SOLIDS OF REVOLUTION

Procedures for tangencies

Joaquín Fernández
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Translation review and voice: Alba Ramos Cabal

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

ETSEIB

Contents

INTRODUCTION

Legend

In this document the following colour code has been used to differentiate the data (what is known), the operations (procedures that must be executed to obtain the result) and the solutions (what is sought):

Black or White	DATA
Blue	OPERATIONS
Red	SOLUTIONS
SDW	ACCESS TO SOLIDWORKS SOLUTION FILE
STEP	ACCESS TO SOLUTION FILE IN STEP FORMAT
VIDEO	ACCESS TO THE VIDEO RECORDING OF THE PROCEDURE

Instructions for reading the graphics:

BLACK COLOR = Fixed elements (they do not move or transform).
RED COLOR = Variable elements (those that modify their position in the space after the data is entered based on the fixed elements).
BLUE COLOR = Elements that contain the construction data.

Tangency between solids of Revolution

General concept

Tangency

Two bodies of revolution are tangent when they share the same tangent plane and a point.

VIDEO

Bi-tangency

Two bodies of revolution are bi-tangent when they are tangent and their axes are cut.

Methods for resolving tangency relations

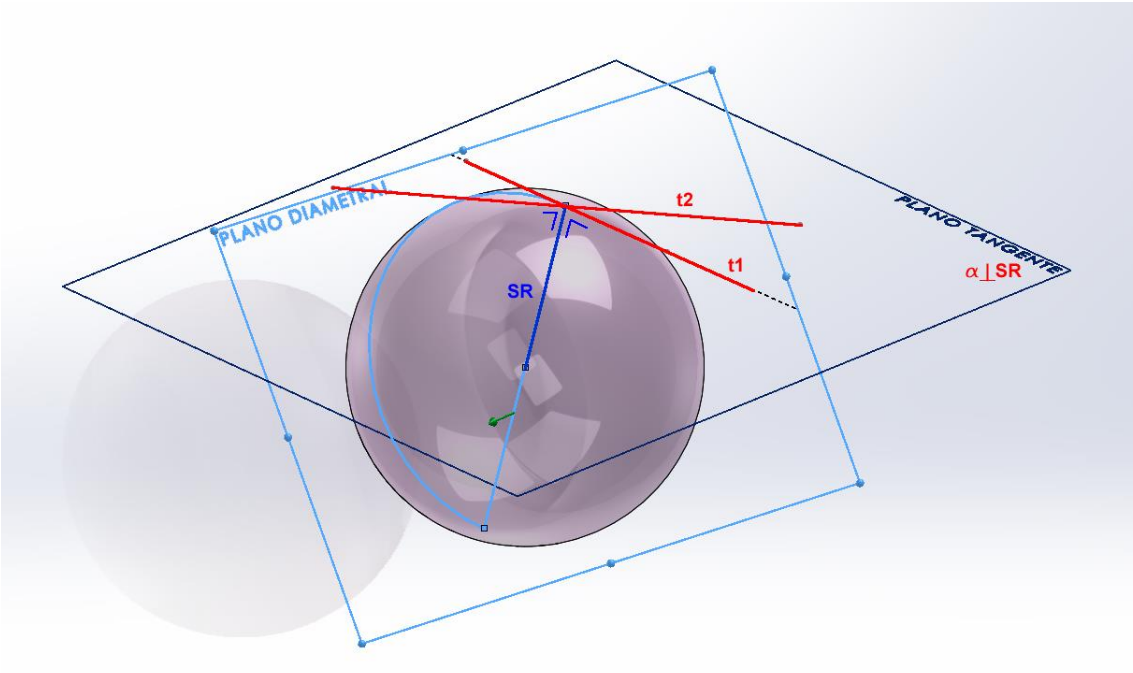
There are different methods to solve tangency relations. The method used will depend on the initial conditions.

The methods used in this document are:

1. SUBSTITUTION method. It is based on the following sequence of substitutions:
    a. Substitution of the solid by an inscribed sphere.
    b. Substitution of the sphere by a plane tangent to it and a point (the point of tangency must be on the surface of the solid to be replaced).
2. PROJECTION method. It is applied in case the direction of the axis of a CYLINDER is known. The result of the tangency is visible in the projection of the assembly in a plane perpendicular to the direction of the axis of the cylinder.
3. SECTION method. It is applied when the axes of the solids intersect. The tangency occurs in the plane defined by the axes of revolution.
4. EXTENSION method. It is applied when the radius of the generating sphere of the solid is known. The area of space where the centres of the spheres that generate the solid will be delimited by the expansion of the other solids in the radius of the generating sphere.
5. APPROXIMATION method by RELATIONS between solids. It is applied using the relations between the parts that are provided by the CAD software. It partially solves the problem facilitating its modelling. Then, it is necessary to continue with the resolution by using at least one of the first 4 methods.

TANGENT PLANE TO A SOLID OF REVOLUTION

27 Plane tangent to a sphere



GRAPHIC LEGEND

PLANO DIAMETRAL = DIAMETRAL PLANE

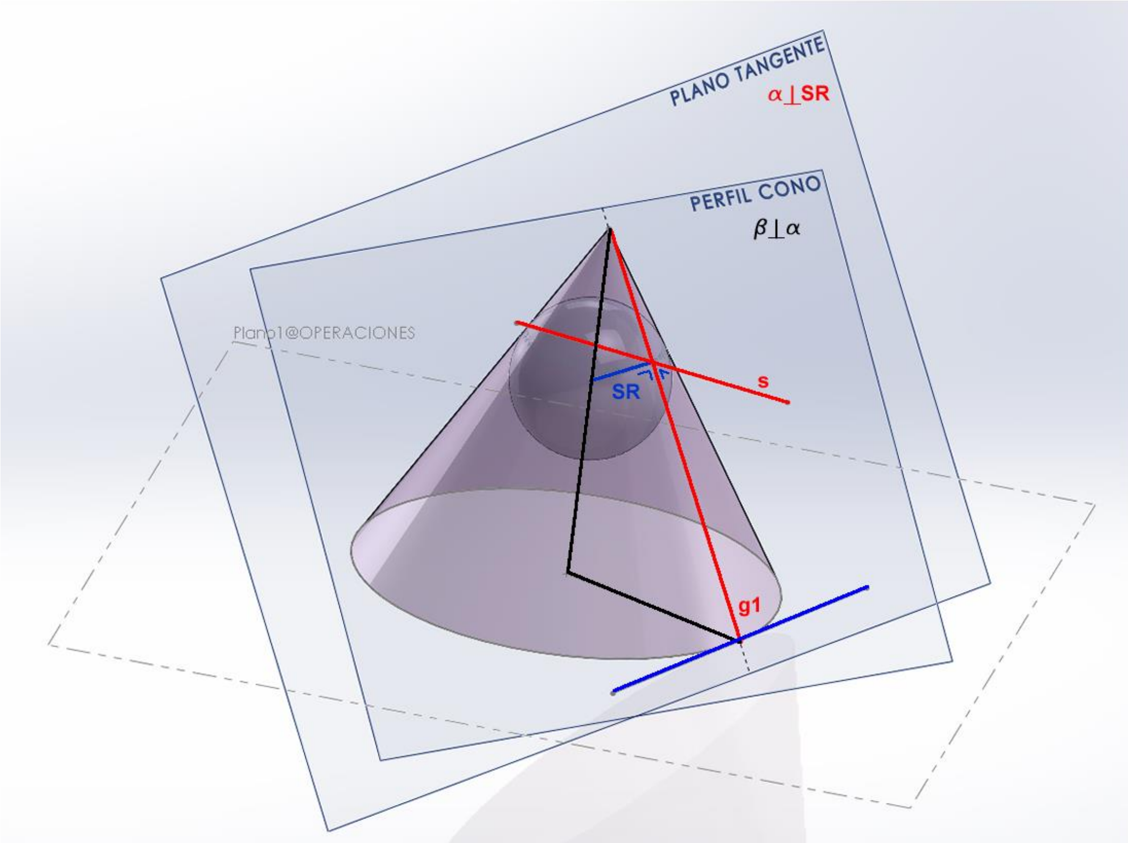
PLANO TANGENTE = TANGENT PLANE

SDW | VIDEO

Construction:

1. Draw a sphere radius **SR.**
2. Create plane **α** perpendicular to **SR** (the plane could be defined by two lines **t1** and **t2**. In this case **SR** must be perpendicular to each one of them).

28 Plane tangent to a cone



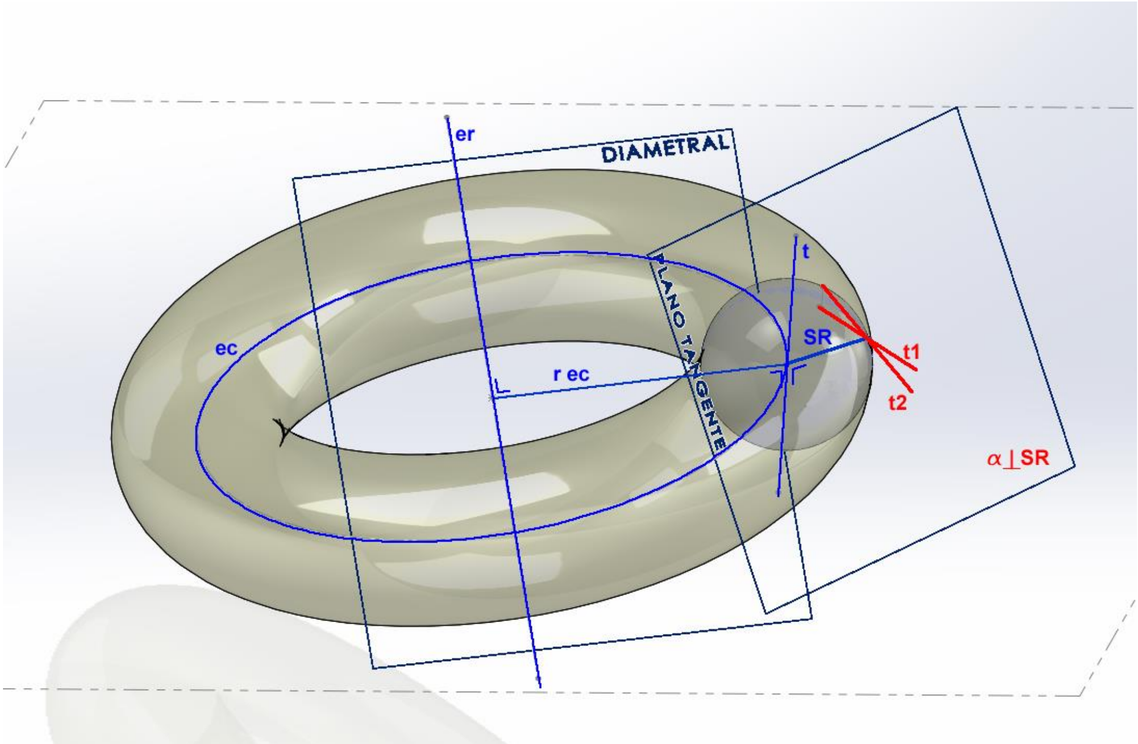
GRAPHIC LEGEND
PERFIL CONO = CONE PROFILE
PLANO TANGENTE = TANGENT PLANE

SDW | VIDEO

Construction:

1. Draw a sphere radius **SR** perpendicular to the generatrix **g1.**
2. Create plane **α** perpendicular to **SR** (the plane could be defined by **g1** and any other line **s**. In this case **SR** must be perpendicular to each one of them).

29 Plane tangent to a torus



GRAPHIC LEGEND
DIAMETRAL = diametral plane
PLANO TANGENTE = TANGENT PLANE

SDW | VIDEO

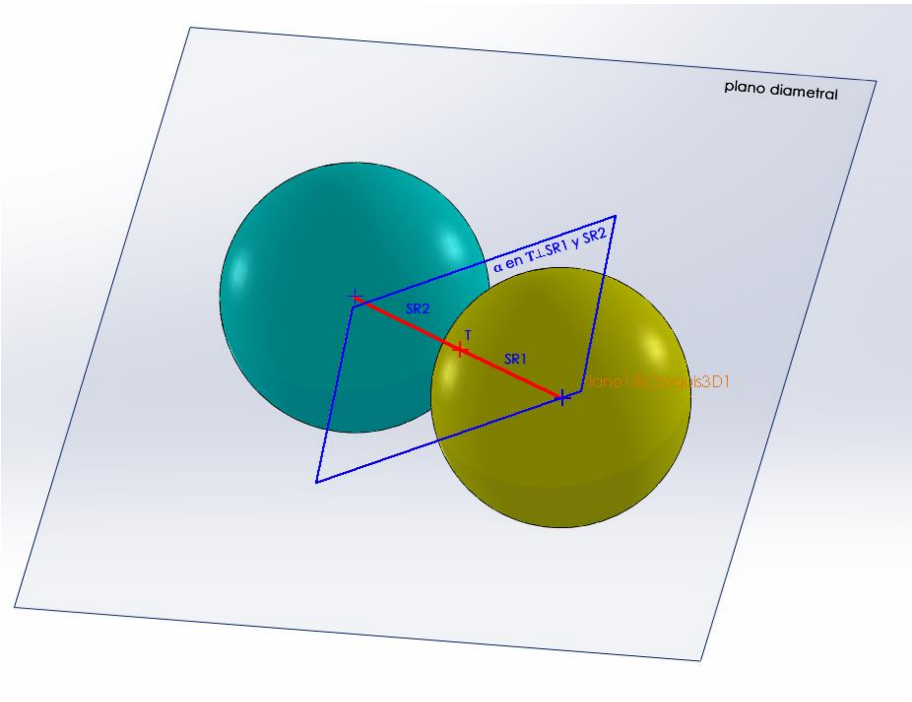
Procedure:

1. Draw a sphere radius **SR.**
2. Create a diametral plane containing **SR** and **er.**
3. Create the plane **α** perpendicular to **SR** (this plane should be defined by two lines **t1** and **t2**. In this case **SR** must be perpendicular to each one of them).

METHOD 1: SUSTITUTION

SPHERE

30 Tangency between spheres



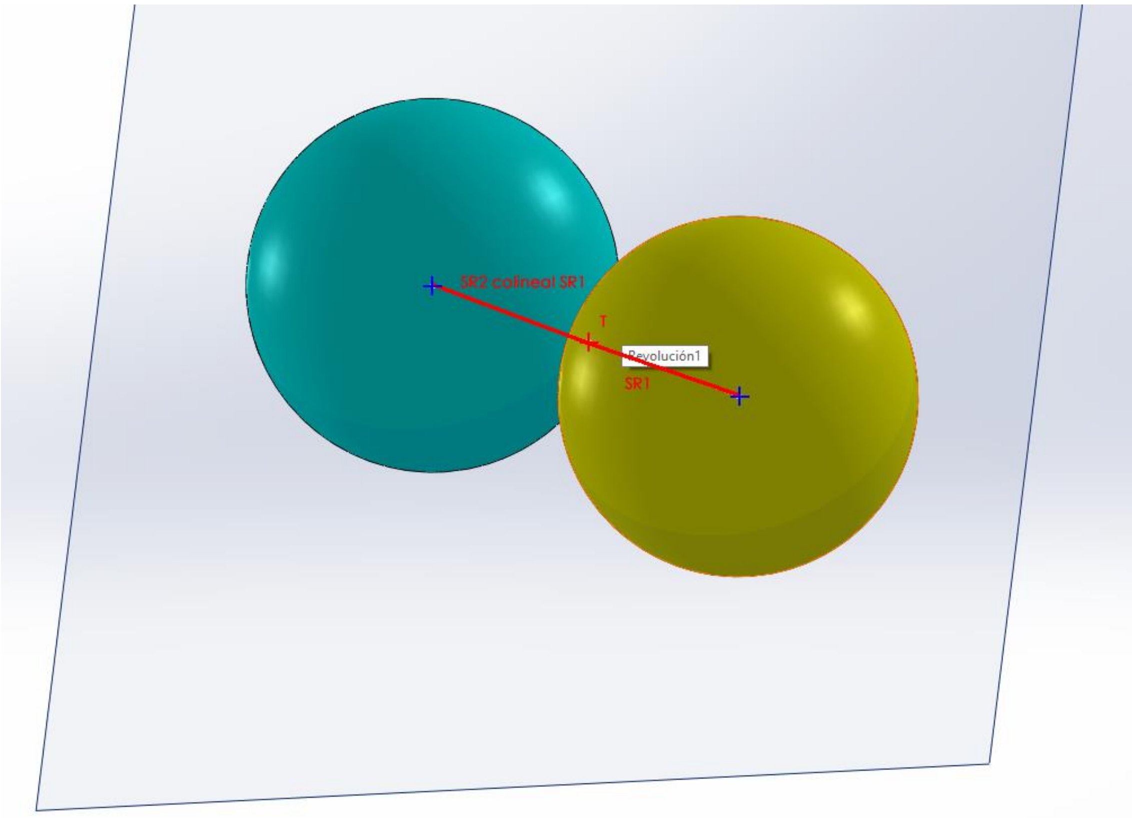
GRAPHIC LEGEND
plano diametral = diametral plane

SDW

Procedure:

1. Draw the radius **SR1** and **SR2** being coincident at **T**.
2. Create the plane α perpendicular to **SR1** and **SR2**.

Simplification without a tangent plane



GRAPHIC LEGEND
colineal = collinear

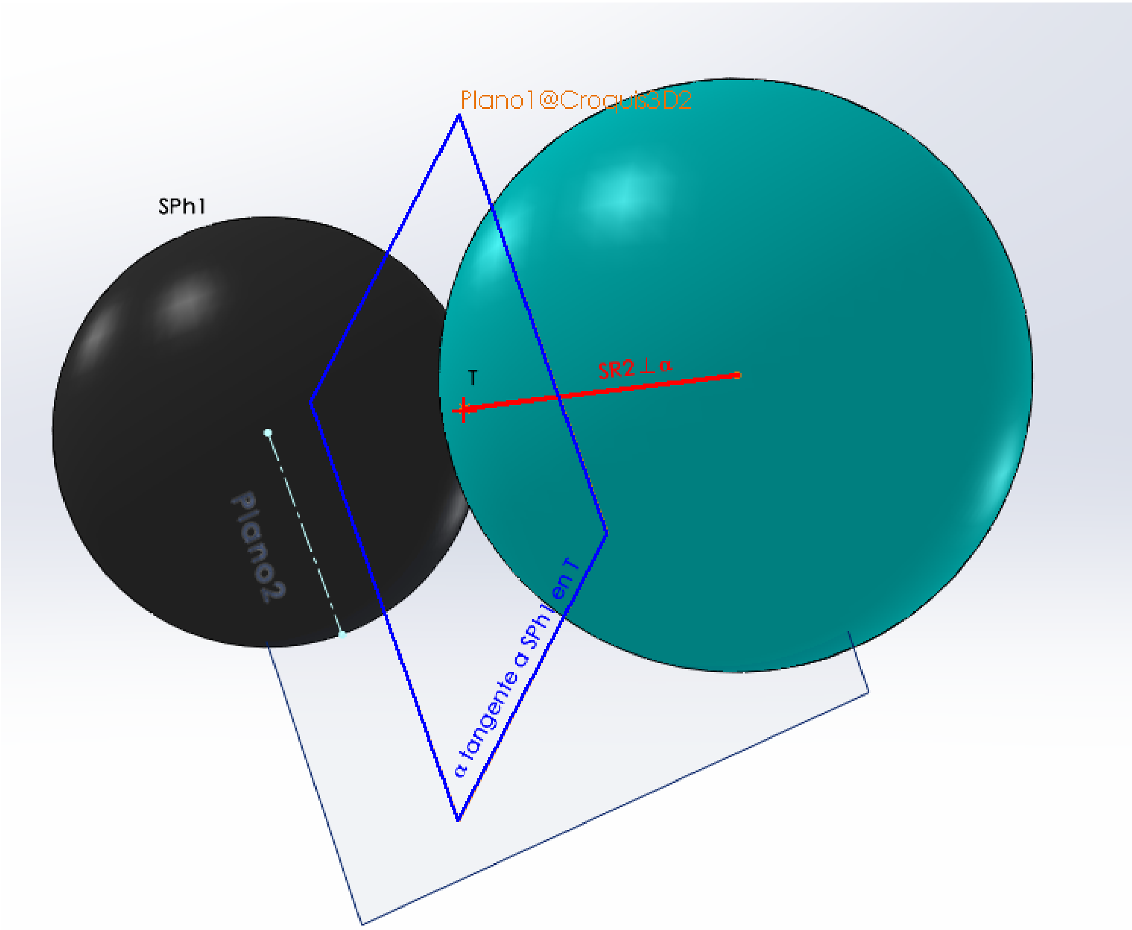
SDW

Procedure:

1. Draw the radius **SR1** and **SR2**, being collinear and sharing point **T**.

31 Simplification with a solid sphere

V1 with tangent plane



GRAPHIC LEGEND

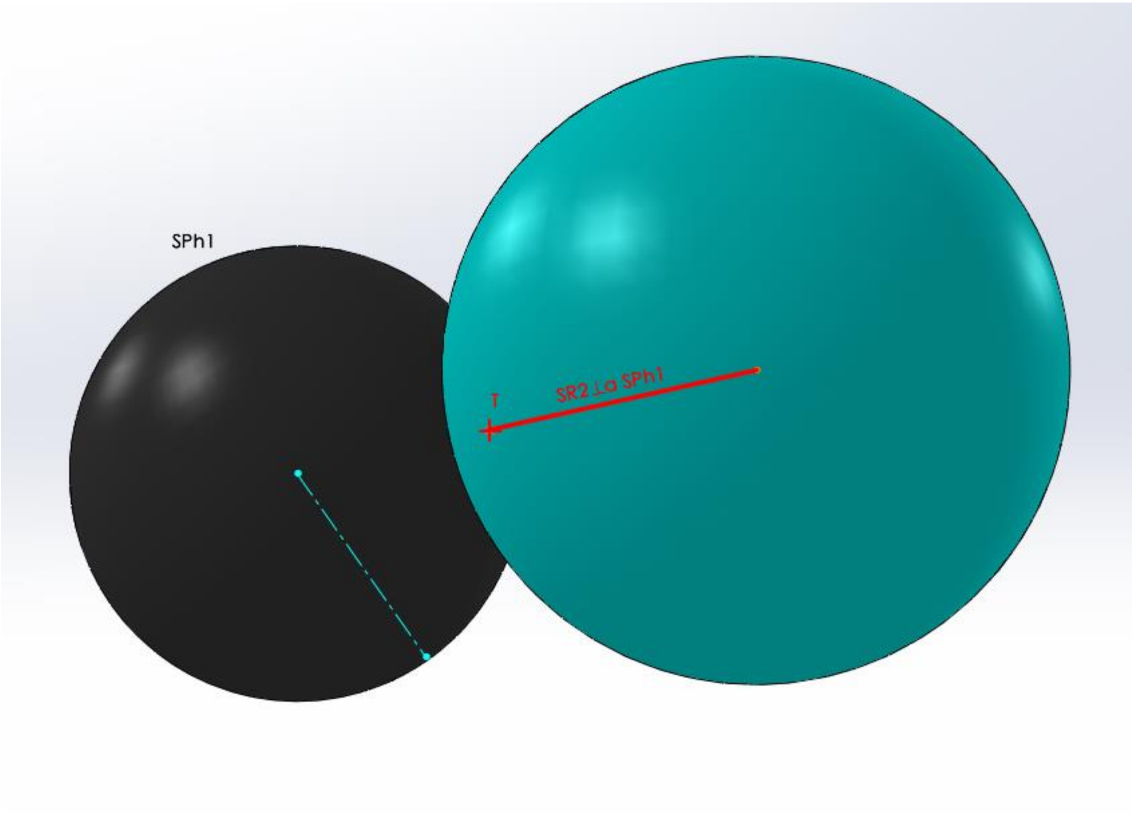
tangente a = tangent to

SDW

Procedure:

1. Create the point **T** on the Surface of the solid sphere **Sph1.**
2. Create the plane α tangent to the sphere **Sph1** at point **T**.
3. Draw the sphere radius **SR2** ending at point **T** and perpendicular to the plane α.

V2 normal to a surface



SDW

Procedure:

1. Create the point **T** on the surface of the solid sphere **Sph1.**
2. Draw the sphere radius **SR2** ending at point **T** and normal to **Sph1.**

V3 alignment with the centre

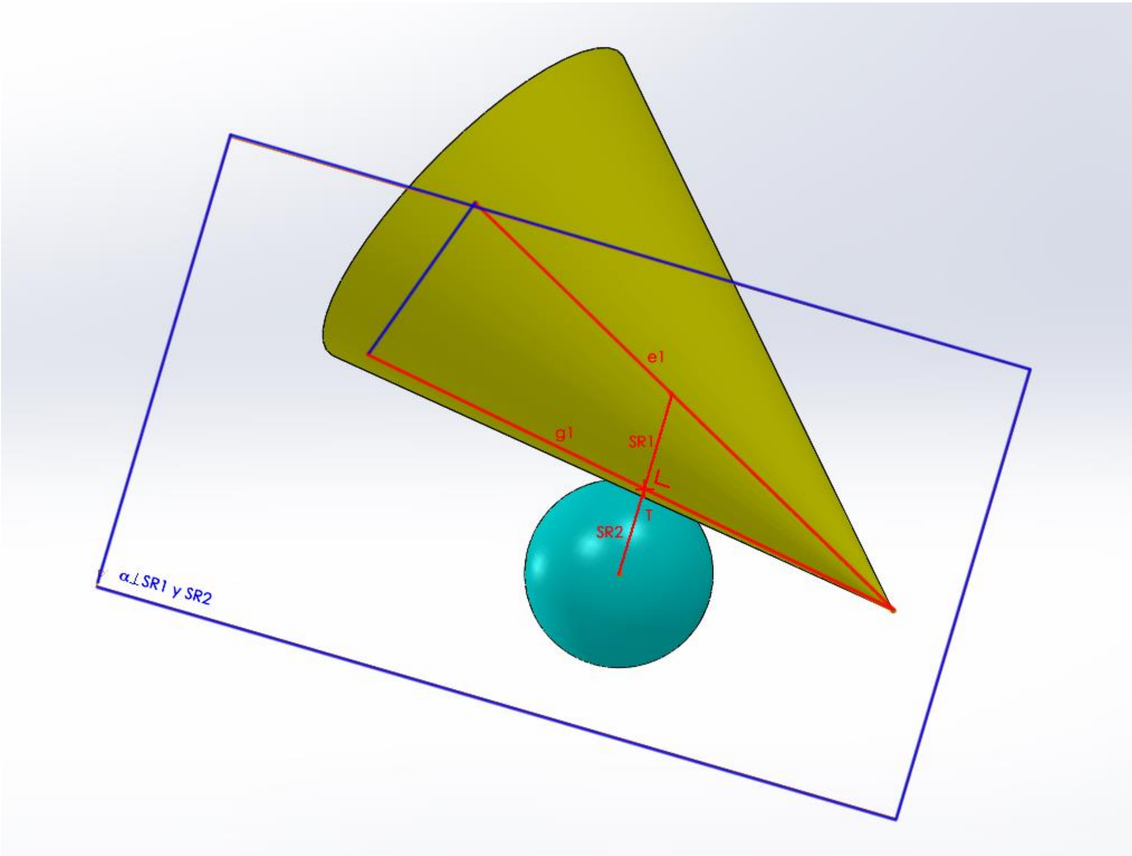
GRAPHIC LEGEND
coincidente con = coincident with

Procedure:

1. Draw point **T** on the surface of the solid sphere **Sph1.**
2. Draw the sphere radius **SR2** ending at point **T** and coincident with the centre **O1**.

CONE

32 Tangency between a cone and sphere

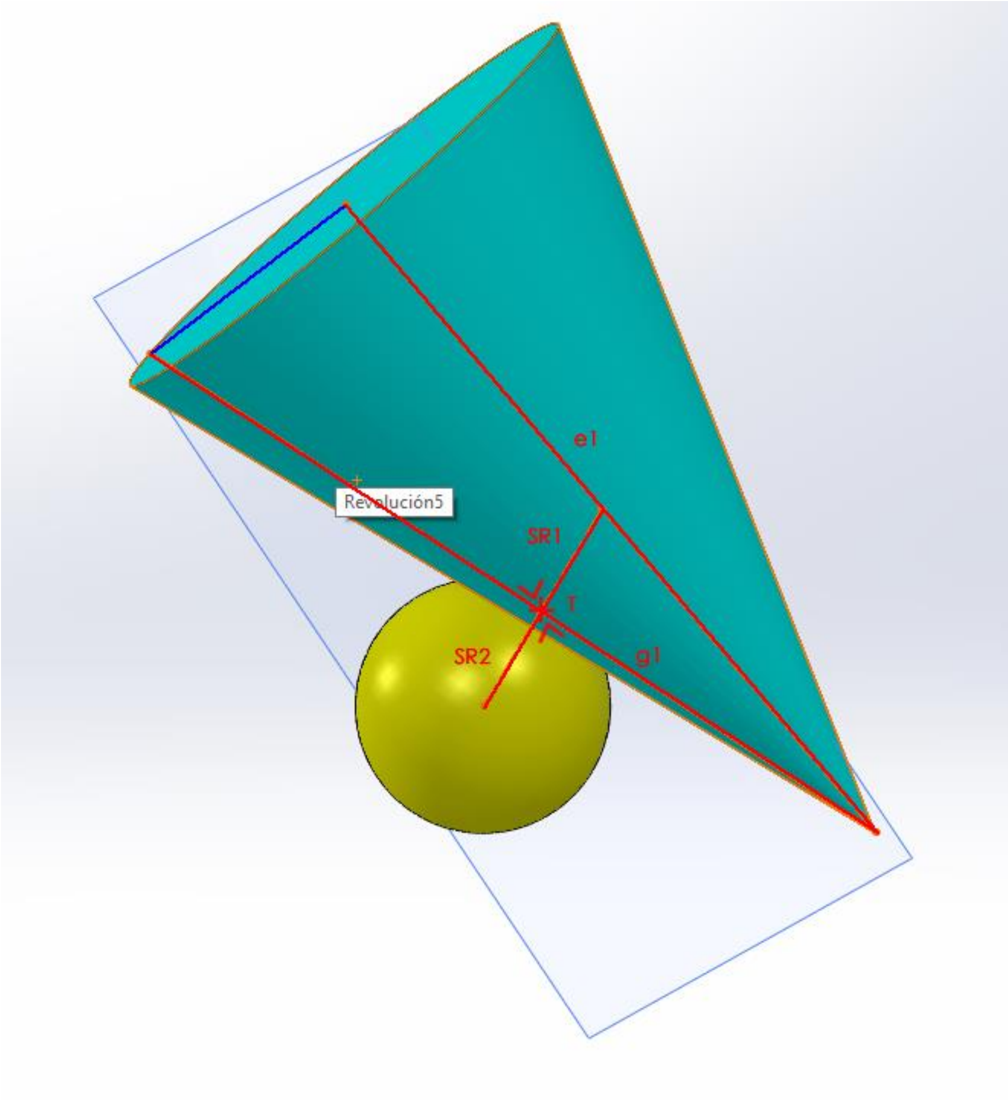


SDW

Procedure:

1. Draw the sphere radius **SR1** perpendicular to the generatrix **g1** at point **T.**
2. Create the plane α containing point **T** and being perpendicular to **SR1.**
3. Draw the sphere radius **SR2** ending at point **T** and perpendicular to the plane α.

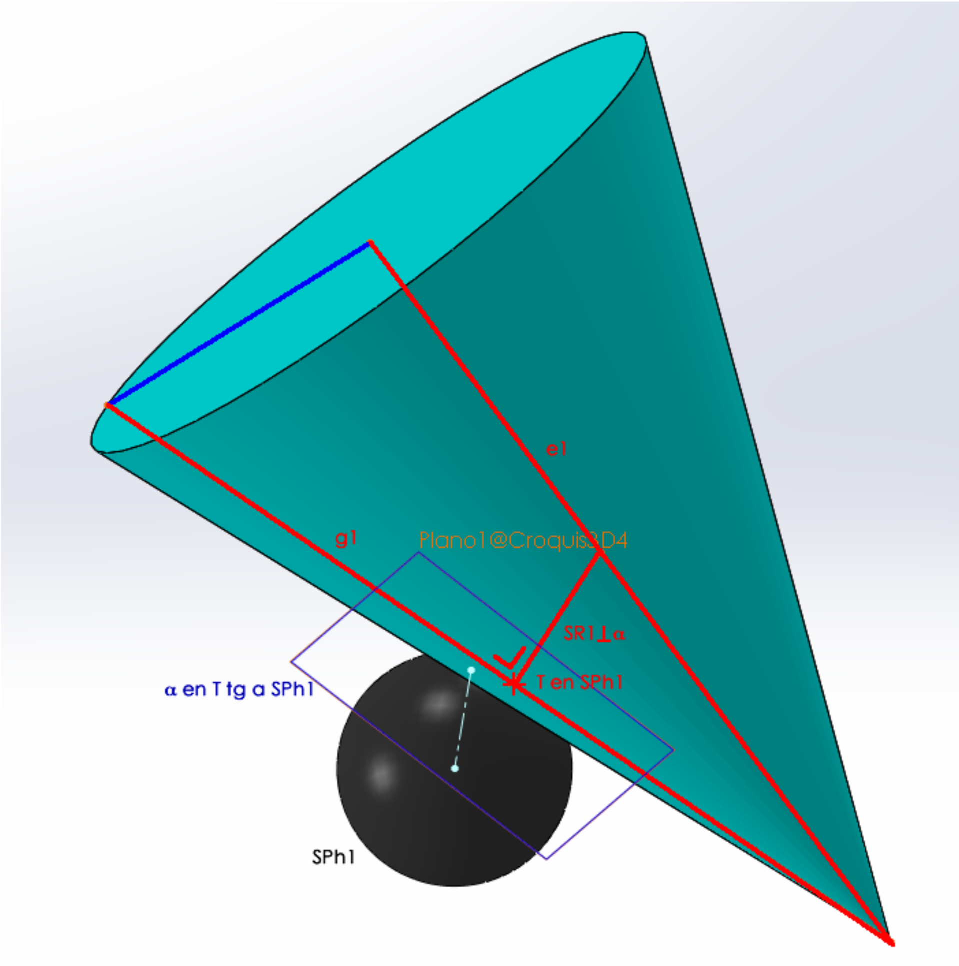
Simplification without a tangent plane



Procedure:

1. Draw the sphere radius **SR1** perpendicular to the generatrix **g1** at point **T.**
2. Draw the sphere radius **SR2** ending at point **T** and being collinear with **SR1**.

33 Simplification with a solid sphere

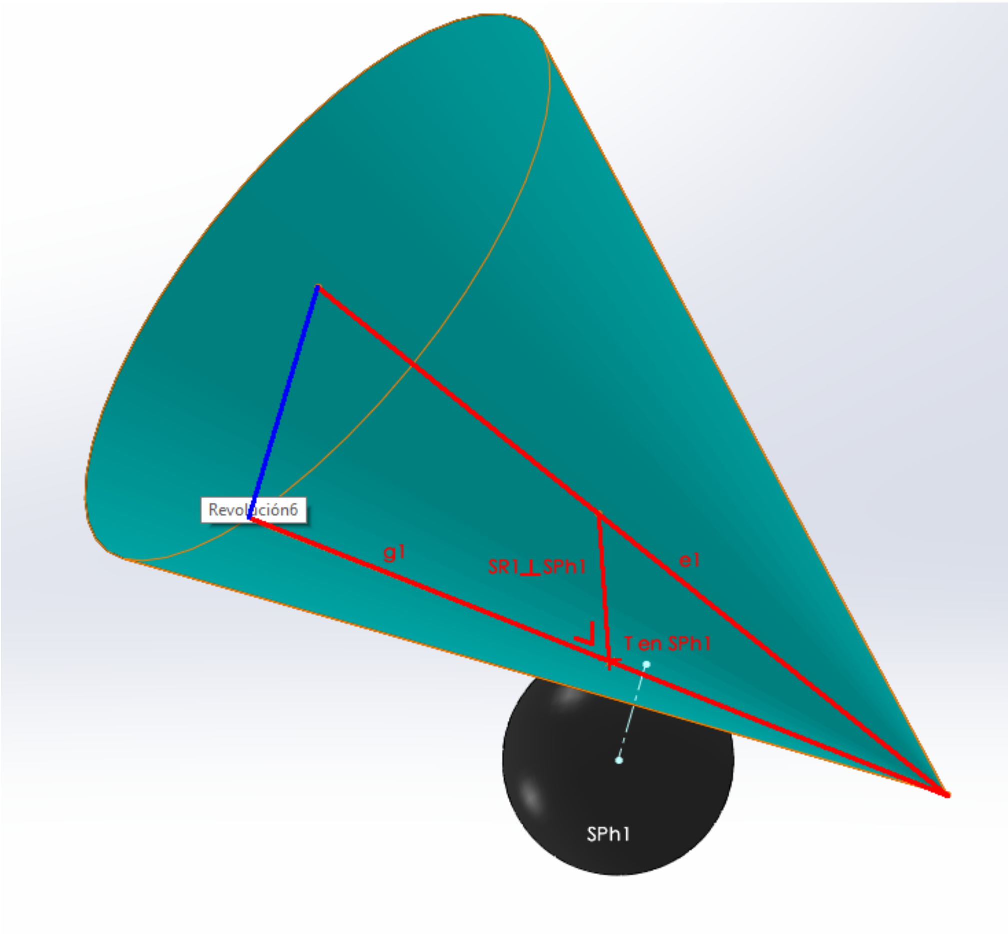


SDW

Procedure:

1. Create the point **T** on the surface of the solid sphere **Sph1.**
2. Create the plane **α** tangent to the sphere **Sph1** at point **T**.
3. Draw the sphere radius **SR1** perpendicular to the generatrix **g1** at point **T** and perpendicular to plane **α.**

Without a tangent plane

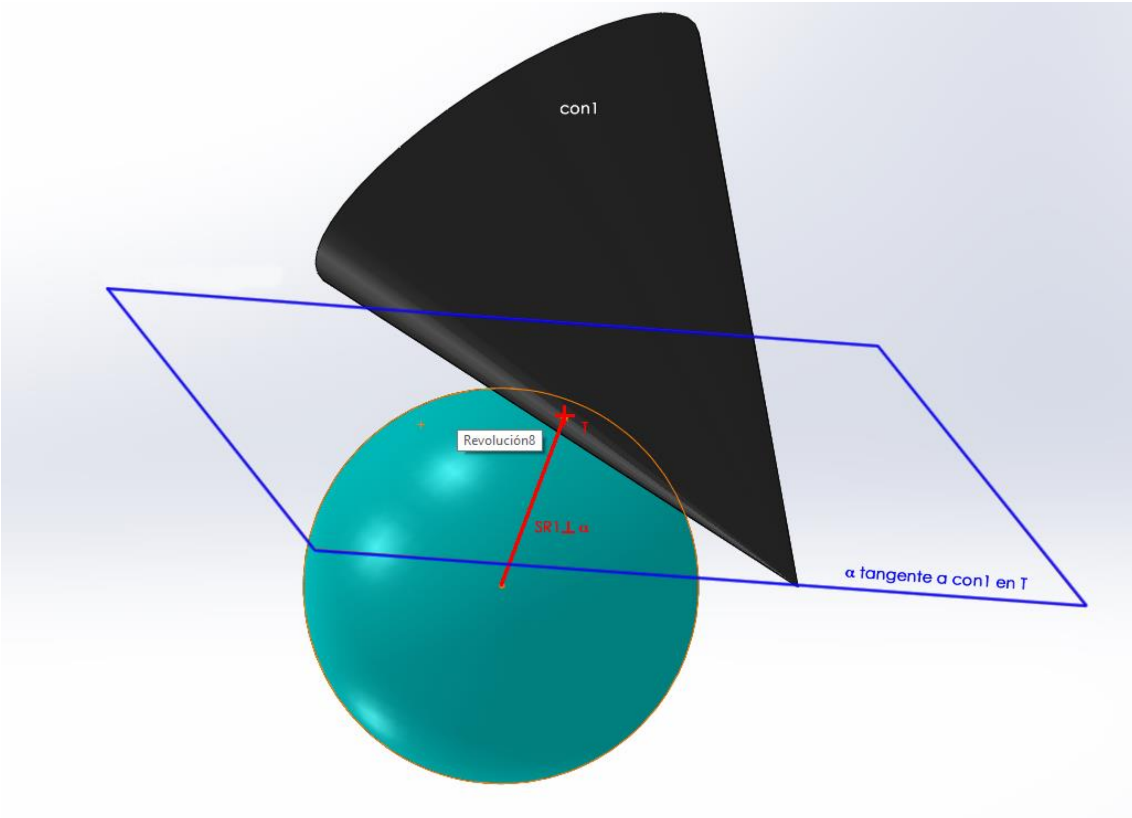


SDW

Procedure:

1. Create the point **T** on the surface of the solid sphere **Sph1.**
2. Draw the sphere radius **SR1** perpendicular to the generatrix **g1** at point **T** and normal to the surface of **Sph1**.

34 Simplification with a solid cone



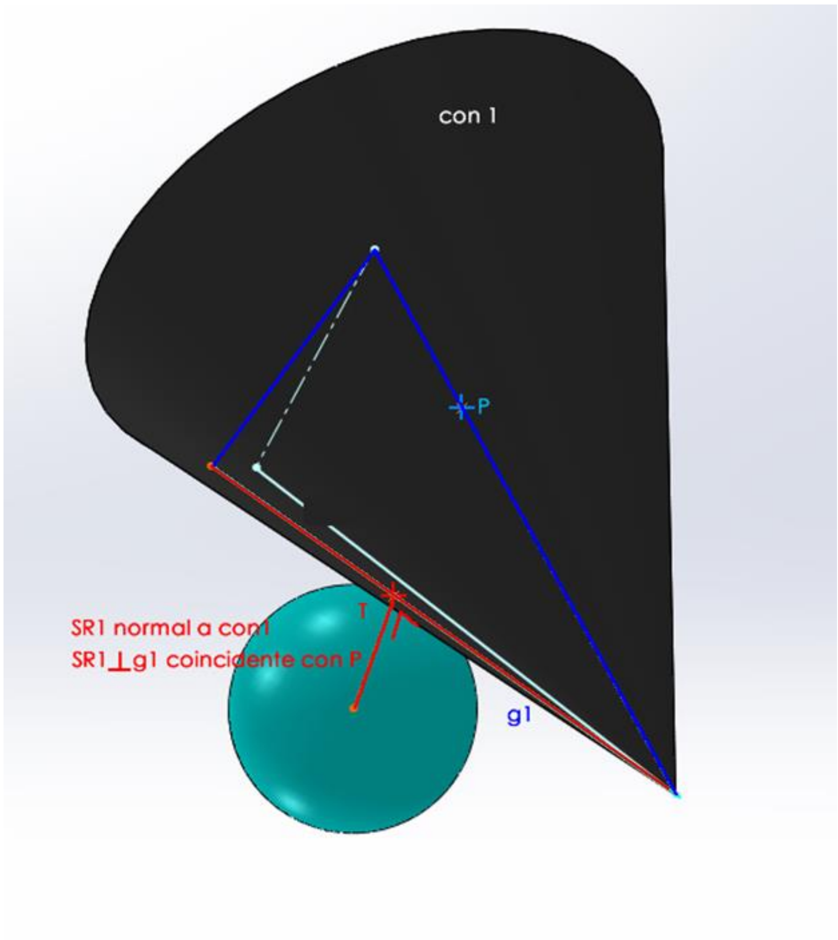
GRAPHIC LEGEND
tangente a = tangent to

SDW

Procedure:

1. Create the point **T** on the surface of the solid cone **con1.**
2. Create the plane α at point **T** and tangent to **con1.**
3. Draw the sphere radius **SR1** ending at point **T** and being perpendicular to **α.**

Without tangent plane



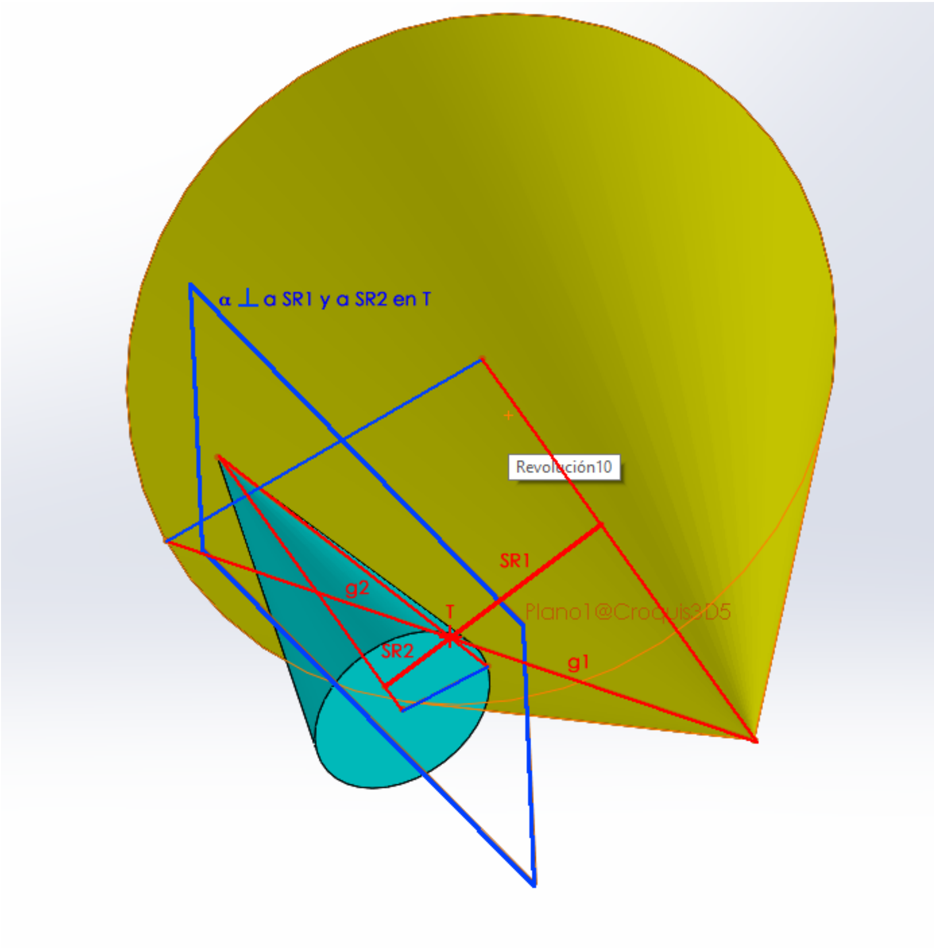
GRAPHIC LEGEND
normal a con1 = normal to con1
coincidente con P = coincident with P

SDW

Procedure:

1. Create the point **T** on the surface of the solid cone **con1.**
2. Option 1: Draw the sphere radius **SR1** ending at point **T** and perpendicular to **con1.**
3. Option 2: Draw the sphere radius **SR1** ending at point **T,** perpendicular to **g1** and coincident with point **P.**

35 Tangency between cones

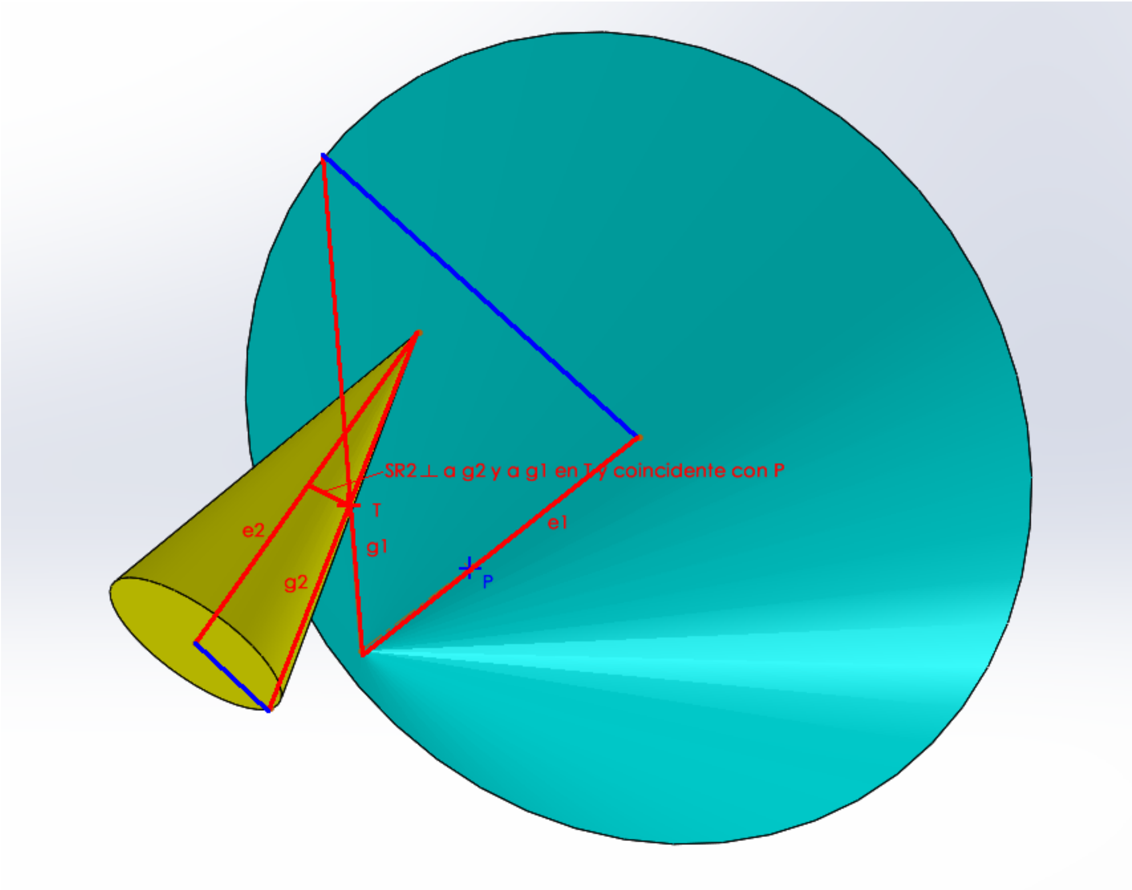


SDW

Procedure:

1. Draw **SR1** perpendicular to **g1** and containing point **T.**
2. Create the plane **α** perpendicular to **SR1** at **T.**
3. Draw **SR2** perpendicular to **g2** containing point **T** and perpendicular to **α.**

Simplification without tangent plane



GRAPHIC LEGEND

en T = at T

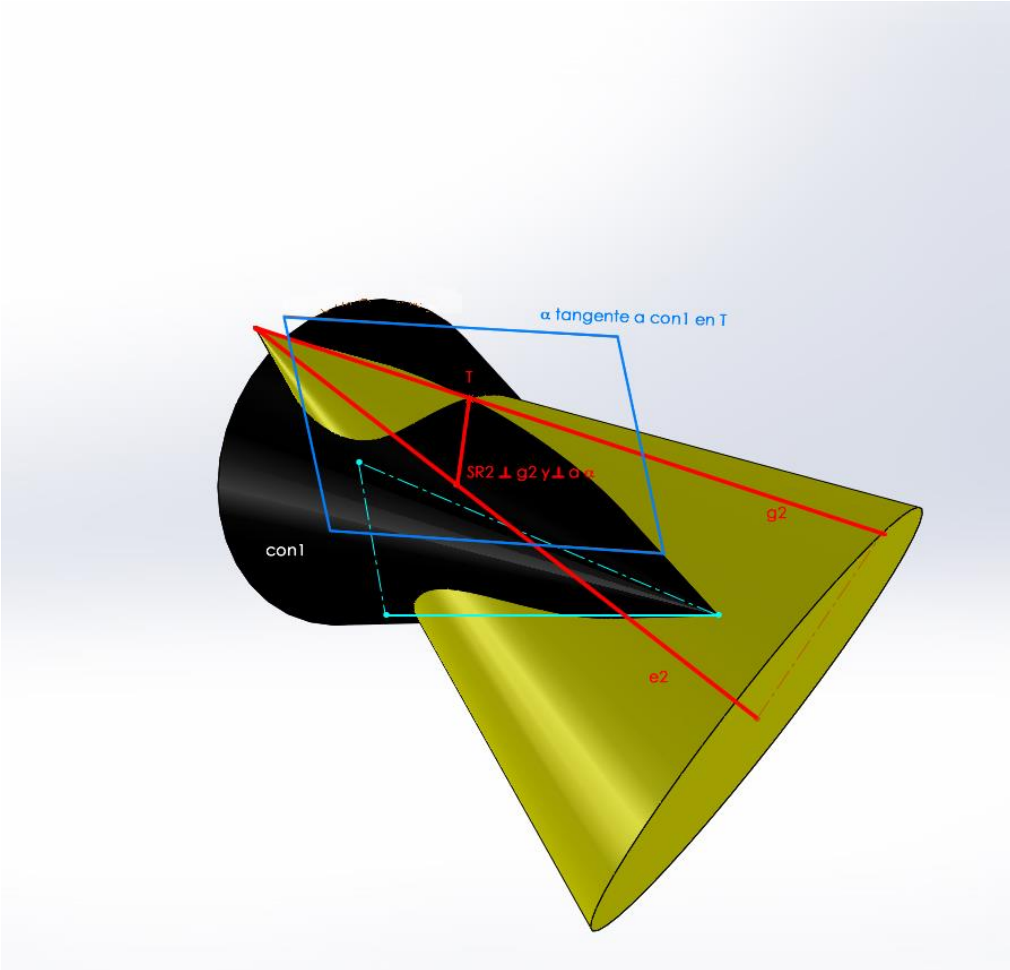
coincidente con P = coincident with P

SDW

Procedure:

1. Draw **SR2** perpendicular to **g1** and to **g2** ending at point **T** and containing point **P.**
   (It is also possible to draw **SR1** from point **P** to **T** and make **SR1** collinear with **SR2**)

36 Tangency between cones with one cone being solid



GRAPHIC LEGEND

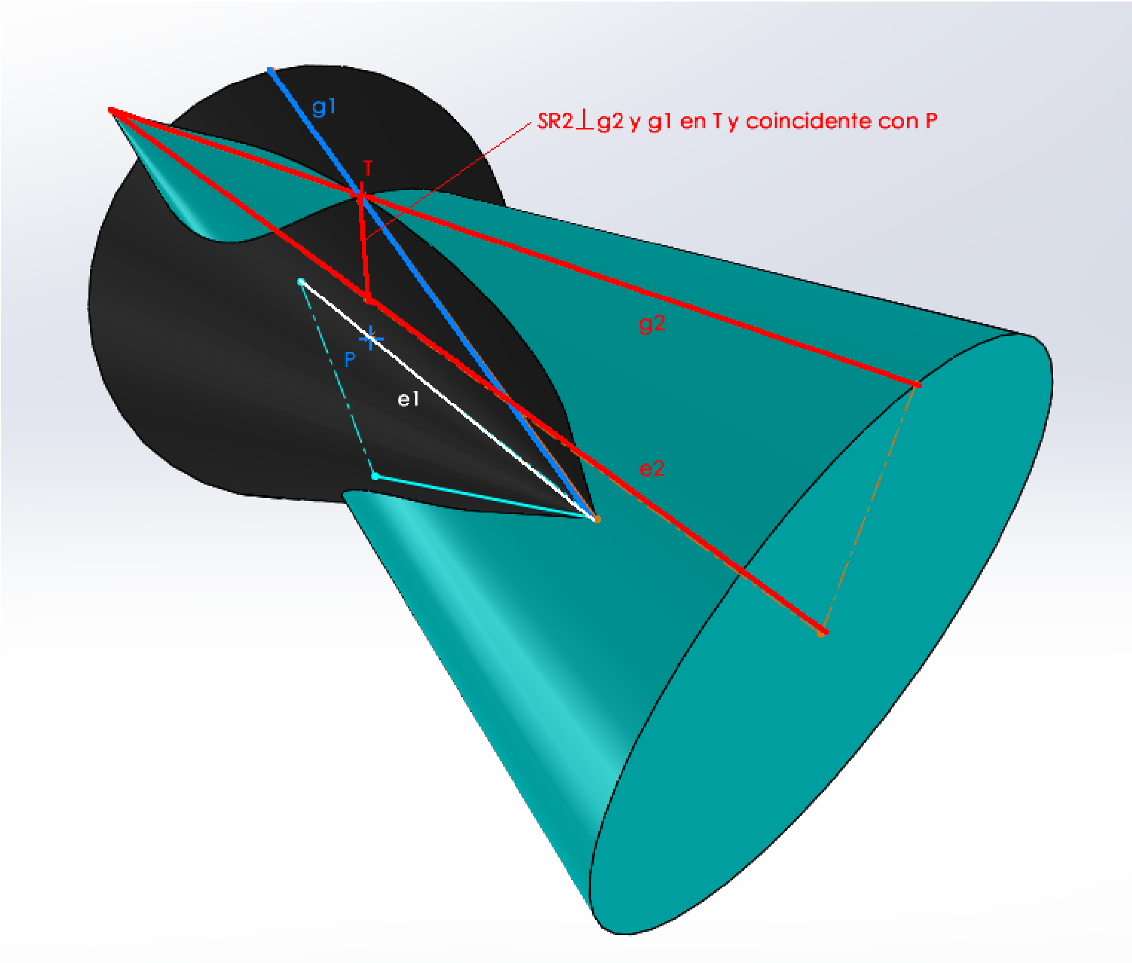
tangente a con1 en T = tangent to con1 at T

SDW

Procedure:

1. Create point **T** on the surface of the solid cone **con1.**
2. Create the plane α tangent to **con1** on **T.**
3. Draw **SR2** being perpendicular to **g2** at point **T** and perpendicular to α.

Simplification with a solid cone and without a tangent plane v1



GRAPHIC LEGEND

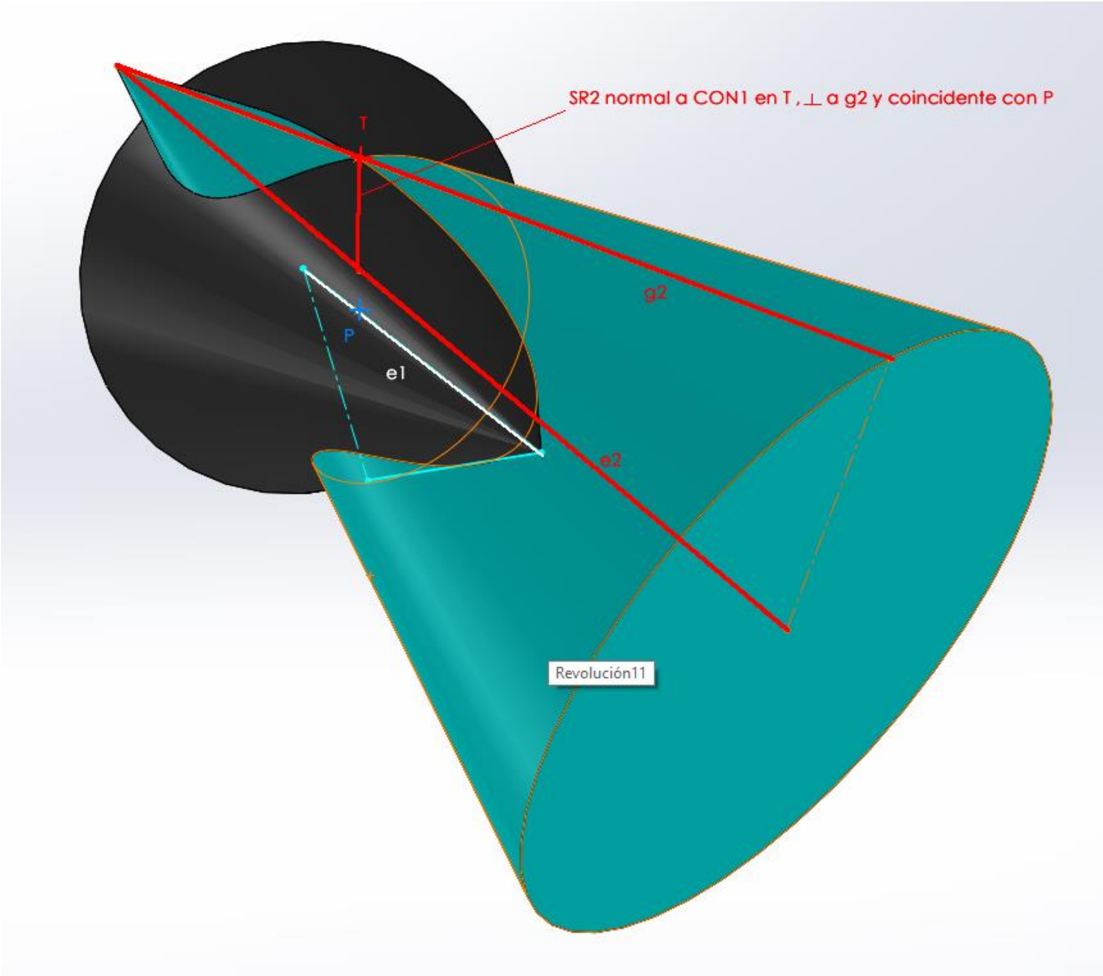
en T y coincidente con P = at T and coincident with P

SDW

Procedure:

1. Draw **SR2** perpendicular to **g1** and **g2** at point **T** and coincident with **P**. (It is also possible to draw **SR1** from point **P** to **T** and make it collinear with **SR2**)

Simplification with a solid cone without a tangent plane v2



GRAPHIC LEGEND

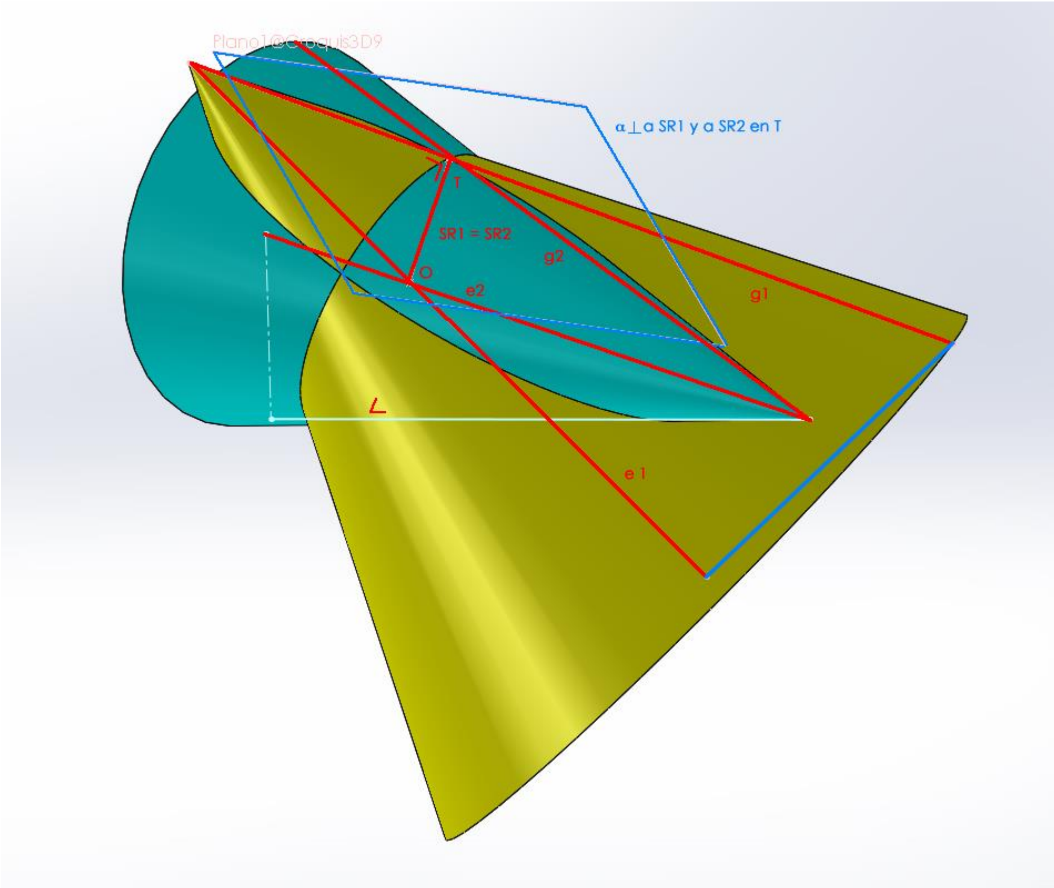
y coincidente con P = and coincident with P

SDW

Procedure:

1. Draw **SR2** perpendicular to **g2** at point **T**, normal to **con1** and coincident with **P**.
   (It is also possible to draw **SR1** from point **P** to **T** and make it collinear with **SR2**)

37 Bi-tangency between two cones



GRAPHIC LEGEND

y coincidente con P = and coincident with P

SDW

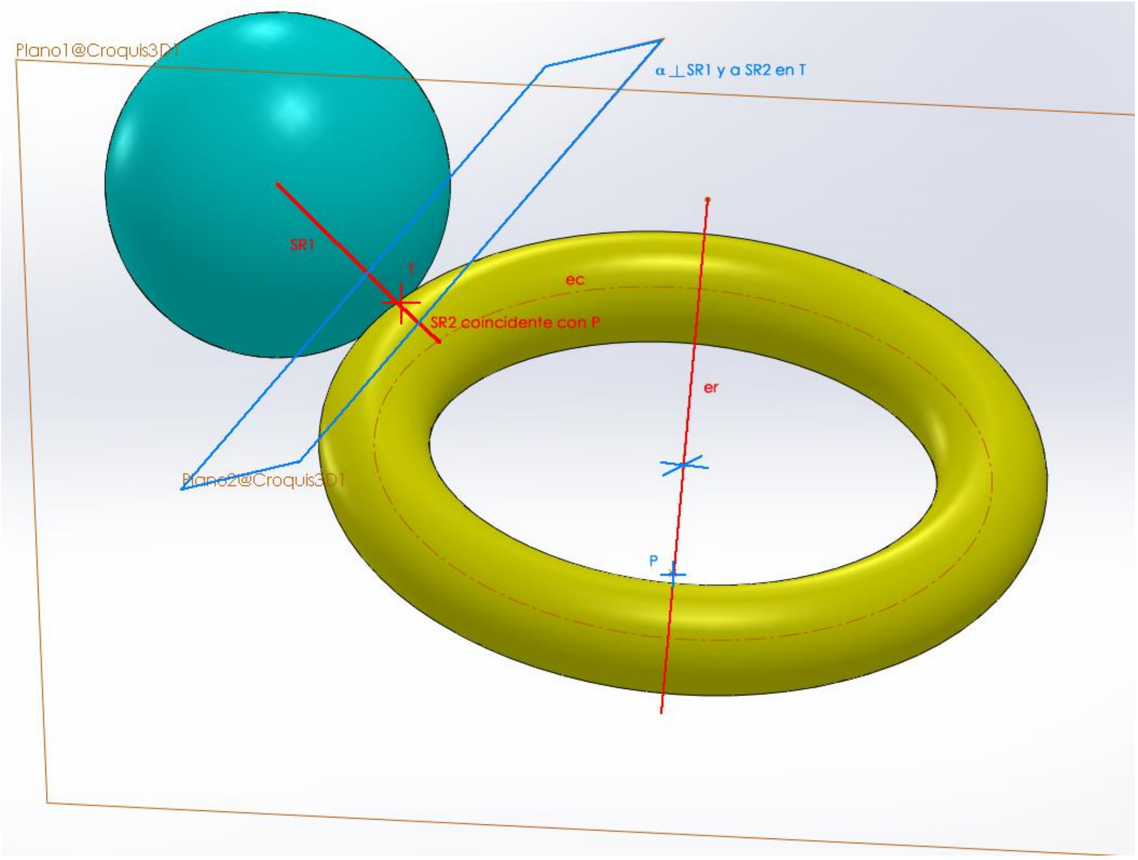
NOTE: Two solids of revolution are bi-tangent when they are tangent and their axes intersect (i.e. point O).

Construction:

1. Draw **e1** and **e2** intersecting at point **O**.
2. Draw **g1** and **g2** intersecting at point **T**.
3. Create the plane **α** with lines **g1** and **g2**.
4. Draw **SR1=SR2** from point **O** to **T** (**SR1**and **SR2** are coincident at T) and perpendicular to **α**.

TORUS

38 Tangency between a torus and a sphere



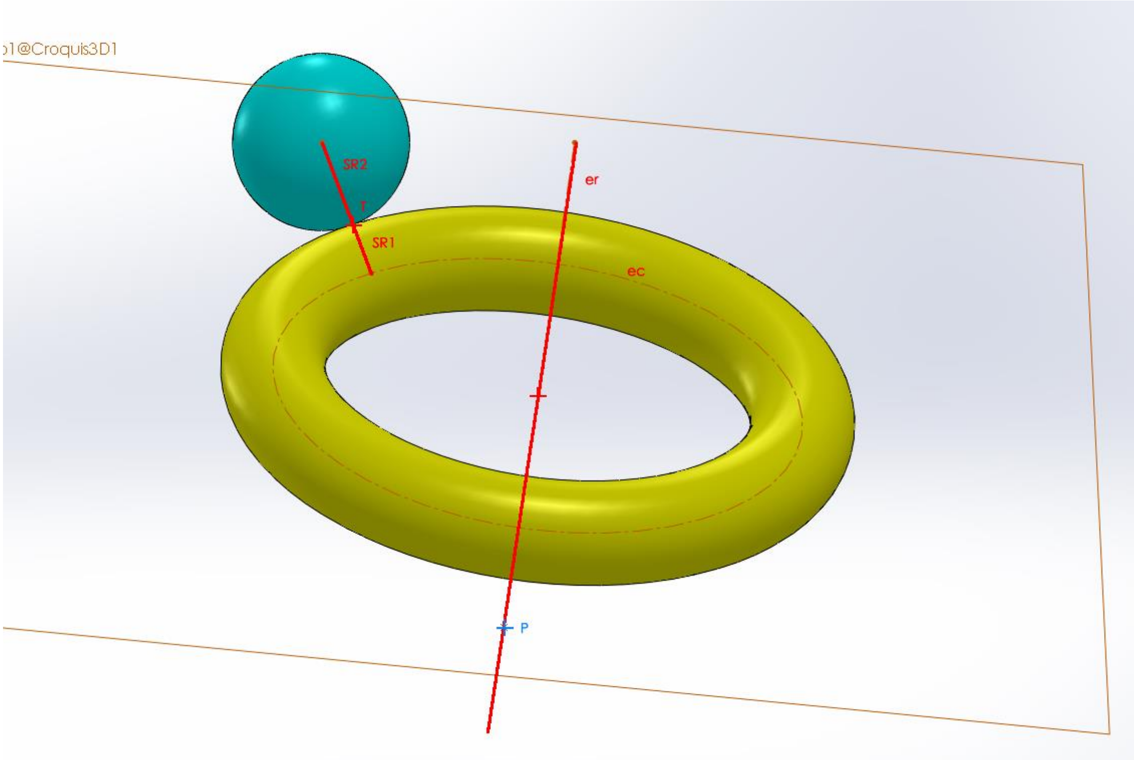
GRAPHIC LEGEND
coincidente con P = coincident with P

SDW

Procedure:

1. Draw **SR1** ending at point **T** on the surface of the sphere.
2. Create the plane α perpendicular to **SR1** at **T**.
3. Draw **SR2** ending at **T**, being perpendicular to α and coincident with **P**. (The coincidence with **P** defines a plane β which contains the lines **SR2** and **er**).

Simplification without a tangent plane

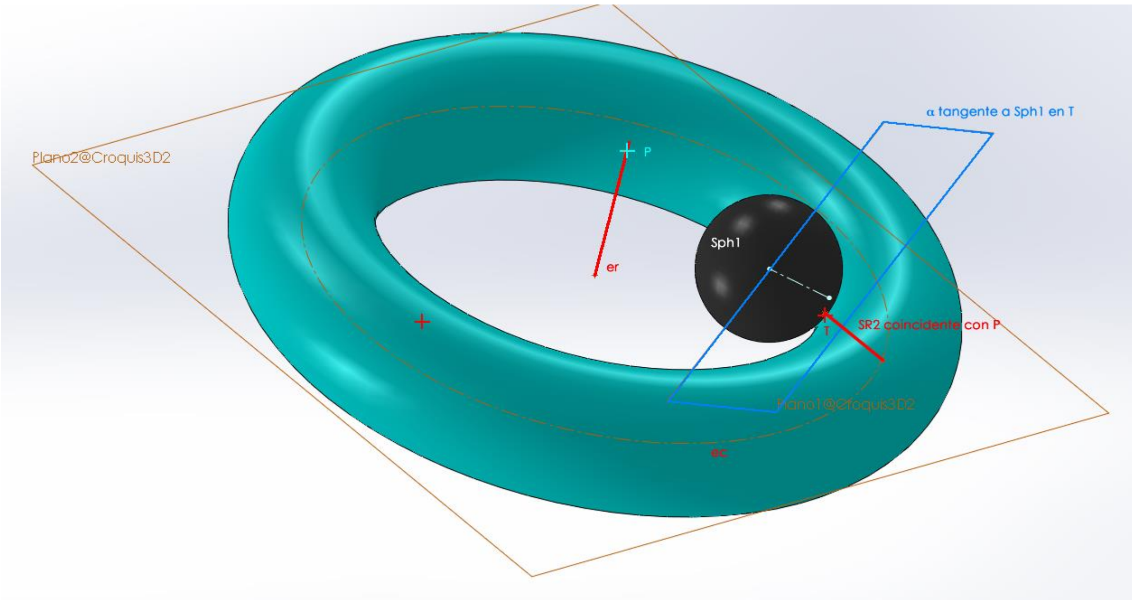


SDW

Procedure:

1. Draw **SR1** ending at point **T** on the surface of the sphere.
2. Draw **SR2** containing point **T,** being coincident with **P and** collinear with **SR1**.
   (The coincidence with **P** defines a plane β which contains the lines **SR2** and **er**).

39 Simplification with a solid sphere



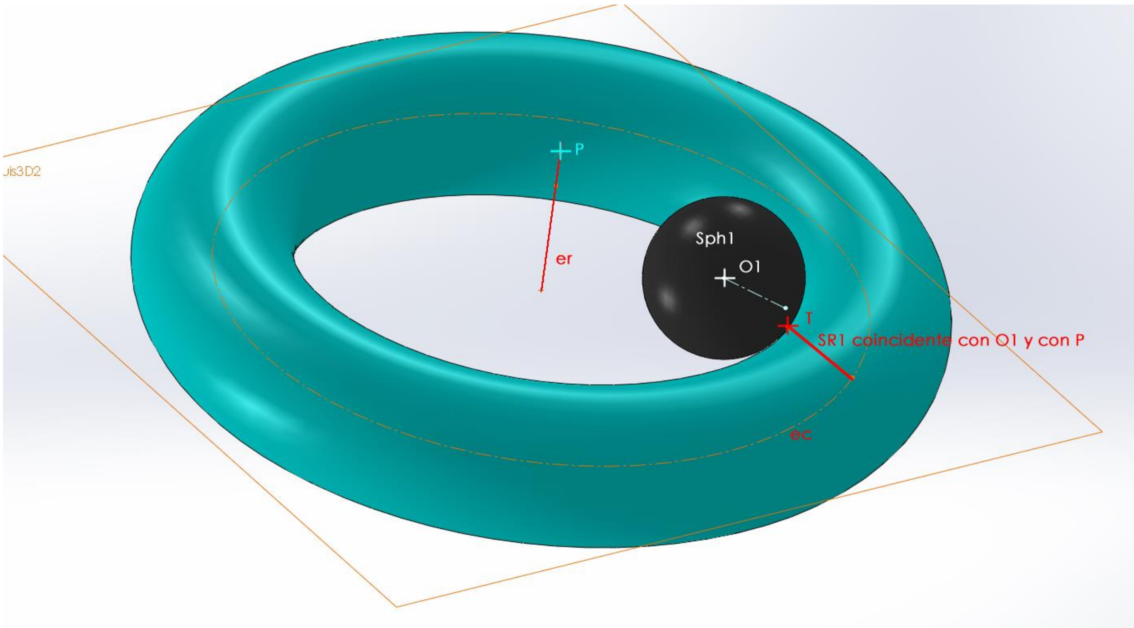
GRAPHIC LEGEND
coincidente con P = coincident with P
tangente a = tangent to

SDW

Procedure:

1. Create point **T** on the surface of the sphere **Sph1**.
2. Create the plane α at **T** and tangent to **Sph1**.
3. Draw **SR2** ending at **T**, coincident with **P** and perpendicular to α.
   (The coincidence with **P** defines the plane β which contains the lines **SR2** and **er**).

Without a tangent plane



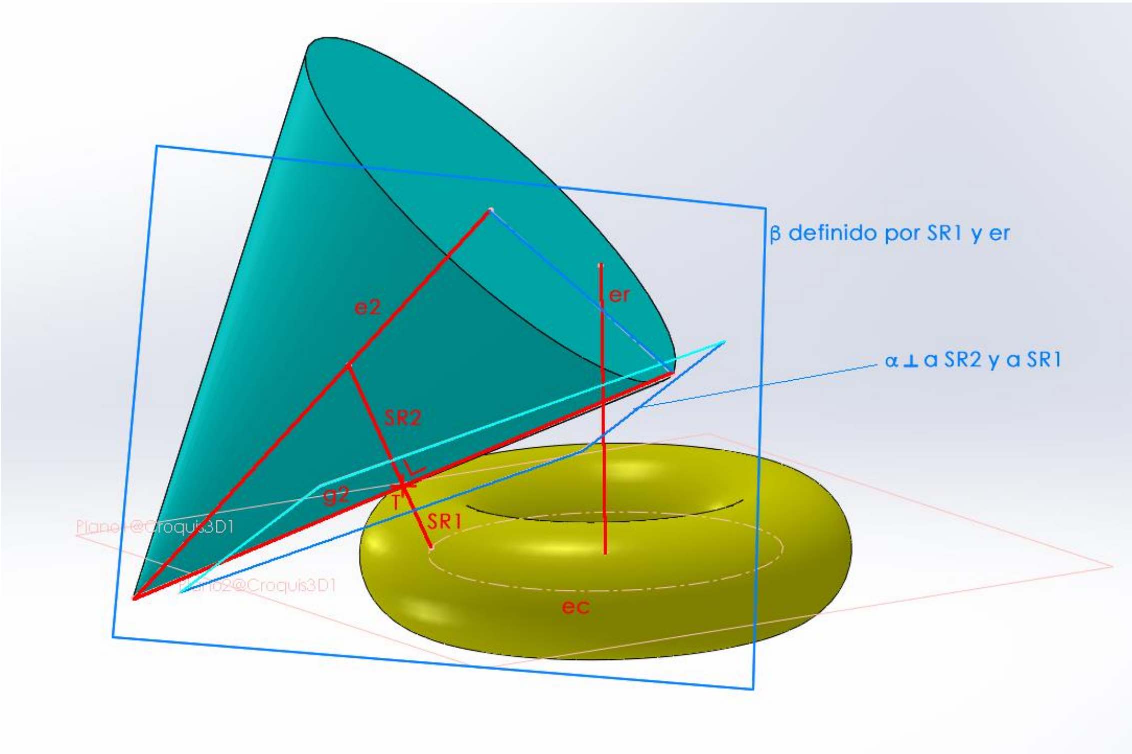
GRAPHIC LEGEND
coincidente con P = coincident with P

SDW

Procedure:

1. Create the point **T** on the surface of the sphere **Sph1**.
2. Draw **SR2** ending at **T** and being coincident with **P** and **O1.**
   (It could be also solved replacing the coincidence with **O1** with a perpendicular relation with the surface of the sphere **Sph1**).
   (The coincidence with **P** defines a plane β which contains the lines **SR2** and **er**).

40 Tangency between a torus and a cone



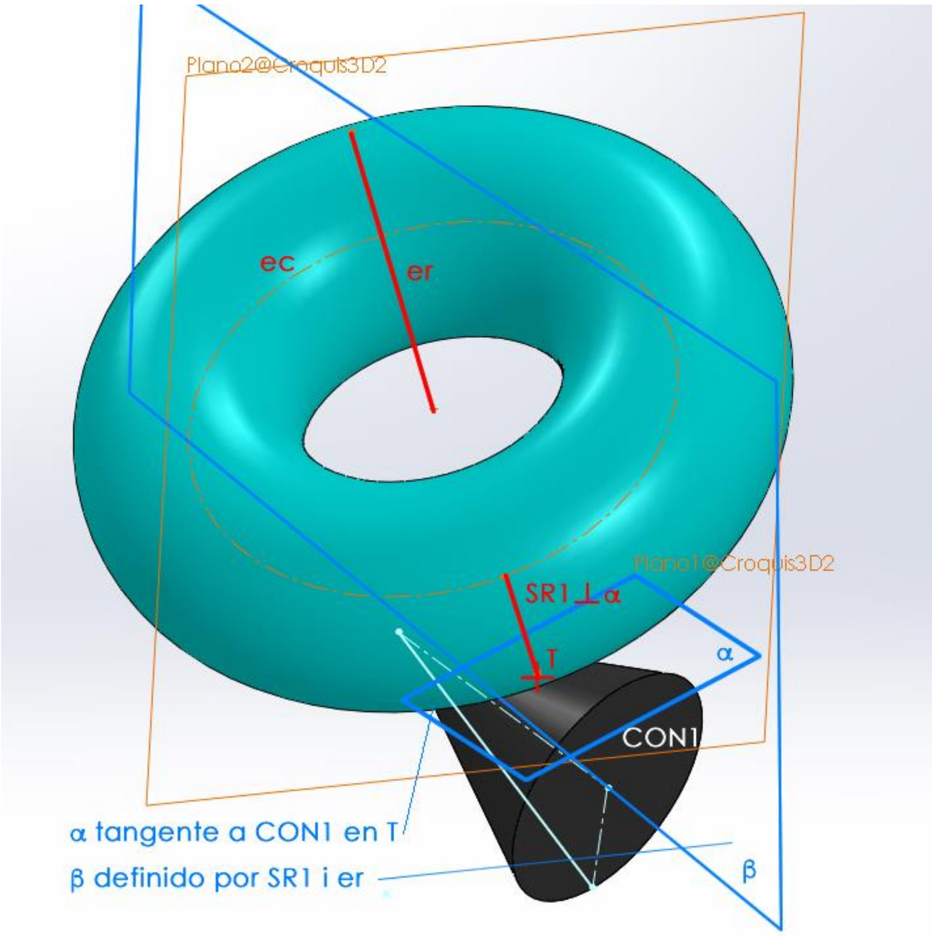
GRAPHIC LEGEND
definido por = defined by

SDW

Construction:

1. Draw **SR1** ending at point **T** on the surface of the torus.
2. Create the plane β defined by **SR1** and **er**.
   (It is possible to skip drawing plane β making **SR1** and **er** coplanar By creating an intersection point **P** between both lines).
3. Create the plane α perpendicular to **SR1** at point **T**.
4. Draw **SR2** ending at point **T**, perpendicular to **g2** and perpendicular to α.

41 Simplification with a solid cone



GRAPHIC LEGEND
tangente a = tangent to
definido por = defined by

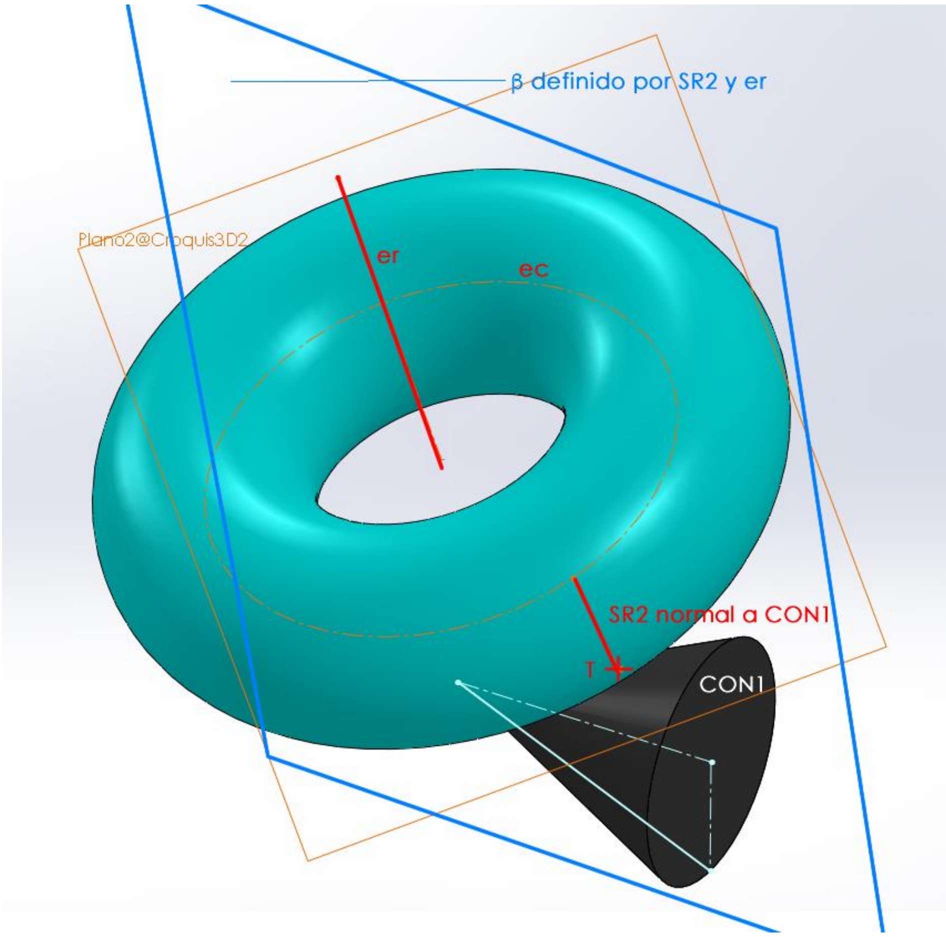
SDW

Procedure:

1. Create the point **T** on the surface of the solid cone **con1**.
2. Create the plane α at point **T** and being tangent to cone **con1**.
3. Draw **SR1** ending at point **T** and perpendicular to α.
4. Create the plane β defined by **SR1** and **er**.

   (It is possible to skip drawing plane β making **SR1** and **er** coplanar by creating an intersection point **P** common to both lines).

Without tangent plane



GRAPHIC LEGEND
normal a = normal to
definido por = defined by

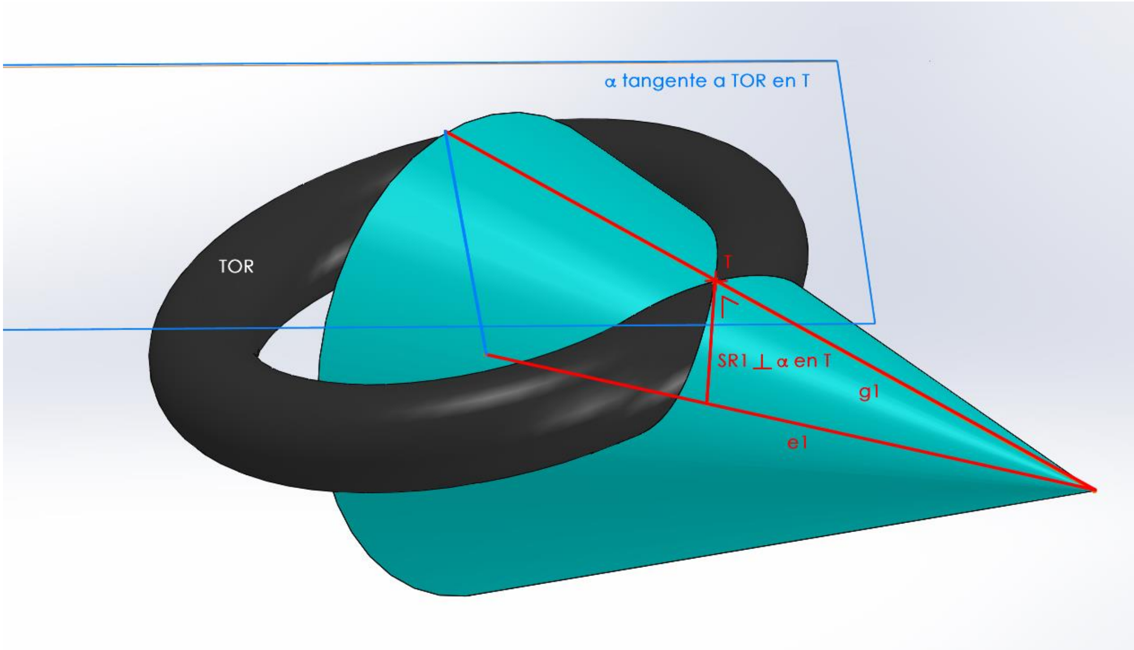
SDW

Procedure:

1. Create the point **T** on the surface of the solid cone **con1**.
2. Draw **SR2** ending at point **T** and being perpendicular to the surface of **con1**.
3. Create the plane β defined by **SR1** and **er**.

(It is possible to skip drawing plane β making **SR1** and **er** coplanar by creating an intersection point **P** common to both lines).

42 Simplification with a solid torus



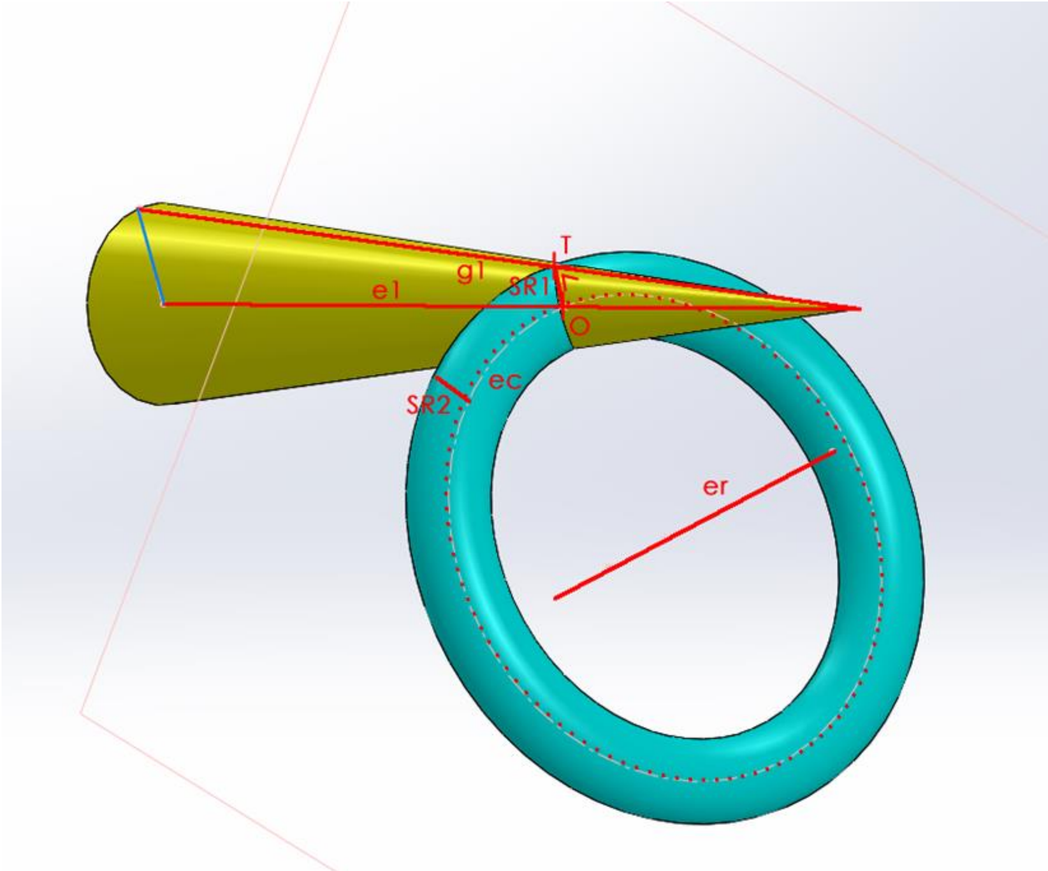
GRAPHIC LEGEND
tangente a T = tangent to

SDW

Construction:

1. Create the point **T** on the surface of the solid torus **TOR**.
2. Create the plane **α** tangent to **TOR** at point **T**.
3. Draw **SR1** ending at point **T,** perpendicular to **g1** and perpendicular to **α**.

43 Simplification of the bi-tangency between a torus and a cone

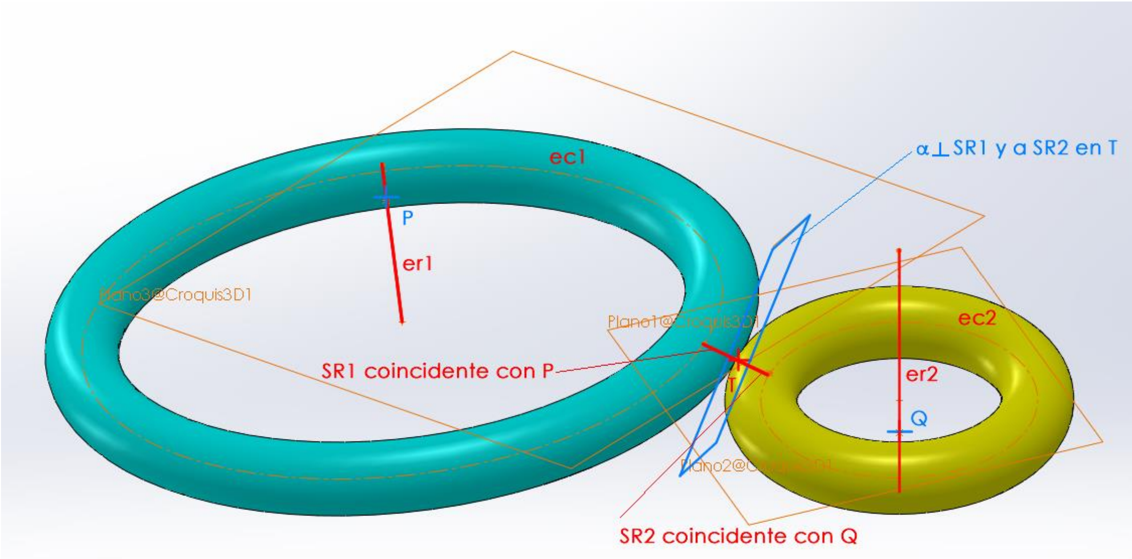


NOTE: The bi-tangency between a torus and a cone or a cylinder is based on the matching of a sphere inscribed in the cone (or the cylinder) with a sphere inscribed in the torus. This common sphere to both solids must have its centre at the intersection of the circular axis of the torus and the axis of revolution of the cone and the radius SR must be equal to the radius of the sphere inscribed in the torus.

Procedure:

1. Create the point **O**, intersection point between the circular axis of the torus **ec** and the revolution axis of the cone **er**.
2. Draw **SR1** from point **O** to **T** perpendicular to **g1**.
3. Draw **SR2** from **ec** to the surface of the torus.
4. Match the length of **SR1** and **SR2**.

44 Tangency between torus



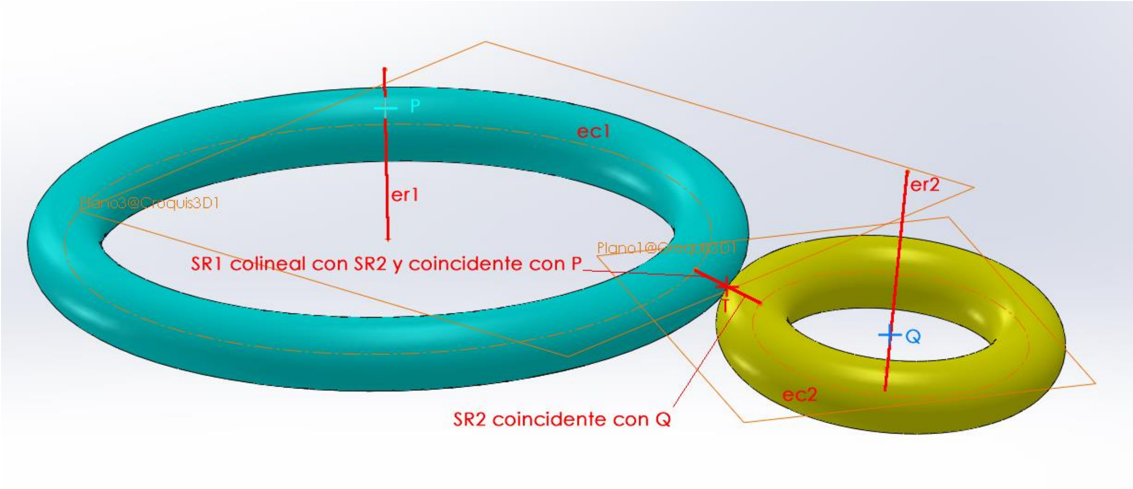
GRAPHIC LEGEND
coincidente con = coincident with

SDW

Procedure:

1. Draw **SR1** ending at point **T** on the surface of **toro1** and being coincident with point **P**.
2. Create the plane **α** perpendicular to **SR1** at **T**.
3. Draw **SR2** ending at point **T**, coincident with point **Q** and perpendicular to **α**.

Simplification without a tangent plane



GRAPHIC LEGEND

coincidente con = coincident with

colineal con = collinear with

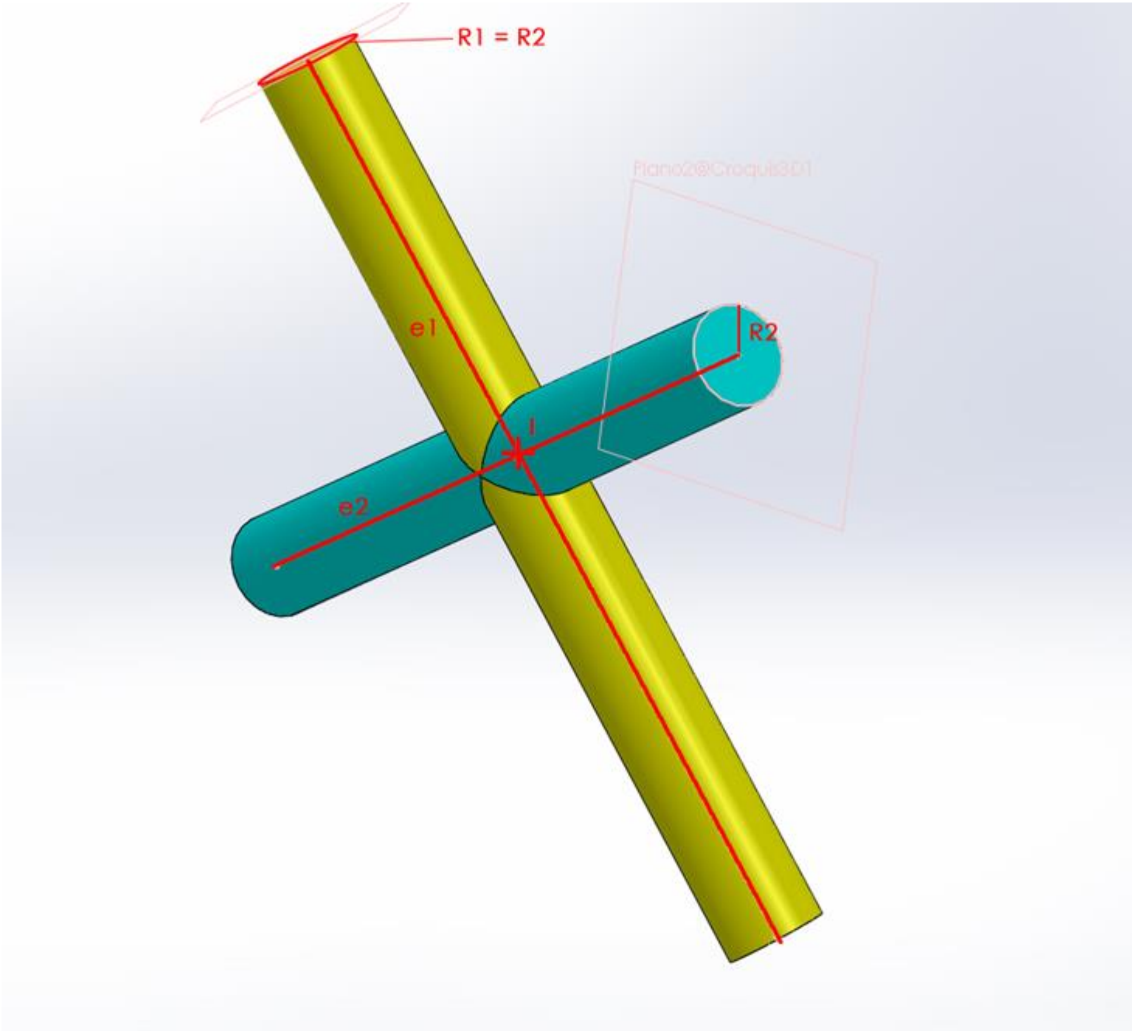
SDW

Procedure:

1. Draw **SR1** ending at point **T** on the surface of the torus **toro1** and being coincident with **P**.
2. Draw **SR2** ending at point **T,** coincident with point **Q** and collinear with **SR1**.

CILINDER

45 Simplification in the bi-tangency between cylinders

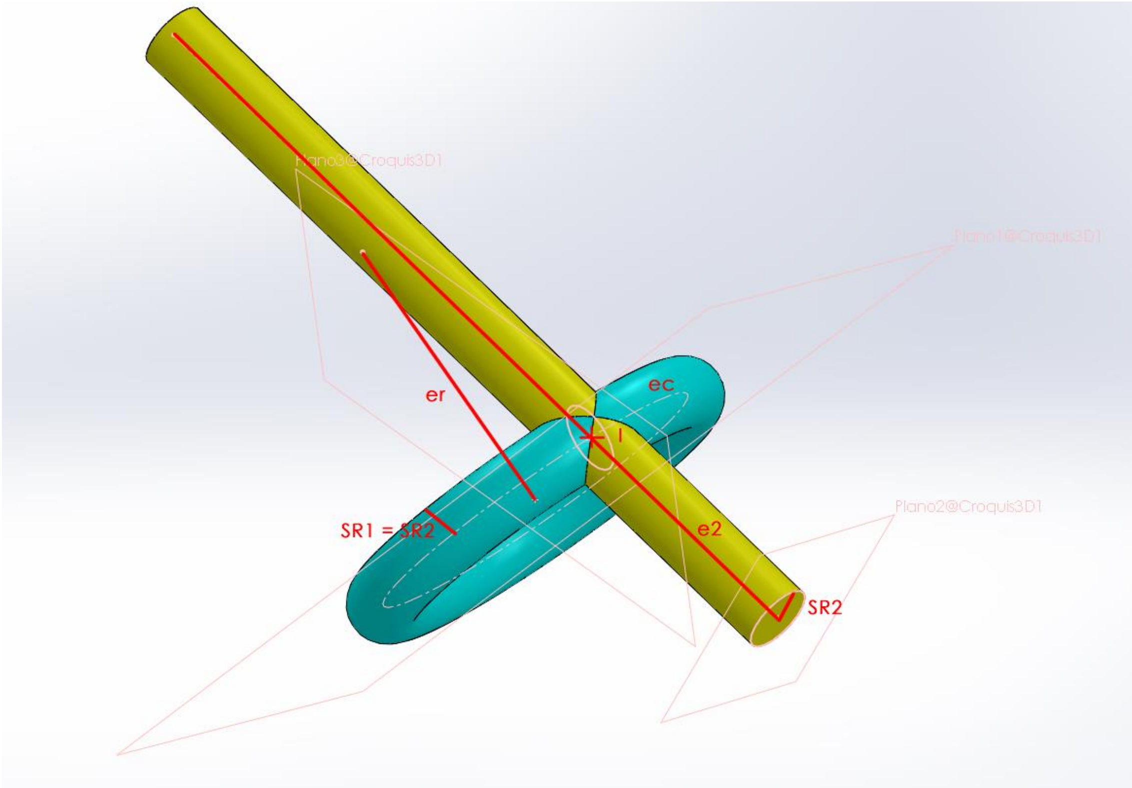


SDW

Procedure:

1. Create the point **I** intersection between the axis of both cylinders.
2. Match the radius **SR1** and **SR2** of the cylinders.

46 Simplification in the bi-tangency between the torus and the cylinder



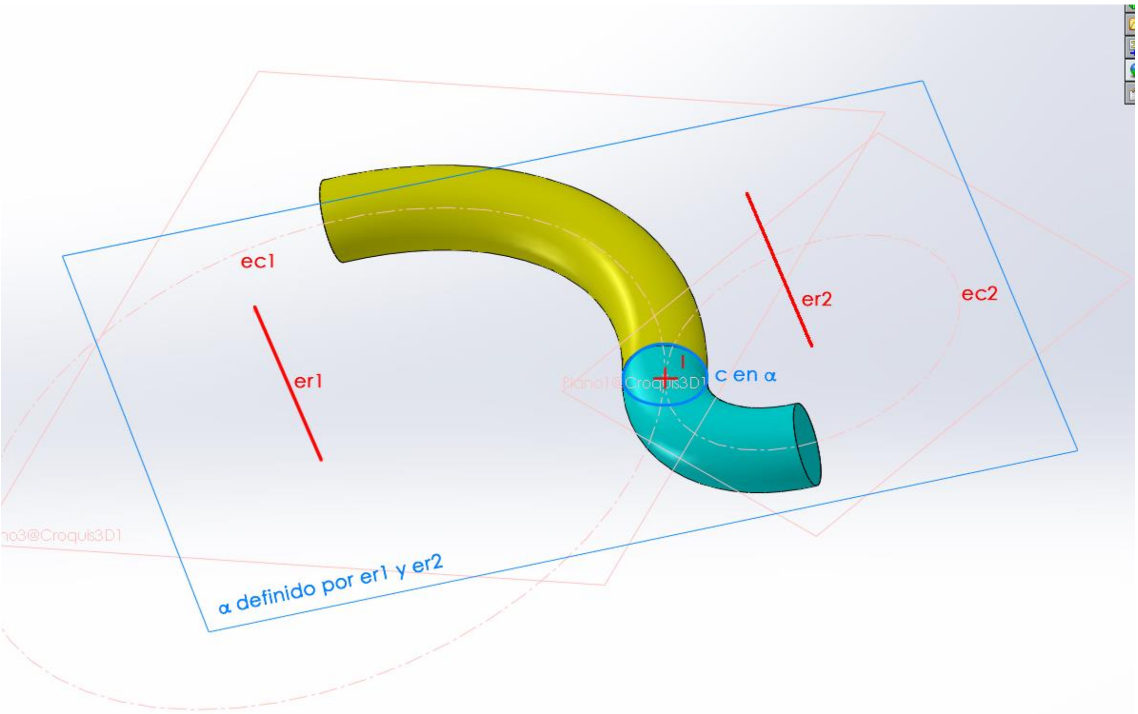
SDW

Procedure:

1. Create the point **I** intersection between the circular axis of the torus **ec** and the revolution axis of the cylinder **e2**.
2. Match the radius **SR1** of the torus and the radius **SR2** of the cylinder.

PARTICULAR CASES

47 Connection between torus by a circular base



GRAPHIC LEGEND
definido por = defined by

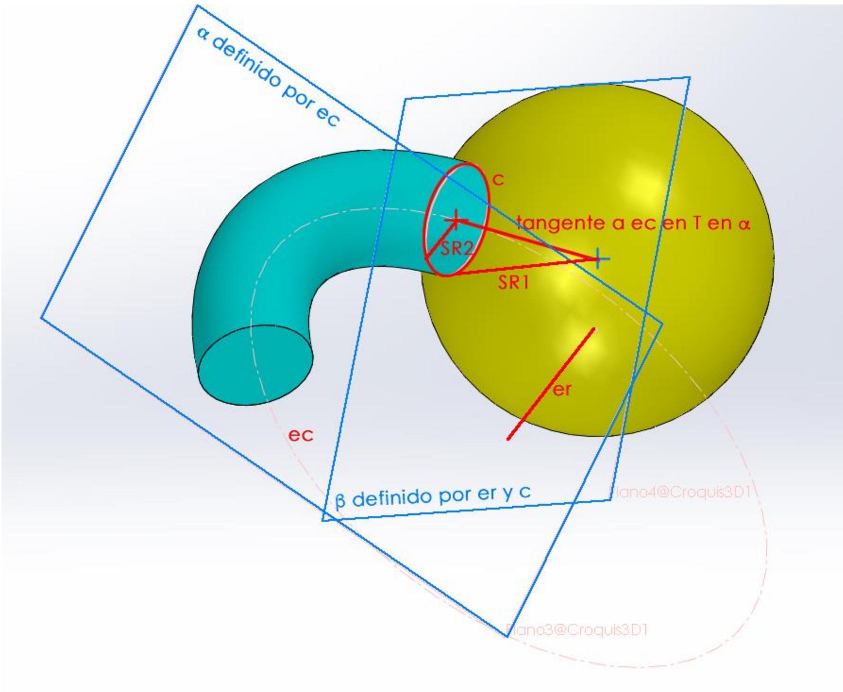
SDW

NOTE: In order to connect two torus at the same circle, it is necessary that the circle by which they connect and the two axes of revolution be coplanar.

Procedure:

1. Create the point **I** intersection between the circular axis **ec1** and **ec2**. The two circular axes should not have any other intersection point.
2. Create the plane **α** defined by **er1**, **er2** and **I**.
3. Create the circle **c** with centre at **I** on plane **α**.

48 Connection between a torus and a sphere by a circle



GRAPHIC LEGEND

definido por = defined by

tangent a = tangent to

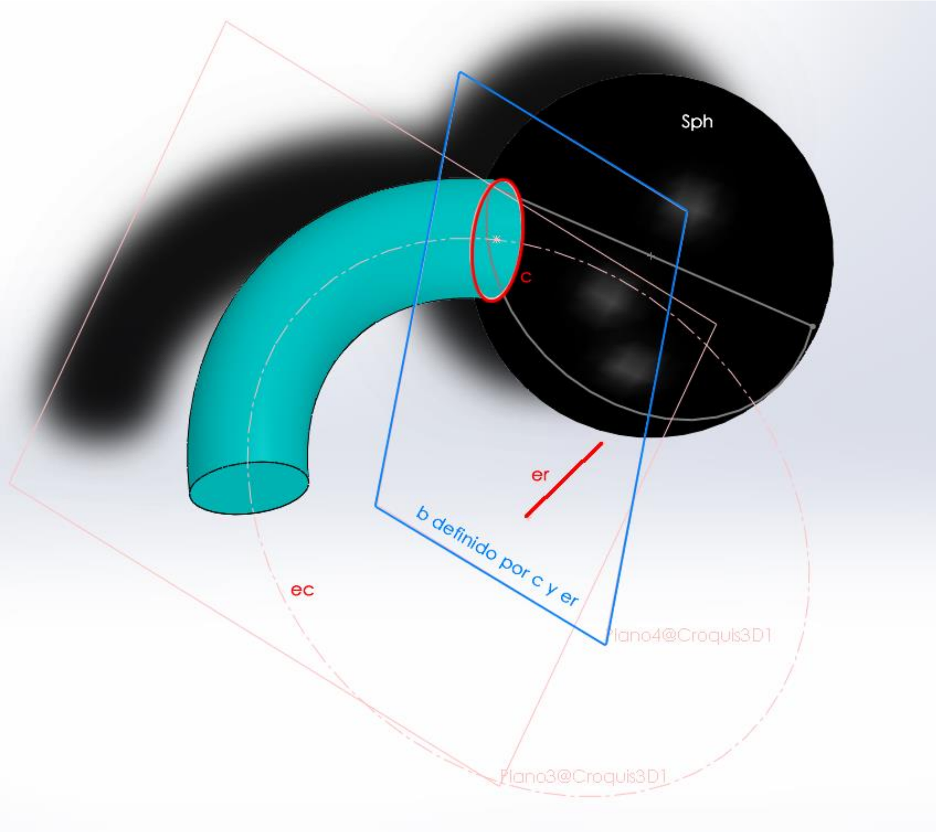
SDW

NOTE: If a sphere and a torus are to be connected by the same circle, it is necessary to ensure that the circle is the generator of the torus (it must be coplanar with its axis of revolution) and that it is located on the surface of the sphere (the perpendicular line to the plane of the circle from its centre must be coincident with the centre of the sphere).

Procedure:

1. Create the circle **c** and the radius **SR2** of the torus, starting the latter at one of the points of the circular axis **ec**.
2. Make the plane β of the circle coincident with **er**.
3. Draw from the centre of the sphere the radius **SR1** to one of the points of the circle **c**.
4. Draw on plane α the tangent line to the circular axis **ec** from the centre of the circle **c** and make it coincident with the centre of the sphere.

49 Simplification with a solid sphere



GRAPHIC LEGEND
definido por = defined by
tangent a = tangent to

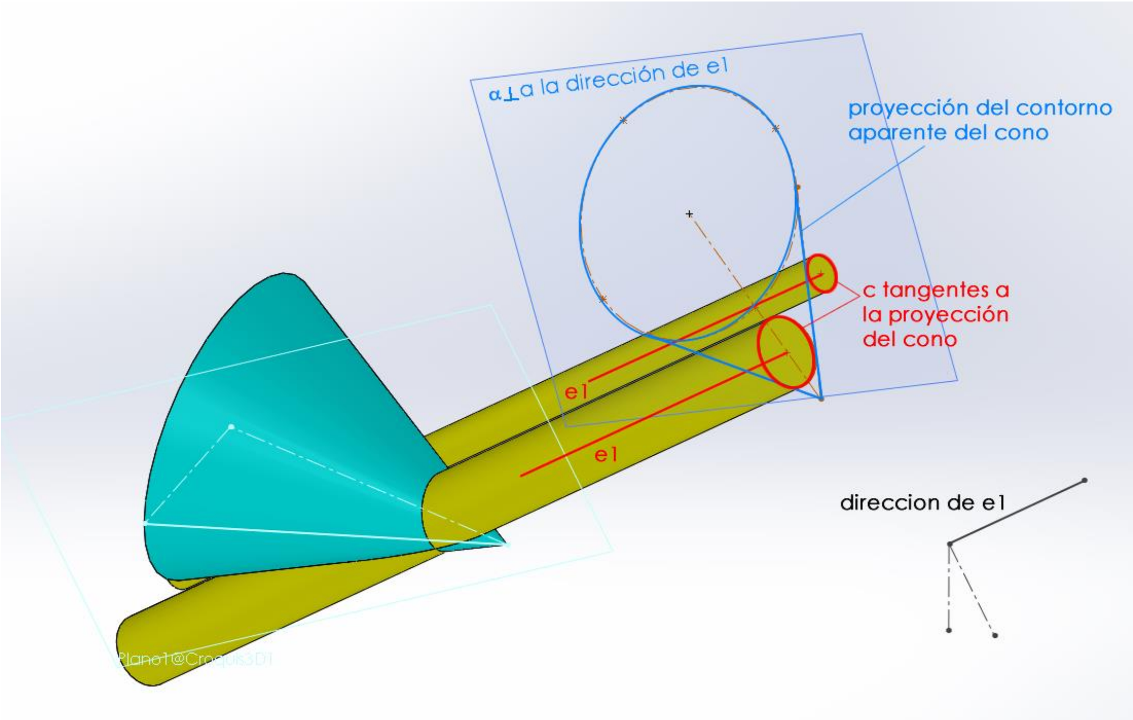
SDW

Procedure:

1. Create the circle **c** on the surface of the sphere **Sph**.
2. Make the plane **β** of the circle **c** coincident with **er**.

Method 2: PROJECTION

Tangency when the direction of the axis of the cylinder is known



GRAPHIC LEGEND

a la dirección de = to direction of

proyección del controno aparente del cono = apparent contour projection

tangentes a la proyección del cono = tangents to the projection of the cone

dirección de = direction of

SDW

Procedure:

1. Create a plane **α** perpendicular to the direction of **e1**.
2. Project on plane **α** the contour of the other solid(s) to those which the cylinder is tangent.
3. Create the circle **c** (projection of the cylinder on plane **α**) tangent to the contours of other projected solid(s).

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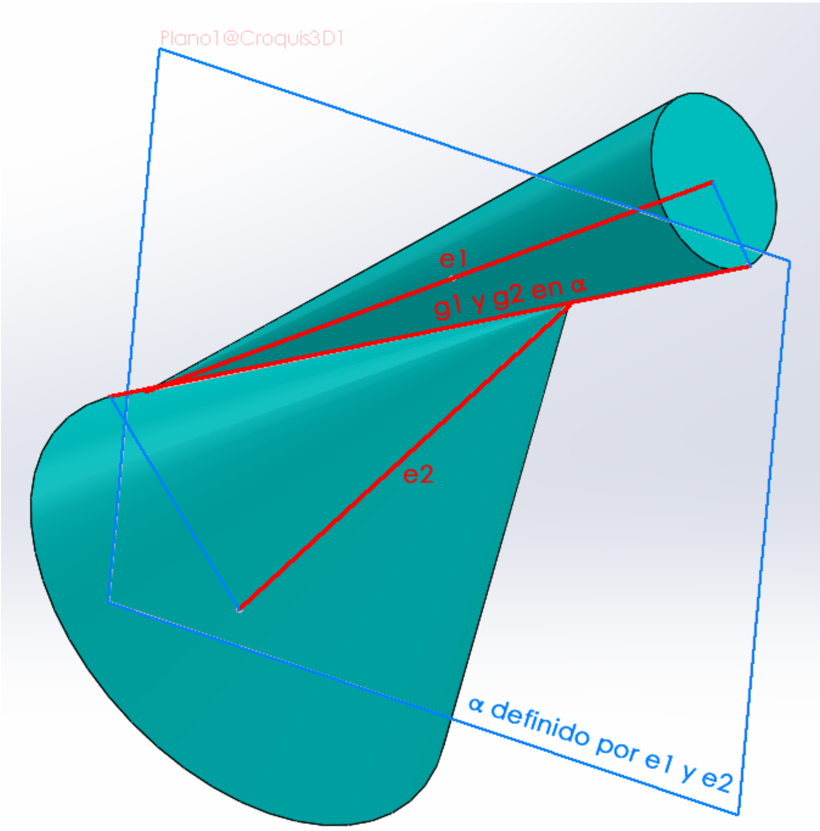
Method 3: SECTION. TANGENCY BETWEEN REVOLUTION SOLIDS WHIT COPLANAR AXES

Two cones



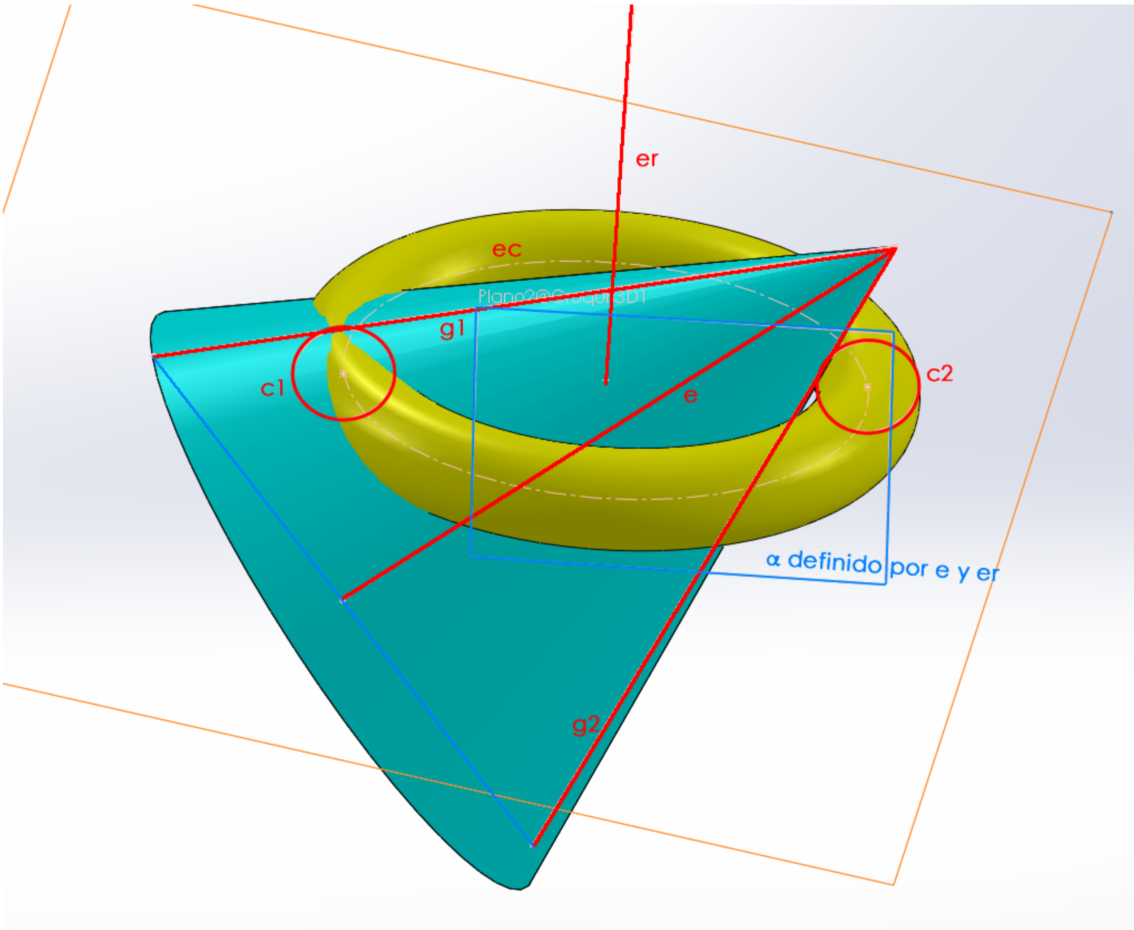
GRAPHIC LEGEND
definido por = defined by

SDW

Procedure:

1. Create the plane **α** defined by **e1** and **e2**.
2. Create on plane **α** the generatrixes **g1** and **g2** being coincident.

Cone and torus



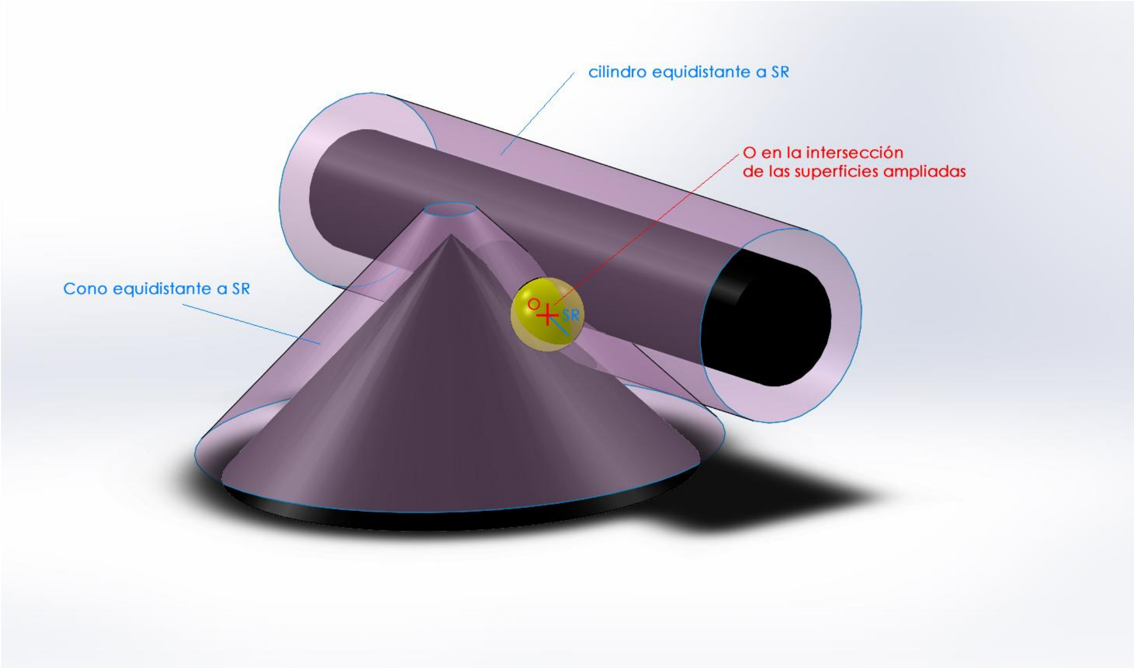
GRAPHIC LEGEND
definido por = defined by

Procedure:

1. Create the plane **α** defined by **er** and **e**.
2. Create on plane **α** the generatrix **g1** and the generatrix **g2** of the cone.
3. Create on the plane **α** the circles **c1** and **c2** of the torus (which must be symmetrical about **er**) tangent to the generatrixes of the cone **g1** and **g2**.

Method 4: EQUIDISTANCE (GEOMETRIC PLACES). TANGENCY WHEN THE RADIUS OF THE GENERATOR SPHERE IS KNOWN.

Sphere



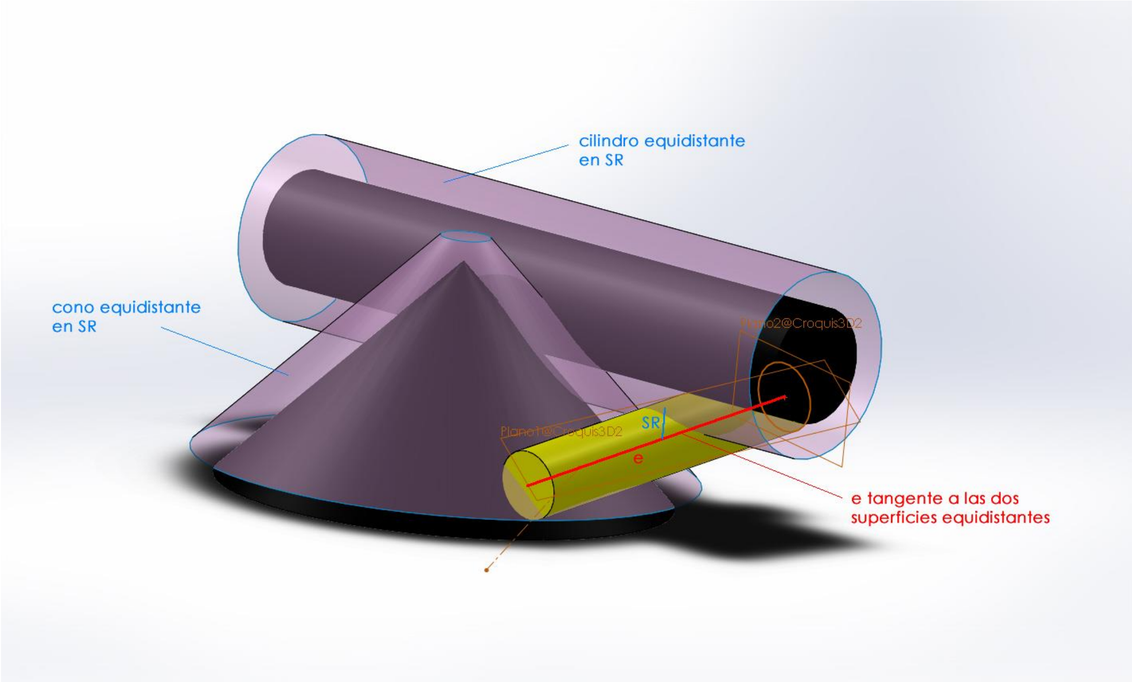
GRAPHIC LEGEND
cilindro equidistante = equidistance cylinder
cono equidistante = equidistance cone
en la intersección de las superficies ampliadas = at the intersection of the extended surfaces

SDW

Procedure:

1. Create the extended surfaces (equidistant to the solids or planes to which the sphere must be tangent) at the distance of the sphere radius **SR**.
2. Obtain the intersection curve of the extended surfaces.
3. Place the centre of the sphere **O** at the intersection point of these curves.

Cylinder



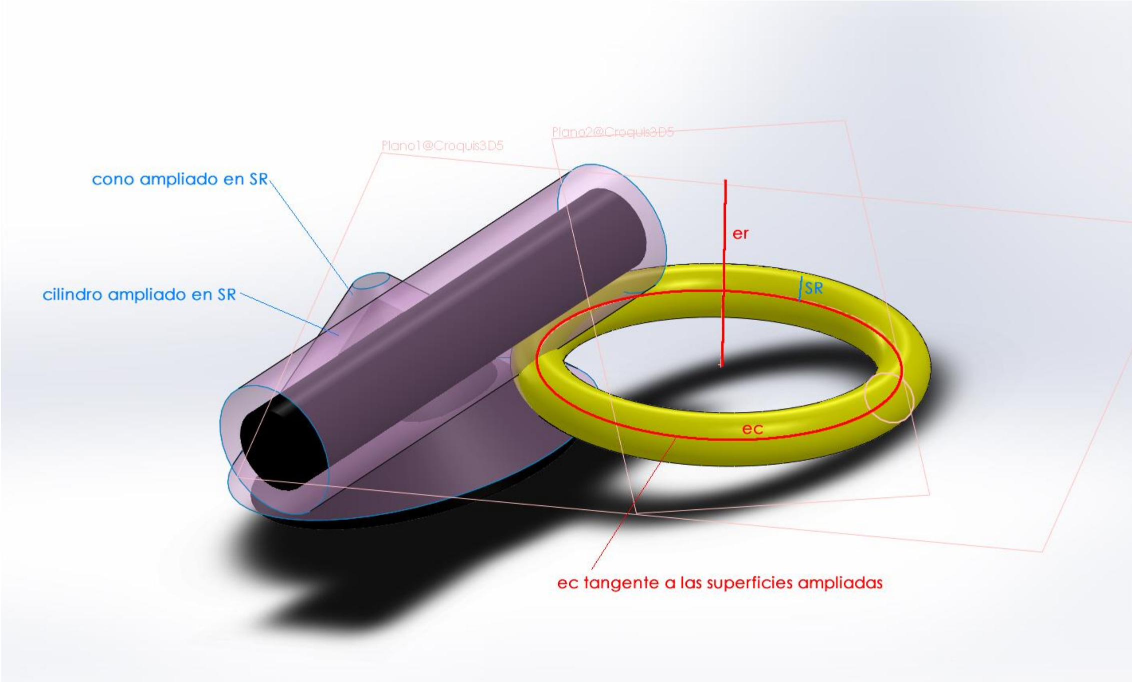
GRAPHIC LEGEND
cilindro equidistante en = equidistance cylinder at
cono equidistante en = equidistance cone at
tangente a las superficies ampliadas = tangent to the extended surfaces

SDW

Procedure:

1. Create the extended surfaces (equidistant to the solids or planes to which the cylinder must be tangent) at the distance of the radius of the cylinder **SR**.
2. Create the revolution axis of the cylinder **e** tangent to the extended surfaces.

For a torus



GRAPHIC LEGEND
cono ampliado en = extended cone at
cilindro ampliado en = extended cylinder at
tangente a las superficies ampliadas = tangent to the extended surfaces

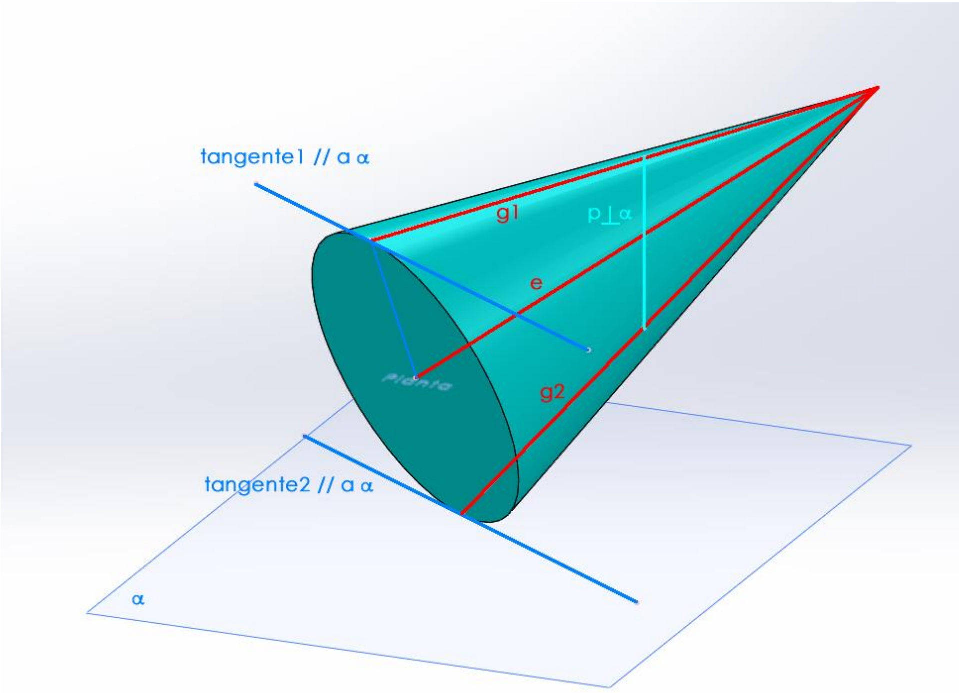
SDW

Constructi0n:

1. Create the extended surfaces (equidistant to the solids or planes to which the torus must be tangent) at the distance of the radius of the torus **SR**.
2. Create the circular axis of the torus **ec** tangent to the extended surfaces.

LIMIT POSITIONS

50 Limit generatrix
(farther or closer to a plane)



GRAPHIC LEGEND
tangente = tangent

SDW

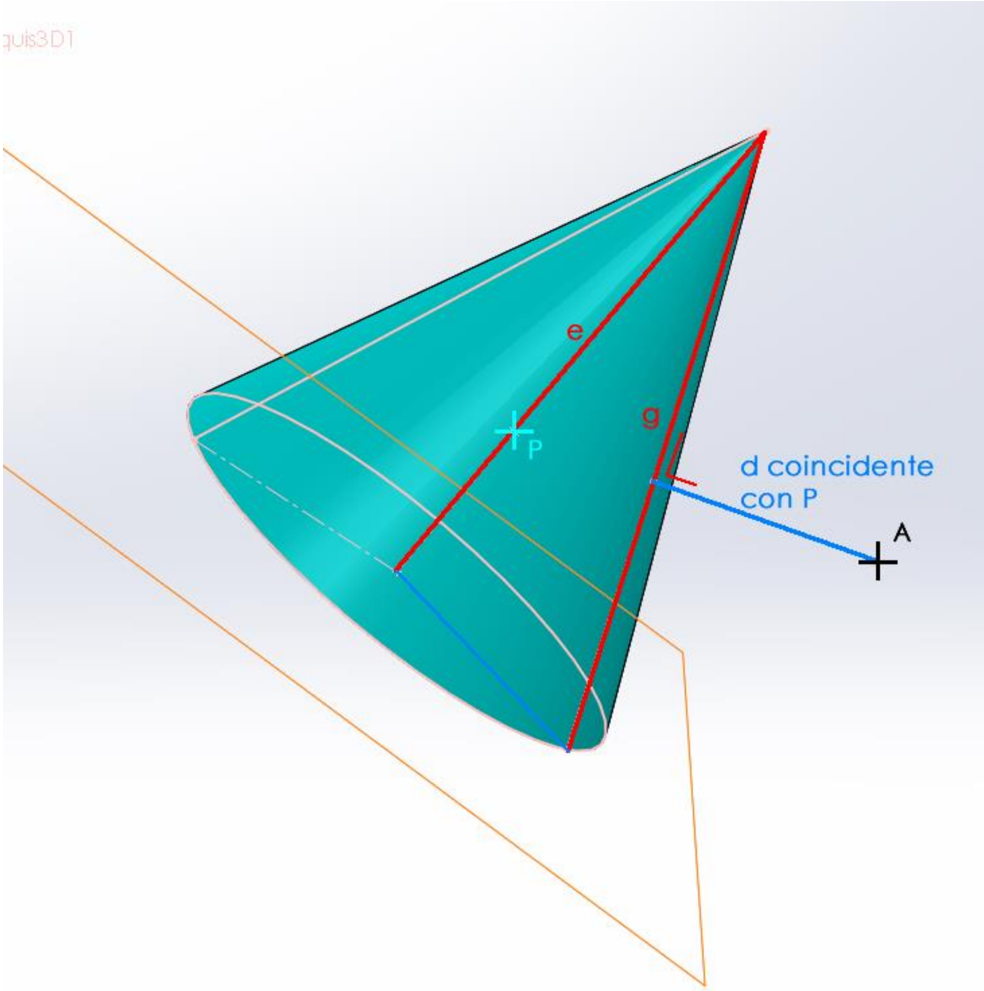
Procedure:

1. Create the tangents **t1** and **t2** to the base of the cone and parallels to the reference plane **α**.
2. Create the generatrixes **g1** and **g2** coincident with one point of the tangents **t1** and **t2**.

Simplified Method:

1. Create the generatrixes **g1** and **g2**.
2. Draw the line **p** which intersects **g1**, **e** and **g2**, and is perpendicular to the reference plane **α**.

51 Relation with a point (distance from point to surface)



GRAPHIC LEGEND
coincidente con = coincident with

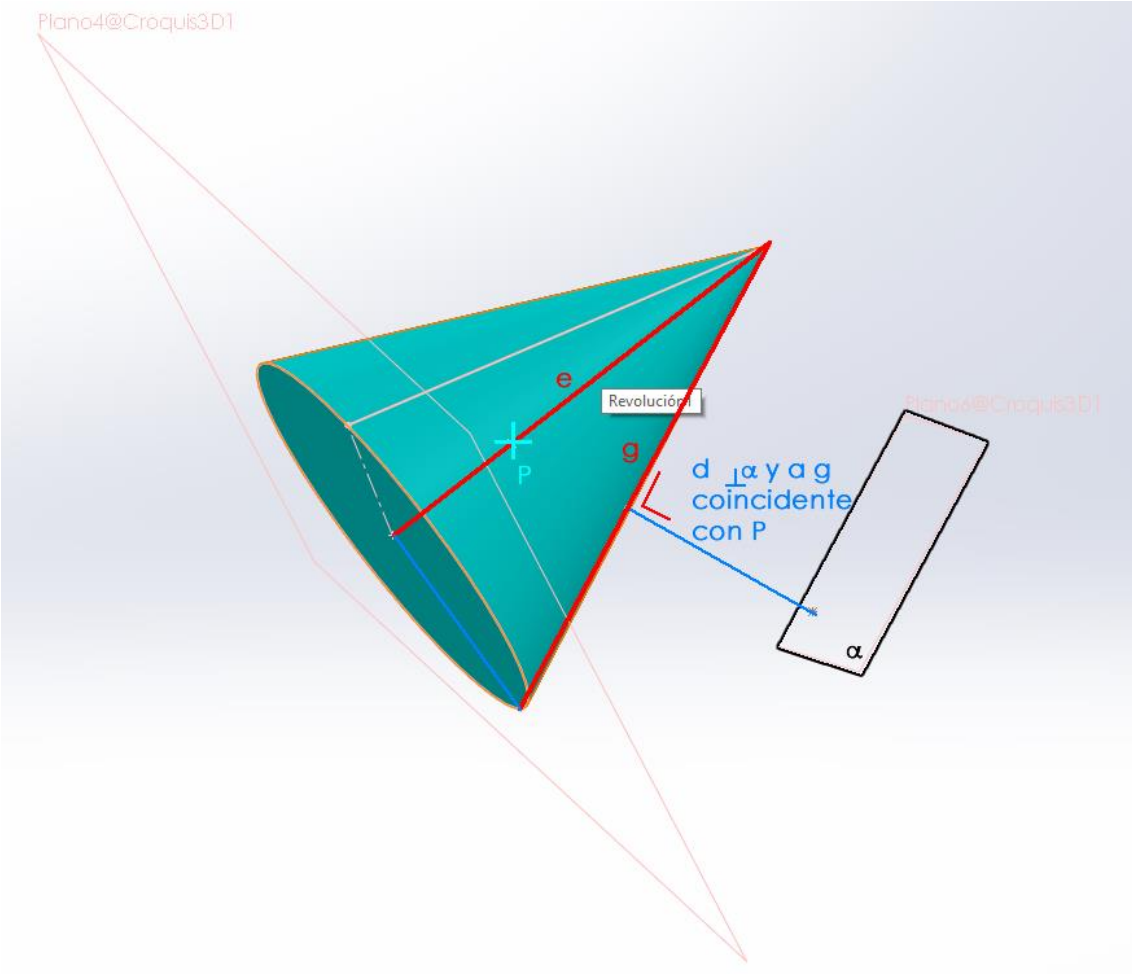
SDW

Procedure:

1. Create the line **d** perpendicular to one generatrix **g** and coincident with **P**.

   NOTE: **d** also could be normal to the solid surface at one of its points (only to be applied when the solid is known).

52 Relation with a plane (distance from plane to surface)



GRAPHIC LEGEND
coincidente con = coincident with

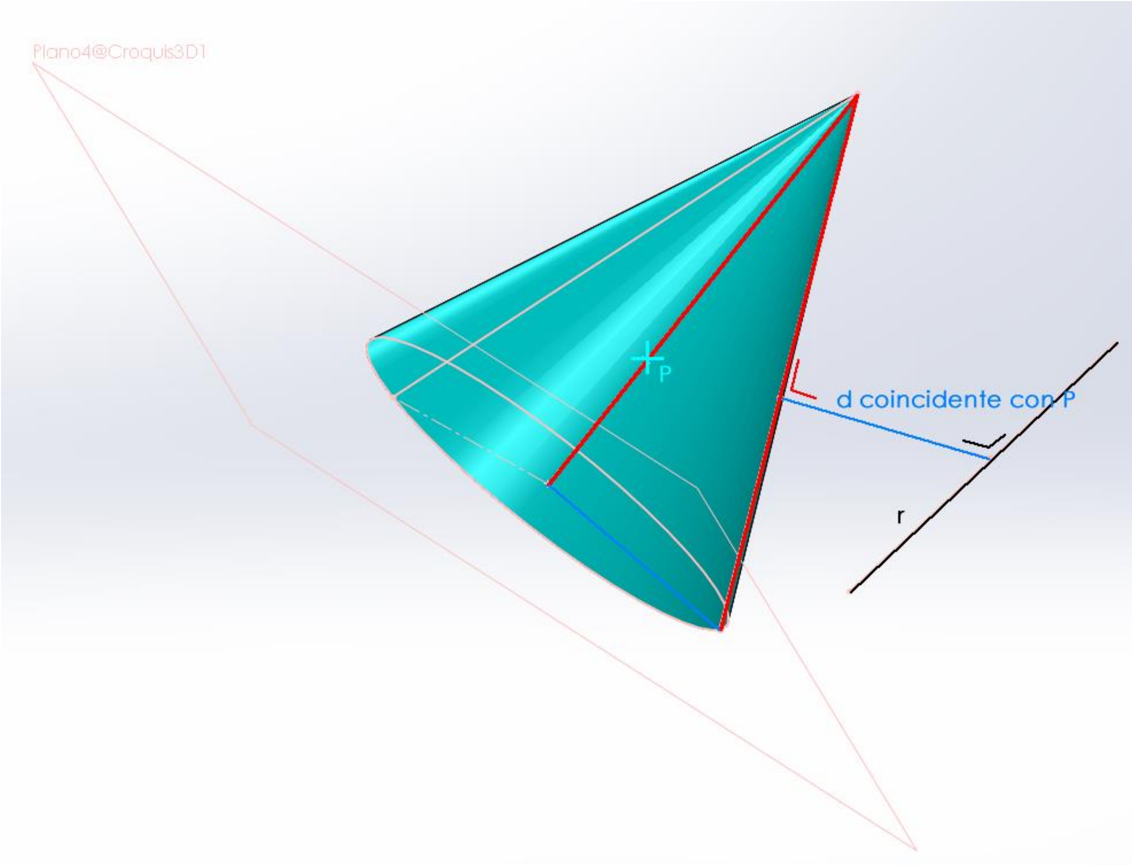
SDW

Procedure:

1. Create the line **d** perpendicular to generatrix **g**, perpendicular to reference plane **α** and coincident with **P**.

   NOTE: Coincidence between **d** and **P** can be avoided making **d** normal to the solid surface at one of its points.
   (only to be applied when the solid is known).

53 Relation with a line (distance from line to surface)



GRAPHIC LEGEND
coincidente con = coincident with

SDW

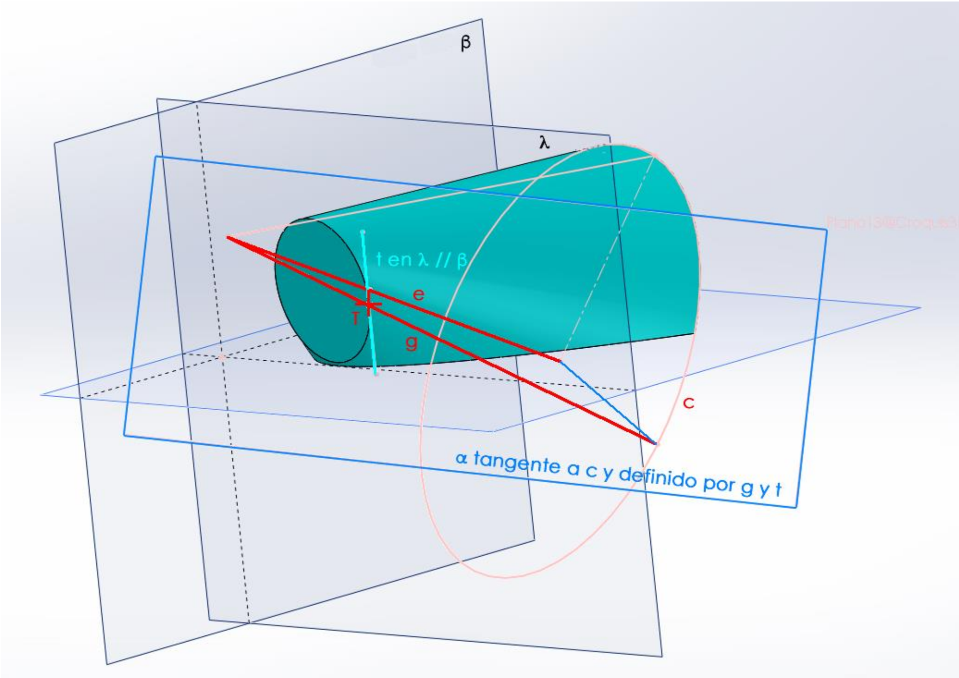
Procedure:

1. Create the line **d** perpendicular to generatrix **g**, perpendicular to reference line **r** and coincident with **P**.

   NOTE: Coincidence between **d** and **P** can be avoided making **d** normal to the solid surface at one of its points. (only to be applied when the solid is known).

54 Limit point of a section

(farther or closer to a plane)



GRAPHIC LEGEND

tangente a = tangent to

definido por = defined by

SDW

NOTE: The method is based on the fact that the limit points of the conic sections of a solid of revolution must be at the intersection of a plane tangent to the solid and the plane that produces the section.

Procedure:

1. Draw the line **t** on the section plane λ and parallel to the reference plane β (plane from which it can be far or close).
2. From point **T** on line **t** draw a generatrix **g** of the solid of revolution.
3. Create the plane α defined by **t** and **g**.
4. Force plane α to be tangent to the cone (it could be done with the radius of an inscribed sphere in the cone or making the plane α tangent to the circular base **c** of the cone).

Method 5: APPROACH BY RELATIONS BETWEEN PIECES

This one may be the method that most closely resembles the design process of a set of pieces.

The design process usually begins with the definition of the needs that the new product must fulfil and its limitations. Both are usually a combination of economic, functional and aesthetic aspects. The set of needs and limitations is usually known by the name of the "requirements" or the "design requirement. The "design requirements" are incorporated to the process by different actors, among which are members of the contracting company and/or the contracted company itself, among others.

The design team must actively participate in the early stages of the design and evaluate the requirements since it is responsible of defining all aspects of the design in order to meet the ''design requirement''.

In general, the design of a product that has to end up satisfying multiple parties is carried out intermittently, iteratively and by successive approximations. Which is translated into the existence of a first draft version to which, in the successive phases, concreteness is added. Each of the phases must be approved by the actors involved in the design. In the end, the result should be described quantitatively. That is to say, exact composition of the materials, sizes of the pieces that compose it, surface roughness, weight, tensile, pressure resistance etc.

CAD assisted design programs facilitate the design process and thus the phased approach to the final result.

When the bodies with which we are working to obtain a final result are based on regular or semi-regular polyhedra and solids of revolution, the CAD tool is likely to help us to solve a part of the problems. If we look at the objects that surround us, we can easily observe that most of them consist of different combinations of regular or semi-regular polyhedra and solids of revolution.

The approximation method for solids of revolution is based on the fact that these solids are always regular and therefore the capabilities of the CAD program can be applied to partially solve the conditions imposed on the requirements. From this first step, are the skills and knowledge of the people involved in the design that guarantees the expected quantitative final result.

References

YouTube channels and lists:

- Geometría Métrica
- Metric Geometry
- Diseño y Tecnología

Knowing authors and teams

- Joaquin Fernandez
- Alba Ramos Cabal
- LAM
- ETSEIB
- UPC

Contents of the same programme

1. Basic Metric Geometry. UPC 2019.
2. Solids of Revolution. Procedures. UPC 2019.
3. Exercises, Problems and Practices. UPC 2019.

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Joaquín Fernández and Alba Ramos Cabal, Barcelona, August 2019.