

Transmission Loss formulas for junctions taking into account in-plane waves

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ABSTRACT

In the transmission of vibrations through structural junctions at high frequencies, the distribution of energy between wave types can suffer important variations. It means, for example, that incoming energy concentrated in a bending wave (out-of-plane vibration) can be split in transmitted energy of other wave types: bending, quasi-longitudinal or transverse shear waves. The goal of the research is to provide simple formulas to include this phenomenon in a Statistical Energy Analysis (SEA) model through the Coupling Loss Factor (CLF) coefficients. A large number of simulations of a population of junctions is generated by means of a numerical model. Afterwards, this data is post-processed in order to obtain the Transmission Loss (TL) between the different wave types by means of a procedure that mimics the Experimental SEA (it is ESEA with some modifications because in the numerical simulations, the internal loss factor is imposed a priori). Each junction is characterised by means of a parameter that minimises the scattering of the TL data with respect to the approximation formula. Some application examples to building structures are shown.

Keywords: K_{ij} , in-plane, vibration

I-INCE Classification of Subject Number: 43 Propagation in structures

(structure-borne noise)

(see <http://i-ince.org/files/data/classification.pdf>)

1. INTRODUCTION

The acoustic design of buildings according to the regulation EN-12354 [1] requires the consideration of flanking transmissions. A key parameter is the vibration reduction index K_{ij} [2]. It characterises the structure-borne transmission. Some methodologies have been developed in order to estimate K_{ij} from the junction parameters by means of simple formulas. The evaluation of a formula is faster than the simulation of the junction by

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means of complex numerical models or to perform experiments [3]. The K_{ij} has been estimated by means of wave-based model in [4, 5], the finite element method (FEM) in [6] or the spectral element method (SFEM) in [7]. The final output of a large number of simulations and parametric analyses is often a set of simple formulas that can be used to simplify the design process [8, 9].

Even if based on three-dimensional models that include all possible wave types (transverse, quasi longitudinal and bending waves), the mentioned formulations focus the interest and outputs on the bending-bending transmission. Considering only the bending waves can be enough for a large number of acoustic designs (out-of-plane displacement and noise generation from walls is mainly caused by bending waves). However, there are some evidences in building acoustics that long flanking transmission paths are largely influenced by in-plane waves [10, 11]. Moreover at mid and high-frequencies the bending behaviour can be non dominant as shown in [7]. Also in the naval industries the effect of in-plane waves influences the noise control inside the ship cabins [12]. It is also clear that when formulating Statistical Energy Analysis (SEA) models, the energy balance is incomplete if only bending-wave subsystems are considered. In general, in the complete modelling of a structure, all wave types need to be considered.

The goals of the present research are:

- To obtain simplified formulas for the estimation of K_{ij} taking into account all the transmission mechanisms and the interaction of all wave types
- Find good descriptor parameters in any of the transmission mechanisms and depending on the plate / shell theory considered

The analysis presented here is restricted to high-frequencies because this is the frequency range where in-plane waves (transverse and quasi longitudinal) are important with respect of out-of-plane waves (bending).

2. DATA ANALYSIS AND APPROXIMATED FORMULAS

The methodology used here is based on the following steps:

1. Consider a set of junctions. The material properties and dimensions are typical of heavy constructions and building acoustics. The damping is constant here with the internal losses of 0.03. For more details, they can be found in [8, 9]
2. Make the computation of the vibration field due to the excitation by means of a point force. The simulations are done by means of FEM [13] and SFEM [7]. As output, the averaged energy on each junction zone and the input power are obtained.
3. This data is used to feed up an Inverse SEA (ISEA) procedure. As output, the CLF are obtained. Details on the ISEA procedure can be found in [14].
4. The Transmission Loss (TL) of each junction is computed as explained in the Section 2.2. A general-purpose TL is obtained for each junction by averaging the results obtained for junctions having different dimensions.
5. An statistical analysis is performed in order to obtain simple expressions that could approximate the data base of TL for each junction.

2.2.1. Description of the SEA model

A general sketch of the general SEA model for the L junction is shown in Fig. 1. It considers two subsystems per plate: out-of-plane and in-plane (or bending, and grouped transverse and quasi longitudinal waves). The couplings between subsystems are multiple (12). Essentially the following types are distinguished:

- Coupling between out-of-plane and in-plane subsystems in the same junction zone (pink). This is neglected due to the very soft interaction between subsystems.
- Coupling between out-of-plane and out-of-plane subsystems in different junction zones (green). Both senses of transmission will be studied together (symmetry in the transmission is assumed), leading to a single formula.
- Coupling between in-plane and in-plane subsystems in different junction zones (red), Both senses of transmission will be studied together (symmetry in the transmission is assumed), leading to a single formula.
- Coupling between out-of-plane and in-plane subsystems in different junction zones (blue). Both senses of transmission will be studied separately (asymmetry in the transmission is assumed), leading to two different sets of formulas.

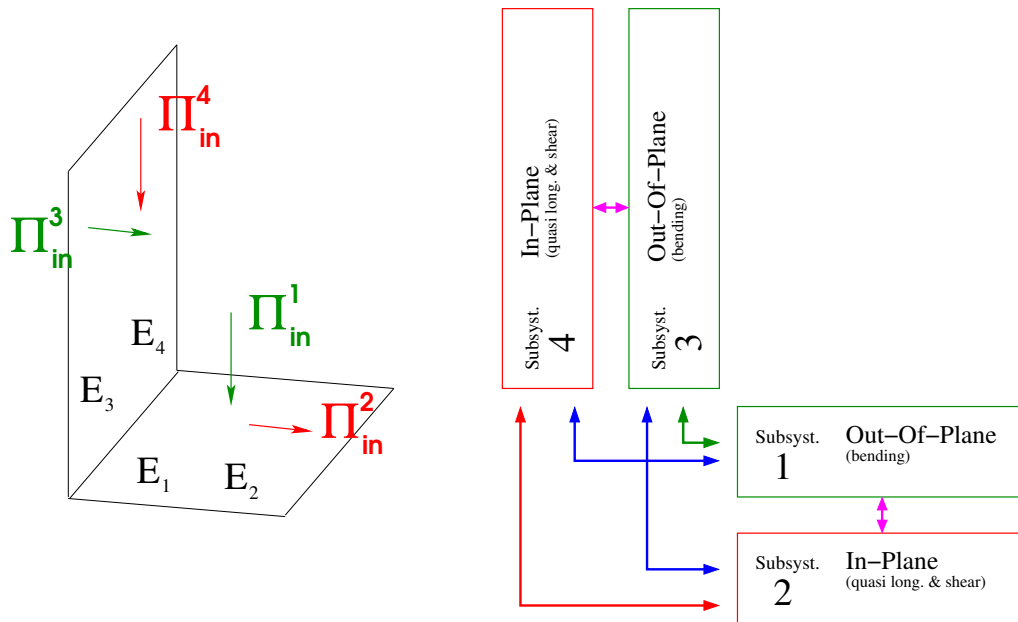


Figure 1: General SEA model for the L junction. It can consider four different subsystems: out-of-plane (subsystems 1 and 3) and in-plane (subsystems 2 and 4) for each region. All the possible connections between them are taken into account.

2.2.2. Compute the TL from the CLF

The assumptions on the SEA model and the definition of subsystems are based on [10]. It is found that there is close to equipartition of energy between longitudinal and transverse modes. This allows the model to be simplified by combining longitudinal and transverse subsystems into a single in-plane subsystem.

The SEA coupling loss factor (from the subsystem i to the subsystem j) is computed as

$$\eta_{ij} = \frac{c_G L_{ij}}{\pi \omega S_i} \tau_{ij} \quad (1)$$

where c_G is the group velocity of the waves in the subsystem i , S_i is the surface of the plate that is represented in the subsystem i , and L_{ij} is the length of the junction between subsystems i and j . It is assumed that Equation 1 is valid whatever the combination (or coupling) of subsystems is considered: out-of-plane with out-of-plane; out-of-plane with in-plane; in-plane with out-of-plane; in-plane with in-plane. Only the group velocity c_G of the waves must be chosen properly. The output to perform the statistical analysis is the transmission loss

$$TL_{ij} = 10 \log_{10} \left(\frac{1}{\tau_{ij}} \right) \quad (2)$$

because it tends to be less variable with frequency than the CLF and it is a more general (applicable to other models) quantity with well known physical meaning.

The wave speeds used to compute the TL from the CLF are as follows. For the bending waves (out-of-plane subsystems)

$$c_B = \sqrt[4]{\frac{\omega^2 E h^3}{\rho_V h 12 (1 - \nu^2)}} = \sqrt{\frac{\omega h c_L}{2 \sqrt{3}}} \quad (3)$$

and the group velocity

$$c_G = 2c_B \quad (4)$$

This is enough for the out-of-plane subsystem. For the quasi-longitudinal wave speed

$$c_G = c_L = \sqrt{\frac{E}{\rho_V (1 - \nu^2)}} \quad (5)$$

And the phase velocity of transverse shear waves is equal to group velocity

$$c_G = c_T = c_L \sqrt{\frac{1 - \nu}{2}} \quad (6)$$

In order to obtain a combined group velocity for the quasi longitudinal and transverse waves (in-plane subsystems), it is assumed like in [10] that everything is proportional to the modal densities of the quasi longitudinal n_L and transverse n_T waves. The relation between modal densities of both wave types is

$$\frac{n_T}{n_L} = \frac{2}{1 - \nu} \quad (7)$$

And the averaged group velocity in the ‘in-plane’ subsystem is

$$c_{LT} = \frac{c_L n_L + c_T n_T}{n_L + n_T} \quad (8)$$

$$c_{LT} = c_L \left(1 + \sqrt{\frac{2}{(1 - \nu)}} \right) \frac{1}{(1 + (2/(1 - \nu)))} \quad (9)$$

With all this we can compute the TL from the CLF.

2.2.3. Find the proper parameters

After the generation of a TL database, the next task is to perform the post-process of data and to be able to find simplified formulas that could approximate all the analysed junction types. The goal is to find a descriptive parameter for each transmission type and junction that distinguish between cases and put them aligned, following some shape that allows ‘easy’ description. The scatter of TL values (groupings in the shape of clouds) should be reduced by a proper choice of this parameter.

A $\frac{\Psi}{\chi}$ parameter was proposed in [6] for the out-of-plane with out-of-plane connection (bending-bending transmission). However, in general it is not the best choice for every transmission type. Some alternatives have been tried. The first one is to define a generalised $\frac{\Psi}{\chi}$ as follows

$$\Upsilon = \left(\frac{E_{\perp}}{E_i} \right)^{\epsilon} \left(\frac{\rho_{V\perp}}{\rho_{Vi}} \right)^{\varrho} \left(\frac{h_{\perp}}{h_i} \right)^{\sigma} \quad (10)$$

where the exponents can be chosen depending on the analysed junction and transmission type. Note that for $\epsilon = 0.75$, $\varrho = 0.25$ and $\sigma = 2.5$, then $\frac{\Psi}{\chi} \equiv \Upsilon$.

The transmission loss is then approximated as

$$TL_{ij} = C_0 + C_1\Upsilon + C_2\Upsilon^2 + C_3\Upsilon^3 \quad (11)$$

The optimal values of the coefficients ϵ , ϱ and σ that reduce the scattering of data are chosen by means of an iterative procedure. They lead to a best fitting curve (in terms of R^2 coefficient for example). This is the procedure used in the results of Section 3.1.

The second possibility is to make some of the coefficients in Equation 11 variable with frequency. The transmission loss is then approximated by an equation of the type

$$TL_{ij} = (C_{0,0} + C_{0,1}f + C_{0,2}f^2) + (C_{1,0} + C_{1,1}f + C_{1,2}f^2)\Upsilon + C_2\Upsilon^2 + C_3\Upsilon^3 \quad (12)$$

This is adequate for frequency-dependent outputs.

Finally, another possibility is to consider some of the parameters defined in [15] to study these type of junctions by means of semi-infinite plates and wave theory:

$$\beta = \frac{c_{B\perp}}{c_{Li}} \quad \text{or} \quad \frac{c_{Bi}}{c_{L\perp}} \quad (13)$$

which are not dimensionless parameters.

For the results presented in this document, only the first option has been explored.

3. RESULTS: L-JUNCTION

The analysis of an L junction is considered here as an example. In all the results shown here, the SFEM uses the Kirchhoff-Love theory to model the bending behaviour. The main difference with the Uflyand–Mindlin theory ([16, 17]) is the frequency limit of physical validity of the model and the optimal values of the parameters ϵ , ϱ and σ in order to obtain a best fitting.

Two kind of outputs are shown. In Section 3.1 the TL values averaged in the third octave bands between 1000.0 Hz and 5000.0 Hz. In Section 3.2 the dependence with frequency is shown.

The simulations are divided in two groups depending on the transmission sense. For example, the bending-bending TL can be computed from the CLF $\eta_{1,3}$ or $\eta_{3,1}$. The data is approximation by means of a least squares fitting curve that is based on Equation 11.

3.3.1. Averaged high-frequency outputs

Fig. 2 shows the four different type of transmissions considered: $\tau_{i,i}$ (red); $\tau_{b,i}$ and $\tau_{i,b}$ (blue) and $\tau_{b,b}$ (green) connections according to Fig. 1 ('*i*' is for 'in-plane' and '*b*' is for out-of-plane or bending). Each set of points (each of them represents a junction) is plotted using an optimal set of parameters ϵ , ϱ and σ to define Υ . There are usually several sets of values that provide similar curve fitting. So, the optimal solution would not be strictly unique and at the end, it is an balanced solution between a good value of R^2 and a set of ϵ , ϱ and σ that could have some physical meaning.

For the $\tau_{b,b}$ in Fig. 2(a), the results are compared with those obtained with an ISEA procedure that accounts only for bending subsystems. The differences are not very large, especially for $\Upsilon > 0.1$.

The case where the scattering of data is less reduced with the optimal choice of ϵ , ϱ and σ is the $\tau_{i,i}$ in Fig. 2(b).

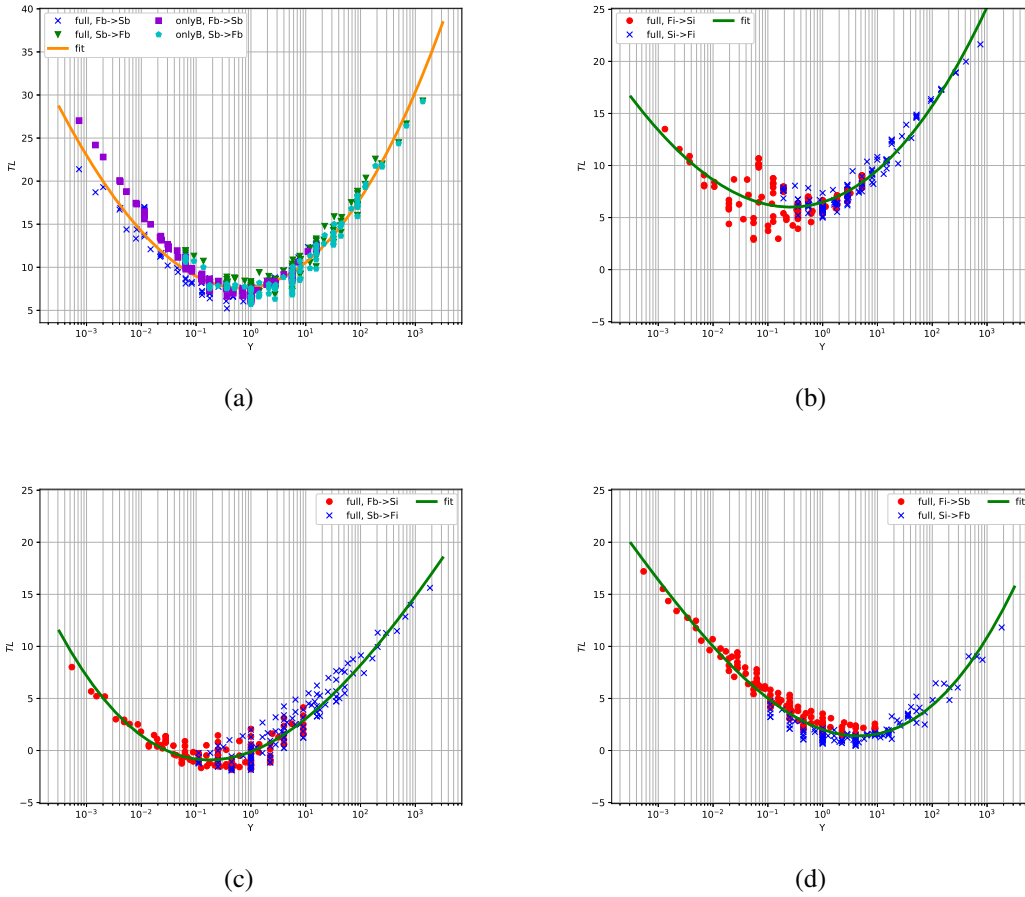


Figure 2: *L*-junction, results averaged for all the bands between 1000.0 Hz and 5000.0 Hz: (a) $\tau_{b,b}$, in the definition of $\frac{\Psi}{\chi} \equiv \Upsilon$: $\epsilon = 0.75$, $\varrho = 0.25$, $\sigma = 2.5$; (b) $\tau_{i,i}$, in the definition of Υ : $\epsilon = 0.5$, $\varrho = 3.0$, $\sigma = 2.0$; (c) $\tau_{b,i}$, in the definition of Υ : $\epsilon = 1$, $\varrho = 0.5$, $\sigma = 2$; (d) $\tau_{i,b}$, in the definition of Υ : $\epsilon = 1$, $\varrho = 0.5$, $\sigma = 2$.

3.3.2. Frequency-dependent outputs

The results of Section 3.1 are shown here without frequency average. The fitting curves are those obtained in Section 3.1. They are included to have a reference in order to see if the trend and the scatter of data are frequency-dependent.

The curves for $\tau_{b,b}$ in the Fig. 3 are almost invariable with frequency. The minimum remains more or less constant in the position $\Upsilon = 1$.

$\tau_{i,i}$ in the Fig. 4 remains more or less frequency invariable in its general trend. However, it suffers a strong reduction of the scatter when the frequency increases.

Finally, the values of $\tau_{b,i}$ in the Fig. 5 and $\tau_{i,b}$ in the Fig. 6, exhibit a clear dependence with frequency. The TL values are smaller for higher frequencies. The scatter of data is in general reduced for all four cases when the frequency increases.

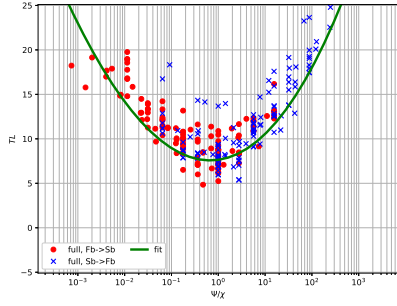
4. CONCLUSIONS

It has been shown how simple curves can be obtained in order to describe the TL of an L-shaped junction between different wave types. However, some tasks are pending on this research:

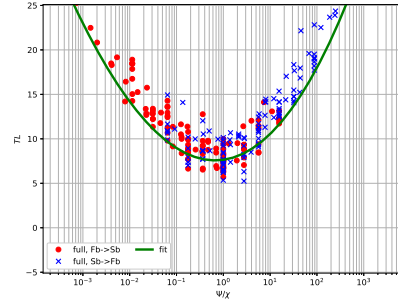
- In addition to the L-shaped junction, consider the other most common junction types: T-shaped and X-shaped. And derive analytical expressions for the fitting curves.
- Find the best parameter Υ for each junction type, plate theory and wave interaction. Try to reduce the scatter of data. Define also for which junctions the behaviour need to be considered frequency-dependent.
- Study the influence of damping in the determination of the simplified formulas
- Test the reliability of the proposed formulas in large problems (i.e. a full building frame). Verify that the provided SEA coefficients are valid to predict the high-frequency limit of a deterministic model/simulation (FEM, SFEM)
- Comparison with some available models: indications in [15, 18] and complex models based on wave theories and infinite junctions such as [19].

5. REFERENCES

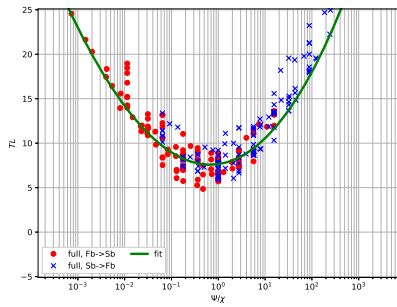
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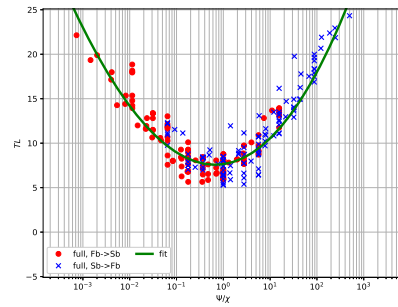
(a) 1000 Hz



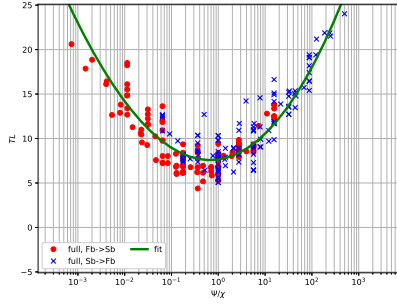
(b) 1250 Hz



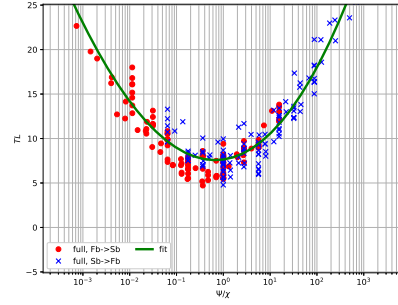
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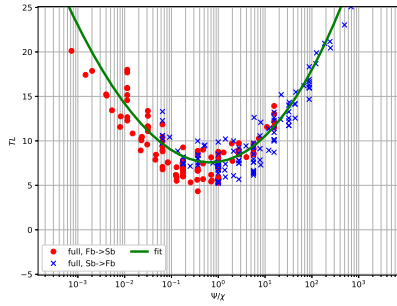
(d) 2000 Hz



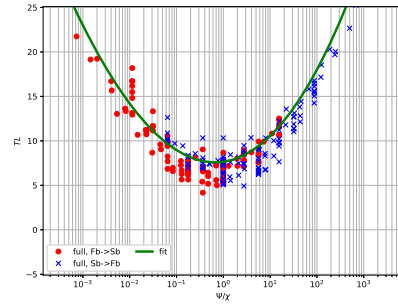
(e) 2500 Hz



(f) 3150 Hz

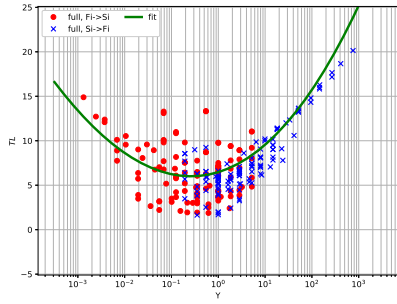


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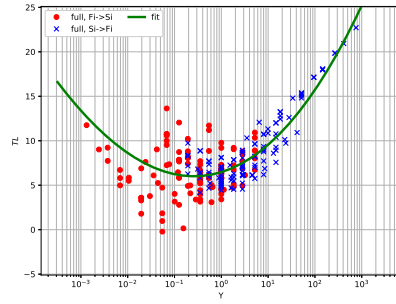


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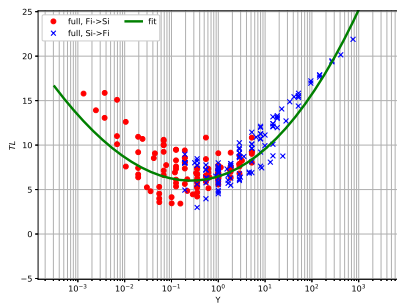
Figure 3: L -junction. $\tau_{b,b}$. In the definition of $\frac{\Psi}{\chi} \equiv \Upsilon$: $\epsilon = 0.75$, $\varrho = 0.25$, $\sigma = 2.5$.



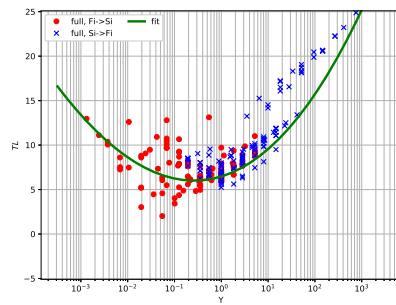
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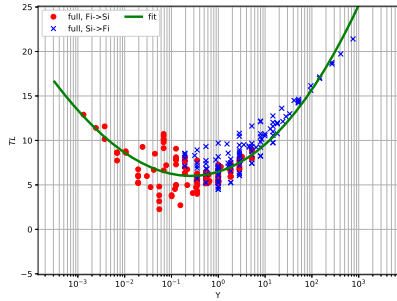
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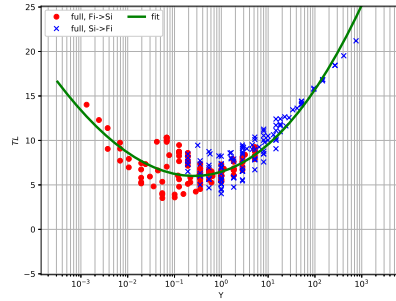
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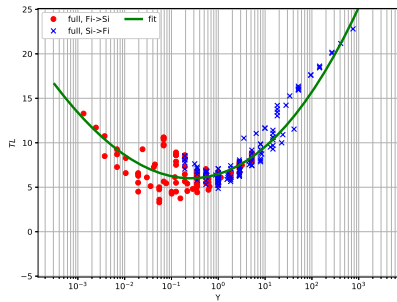
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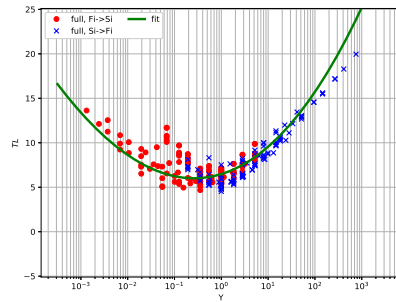
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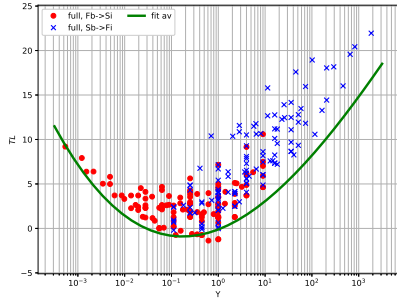


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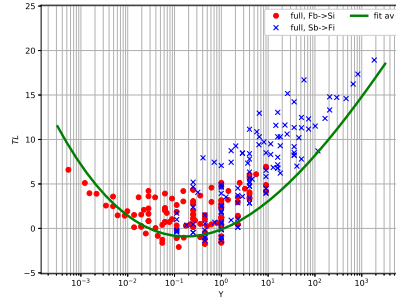


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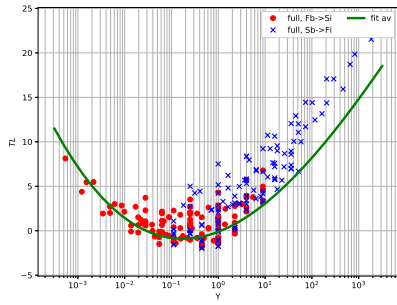
Figure 4: *L*-junction. $\tau_{i,i}$. In the definition of Υ , $\epsilon = 0.5$, $\varrho = 3.0$, $\sigma = 2.0$.



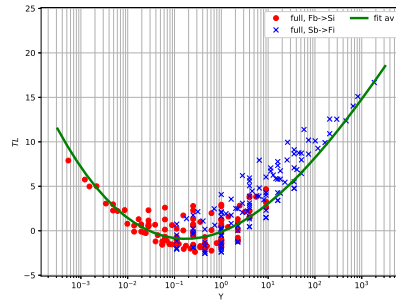
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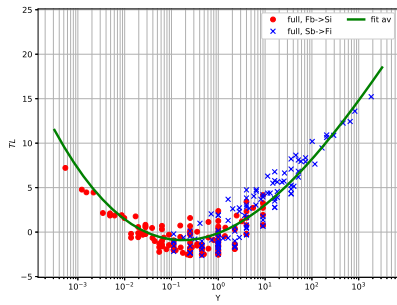
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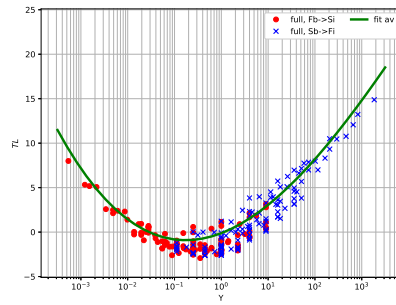
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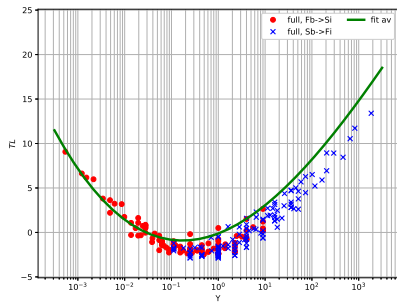
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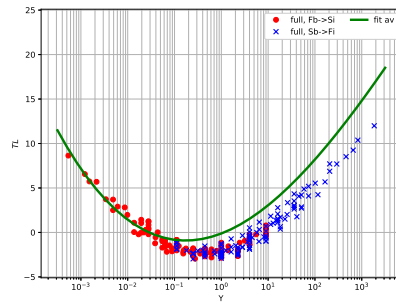
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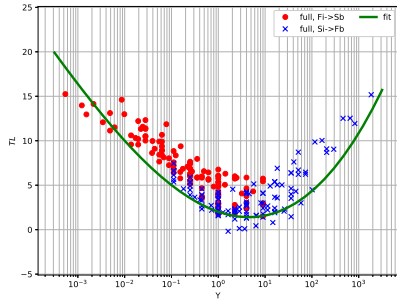


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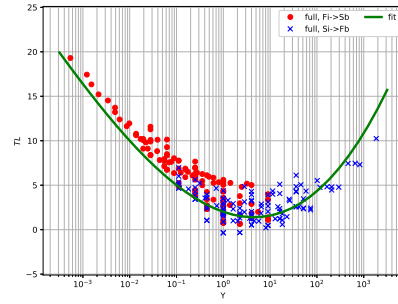


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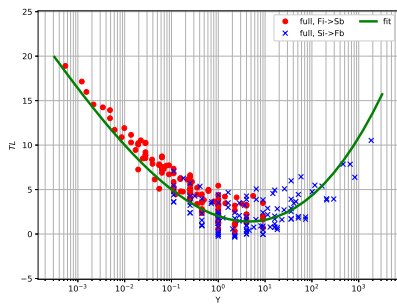
Figure 5: L -junction. $\tau_{b,i}$. In the definition of Υ , $\epsilon = 1.0$, $\varrho = 0.5$, $\sigma = 2.0$. Only in this case, a frequency-dependent fitting based on Equation 12 has been included ('fit f ').



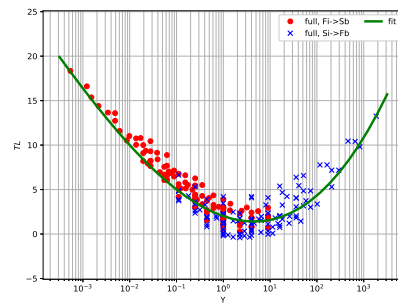
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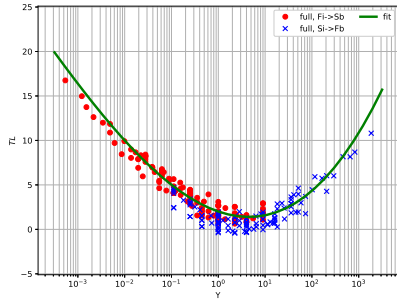
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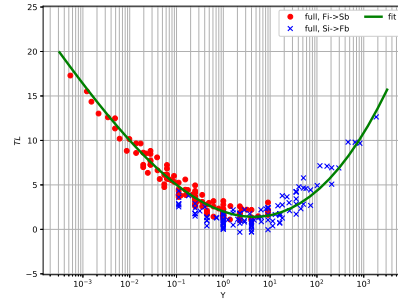
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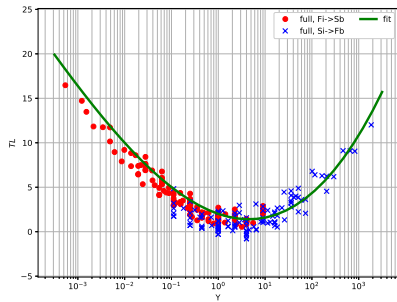
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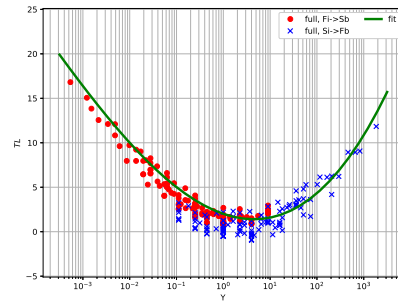
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(g) 4000 Hz



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Figure 6: L -junction. $\tau_{i,b}$. In the definition of \mathcal{Y} : $\epsilon = 1$, $\varrho = 0.5$, $\sigma = 2$.

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