frac{4}{9} - c < \frac{1}{9}$. However, if c is slightly below $\frac{1}{3}$, worker B sends out two applications, and worker C sends three applications (see fourth row of the table). The utility is $1 - 2c > \frac{1}{3}$ for worker B, and $1 - 3c > 0$ for worker C. For c close enough to $\frac{1}{3}$, worker B gains from a lower cost, while worker C is worse off. Next, we provide a formal analysis using the full model.

2.4.1. Market Tightness

Figure 11 depicts the marginal benefits for two sizes of job pools, $M = 1$ and $M' = 0.8$, represented by the blue and red graph respectively. The marginal benefit of the first application $1 - \frac{A(x)}{M}$ is uniformly lower for smaller M, while the marginal benefits of additional applications increase for small A, and decrease for large A.

This suggests that, for fewer available jobs, higher type workers might increase and lower types might decrease their number of applications. Indeed we demonstrate that this is the case by showing that equilibrium objects can be normalized with respect to M. The normalization further allows us to determine that the expected quality of a worker hired by firms in equilibrium is decreasing in the M.

Define $\hat{A}(\frac{1-x}{M}) = \frac{A(x)}{M}$. The equilibrium can be restated with the new transformed variable
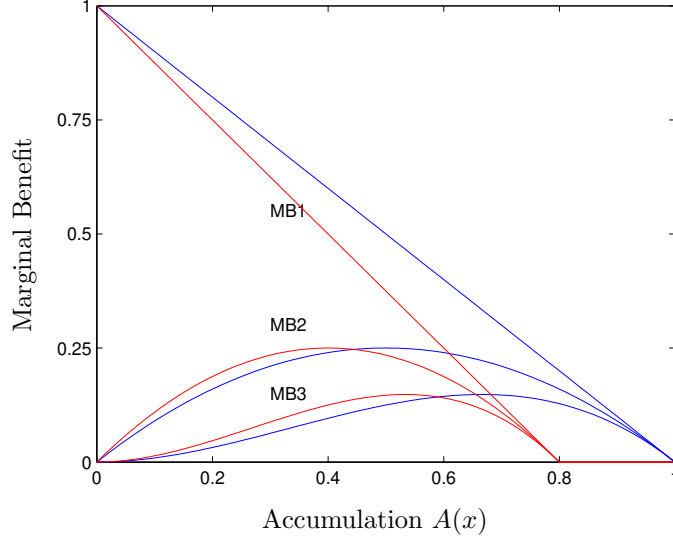
Figure 11: Marginal Benefit for Applications 1-3

$\hat{x} = \frac{1-x}{M}$ and equilibrium functions $(\hat{A}, \hat{K})$, $\hat{A} : [0, \frac{1}{M}] \to \mathbb{R}_+$, and $\hat{K} : [0, \frac{1}{M}] \to \mathbb{Z}_+$:

$$\hat{A}'(\hat{x}) = 1 - \hat{A}(\hat{x})^{\hat{K}(\hat{x})}, \quad \hat{A}(0) = 0$$

$$\hat{K}(\hat{x}) \in K(\hat{A}(\hat{x}))$$

where $K(\cdot)$ maps from the accumulation that a worker faces to the set of the worker's best response.

These equilibrium objects do not depend on M except through changes in the domain. Figure 12 depicts an example application schedule for transformed types and different M, with $\hat{x} = 0$ as the highest type $x = F(1)$. A high M corresponds to a smaller domain, since the red lines indicate the number of applications sent by the lowest type worker, $\hat{x} = \frac{1}{M}$, which is weakly higher for larger M. The observed application pattern (represented by the blue line) is cropped by the red lines. We further see that the maximum number of applications sent out by any worker can only decrease with larger M.

The equivalence of the original and the newly transformed equilibrium definition implies that the objects of interest are scaled by M. Let us denote by a 'cutoff type' x^j who is
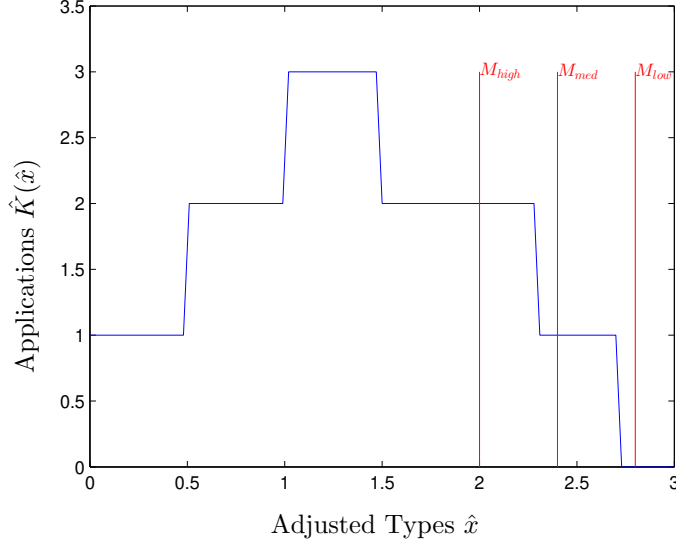
Figure 12: Observed Investment Behavior for Different M

indifferent between sending out j and $j+1$ applications. In particular, on the rescaled type space, the cutoff types $\hat{x}^j$ and accumulations $\hat{A}(\hat{x}^j)$ are invariant in M. Therefore, for two different levels of market tightness M_0, M_1, with $\gamma = \frac{M_1}{M_0}$ we have

$$\hat{x}_0^j = \frac{1 - x_0^j}{M_0} = \frac{1 - x_1^j}{M_1} = \hat{x}_1^j, \quad \forall j,$$

$$\gamma(1 - \hat{x}_0^j) = 1 - \hat{x}_1^j.$$

We now describe how M relates to total employment and firms' hiring probability defined as total employment over job pool size. Without loss of generality assume $\gamma < 1$. The following proposition follows immediately from the normalization and Corollary 1.

Proposition 15. If $K_0(0) = 0$ and $K_1(0) = 0$, then $A_1(0) = \gamma A_0(0)$, that is, total employment is proportional to M. Firms' hiring probability is the same regardless of M.

If $K_0(0) > 0$ or $K_1(0) > 0$ or both, then $\gamma A_0(0) < A_1(0)$, that is, the difference in total employment is less than proportional to the difference in vacancy pool size. Firms' hiring probability is lower for higher M.

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How application behavior differs by M is not obvious. Given a tighter market some workers might try harder to secure a job and some workers might be discouraged and apply less. It turns out that for smaller M, high types tend to apply more and lower types less:

Proposition 16. There exist a cutoff type such that all workers with higher types apply weakly more with M_0 compared to M_1 and the converse is true for lower types.

Proof. Take the type to be the lowest type that applies strictly more for $M = M_1$. □

Since all workers' normalized type $\hat{x}$ is higher for smaller M, this result reflects the non-monotonicity of application behavior in type from our main theorem.

Note that worker types who apply more with small M do not need to have a higher employment probability compared to the case with large M, since the fraction of available jobs is also smaller.

Worker Quality and Firm Entry We now show that the average quality of hired workers decrease with M. The unconditional expected quality of a worker hired by a firm is given by:

$$E[x|\text{hired}]Pr(\text{hired})\frac{1}{M} = \int_{\max\{\underline{x},0\}}^{1} Pr(x \text{ is hired})x \, dx \frac{A(0)}{M} = \int_{\max\{\underline{x},0\}}^{1} -\frac{A'(x)}{M}x \, dx.$$

It turns out that first order stochastic dominance in the worker type distribution over M carries over to $\frac{A(x)}{M}$ and we can establish the following:

Proposition 17. For equilibria of two economies with (M_1, F_1) and (M_2, F_2), if $\frac{1-F_1(x)}{M_1} \leq \frac{1-F_2(x)}{M_2}$ for all x, then the expected quality of hired workers in economy 2 is weakly higher than in economy 1.

The proof can be found in 2.7.4.

Therefore, if either $1 - F_2$ first order stochastically dominates $1 - F_2$ and $M_1 = M_2$, or $F_1 = F_2$ and $M_2 < M_1$, the expected quality of hired workers will be higher in economy

2. If we consider an initial stage in which firms can choose to enter at some cost $v > 0$, it immediately follows that there is a unique M that solves the free entry condition:

$$E[x] - v = E[x|\text{hired}]Pr(\text{hired})\frac{1}{M} - v = 0$$

2.4.2. Application Costs

Now we examine the relationship between application behavior and cost c. Note that a lower c is equivalent to a higher wage since the application decision is determined by the condition $w\left(\frac{A(x)}{M}\right)^k(1 - \frac{A(x)}{M}) \geq c$, which only depends on the ratio of cost to wage. There are two forces to consider for a lower c. On the one hand, workers have a higher net-return on each application. On the other hand, because higher-rank workers might apply more, low-rank workers might have a lower per-application success rate. Hence, it is not clear how a worker's application choice and utility relate to the cost. Counter-intuitively some workers might apply fewer times and be worse off with lower cost.

This can be seen from the results of a numerical exercise for $c \in \{0.148, 0.198, 0.248\}$ and $M = 0.7$. Figure 13 depicts equilibrium applications for different values of c. Some worker types around 0.1 do not apply for $c = 0.198$ but apply for the other cost values. We see from Figure 14 which plots worker utility, that utility for these workers is higher for $c = 0.248$ than $c = 0.148$.

Alternatively, we can consider a frictionless market with $c \approx 0$ and perfectly assortative matching. All workers in the top M-percentile find a job with certainty. So if $M < 1$, then workers of types $x < 1 - M$ remain unemployed. Now turn to the case with a slightly higher cost. We can conjecture that the workers in the top M-th percentile get hired with probability slightly less than one and some workers with types just below $1 - M$ now apply and have a strictly positive employment probability.
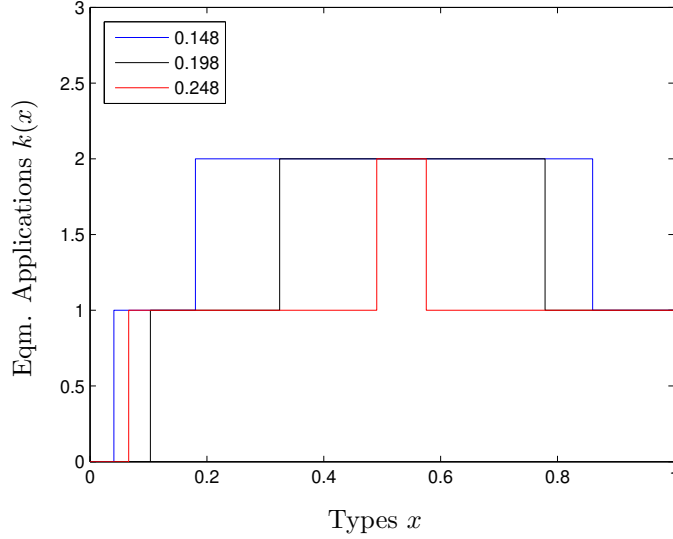
Figure 13: Equilibrium Applications for Different Levels of c
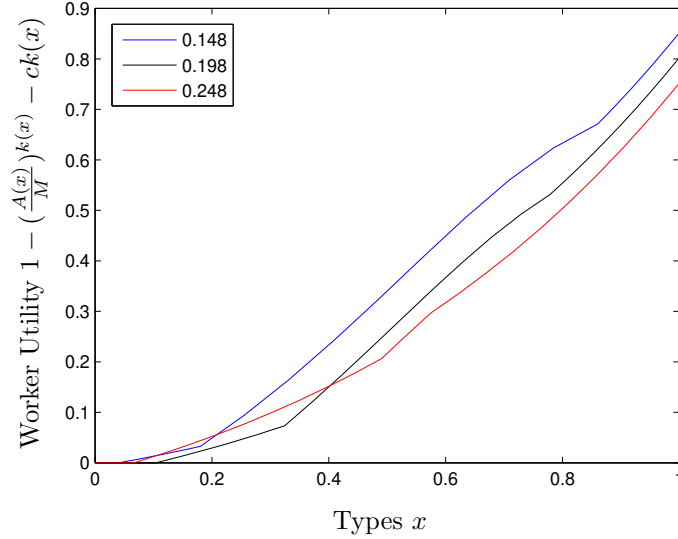
Figure 14: Worker Utility for Different Levels of c

2.5. Efficiency

Note that each worker's search imposes a negative externality on lower type workers, since they 'take away' jobs from the vacancy pool. When evaluating whether to make an additional application, workers weigh the increase in employment probability against c but do not consider the effect on lower-rank workers, or specifically, that a newly found job could have otherwise been taken by another lower type worker.

To illustrate the discrepancy between individual and social welfare, we can consider the problem of a social planner whose objective is to maximize total worker utility, which is equal to total employment minus total application costs. Assume that the planner can dictate each worker type to send out a particular number of applications but is still constrained by the anonymity assumption and application friction. The planner solves:

$$\max_{k \in \mathcal{K}} \int_0^1 1 - \Big(\frac{A(x)}{M}\Big)^{k(x)} - ck(x)\, dx$$
$$s.t. \quad -A'(x) = 1 - \Big(\frac{A(x)}{M}\Big)^{k(x)}, \quad A(1) = 0, \quad \forall x.$$

The planner's solution satisfies two conditions:

Proposition 18. In the planner's solution:

1. If all workers send out applications, the number of applications is decreasing in type.

2. If some workers send zero applications, then all workers send at most one application.

The formal proof can be found in the Appendix 2.7.5. We show that any problem that maximizes total employment for a fixed cost has a solution of this form. For a fixed number of total applications (and hence fixed total cost), if some interval of worker types sends more applications than an adjacent interval of lower types, then employment can always be increased by switching the number of applications for these two type intervals.

Note that this result must not apply if the planner's objective depends on cardinal worker types. However, our formulation of the planner's objective can be seen as the limit case when worker types are arbitrarily close to each other while preserving the rank. In the limit, the match quality is irrelevant for the social point of view.

Numerical Example Revisited   We demonstrate that the market solution in our numerical example from Section 3.1 is not welfare maximizing. Figure 15 depicts a solution (in red) that achieves the same level of employment as the market equilibrium (in blue)

with lower costs. From the upper graph, we see applications decrease from 2 to 1 at around $x = 0.5$, and everyone sends out nonzero applications. In the lower graph we see that $A(0)$ is the same for both solutions, but the employment accumulation is higher for most types in the market equilibrium. In the full solution to the planner's problem, total employment would be lower than in the cost-minimizing solution given here.
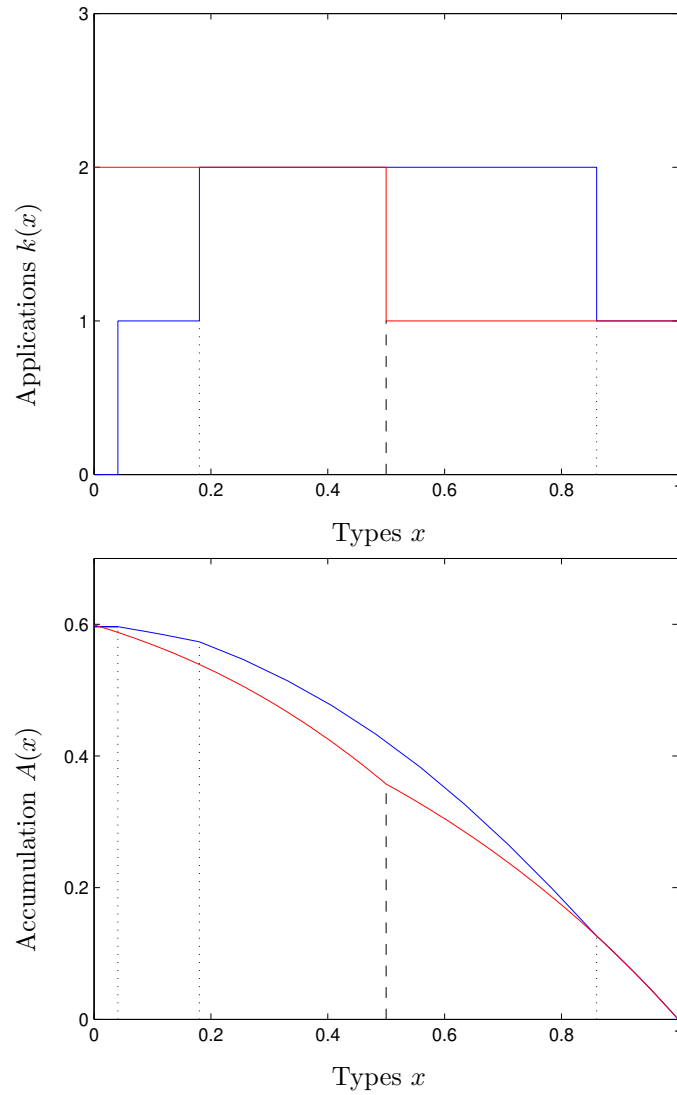

Figure 15: Cost-Min.(Market) Sol. in red(blue) with Long(Short) Dashed Lines at Cutoff Types

2.6. Conclusion

We examined how heterogeneity in ranking of workers by firms, translates into differences in search effort and employment probability. We find that if a worker's rank is sufficiently high, he applies more than higher-ranked competitors to insure against the event of firms choosing someone else. However if a worker's rank is rather low, his poor chances of success per application discourage him from applying as much as more highly-ranked counterparts. In this case, the disadvantage from the lower per-application success rate is amplified by lower search effort. This offers a novel insight to unequal labor market matching outcomes: higher unemployment among low-skilled workers can arise due to search effort choices.

We further showed that workers' equilibrium number of applications and utility can depend ambiguously on the size of the vacancy pool and application costs. Lastly, we showed that in a solution that maximizes worker utility the number of applications must decrease in type.

2.7. Appendix to Chapter 2

2.7.1. Alternative Derivation of DE

Assume a large economy, and then let $\varphi(x) \in [0, 1]$ be the deterministic probability that type x's application results in an offer. We now derive $\varphi(x)$ by examining under which conditions a particular firm is willing to hire worker x.

Fix a worker type x, and assume that there are N different segments in the type space $[x, 1]$, and all types in one segment choose identical number of applications. That is, the type space $[x, 1]$ can be segmented by N different numbers $x = x_0 < x_1 < x_2 < \ldots < x_{N-1} < x_N = 1$ such that all types in (x_{i-1}, x_i) send out the same number of applications. This restriction is not necessary for the proof, but does hold in any equilibrium.

We refer to the types in (x_{i-1}, x_i) as the i-th group. Denote by $k_i \in \mathbb{Z}_+$ the applications sent out by group i, and $\lambda_i = F(x_i) - F(x_{i-1})$ the mass of group i. Since the applications

are uniformly randomly distributed over firms, the total number of applications received by a single firm is a random Poisson variable. For the collection $(k_i, \lambda_i)_{i=1}^N$, the Poisson arrival rate of applications from types higher than x is given by $\frac{\lambda(x)}{M}$, where

$$\lambda(x) = \sum_{i=1}^N k_i \lambda_i = \sum_{i=1}^N k_i (F(x_i) - F(x_{i-1}))$$

This object, divided by the mass of firms M, is the 'gross queue length' of competing applications from higher types for an individual job.

An application by x generates an offer from the firm when either (1) the firm has no other applications or (2) there are other applications but all competitors choose other offers. For (2), the probability that a competing application is from i-th group is $\frac{k_i \lambda_i}{\sum k_i \lambda_i}$. A competitor $z_i \in (x_{i-1}, x_i)$ randomizes equally over his offers if there are multiple. The probability that z_i picks an offer other than the one at the firm considered is given by:

$$P_i(z_i; \varphi) = \sum_{j=0}^{k_i-1} \frac{j}{j+1} \binom{k_i - 1}{j} \varphi(z_i)^j (1 - \varphi(z_i))^{k_i-1-j}.$$

The term within the summation states the following. For the $k_i - 1$ applications the competitors sends out to other firms, the worker generates j offers with a probability that depends on *his* probability of success $\varphi(z_i)$. The worker chooses on of the j offers with probability $\frac{j}{j+1}$.

This is true for any $z_i \in (x_{i-1}, x_i)$, the probability that the competitor does not take the job at this specific firm conditional on there being one from group i is given by:

$$\int_{x_{i-1}}^{x_i} P_i(z; \varphi) \frac{f(z)}{\lambda_i} dz := p_i(\varphi)$$

Therefore, the overall probability that the job is not taken by a competitor from any group is:

$$J(x) = \sum_i \frac{k_i \lambda_i}{\lambda(x)} p_i(\varphi) = \sum_i \frac{k_i}{\lambda(x)} \int_{x_{i-1}}^{x_i} P_i(z; \varphi) f(z) dz$$

From the large market assumption, the probability is independent across competitors and with $n > 1$ competitors, the probability that the job is not taken is simply $J(x)^n$.

Since the number of competitors (random variable N) is a Poisson random variable with gross queue length as parameter, the expectation summarizes to:

$$\varphi(x) = \mathbb{E}_N\left[J(x)^N\right] = \sum_{n=0}^{\infty} \frac{(\frac{\lambda(x)}{M})^n e^{-\frac{\lambda(x)}{M}}}{n!} J(x)^n = e^{-\lambda(x)(1-J(x))}$$

The last equality follows from the fact that $E[a^X] = \exp(\lambda(a-1))$ for $X \sim Pois(\lambda)$ and $a > 0$.

This expression is similar to the one in Kircher (2009). The difference is that in our case, the probability of generating an offer is defined recursively as an expectation over the probability of types higher than x.

If $\varphi(x)$ is differentiable at x, taking the log and differentiating with respect to x yields:

$$\frac{\varphi'(x)}{\varphi(x)} = \frac{d}{dx}\left(-\frac{\lambda(x)}{M}(1 - J(x))\right)$$

Note that from the definition of $J(x)$,

$$\lambda(x)J(x) = \sum_{i=2}^{N} k_i \int_{x_{i-1}}^{x_i} P_i(z;\varphi)f(z)\,dz + k_1 \int_{x}^{x_1} P_1(z;\varphi)f(z)\,dz$$

so that

$$\frac{d}{dx}\left(\frac{\lambda(x)}{M}J(x)\right) = -k_1 P_1(x;\varphi)\frac{f(x)}{M}.$$

Furthermore:

$$\frac{d}{dx}\left(\lambda(x)\right) = -k_1 f(x),$$

We can rewrite:

$$\frac{\varphi'(x)}{\varphi(x)} = -k_1 \frac{f(x)}{M}(P_1(x;\varphi) - 1)$$

Taking the complement of the binomial sum:

$$P_1(x; \varphi) = \sum_{j=0}^{k_1-1} \frac{j}{j+1} \binom{k_1-1}{k_1-1-j} \varphi(x)^j (1 - \varphi(x))^{k_1-1-j}$$

$$= 1 - \sum_{j=0}^{k_1-1} \frac{1}{j+1} \binom{k_1-1}{k_1-1-j} \varphi(x)^j (1 - \varphi(x))^{k_1-1-j}$$

Using this result and changing indices, we get the sum:

$$\varphi(x) k_1 (P_1(x; \varphi) - 1) = - \sum_{j=0}^{k_1-1} \frac{k_1}{j+1} \binom{k_1-1}{k_1-1-j} \varphi(x)^{j+1} (1 - \varphi(x))^{k_1-1-j}$$

$$= - \sum_{l=1}^{k_1} \binom{k_1}{l} \varphi(x)^l (1 - \varphi(x))^{k_1-l}$$

$$= -(1 - (1 - \varphi(x))^{k_1})$$

So we see that:

$$\varphi'(x) = \frac{f(x)}{M} (1 - (1 - \varphi(x))^{k_1}))$$

$1 - \varphi(x)$ the proportion of jobs that are already taken up by types higher than x. That is, by definition $1 - \varphi(x) = \frac{A(x)}{M}$, and we arrive at the DE defined in the main part of the paper:

$$-A'(x) = f(x)(1 - (\frac{A(x)}{M})^k).$$

2.7.2. Proof of Theorem 1

Consider the following possibilities:

- $A(x) < M(1 - c)$ for all x: In this case, everyone sends out 1 or more applications and A is strictly decreasing everywhere.

  Define A_l as the cutoff accumulation for which $MB_{l+1}(\frac{A_l}{M}) = (\frac{A_l}{M})^l (1 - \frac{A_l}{M}) = c$ for $l \in \mathbb{Z}_+ \setminus \{0\}$. In general, A_l has either two or zero real solutions due to single-peakedness of the marginal benefits. Denote the two solutions $(\underline{A_l}, \bar{A_l})$. The types x_l for which

$A(x_l) = \underline{A}_l$ or $A(x_l) = \bar{A}_l$ are indifferent between sending out $l+1$ applications and l applications. A is continuous, strictly decreasing, and we assumed that types that are indifferent are choosing the smaller number of application. Therefore, the set of types who send out l or more applications

$$X_l = \{x \in [0,1] : A(x) \in (\underline{A}_l, \bar{A}_l]\}$$

is a connected interval on $[0,1]$. Denote this interval by $[x_l^{low}, x_l^{high}]$. From the decreasing marginal benefit, it is true that the sets are nested: for all $l \geq 1$

$$X_l = [x_l^{low}, x_l^{high}] \supseteq [x_{l+1}^{low}, x_{l+1}^{high}] = X_{l+1}$$

Mapping the nested set of intervals to the respective number of applications, we get a single-peaked graph.

- $A(x) = M(1-c)$ for some x: In this case, there is an interval $[0, \bar{x}]$ where $K(\frac{A(\bar{x})}{M}) = 0$. For $x > \bar{x}$, $K(\frac{A(x)}{M}) > 0$ and $A'(x) < 0$; this implies that the above observation applies and the equilibrium applications is single-peaked. For $x < \bar{x}$, the applications is 0.

2.7.3. Proof of Normalization

We change variables and define $\tilde{x} = F(x)$, and $F^{-1}(\tilde{x}) = x$. Assuming F is continuous, differentiable and one-to-one, its inverse is well-defined and the equilibrium conditions can be rewritten as

$$-A'(F^{-1}(\tilde{x})) = F'(F^{-1}(\tilde{x}))(1 - \left(\frac{A(F^{-1}(\tilde{x}))}{M}\right)^{k(F^{-1}(\tilde{x}))}),$$

$$k(F^{-1}(\tilde{x})) = K(A(F^{-1}(\tilde{x})))$$

$F'(F^{-1}(\tilde{x})) = \frac{1}{\frac{dF^{-1}(\tilde{x})}{d\tilde{x}}}$, so that

$$-A'(F^{-1}(\tilde{x}))\frac{dF^{-1}(\tilde{x})}{d\tilde{x}} = 1 - \left(\frac{A(F^{-1}(\tilde{x}))}{M}\right)^{k(F^{-1}(\tilde{x}))}.$$

Let $A(F^{-1}(\tilde{x})) = \tilde{A}(\tilde{x})$, and $k(F^{-1}(\tilde{x})) = \tilde{K}(\tilde{x})$. Then:

$$-\tilde{A}'(\tilde{x}) = 1 - \left(\frac{\tilde{A}(\tilde{x})}{M}\right)^{\tilde{K}(\tilde{x})},$$

$$\tilde{K}(\tilde{x}) \in K\left(\frac{\tilde{A}(\tilde{x})}{M}\right)$$

2.7.4. Proof for Proposition 17

We show that $\frac{1-F_1(x)}{M_2}$ FOSD $\frac{1-F_1(x)}{M_1}$ implies that $\frac{A_1(x)}{M_1} \le \frac{A_2(x)}{M_2}$ for all x:

$$\frac{A_1(x)}{M_1} \le \frac{A_2(x)}{M_2} \iff \int_x^1 -\frac{A_1'(y)}{M_1} \, dy \le \int_x^1 -\frac{A_2'(y)}{M_2} \, dy$$

$$\iff \int_x^1 \left(1 - \left(\frac{A_1(y)}{M_1}\right)^{k_1(y)}\right)\frac{f_1(y)}{M_1} \, dy \le \int_x^1 \left(1 - \left(\frac{A_2(y)}{M_2}\right)^{k_2(y)}\right)\frac{f_2(y)}{M_2} \, dy$$

$$\iff \int_0^{\frac{1-F_1(x)}{M_1}} (1 - \tilde{A}(z)^{\tilde{K}(z)}) dz \le \int_0^{\frac{1-F_2(x)}{M_2}} (1 - \tilde{A}(z)^{\tilde{K}(z)}) dz$$

Last line by the change of variables $z = \frac{1-F_i(x)}{M_i}$.

It is then straightforward to show that first order stochastic dominance in $\frac{A(x)}{M}$ implies higher expected quality. Since the endpoints are the same for all equilibria: $A_1(1) = A_2(1) = 0$, by the fundamental theorem of calculus:

$$\frac{A_1(x)}{M_1} \le \frac{A_2(x)}{M_2} \iff 0 \le \int_x^1 -\frac{A_1'(y)}{M_1} dy \le \int_x^1 -\frac{A_2'(y)}{M_2} dy$$

Applying integration by parts, for all x, we obtain that

$$\int_x^1 -\frac{A_1'(y)}{M_1} y \, dy = \frac{A_1(x)}{M_1} x + \int_x^1 \frac{A_1(y)}{M_1} \, dy$$

$$\leq \frac{A_2(x)}{M_2} x + \int_x^1 \frac{A_2(y)}{M_2} \, dy$$

$$= \int_x^1 -\frac{A_2'(y)}{M_2} y \, dy$$

2.7.5. Proof for Section 6

First of all, it is without loss of generality to focus on piecewise continuous k, since any measure zero change of k's does not affect the objective. Secondly, we can also rule out any nonzero gap in the support of $k(x)$, where $k(x)$ takes a nonzero value, because one can find a k with connected support that obtains the same value. Formally:

Lemma 3. For $n > m > 0$ number of applications, assume that in an equilibrium, there is a lower type applies less than a higher type, i.e., there are types $x' > x$ such that $k(x') = n$ and $k(x) = m$. Then the planner can increase the total employment without affecting the cost.

Proof. There exists small interval $\Delta > 0$ and threshold x such that k takes value $n > m > 0$ for $z \in (x, x+\Delta]$ and $m > 0$ for $z \in [x-\Delta, x)$. For any value $A = A(x+\Delta)$, the accumulation up until type $x + \Delta$, the choice of k implies

$$A(x - \Delta) = \int_{x-\Delta}^x 1 - \left(\frac{A(z)}{M}\right)^m dz + \int_x^{x+\Delta} 1 - \left(\frac{A(z)}{M}\right)^n dz + A(x + \Delta)$$

Now consider an application schedule of the planner k^* which differs from k only for types in $[x - \Delta, x + \Delta]$. In particular, $k(z) = m$ for $z \in (x, x+\Delta]$ and $k(z) = n$ for $z \in [x - \Delta, x)$. Since $k = k^*$ over $[x + \Delta, 1]$, the accumulations A and A^*, are also equivalent for these

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types, however,

$$A^*(x - \Delta) = \int_{x-\Delta}^{x} 1 - \left(\frac{A(z)}{M}\right)^n dz + \int_{x}^{x+\Delta} 1 - \left(\frac{A(z)}{M}\right)^m dz + A(x + \Delta)$$

Let $A = A(x + \Delta)$. For small Δ, the integrals are approximated by:

$$\left(1 - \left(\frac{A + (1 - (\frac{A}{M})^n)\Delta}{M}\right)^m\right)\Delta + \left(1 - \left(\frac{A}{M}\right)^n\right)\Delta + A,$$

$$\left(1 - \left(\frac{A + (1 - (\frac{A}{M})^m)\Delta}{M}\right)^n\right)\Delta + \left(1 - \left(\frac{A}{M}\right)^m\right)\Delta + A$$

We show that we can find small Δ around x such that the second term is greater than the first. If so, then by switching the numbers of application for these Δ interval of types, the planner achieves higher total employment without affecting the cost. Expanding the two sums of polynomials, $1 - (\frac{A}{M})^m + 1 - (\frac{A}{M})^n$ term is common for both sums. Cancelling them out, we have to compare:

$$-\left(\frac{A + (1 - (\frac{A}{M})^n)\Delta}{M}\right)^m + \left(\frac{A}{M}\right)^m \tag{a}$$

$$-\left(\frac{A + (1 - (\frac{A}{M})^m)\Delta}{M}\right)^n + \left(\frac{A}{M}\right)^n \tag{b}$$

We can compare the coefficients of the same order of $\frac{A}{M}$. Expansion yields that the k-th order terms $(k \leq m < n)$ are

$$-\binom{m}{k}\left(\frac{A}{M}\right)^{m-k}\left(1 - \left(\frac{A}{M}\right)^n\right)^k\left(\frac{\Delta}{M}\right)^k \tag{a-k}$$

$$-\binom{n}{k}\left(\frac{A}{M}\right)^{n-k}\left(1 - \left(\frac{A}{M}\right)^m\right)^k\left(\frac{\Delta}{M}\right)^k \tag{b-k}$$

respectively. Division yields their ratio:

$$\frac{m(m-1)\ldots(m-k+1)\left(\frac{A}{M}\right)^{m-1}}{n(n-1)\ldots(n-k+1)\left(\frac{A}{M}\right)^{n-1}}\left(\frac{1-\left(\frac{A}{M}\right)^n}{1-\left(\frac{A}{M}\right)^m}\right)^k$$

Collecting terms, this ratio can be written as:

$$\underbrace{\frac{\frac{m\left(\frac{A}{M}\right)^{m-1}}{1-\left(\frac{A}{M}\right)^m}}{\frac{n\left(\frac{A}{M}\right)^{n-1}}{1-\left(\frac{A}{M}\right)^n}}}_{(1)}\underbrace{\left(\prod_{z=1}^{k-1}\frac{m-z}{n-z}\right)\left(\frac{1-\left(\frac{A}{M}\right)^n}{1-\left(\frac{A}{M}\right)^m}\right)^{k-1}}_{(2)}$$

Note that for the first order term $(k=1)$, the part (2) vanishes. The following claim shows that (1) is greater than 1, from which follows that the magnitude of the first order effect in (a) is greater than the effect in (b).

Claim 3. For $A < M$, part (1) is greater than 1.

Proof of claim. For simplicity, denote $\frac{A}{M}$ by $x \in [0,1]$. Since $n > m$, it is true that $\frac{mx^{m-1}}{nx^{n-1}} = \frac{m}{n}\frac{1}{x^{n-m}}$ is strictly decreasing in x. Hence, for all $y > x$,

$$mx^{m-1}ny^{n-1} > nx^{n-1}my^{m-1},$$

Integrating both sides from x to 1:

$$\frac{mx^{m-1}}{1-x^m} > \frac{nx^{n-1}}{1-x^n}. \tag{A}$$

Note that this is true for any $x < 1$, and in the limit as x goes to 1, the two expressions are equal. $\square$

Therefore, for $k = 1$, (a-k) is strictly smaller than (b-k). The signs of the differences of higher order terms are ambiguous, but their effects are dominated by the first order effect, for small Δ.

Formally, if we denote by $B = \frac{mx^{m-1}}{1-x^m} - \frac{nx^{n-1}}{1-x^n} > 0$, the difference in first order term, then, (b)-(a) is given by:

$$B\frac{\Delta}{M} - O((\frac{\Delta}{M})^2)$$

Furthermore, the big-O term is bounded above by (the most conservative bound) $n!\left(\frac{\Delta}{M}\right)^2$, hence, the difference is

$$B\frac{\Delta}{M} - o(\frac{\Delta}{M}).$$

In fact, this is true independent of A as long as $\frac{A}{M} < 1$. Hence, even if A varies with Δ the result holds. For small enough Δ, the expression is strictly positive and the planner's solution outperforms the market equilibrium. $\square$

For the second part of the proposition, we prove the following lemma:

Lemma 4. In the planner's solution, if some workers send out more than one application, then all workers send out at least one application.

Proof. Suppose, on the contrary, that there are some workers who send out $m \geq 2$ applications, while some other workers send out zero applications. From the previous lemma, we see that the planner's solution has to be monotone. Hence, the only possibility is that the workers increase the number of applications up until some threshold type, and all types below the threshold do not apply.

We show that the planner can improve this outcome. Denote by A the total accumulation up until the cutoff type x. Then, there exists an interval $[x, x + \Delta]$ of types who send out m applications. Take a small neighborhood $\varepsilon < \Delta$ of x and let $[x, x + \varepsilon]$ workers instead send $m - 1$ applications and let the $[x - \varepsilon, x]$ workers send one application. By letting $\tilde{A} = A(x + \varepsilon)$, the high types $[x, x + \varepsilon]$ lose

$$(\frac{\tilde{A}}{M})^{m-1}(1 - \frac{\tilde{A}}{M})\varepsilon.$$

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The gain by the low types $[x - \varepsilon, x]$ is

$$\left(1 - \frac{\tilde{A} + (1 - (\frac{\tilde{A}}{M})^{m-1})\varepsilon}{M}\right)\varepsilon = (1 - \frac{\tilde{A}}{M})\varepsilon - (1 - \frac{\tilde{A}}{M})^{m-1}\frac{\varepsilon^2}{M}$$

The difference between them is:

$$(1 - (\tfrac{\tilde{A}}{M})^{m-1})(1 - \tfrac{\tilde{A}}{M})\varepsilon - (1 - (\tfrac{\tilde{A}}{M})^{m-1})\tfrac{1}{M}\varepsilon^2.$$

Again, as long as $\frac{\tilde{A}}{M}$ remains strictly bounded away from 1, for ε small enough, the switch increases employment. $\qquad\square$

CHAPTER 3 : A Critical Literature Review on Markets with Search and
Information Frictions

3.1. Introduction

In an introductory economics framework, it is assumed that there is 'Walrasian auction-eer' who magically and impartially sets the price that clears the market. Although this framework usually captures the basic underlying forces of an economy, there have long been attempts to provide a more concrete foundation for how a market functions, and whether neglecting such factors would lead to a drastically different conclusion. Most notably, it is reasonable and realistic to think about a decentralized market, in which it takes time and effort to find a trading partner (search friction) and some agents know more about the good to be traded than others (information friction). Adding these frictions into the model seem to yield a very different outcome from the Walrasian benchmark. Diamond (1971) has famously noted that the presence of very small search costs, which an agent incurs in order to search for an alternative, will raise equilibrium prices to monopoly levels. Akerlof (1974) has also shown that information asymmetry can lead to a perfect collapse of profitable market transactions. Spence (1974) has also shown that a market outcome may involve inefficiency when the agents try to overcome such information asymmetry. The objective of this article is to review the recent literature on such market frictions and extend its modeling implications to the search equilibrium in an applied sense.

Most importantly, this article will be interested in the labor market with an on-the-job search. On-the-job search is known to be an important channel through which workers raise income, and it is also known to represent a large flow in the labor market. Although it is an interesting question to disentangle the underlying forces behind a job transition (promotion or demotion), or how the career concerns feed into the equilibrium job transition patterns, addressing these questions can be tricky once we incorporate informational components into the model.

This article proceeds as follows. The first section starts with a review of the theoretical treatment of markets with search and information frictions. I start out from the article by Lester et al. (2019) to highlight the underlying economic forces behind trading in such markets. I highlight the role of screening and how such elements can generate complex market dynamics. Next, I briefly discuss an important innovation by Guerrieri et al. (2010) which is the most well-known and tractable framework to model markets with the asymmetry of information. Most importantly, I argue how the robustness of separating equilibrium comes about and show why it useful. Later I move on to the topics related to the search equilibrium and discuss two papers regarding communication in such settings. The search and information frictions combined are shown to affect the informativeness of communication in an equilibrium, through two papers: Menzio (2007), and Harrington Jr and Ye (2017). In the second section, I move on to the topic of the labor market. I first discuss Golan (2009) which is an illustrative example of how the forces of information revelation work in the intra-firm wage bargaining. Next, I discuss Doppelt (2016) which is written in the symmetric information framework but provides a very good characterization of how incomplete information and reputation concern shape equilibrium wage profile and unemployment hazard. Carrillo-Tudela and Kaas (2015) shows how adverse selection element shape steady-state job-to-job flows and the wage profile of a worker. Depending on the parameters, it may be that the equilibrium is perfectly separating (a market for low productivity workers operates separately from the market for high productivity) or it may be that the equilibrium involves pooling. The latter equilibrium exhibits demotion of low type workers on the equilibrium path. Lastly, I discuss Visschers (2007) which characterizes the optimal wage-tenure contract when the firm is privately informed about the worker's quality, and the workers can always quit to a wage contract which offers higher value. The last section summarizes and concludes.

3.2. A Review on Theoretical Results

The two backbone papers discussed in this section derives from the characterization in Rothschild and Stiglitz (1978). In a competitive insurance market, Rothschild and Stiglitz (1978) has shown that the pooling contract is always susceptible to a cream-skimming deviation while a conjectured equilibrium with only separating contracts may also be disrupted by a pooling contract if the fraction of bad types is small. Lester et al. (2019) explores an intermediate case when the insurance market may not have perfect competition, that is, some consumers looking for insurance may be 'captive' who has access to only one insurance provider, while others may be 'non-captive' and chooses the best deal among the two insurance providers as in the Rothschild-Stiglitz world. Guerrieri et al. (2010) argues the robustness of the separating contract by imposing capacity constraints – a cream-skimming deviation is ruled out if it instead leads to a continuous adjustment of match probability, rather than a discontinuous jump in payoffs.

3.2.1. Disperson of Contracts: Lester et al. (2019)

Lester et al. (2019) considers a market in which sellers, who know the quality of the good they possess (good or bad), sells to buyers. Buyers individually post a menu of contracts, which trades off the trading probability and the terms of trade, in order to screen the sellers. The problem is a well-known mechanism design problem if the buyer is a monopolist, in which the buyer optimal menu exhibits the following:

- Good type sellers earn zero rent (binding IR constraint for the low type)

- A bad type seller always trades in the equilibrium (no distortion at the top)

- If there are sufficiently many good types, the good-type seller trades with positive probability, in which case, bad types are cross-subsidized with positive information rent.

- If there are not enough good types, good types do not trade at all and only the

bad-type seller trades (lemons market).

However, assume now that there are two buyers who are not bound by capacity constraints. Since they are not capacity-constrained, the buyers may compete by undercutting the other's offered contract more favorable to the seller. A seller who has access to both buyers chooses the buyer who offers higher utility.

In case all sellers have access to both buyers, this force reduces to the perfect competition similar to that in the Rothschild and Stiglitz (1978). In case the pure strategy equilibrium exists, it is the least cost separating equilibrium; in case there is no pure strategy equilibrium, Rosenthal and Weiss (1984) have shown that there is a unique mixed strategy equilibrium over a continuum of support. In the equilibrium, buyers receive a negative surplus from the low types and compensate it from the high types (cross-subsidization). Similar to the Burdett-Judd (1983) equilibrium, it is shown that the buyers randomize on the continuum of an interval of contracts, over which the buyers are indifferent due to the terms and trade probability trade-off.

In these two polar cases, the buyers either offer a perfectly pooling menu or a perfectly separating menu in an equilibrium. The authors proceed to show that in the intermediate case with imperfect competition, there may be a region of parameters such that partial pooling occurs; a buyer aware of the probability that the competitor exists mixes between offering a pooling contract and offering a separating contract. In this sense, the paper provides a complete characterization of imperfect competition in the market with adverse selection, which can provide a richer set of forces underlying comparative statics.

For instance, the authors consider the role of increased competition in the market, manifest in a higher probability that a seller is non-captive. Increased competition can distort trading probability for high types in two ways: through the intensive margin in terms of separating low types from high types, and through the extensive margin (the effect of competition) that affects equilibrium utility provided to the sellers. It is shown that for some range of

parameters, these two forces can work in the opposite direction so that increased competition can decrease the probability of trade.

The authors also consider the role of information revelation in such economy to show that providing more information (decreasing the adverse selection problem) may some times lower the social welfare. Viewing 'more information' as the dispersion in posteriors, the authors show that the social welfare function with respect to the composition parameter may be concave so that ex-ante social welfare is lower compared to no dispersion in the posterior.

3.2.2. Competitive Search: Guerrieri, Shimer and Wright (2010)

Guerrieri et al. (2010) studied the market with adverse selection using the framework of competitive search. In the competitive search framework, principals post contracts, while privately informed agents direct their search to the posted contracts. Assuming that the principals compete for the agents by posting contracts, zero profit condition for the principal pins down the matching probability and the composition of agents attracted by the contract. By adding in the adverse selection element into the competitive search model, the authors characterize the equilibrium trade-off between the extensive margin (probability of trade) and the intensive margin (terms of trade).

To briefly outline the model, the authors consider a one-to-one matching environment in which there is potentially large continuum number of homogeneous principals and a measure 1 of heterogeneous agents. The principals incur the cost of $k > 0$ for posting a vacancy, and the equilibrium measure of principals in the market is pinned down by the zero profit condition for the principals. Since there is asymmetry of information (agents privately know their types), in general, the principals' problem is that of posting a mechanism; using the large market assumption, the authors instead focus on the problem of a principal choosing a particular contract to offer, which is later shown to be outcome equivalent with the equilibrium with mechanism posting.

Denote the set of contracts offered in the equilibrium by $C = \{y_1, \ldots, y_I\}$, where the index i means that it is a contract intended for a i-type agent to accept. The contract then must satisfy the incentive compatibility constraints for all types. Since the matching market is one-to-one, a competitive search equilibrium is defined by the vector of agent utilities $\{\bar{U}_i\}_{i \in I}$, market tightness measure Θ that maps from all contracts (including those not offered in equilibrium) to the space of $[0, \infty]$, and a belief function Γ that coincides with the distribution of agent types attracted by any contract, which satisfy the optimality conditions:

- Principals' profit maximization and free-entry: for all $y \in Y$, [match probability given Θ] times the [expected value from the set of agents attracted] is equal to k for y offered in equilibrium, and less than or equal to k for y not offered in equilibrium.

- Agents' optimal search: for an agent with positive expected value in equilibrium, $\bar{U}_i > 0$, incentive compatibility must be satisfied for all contracts offered in equilibrium, and for contracts that are not offered in equilibrium, in conjunction with the off-the-equilibrium meeting probability dictated by Θ. The indifference condition must satisfy for all contracts that an agent i chooses with positive probability.

- Market clearing: The measure of agents present in markets for any contract has to add up to the total number of agents in the economy. The beliefs that firms hold about the type of agents present in the market coincides with the correct measure of agents.

In order to solve for the competitive equilibrium, the authors break down the problem into a sequence of problems in which type i's incentive compatibility binds for all its neighborhood type $i + 1$, while the incentive compatibility for $i + 1$ for masquerading to be a lower type i is slack. Using the indifference condition for upward deviation, the authors can pin down the market tightness Θ even for the contracts that are not offered in equilibrium. The single-crossing condition for the two types guarantees that no deviation is profitable for the

off-the-equilibrium beliefs about matching probability Θ.

In general, the competitive equilibrium exhibits full separation. Intuitively, this resolves the non-existence issue in Rothschild and Stiglitz (1978) by assuming capacity constraints, and by imposing appropriate beliefs (on matching probabilities) that deters any arbitrary deviations to off-path contracts. The authors show the existence and uniqueness of the competitive equilibrium and revisit a few earlier examples of markets with adverse selection. Applying their algorithm, they calculate the equilibria and show that the equilibrium properties such as Pareto inefficiency, and distorted extensive and intensive margins still continues to hold. Overall, it can be thought of as a unifying framework to treat the competition with an adverse selection problem, using the tools of competitive search.

In another paper (Guerrieri and Shimer (2014)) this model is extended to a dynamic setting in which the values of holding assets is also endogenously determined (by the resale value of a good that one holds). When the seller has better knowledge about the quality of the asset that he/she holds, this increases the adverse selection problem in the market and can lead to low liquidity, which depresses the overall asset prices. At the same time, buyers shifting to the market with less adverse selection leads to the episode of flight-to-quality in times of financial crisis.

3.2.3. Informativeness of Communication: Menzio (2007), Harrington and Ye (2019)

The literature on directed search has focused on the posting of terms of trade or contracts that are enforceable. Menzio (2007) instead focused on posting a message to attract the searchers, while the message itself does not carry any contractual obligations. With the presence of asymmetry of information, the author shows that the communication can be partially informative if a more informed party posts cheap talk messages in order to attract searchers. Important examples of such messages are, help wanted ads posted by an employer, or list prices posted by intermediaries while the prices are often subject to discounts offered at the actual transaction stage.

In Menzio (2007), firms privately draw their productivity y and post a cheap talk message toward which unemployed workers direct their search. Once a worker applies to the firm, the firm and the worker negotiate wage through alternating offers bargaining. In this alternating offers bargaining, there is a time interval of Δ between rejection and the next counter-offer. As usual, the cost of delay is captured by the parameter β standing for the breakdown rate after the firm rejects the worker's offer. Therefore, when the firm rejects an offer, with probability $1 - \exp(-\beta\Delta)$ the game ends. Similarly, when the worker rejects an offer, the negotiation breaks down with probability $1 - \exp(-(1 - \beta)\Delta)$.

The bargaining is between an informed firm who knows the productivity and the worker who only has an expectation based on the posted message. The author focuses on the sequential equilibrium where the firm's strategy is stationary in the sense that history matters only through the worker's beliefs. In equilibrium, delay signals that the firm is of low productivity, and it can be shown that no firms with productivity y can gain more than the perfect information case, and no firms are worse off than the firm with the lowest productivity in the support, bargaining in perfect information. As the interval between offers vanishes $\Delta \to 0$, the unique equilibrium of the asymmetric information bargaining game converges to the first round offer and acceptance of the wage that corresponds to the perfect information outcome of the lowest productivity firm in the support. That is, if the firm's productivity lies in the interval $[\underline{y}, \bar{y}]$, the bargaining outcome is the wage $\beta\underline{y}$.

Using this equilibrium in the second stage, firms trade off the probability of a hire (queue length) with the margin (product net wage to be paid to the worker after the bargaining stage). Since a firm's incentive to post a higher message (and trade more frequently) is increasing in the firm's productivity, while the cost remains independent, there is a natural single-crossing property in the payoffs that separates the firms' equilibrium messages. Overall, given the worker's indifference curve along with the market tightness measures, firms choose which market to go to; single-crossing guarantees that as we move up the productivity from the lowest productivity firm, firms would like to move up the announcement. Since

there is a continuum of firms, firms would like to pool with lower productivity firms for small differences in productivity. Using the indifference conditions, one can identify all the cutoff types who are indifferent between pooling with lower types and posting a message that reveals its type.

Harrington et al. (2019) examine how the announcement of list prices can serve as a means for collusion among competing sellers. Similar to the line of Lester et al. (2019), the authors consider an environment with imperfect competition, in which there are two competing sellers, with a seller acting as a monopoly over a fraction $b \in [0, 1]$ of buyers who is in contact with him/her. However, there are two important differences with the Lester et al. (2019). First of all, unlike the screening problem using posted contracts in Lester et al. (2019), the authors consider a bargaining stage after a buyer chooses the seller to go to. In the bargaining round, the seller bargains knowing whether the buyer is captive (the seller is a monopoly over the buyer) or the buyer is non-captive (the seller is in competition with another seller.) Secondly, a captive buyer plays a more active role in the game by choosing which seller to go to. This assumption delivers a non-trivial role of price announcement from the firm's part; by announcing a low price announcement, the seller can attract more buyers despite the fact that the low price announcement may hurt the firm in the bargaining stage.

The bargaining is modeled as a second price auction with a reserve price set by a buyer. The reserve price is meant to capture the instances of negotiation breakdown when the seller has private value over the good to be traded. The seller simultaneously draws his/her cost, and the good is traded if and only if the cost is below the reserve price set by the buyer. A buyer who draws a high private value sets a high reserve price and hence admits a higher probability of trade with the higher average price paid to the seller.

An informative announcement by the seller influences the negotiation outcome by altering the buyer's belief about the seller's distribution of private cost, which gets reflected in the buyer's choice of the reservation price. For instance, if the seller convinces the buyer

that the seller's cost is high, ceteris paribus, the buyer sets a higher reservation price (less aggressive in bargaining) from which the seller benefits. This is basically the force that incentivizes the sellers to collude to a high-cost announcement. However, the presence of captive buyers ($b > 0$) gives sellers incentives to cheat on the other firm, by making a low-cost announcement and stealing the market from the other firm. Therefore, an equilibrium with truthful cost announcement can be sustained with a high fraction of captive buyers (low incentive to collude).

The authors formulate a repeated version of this 'cost announcement game' to show that a standard grim-trigger type of strategy can be used to sustain the collusive outcome on the equilibrium path. Furthermore, using a parametrized version of the model, the authors also show the possibility that social welfare (measured by the expected trade surplus) increase under collusion through the increased reservation price.

3.3. Application to On-the-Job Search and Bargaining

In this section, I review a few papers examining the asymmetric information bargaining problems in the labor market setting. The first paper by Golan (2009) examines the channel through which bargaining can be affected when its outcome (wage) signals outside firms about the worker's quality and affects bargaining outcome in the next period.

3.3.1. Asymmetric Learning and Wage Bargaining: Golan (2009)

In this paper, the author considers the wage dynamics of a worker whose productivity is learned by the current employer. The worker lives for three periods. At period 1, the worker does not produce anything but the worker and the employer jointly learn the productivity of the worker θ, which is drawn from an interval $[\underline{\theta}, \bar{\theta}]$ according to the distribution f. Although outside firms do not know the exact realization of the θ, the information is indirectly revealed to the outside firms through the past history of wage that the worker was paid. The wage history itself is shaped by the outside offer of expected productivity, and the renegotiation that can be triggered by a worker at periods 2 and 3. Since the bargaining outcome – wage

– sends information to the outside firms at the beginning of period 3, this change in belief is fed into the bargaining problem when the renegotiation is triggered at period 2.

The bargaining protocol is an ultimatum game that lasts for two rounds, $t = 1, 2$, in which the proposer is chosen randomly by a toss of a fair coin. In the renegotiation of period 3, it can be easily seen that the bargaining outcome will be a split of $\frac{\theta}{2}$ offered and accepted at the first round of the bargaining. In the period 2 renegotiation, however, each round of rejection delivers a different message to the outside firms, which means an acceptable offer at the previous round has to take into account these changing outside options in period 3. For instance, in round 2 of period 2, if an offer is rejected, nothing is produced and it does not generate any information to the outside firms at the beginning of period 3. Taking this into account, the worker's offer has to compensate for the firm's loss in surplus in the next period if the firm is to agree on a type-revealing wage. Therefore, an agreed upon wage at period 2 renegotiation is lower than $\frac{\theta}{2}$, and the firm expects to earn information rent at this point.

Given these bargaining outcomes, the author characterizes the threshold productivity below which workers choose not to renegotiate wage. The threshold worker's type θ^* is given by the indifference condition between the revealing wage profile, which the worker can obtain by triggering wage renegotiation, and accepting the stream of constant wage $\underline{\theta}$ for the rest of the period. The threshold is interior because a worker has to give up some of the surpluses in period 2 bargaining if he/she is to receive a fully revealing competitive wage in period 3. For this conclusion, it is also important that the workers' outside options, without a revealing wage, at periods 2 and 3 are given by $\underline{\theta}$, as dictated by the force of extreme adverse selection faced by outside firms making an offer. Otherwise, the workers might find it profitable to take an outside offer greater than $\underline{\theta}$ at period 3.

The outcome wage profile exhibits a two-tier structure. Below the threshold productivity θ^*, workers receive a constant wage and do not reveal their types nor trigger renegotiation. Above the threshold, workers bargain to a low wage at period 2 in order to receive a fully

revealing, competitive level of compensation at period 3.

3.3.2. Résumé Dynamics and Unemployment Hazard: Doppelt (2016)

Data from the Current Population Survey suggests that a job-finding probability for the next month can be as low as 30 percentage points for the workers who were unemployed for a year, compared to the newly unemployed. One hypothesis is that the longer duration of unemployment serves as a signal about the worker's quality – a prospective employer cannot see the worker's past rejection during a search and is likely to infer that the duration is because of the worker's bad quality.

Doppelt (2016) analyzed a dynamic search equilibrium in which a worker's duration of unemployment affects his/her future employment probability through the learning effect. The paper is focused on the job finding rate that is determined both by the worker's reservation value, and the firm's value over the worker with a particular job history (résumé). In order to focus on the effect of learning, any element that leads to asymmetry of information is ruled out – the environment is kept symmetric despite uncertainty about the worker's type.

The author considers a model of heterogeneous workers, low and high quality, who are distinguished by the stochastic process of productivity. The productivity process is Markov; the high-quality worker's productivity draw is higher than the low-quality worker's in the sense of stochastic dominance. The sufficient statistic for a worker's experience in the labor market is the posterior probability about the worker's type. This is the worker's state variable which evolves according to an update rule that is generated endogenously in equilibrium.

The worker's history in the labor market is public information, including productivity draws during employment. When a worker is hired from unemployment, productivity is assumed to be an inspection good; when the worker and a firm meets, they draw the productivity and choose whether to match or not. Due to the selective exit from the unemployment pool, the worker's state variable changes with the duration of unemployment – a worker

may be unemployed for a long time because he/she is unlucky (no meetings) or because he/she drew bad productivity in all the meetings.

Since a worker's history diverges when the worker switches from unemployed to employed, (for the other case, employed to unemployed, it is assumed that the productivity draw that triggered match dissolution is public) there is résumé effect in the wage determination and match consummation through the unemployment value with a counterfactual résumé.

A parametrized version of the steady state equilibrium (new inflow of résumé is assumed to follow a beta distribution) is calibrated to match the empirical job-finding hazard in the CPS data. Two interesting findings emerge. First of all, the worker's job finding rate is single-peaked in his/her résumé. This is because there is more entry in the market for mid-level résumés than the resume close to one; due to the fact that the workers accept wage cuts to be hired in this region and the firms earn higher profit. As a matter fact, due to this 'information wedge', initial wage varies significantly with résumé, and the worker who receives the lowest starting wage is the worker with an intermediate resume facing the largest information wedge. The author also suggests this as an explanation for increasing wage-tenure relationship – the wage is expected to catch up very quickly when the following productivity draws are observed during the employment.

Secondly, the model provides new insight into the interpretation of a negative correlation between the job finding rate and the unemployment duration. In the micro level, the negative correlation in the aggregate level may either be because of decreasing job-finding rate, or heterogeneous but constant job finding rates. The incomplete information framework can account for both mechanisms and serves to amplify the effect (through both the change in types and the change in hazard rate) compared to having only one mechanism. Making productivity deteriorate during unemployment flattens the information value of unemployment and decreases the variability in job finding rates among different résumés.

3.3.3. Worker Mobility with Adverse Selection: Carrillo-Tudela and Kaas (2015)

The paper is interested in worker mobility and wage dynamics when the firms post wage contracts to screen workers of unknown quality. The authors study steady-state equilibrium in a continuous time economy with two types of workers, H and L, with productivities $p_H > p_L$, which are known to workers, but is unknown to the firms at the time of hiring. It is assumed that the type is perfectly learned by the employer firm at the Poisson rate of $\rho > 0$.

The learned productivity is verifiable and contractible at the point of hiring. The firm posts a menu of contracts, intended for either H or L types, which specifies three wages: starting wage, promotion wage in case the realized ability is true, and the demotion wage in case the realized ability is false. In order to maximize the truth-telling incentives, the firms optimally choose the promotion wage to be the full productivity of a worker, p_i, while demoting a misreported worker to the reservation wage (unemployment benefit b). The authors solve for the equilibrium under these contractual assumptions, and later show that there are no deviation incentives for a large set of reasonable parameters.

The form of the contract admits restricting attention only to the pair of wages in the probation stage, (w_H, w_L) intended for high and low types. Focusing on the rank preserving equilibrium, in which there is one to one mapping from w_H and the corresponding w_L posted by a single firm, the wage dispersion equilibrium as in Burdett and Mortensen (1998) arises for job transitions in the probation stage. The equilibrium is calculated starting from the pair of reservation wages (R_H, R_L), the lowest offered in the equilibrium, and by working up to the top wage pairs $(\bar{w}_H, \bar{w}_L)$ using the firms' equal-profit condition. It is shown that the incentive constraints are slack at the lowest contract offered. However, the constraint might become binding for higher-ranked contracts.

Depending on the learning parameter ρ, there are two cases. When ρ is high enough, there is a fully separating equilibrium: the high and low markets are perfectly segregated, and

there is no demotion in equilibrium. When the learning rate is too low, it is difficult to sustain the incentive compatibility constraint, and it might be that the low types want to shift to a high-type probation wage despite the possibility of demotion. Due to the presence of demotion in equilibrium, the low type workers exhibit both within-firm wage cuts (by demotion) and between-firm wage cuts (when they shift to another firm after being demoted). The equilibrium also exhibits the presence of pooling firms at the top of the contract distribution, that kicks out a proportion of low type workers and sustains a large proportion of high type workers (compared to other firms offering separating contracts.)

The authors test the quantitative implications of the model by setting the four parameters $(\lambda, p_H, p_L, \alpha)$ to match the targets (1) monthly job-to-job transition rate, (2) average promotion gain, (3) average quit gain, and (4) average layoff loss observed in the NLSY and Current Population Survey data. In the model, the low type workers transit jobs more frequently because they are sometimes demoted after taking a pooling contract. This is in line with OLS regressions on data (similar to the study by Light and McGarry (1998), but with additional data on job-to-job transitions) which shows that both the counts of non-employment spells and job-to-job transitions are negatively correlated wage. Although the average wage gain from transition is calibrated to be positive, the negative link between the counts is driven by worker heterogeneity. Since an IV regression accounting for the unobserved heterogeneity turns out positive coefficients, the authors conclude that the forces of standard job ladder models are at work.

The authors also link their findings with a high level of internal mobility in large (pooling) firms and lower tenure in small (separating) firms. They also suggest that the variation in starting wage for small (separating) firms is larger than large (pooling) firms.

3.3.4. Visschers (2007)

Visschers (2007) solves for optimal wage contracts when neither the firm and the worker are enforced to stay in the contractual relationship forever. Given a wage contract that is

to be implemented whenever the match lasts, the participation risk comes from both sides. The worker side incentives are as standard in on-the-job search literature: the worker may generate an outside offer with a superior starting value, and hence quit the current job. The firm has superior knowledge about the productivity of the match and terminates the match if the worker's productivity is not enough to generate profit given the promised wage payment. The contract, the future sequence of wage payments, is agreed upon before the match productivity is realized. Once the match is formed, the firm is privately informed about the productivity of the worker. Hence, at the hiring stage, the worker correctly anticipates the probability that, depending on the firm's realization of productivity, some of the promised future wages will not be paid.

Using a similar argument as in Stevens (2004), the author shows that the optimal wage, from the firm's retention point of view, is a step contract. Since the step contract always exhibits increasing wages, it is optimal to fire a worker right away when the wage jumps above his/her productivity. However, then it needs to be considered, in the surplus maximization point of view, whether it is optimal to fire the low productivity workers by jumping to a high wage (high productivity) or to keep the workers by jumping to a low wage (low productivity). The two regimes trade off the retention probability of high productivity workers, and the size of the labor force. In a no-firing contract, firms gain positive profit over high productivity workers, but the workers might choose to quit to a better match. In a firing contract, high types are less mobile and contribute to the surplus, but the surplus from the low type match is lost. It is also possible that the wage jumps twice so that there is an intermediate phase where low productivity p_l is paid before finally jumping up to the high productivity.

3.4. Conclusion

This paper has focused on the treatment of asymmetric information on the topics of on-the-job search in the labor market. Market trading, especially in the labor market, is an area that provides a promising avenue for future research: many people are affected

by the market's organizational efficiency, and asymmetry of information is a real concern. For instance, the market for CEOs and sports athletes seem to have a winner-take-all property. The market greatly rewards a few successful characters, although the risk of such a huge reward is also very well known. An interesting question is to address what are the informational elements that serve as a leverage to extract a huge rent. In many markets with imperfect competition, it is difficult to easily identify the 'productivity' or 'use value' of the good itself, while a large part of the rent comes from small differences. This also includes how much others value the good as well as the intrinsic value itself. I argue that it is important to tackle this question in general since we are living in a world where a large volume of trade is carried out directly between producers and consumers. One application would be online marketplaces which serve as platforms but not intervene in the transaction itself.

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