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LOW THRUST MULTI-REVOLUTION TRANSFER DESIGN USING ARTIFICIAL POTENTIAL GUIDANCE IN THE PHASE SPACE

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ABSTRACT

The semi-analytical treatment of spacecraft's perturbed orbits can be used to interpret and evaluate efficiently its orbit evolution. The present study proposes a semi-analytical methodology to design continuous low thrust multi-revolutions manoeuvres in the mean orbital elements phase space introducing an artificial perturbation formulated as disturbing averaged potential function. The method proposed aims to be computationally efficient both for an onboard implementation of the guidance with high autonomy requirements and for preliminary mission design. The low thrust is modelled in the averaged domain, introducing a pre-defined steering law, which will be tuned over many revolutions to obtain the artificial potential perturbation effects. The potentials introduced to perturb the orbit phase space are expressed in function of constant parameters and the transfer problem is translated into a parameter optimisation problem to shape the artificial potential function. The methodology developed is preliminary applied to the orbit planar dynamics reduced to the equatorial plane perturbed by the Earth's oblateness, studying the conservative transfers for a test case, an heliotropic control of a swarm of satellites and a reconfiguration of an elliptic equatorial constellation with equally spaced oriented apogees.

Keywords: Orbit manoeuvres, semi-analytical methods, phase space, low thrust.

1 INTRODUCTION

Multi-revolutions transfers can be cumbersome to design with the traditional methods used to deal with continuous low thrust. In particular, the optimal control methods are characterised by a substantial computational effort needed to compute the low thrust action along the trajectory. For satellites with limited resources and high autonomy requirements, simpler and computationally cheaper methods are often preferred. In literature the semi-analytic formulations of the dynamics over one orbit revolution have been exploited to simplify the low thrust multi-revolutions transfer problem [1][2][3][4]. For example, in Gao [1], a suboptimal steering law is employed and tuned using a direct optimisation method both for the minimum fuel and minimum time cases in the average domain. Blended control laws have been also investigated, both with direct optimisation methods, Kluever and Oleson [4], or with closed-loop tuning based on the errors with the target state, Huang et al. [3]. Closed-loop guidance schemes using Lyapunov based controls have been also explored [2][5] for the robustness to the external perturbations, uncertainties and errors. The present work develops a methodology for planning the low thrust manoeuvre in the orbital elements averaged perturbed phase space, making an extensive use of semi-analytical methods to reduce the computational effort in the trajectory computation. The method is useful as cheap and efficient algorithm to be implemented onboard a highly autonomous and low resources spacecraft or for preliminary mission design considerations on the low thrust manoeuvre.

2 GUIDANCE MODEL

2.1 Methodology description

The dynamics describing the low thrust action is simplified using the averaging procedure over one orbital revolution.

$$\frac{d\bar{c}_i}{dt} = \frac{n}{2\pi}\int_{E_0}^{E_f} g_i(\bar{\boldsymbol{c}}, \boldsymbol{F}(E))\, dE = G_i(\bar{\boldsymbol{c}}, \boldsymbol{steer}) \qquad \text{with} \qquad g_i(\bar{\boldsymbol{c}}, \boldsymbol{F}(E)) = \frac{d\bar{c}_i}{dE} \tag{1}$$

where the rate $d\bar{c}_i/dE$ is obtained applying to the *i*-th Gauss' equation the approximation on the rate of eccentric anomaly $dE/dt \approx na/r$, valid for small perturbing accelerations relative to the primary body spherical gravity [1]. In this equation *n* is the mean motion, *r* is the osculating position of the spacecraft and *a* the orbit semi-major axis. The orbital elements are considered constant during the integral operation. The $G_i(\bar{\boldsymbol{c}}, \boldsymbol{steer})$ function represents the averaged simplification of the Gauss' equation of the *i*-th orbital element, obtained from the knowledge of the acceleration vector $\boldsymbol{F}(E)$ dependence on the fast variable, i.e. the eccentric anomaly, within one revolution with analytical integration or with quadrature methods. In this work, great care is placed upon the introduction of a steering law, $\boldsymbol{F}(E)$, analytically averageable. The $\boldsymbol{steer}$ variables will represent the parameters that define the steering scheme over one revolution, i.e. thrust angles and burn arcs widths.

The key aspect of this work is to use the low thrust averaged dynamics to follow a target artificial potential function evolution, which is then used to perturb the long-term orbit phase space in the desired fashion. It is well known that the perturbed long-term orbit dynamics can be expressed in the Lagrange's form from the averaged disturbing potential function $\bar{\mathcal{R}}$,. Such formulation highlights the property of the disturbing potential function of being an integral of motion, where the iso-surfaces of the potential function in the orbital elements phase space represent the long-term evolution of the orbit in time. In this work, an additional artificial potential function $\mathcal{R}_{LT}$ is used to act on the perturbed orbital element phase space $\mathcal{R}_{nat}$, inducing different evolutions by modifying the orbit phase space topology.

$$\frac{d\bar{c}_i}{dt} = f_i\left(\bar{\boldsymbol{c}}, \frac{\partial \bar{\mathcal{R}}_{TOT}}{\partial \bar{\boldsymbol{c}}}\right) \qquad \text{with} \qquad \bar{\mathcal{R}}_{TOT} = \bar{\mathcal{R}}_{nat} + \bar{\mathcal{R}}_{LT} \tag{2}$$

In Eq. (2), the f_i functions represent the Lagrange's expression of the perturbed dynamics for the i-th element [6]. To obtain an artificial perturbation on the orbit phase space $\mathcal{R}_{LT}$, its dynamic effects will be introduced with a low thrust steering scheme. The mapping of the time evolution of the low thrust steering scheme dynamics to the one defined by the artificial potential function is reduced to the condition in Eq. (3), equating the averaged dynamics of the artificial potential function with the averaged steering scheme dynamics over one revolution.

$$f_i\left(\bar{\boldsymbol{c}}, \frac{\partial \bar{\mathcal{R}}_{LT}}{\partial \bar{\boldsymbol{c}}}\right) = G_i(\bar{\boldsymbol{c}}, \boldsymbol{steer}) \tag{3}$$

Provided that the artificial potential is known, and the average of the low thrust dynamics can be obtained analytically, the condition in Eq. (3) becomes an algebraic problem to be solved each orbit revolution by finding the proper ***steer*** parameters. The methodology proposed includes the search of an artificial perturbation which allows the fulfilment of an orbit transfer. For this purpose, a candidate nonlinear function $\bar{\mathcal{R}}_{LT}(\boldsymbol{k})$ is introduced, dependent on the control parameters vector $\boldsymbol{k}$. The manoeuvre design is then translated into a parametric optimisation problem which shapes the artificial potential to drive the orbit to the desired condition while minimising a trajectory performance index. As performance index, the cumulated $\overline{\Delta V}$ along the manoeuvre is considered, integrating its averaged rate over one revolution as follows.

$$\frac{d\,\Delta V}{dt} = F \quad \Rightarrow \quad \frac{d\,\overline{\Delta V}}{dt} = \frac{n}{2\pi}\int_{E_0}^{E_f} F(E)\frac{r}{na}dE = \frac{1}{2\pi}\int_{E_0}^{E_f} F(E)\,(1 - e\cos E)dE \tag{4}$$

where e is the orbit eccentricity. The general formulation of the optimisation problem approached with a single shooting, shown in Figure 1, is as follows:

$$min_{\{x\}}\, J \tag{5}$$

$$\text{with: } \boldsymbol{x} = [\boldsymbol{k}, T_f] \text{ and } J = \Delta V \qquad \text{subject to: } \bar{\boldsymbol{c}}_f = \bar{\boldsymbol{c}}_t$$

where the low thrust orbit dynamics propagated inside the cost function and constraint function evaluation are driven by the artificial potential introduced $\bar{\mathcal{R}}_{LT}(\boldsymbol{k})$ by imposing the solution of the algebraic nonlinear problem of Eq. (3). The parametric optimisation problem is solved with a multistart approach, using single and multiple shooting methods. The numerical routine employed is the interior-point algorithm embedded in the MATLAB® built-in function *fmincon.m*. Multiple runs of the optimisation problem are performed to improve the optimality and convergence of the gradient based search, notoriously sensitive to the initial guess. In the multiple shooting approach, the artificial potential function is defined piecewise to improve convergence property and optimality of the phase space trajectory. The time discretisation of the phases is done dynamically in function of the $\boldsymbol{\alpha}$ parameters, according to Eq. (6) where T_i stands for the time at the end of the j-th phase, which are then included among the optimisation variables. In the multi-phase approach, the multiple shooting node guesses of orbital elements, $\bar{\boldsymbol{c}}_g$, are also included in the optimisation variables vector which becomes as Eq. (7), and continuation equality constraints are imposed at nodes for the orbital elements.

$$T_j = \alpha_j(T_f - T_{j-1}) \tag{6}$$

$$\boldsymbol{x} = [\boldsymbol{k}, \boldsymbol{\alpha}, \bar{\boldsymbol{c}}_g, T_f] \tag{7}$$

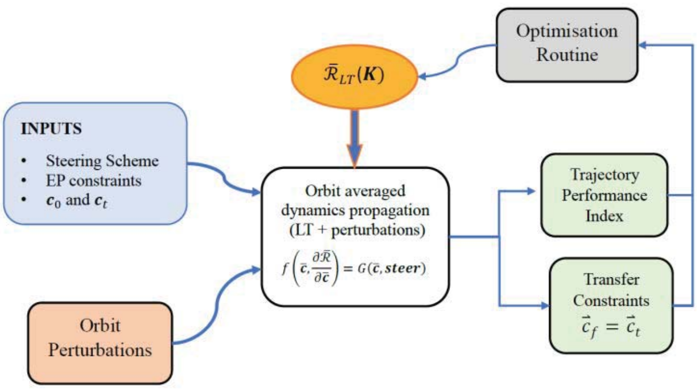


Figure 1: Scheme of the parametric optimisation used for the transfer problem.

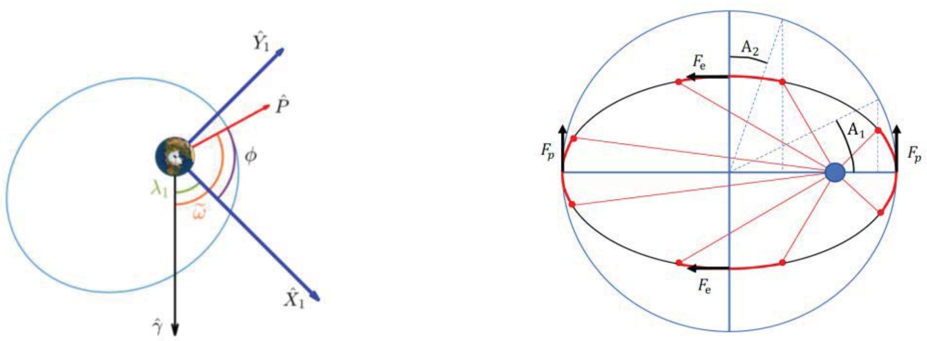
Once the parametric optimisation is solved, the phase space trajectory to reach the target condition corresponds to the iso-surface of the potential function $\bar{\mathcal{R}}_{LT}(\boldsymbol{k})$, which represents the analytic description of the desired orbit long-term dynamics. The phase space trajectory can then be implemented by solving the low thrust mapping condition of Eq. (3) at each revolution, resulting in a cheap and highly autonomous onboard guidance. The methodology is thought to be advantageous for highly perturbed environments where the natural effects can be exploited to reduce the manoeuvre cost. In the present work, the methodology is only applied to a simplified model in a low perturbed environment with the aim of assessing its behaviour.

2.2 Simplified conservative model

The simplified model studied describes the orbit planar dynamics constrained onto the Earth's equatorial plane and subjected only to conservative natural and artificial perturbations. The system is formulated in rotating equatorial reference frame at a constant frequency n_1 with respect to the geocentric inertial equatorial frame. In the orbit model, only conservative perturbation effects are accounted, both describing the the natural perturbations and the artificial control perturbations with low thrust. Therefore, the orbit can be described with two scalar parameters, the eccentricity e and the ϕ angle, which describes the orientation of the orbit pericentre with respect to the rotating frame. The orientation angle is defined in Eq. (8), and it is function of the longitude of perigee $\tilde{\omega}$, defined for equatorial orbits [6], and the frequency of rotation n_1.
According to the methodology presented, a low thrust steering scheme should be introduced to act on the orbit long-term dynamics. The selected steering scheme uses a sequence of burn and coasting arcs to obtain a null effect on the semimajor axis and the desired effect on the magnitude and orientation of the eccentricity vector on the equatorial plane. In this work the simplifications of constant thrust acceleration magnitude and direction within the burn arcs is considered, which allows the analytical simplifications in the average procedure of Eq. (1).

Using symmetric arcs, the conservative condition can be fulfilled, while studies on the suboptimality of the action on the e and $\tilde{\omega}$ discriminated the decision of location and thrust angles within the burn arcs [1]. The proposed conservative steering features a pair of symmetric arcs centred at periapsis/apoapsis and one at E = {90°, 270°} locations, with an "inertial" control acceleration directed respectively perpendicular and parallel to the semi-major axis, as shown in Figure 2b. It is worth noticing that the possibility of other steering schemes to act on the planar conservative dynamics is possible, but in this work the sensitivity of the guidance

$$\phi = \tilde{\omega} - \lambda_1 \qquad \text{with} \qquad \lambda_1 = n_1 t \tag{8}$$



(a) (b)

Figure 2: (a) Rotating reference frame. (b) Conservative inertial low thrust steering scheme.

with respect to different steering scheme definition is not presented. The averaged dynamics of the steering law, computed analytically applying Eq. (1), is reported in Eq. (9).

$$\begin{cases} \frac{de}{dt} = \frac{\sqrt{1-e^2}}{na\pi}(3A_1 + \sin A_1 \cos A_1)F_{p1} \\ \frac{d\tilde{\omega}}{dt} = -\frac{\sqrt{1-e^2}}{nae\pi}(3A_2 + \sin A_2 \cos A_2)F_{e2} \end{cases} \tag{9}$$

where A_1 and A_2 are the half arcs widths shown in Figure 2b, while F_{p1} and F_{e2} are the constant magnitudes of the thrust accelerations involved in the steering scheme.

The class of artificial potentials to perturb the phase space are introduced in a simple manner considering the following nonlinear parametrised function:

$$\bar{\mathcal{R}}_{LT} = a\, k_1 e \cos(\phi + \gamma) + a\, f(\boldsymbol{k}_e, e) \qquad \text{where} \quad f(\boldsymbol{k}_e, e) = k_2 e \tag{10}$$

where the eccentricity function is preliminary considered only as a linear term. The simplicity of the nonlinear function has been favoured at this early stage of research. The control parameters K_1, K_2 and γ univocally define a function topology of the phase space, together with the natural perturbing effects. The natural perturbations on the orbital elements accounted in this work on the orbit are the J_2 effect due the Earth's oblateness and the frame rotation contribution. The total perturbing potential, reported in Eq. (11) results in the dynamics expressed in Eq. (12) applying Eq. (2).

$$\bar{\mathcal{R}}_{nat} = \bar{\mathcal{R}}_{J_2} + \bar{\mathcal{R}}_{n_1} = \frac{n^2 R_\oplus^2 J_2}{2\,(1-e^2)^{3/2}} + na^2 n_1 \sqrt{1-e^2} \tag{11}$$

$$\begin{cases} \frac{de}{dt} = \frac{\sqrt{1-e^2}}{na} k_1 \sin(\phi + \gamma) \\ \frac{d\phi}{dt} = \frac{\sqrt{1-e^2}}{nae}(k_1 \cos(\phi+\gamma) + k_2) + \frac{3}{2}\frac{R_\oplus^2 J_2\, n}{a^2(1-e^2)^2} - n_1 \end{cases} \tag{12}$$

Using the steering law and potential dynamics introduced, the low thrust mapping condition of Eq. (3) reduces to:

$$\begin{cases} (3A_1 + \sin A_1 \cos A_1)F_{p1} = \pi K_1 \sin(\phi + \gamma) \\ (3A_2 + \sin A_2 \cos A_2)F_{e2} = -\pi(K_1 \cos(\phi+\gamma) + K_2) \end{cases} \tag{13}$$

The latter algebraic problem is solved determining the A_1 and A_2 half arcs widths at each orbit evolution along the propagation of the orbit. The constant acceleration magnitude is taken equal to $5 \times 10^{-4}\ m/s^2$, where F_{p1} and F_{e2} are free to assume positive and negative values according to the guiding potential rates required.

3 CONSTRVATIVE TRANSFERS RESULTS

3.1 Test case

The methodology is applied to a transfer test case in the planar equatorial conservative (e, ϕ) phase space, with initial and target conditions reported in Table 1. The rotating reference frame frequency is considered equal to the apparent mean motion of the Sun on the ecliptic n_{Sun}. The rotating frame will then keep approximately a constant orientation with respect to the Sun direction, neglecting the obliquity of the ecliptic. Initial conditions are taken considering the Sun at the spring equinox, and the frame first axis aligned in such direction. Figure 3 shows the phase space trajectory computed with the single and multiple shooting methods of shaping artificial potential perturbation, together with the total potential function isolines computed from the single shooting solution. It can be noticed, from Figure 3, how the multiple shooting algorithm switches between different isolines, while the single shooting phase space trajectory follows a single isoline from initial to target condition defined by the $\bar{\mathcal{R}}_{LT}(\boldsymbol{k})$ integral of motion. Grey shaded area corresponds to critical eccentricity values that results in a perigee altitude below 400 km above the Earth's surface. The piecewise trajectory computed with multiple shooting has been discretised in four phases, and the upper boundary of manoeuvre time is set to half a year for both shooting methods. From Table 1 the improvement of the multiple shooting approach to the optimality of the phase space trajectory in term of performance index is evident. As expected, the capability of defining a piecewise phase space isoline with multiple shooting overcomes the limitation of the finite class of disturbing functions topology defined which may not be able to drive the system for the whole phase space trajectory efficiently. The CPU time for 5 and 10 runs respectively of the multiple and single shooting parametric optimisation problems are reported in Table 1, where two Intel(R) Core(TM) i7-5500U CPU@ 2.40 GHz are used in parallel for the computations. The transfer problem is also solved considering various upper limits for the total manoeuvre time, and the solutions in terms of ΔV and manoeuvre time used are shown in Figure 4.

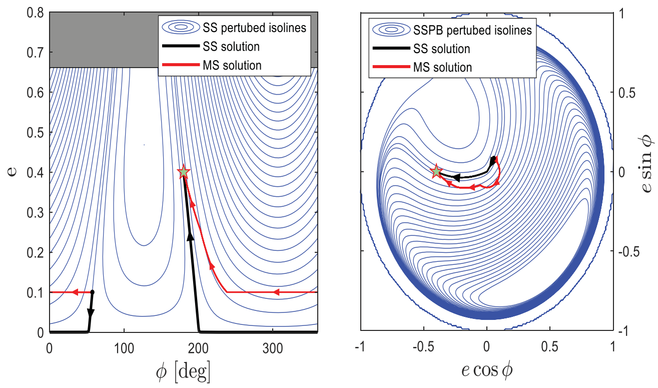


Figure 3: Phase space trajectories of test case transfer.

Initial Conditions	Target Conditions
$a = 20000$ km	$a = 20000$ km
$e = 0.1$	$e = 0.4$
$\phi = 57.3°$	$\phi = 180°$
Performance	
$\Delta V_{SS} = 1.41$ km/s	
$\Delta V_{MS} = 1.15$ km/s	
$T\_CPU_{ss} = 24.1$ min	
$T\_CPU_{MS} = 94.7$ min	

Table 1: Test case transfer conditions and performance.

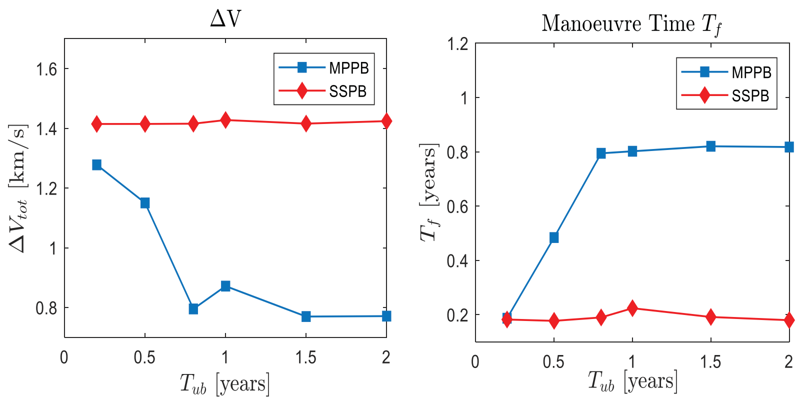


Figure 4: Test case transfer problem solutions for different upper limit of manoeuvre times.

The solutions are influenced by the manoeuvre time limit and show an incremental improvement in the minimisation of the performance index for the solutions obtained with upper time limit below 0.8 years, reported in Figure 4. This characteristic is associated with a greater exploitation of the natural regression on the ϕ element to minimise the low thrust control action using the whole manoeuvre time available. The plateau region corresponds to the situation in which the natural regression of ϕ will drive the orbit orientation passed the desired condition for longer manoeuvre times.

3.2 Heliotropic swarm control

The conservative phase space shaping method for low thrust manoeuvres is here applied to the control of multiple spacecraft starting from different conditions in the (e, ϕ) phase space to the same eccentric heliotropic orbit on the equatorial plane. Stable heliotropic orbits maintain the apogee orientation fixed towards to the Sun direction, guaranteeing a useful coverage of the Earth's surface at the same local hour being advantageous for telecommunications and Earth observations missions [7] [8]. The same approximation done in the transfer test case for the Sun direction on the equator is used. The target heliotropic orbit is defined fixing the eccentricity and retrieving the semi-major axis from the frozen condition with respect to the J_2 effect in Eq. (14), obtained setting to zero the natural rate of ϕ in Eq. (12). The transfer problem is solved with the multiple shooting method using four phases and upper manoeuvre time limit of 0.5 years. The phase space trajectories for a grid of initial conditions are shown in Figure 5, where the colormap represents the required ΔV of each trajectory.

3.3 Elliptic constellation configuration

At last, the conservative guidance is applied to a reconfiguration of an elliptic equatorial constellation with equally spaced apogees from the same injection orbit used to provide enhanced coverage for equatorial countries, [9]. In this case the frequency of rotation of the reference frame is determined from the target orbits eccentricity and semimajor axis, imposing the frozen condition with respect to J_2 of Eq. (15). In such a way, regardless of the time needed to perform the transfer for each satellite, the perigee equispaced condition is guaranteed for a simultaneous control of the whole constellation. Initial and target conditions for the constellation are reported in Table 3, while the phase space trajectories computed with a four phases multiple shooting approach are shown in Figure 6, with an upper limit of manoeuvre time set equal to 0.5 years.

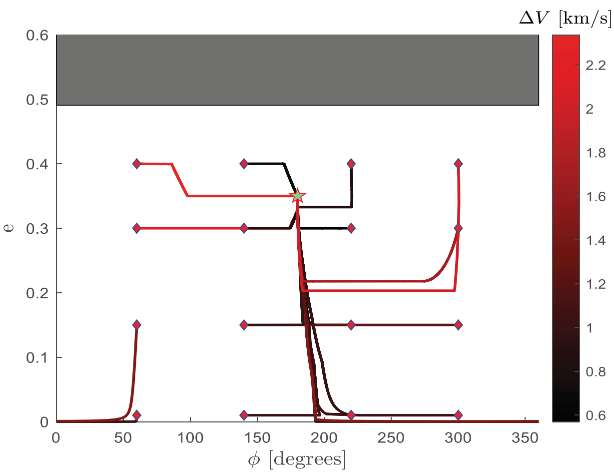


Figure 5: Phase space trajectories of the heliotropic control of multiple spacecraft.

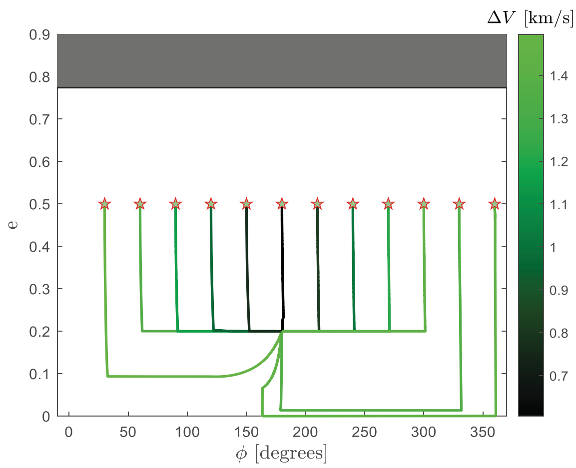


Figure 6: Phase space trajectories of the constellation reconfiguration solutions.

$$a = \left(\frac{3}{2}\frac{J_2 R_\oplus^2 \sqrt{\mu}}{n_{Sun}(1-e^2)^2}\right)^{2/7} \quad (14)$$

$$n_1 = \frac{3}{2}\frac{J_2 R_\oplus^2 \sqrt{\mu}}{a^{\frac{7}{2}}(1-e^2)^2} \quad (15)$$

Target Heliotropic Orbit
$a = 13310$ km
$e = 0.35$
$\phi = 180°$

Table 2: Target heliotropic orbit.

Elliptic Constellation Conditions (N=12 satellites)	
$a = 30000$ km	$n_1 = 1.58 \times 10^{-8}$ rad/s
$e_0 = 0.2$	$e_t = 0.5$
$\phi_0 = 180°$	$\phi_{t,i} = i\, 360°/N$

Table 3: Elliptic constellation initial and target orbits.

4 CONLCUSION

A methodology for designing orbit transfer perturbing the averaged perturbed orbit phase space was defined in this work, using semi-analytical techniques. The method can be the basis for an onboard autonomous guidance implementation or can be used for mission preliminary studies, thanks to its computational efficiency. The applications to a simplified orbit model of conservative planar orbit transfer on the equatorial plane are studied in this paper. Possible future developments will address the extension to the 3D non-conservative orbit dynamics, together with the applications in higher perturbed environments to better exploit the natural perturbing effects reducing the low thrust manoeuvre cost.

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