An investigation of production and transportation policies for multi-item and multi-stage production systems

vom Fachbereich Rechts- und Wirtschaftswissenschaften
der Technischen Universität Darmstadt

zur Erlangung des akademischen Grades
Doctor rerum politicarum
(Dr. rer. pol.)

genehmigte Dissertation von

Dipl.-Wi.-Math. Fabian Günter Beck

Erstgutachter: Prof. Dr. Christoph Glock
Zweitgutachterin: Prof. Dr. Anne Lange

Darmstadt 2018

Beck, Fabian: An investigation of production and transportation policies for multi-item and multi-stage production systems
Darmstadt, Technische Universität Darmstadt,
Jahr der Veröffentlichung der Dissertation auf TUprints: 2019
Tag der mündlichen Prüfung: 03.12.2018

Danksagung

„Das Ende kommt gewisslich“ gilt nun auch für meine Dissertation, die ich am Fachgebiet „Produktion und Supply Chain Management“ an der TU Darmstadt angefertigt habe.

Mein besonderer Dank gilt meinem Doktorvater, Prof. Dr. Christoph Glock, für die Betreuung und die fortwährende Unterstützung bei meinen Forschungsprojekten. Besonders hervorheben möchte ich dabei auch seine Besetzung unseres „PSCM-Teams“, die maßgeblich zur Schaffung eines angenehmen Arbeitsumfeldes beigetragen hat. Vor allem möchte ich mich aber bei ihm für die Unterstützung in der schweren Phase vor meiner Disputation bedanken, in der er mich immer wieder aufgebaut und mir dadurch viel Kraft gegeben hat. Außerdem möchte ich meiner Zweitgutachterin Prof. Dr. Anne Lange danken. Einerseits für die Bereitschaft, das Gutachten zu erstellen, und andererseits für ihre hilfsbereite Art, die für mich bei den Vorbereitungen des Logistics Summits sowie der Forschungssäulentreffen sehr wertvoll war.

Mein Dank gilt außerdem meinen Kollegen an der TU Darmstadt, die mich jeden Tag mit einem Lächeln ins Büro haben kommen lassen. Dabei geht ein großer Dank an meinen Bürokollegen Dr. Eric Grosse, der mir bei unzähligen Formatierungs- sowie Formulierungsfragen mit Rat und Tat zur Seite stand. Viel bedeutender ist aber die sehr gute Freundschaft, die sich zwischen dem Urbayer Eric und mir als Franken entwickelt hat und z.B. bei gemeinsamen Schafkopfabenden intensiviert wurde. Ein besonderer Dank geht auch an meine beiden ehemaligen Kollegen Dr. Jörg Ries und Dr. Konstantin Biel. Mit Jörg kamen Gespräche über fachliche, politische und wirtschaftliche Themen nie zu kurz und er bestach in unserer gemeinsamen Zeit darüber hinaus durch seine nützlichen Praxistipps, z.B. den Einsatz von Goldbarren als Zahlungsmittel. Bei Konstantin möchte ich mich zunächst für die hilfreichen Besprechungen von mathematischen Modellen bedanken. Außerdem werden mir die intensiven Diskussionen mit ihm über Sportereignisse aller Art und die Anwendung möglichst vieler deutscher Redewendungen in unseren Gesprächen in positiver Erinnerung bleiben. Hervorzuheben ist noch die Freundschaft, die sich mit Dr. Hamid Abedinnia entwickelt hat. Neben den fachlichen Gesprächen zu verschiedensten Reihenfolgeproblemen bei Maschinenbelegungen waren insbesondere die Einblicke in die iranische Kultur als auch nähere politische Informationen sehr spannend für mich, um meinen Horizont zu erweitern. Ein besonderer Dank geht darüber hinaus an Sabrina Göttmann, die durch ihre immer fröhliche Art einen großen Anteil an dem positiven Arbeitsumfeld hatte und mit mir die große Leidenschaft für Kaffee geteilt hat. Es ist ihr darüber hinaus gelungen, mich für die Teilnahme an der „bewegten Mittagspause“ zu motivieren, wodurch sie einen positiven Einfluss auf mein körperliches Wohlbefinden hatte. Vielen Dank auch an Kerstin Spiehl, die mir in den vergangenen Jahren immer wieder bei organisatorischen Problemen aller Art unter die Arme gegriffen hat. Ein Dankeschön geht an meine beiden neuen Kollegen Marc Füchtenhans und Heiko Diefenbach, die sich in kurzer Zeit schon sehr gut ins Team integriert haben und mir eine große Hilfe bei der Organisation meiner Disputation waren.

Danken möchte ich auch meinen Würzburger Kommilitonen und Freunden Dr. Daniela Anton, Dr. Andrea Betz, Dr. Thomas Christ, Dr. Kristina Lurz, Dr. Frederike Rüppel und Dr. Franziska Ziemer, die mich im Rahmen des Studiums und der Promotionszeit begleitet und moralisch unterstützt haben, sodass sie einen festen Platz in meinem Freundeskreis gefunden haben.

Besonders danken möchte ich Isabella Zerritsch, die für mich in den vergangenen Jahren immer ein offenes Ohr bei Problemen hatte und eine zuverlässige Anlaufstelle in meinem Leben darstellt und mir immer mit Rat und Tat zur Seite steht. Unvergessen bleiben für mich natürlich zahlreiche lustige oder besondere Erlebnisse, wie unsere Shoppingtour in Lübeck oder das gemeinsame Schauen des ersten Europapokalspiels meines FCs nach 25 Jahren.

Ein ganz besonderer Dank gilt meinen Eltern Maria und Edgar, die mich auf meinem bisherigen Lebensweg vom Kindergarten bis zur Promotion begleitet und in jeder Lebensphase unterstützt haben. Ohne ihre Unterstützung wäre mein Studium und auch die Promotionszeit sicherlich nicht umsetzbar gewesen. Ich freue mich ganz besonders für sie, dass sie jetzt das Leben ohne Arbeit in vollen Zügen genießen können. Unendlich dankbar bin ich auch meinem Tischtennis- und Tennisdoppelpartner, besten Freund und hauptberuflichen Bruder Tobias Beck, der meine Dissertation Symbol für Symbol Korrektur gelesen hat und hoffentlich bald nicht mehr von „batch shipments“ träumt.

Widmen möchte ich meine Doktorarbeit aus gegebenem Anlass zwei ganz besonderen Menschen in meinem Leben: Meiner Oma Gustl und meinem Opa Hans. Leider konnten beide meinen Abschluss nicht mehr miterleben, aber ohne ihre bedingungslose Unterstützung und Hilfe in allen Lebenslagen wären sicher einige Dinge in meinem bisherigen Leben nicht möglich gewesen. Ganz besonders möchte ich ihnen für die Werte danken, die sie mir vermittelt haben. Ich werde sie nie vergessen und immer versuchen, „das Beste draus zu machen“.

Darmstadt, 2019 Fabian Beck

Zusammenfassung

Die vorliegende kumulative Dissertation besteht aus fünf Artikeln, einem Arbeitspapier und vier Artikeln, die in wissenschaftlichen Zeitschriften veröffentlicht wurden. Alle fünf Artikel beschäftigen sich mit der Losgrößenplanung, jedoch mit unterschiedlichen Schwerpunkten. Artikel 1 bis 4 untersuchen das Economic Lot Scheduling Problem (ELSP), während sich der fünfte Artikel mit einer Variante des Joint Economic Lot Size (JELS) Problems beschäftigt. Die Struktur dieser Dissertation trägt diesen beiden Forschungsrichtungen Rechnung und ordnet die ersten vier Artikel dem Teil A und den fünften Artikel dem Teil B zu.

Teil A untersucht Entscheidungen bezüglich der Losgrößen- und Reihenfolgeplanung mit einem besonderen Fokus auf dem ELSP. Das ELSP in seiner ursprünglichen Form betrachtet eine Situation, bei der mehrere Produkte auf einer Maschine produziert werden müssen. Die Zielsetzung besteht darin, einen kostenminimalen Produktionszyklus zu ermitteln, der frei von Überschneidungen ist und die Nachfrage des Konsumenten ohne Unterbrechungen befriedigt. Artikel 1 präsentiert zunächst die Ergebnisse einer Inhaltsanalyse zum ELSP, um die zentralen Forschungsthemen aufzudecken und mögliche Gebiete für zukünftige Forschungsarbeiten zu identifizieren. Die verbleibenden vier Artikel entwickeln mathematische Modelle für ungelöste Problemstellungen des ELSP und schlagen passende Lösungsverfahren vor. Artikel 2 erweitert das ELSP, indem zusätzlich noch Energieverbräuche betrachtet werden, die während des Produktionsprozesses anfallen. Der Beitrag untersucht, wie die Betrachtung von Energieverbräuchen die Reihenfolgebelegung der Produkte auf der Maschine beeinflusst. Artikel 3 und 4 erweitern das klassische ELSP, indem zusätzlich Transportlose betrachtet werden, wobei Artikel 3 einen heuristischen und Artikel 4 einen optimierenden Lösungsansatz (dynamische Programmierung) vorschlägt.

Im Teil B der vorliegenden Dissertation werden Losgrößenentscheidungen in Zulieferer-Käufer-Beziehungen (innerhalb einer Supply Chain) im Rahmen von JELS-Modellen näher betrachtet. Im Allgemeinen untersuchen diese Modelle Losgrößenentscheidungen in Supply Chains und zielen darauf ab, sowohl Losgrößen- als auch Transportpolitiken zu bestimmen, die die Leistung der gesamten Supply Chain optimieren, anstatt sich auf die individuell optimalen Politiken der Unternehmen innerhalb der Supply Chain zu konzentrieren. Artikel 5 erweitert das JELS Problem für den Fall, dass mehrere Zulieferer einen Käufer mit einem Produkt mittels geometrisch ansteigender Transportlose beliefern.

In Artikel 1 wird eine Inhaltsanalyse zur Untersuchung des ELSP einschließlich seiner verschiedenen Problemvarianten und Erweiterungen durchgeführt, um Muster bei Veröffentlichungen, Hauptthemen und Forschungstrends in diesem Bereich zu identifizieren. Die Untersuchung der relevanten Artikel wird in neun verschiedene Kategorien unterteilt: I) Art des Problems, II) Strategien der Reihenfolgeplanung, III) Lösungsverfahren, IV) spezielle Annahmen, V) Strömungsmuster, VI) strukturelle Eigenschaften des ELSP, VII) Zielsetzungen der Modelle, VIII) Erweiterung des ELSP, und IX) sonstige Themen. Aufbauend auf den Ergebnissen der Inhaltsanalyse schließt der Artikel mit der Diskussion zukünftiger Forschungsmöglichkeiten und bildet die Grundlage für die Artikel 2 bis 4 in dieser Dissertation.

Artikel 2 untersucht eine Planungssituation des ELSP, bei der das Produktionssystem beim An- und Abschalten der Maschine sowie während den Stillstandszeiten und Produktionsphasen Energiekosten verursacht. Im ersten Schritt werden Energiekostenfunktionen für die verschiedenen Betriebszustände der Maschine vorgestellt, die anschließend in den Ansatz des gemeinsamen Produktzyklus und den Basisperiodenansatz integriert werden. Für beide Strategien der Reihenfolgeplanung werden zweistufige Optimierungsverfahren entwickelt. Die erste Stufe der Lösungsverfahren fokussiert sich dabei auf die Maschinenbelegungszeit und legt fest, ob sich eine Maschine im Produktions- oder Rüstzustand befinden sollte. In der zweiten Stufe werden die Stillstandszeiten der Maschine näher betrachtet und die Entscheidung getroffen, ob die Maschine in den Standby-Modus versetzt oder ganz ausgeschaltet werden soll. In numerischen Studien wird anschließend gezeigt, dass sich Produktionspläne signifikant ändern, wenn Energieaspekte mit in die Betrachtung einbezogen werden. Die Ergebnisse deuten außerdem darauf hin, dass das entwickelte Lösungsverfahren für den Basisperiodenansatz versucht, Werkzeugwechsel zu vermeiden, und damit im Vergleich zum ursprünglichen Lösungsverfahren die zugehörigen Energiekosten verringert.

Artikel 3 erweitert das klassische ELSP um die Möglichkeit der gleich und ungleich großen Transportlosweiterleitung und vergleicht die Ergebnisse mit dem Fall, dass nur ganze Lose transportiert werden können. Dazu wird die klassische Zielfunktion modifiziert, um Rüst-, Lagerhaltungs- und Transportkosten zu berücksichtigen. Um die Produktionspolitiken anschließend zu ermitteln, werden die unabhängige Lösung, der Ansatz des gemeinsamen Produktzyklus und ein heuristischer Basisperiodenansatz mit zwei Zuordnungsverfahren auf das neue Problem angepasst. Die numerischen Studien zeigen, dass die Aufteilung eines Loses in gleich oder ungleich große Transportlose im ELSP die Gesamtkosten des Produktionssystems signifikant reduzieren kann.

Artikel 4 erweitert Artikel 3, indem er gleich große und geometrisch ansteigende Transportlose im ELSP mit Hilfe eines optimierenden Basisperiodenansatzes untersucht. Zuerst wird die aus der Literatur bekannte mathematische Formulierung eines optimierenden Basisperiodenansatzes modifiziert, so dass gleich große und geometrisch ansteigende Transportlose Berücksichtigung finden können. Dann wird das Lösungsverfahren des optimierenden Basisperiodenansatzes auf die Planungssituation angepasst. Anschließend wird das entwickelte Modell mit seinem Lösungsverfahren mit den alternativen Ansätzen, die in Artikel 3 vorgestellt wurden, verglichen, um Einblicke in die relative Vorteilhaftigkeit des neuen Ansatzes zu gewinnen. Es kann gezeigt werden, dass der relative Performancenachteil des optimierenden Basisperiodenansatzes im Vergleich zu den anderen Lösungsverfahren verbessert werden kann, falls Fertigungslose in gleich große und geometrisch ansteigende Transportlose aufgeteilt werden können.

Artikel 5 beschäftigt sich mit dem JELS-Modell und betrachtet einen Käufer, der ein Produkt von mehreren homogenen Lieferanten bezieht. Das klassische JELS-Problem wird durch die Annahme erweitert, dass die Lieferanten die Möglichkeit besitzen, ihre Lose in geometrisch ansteigenden Transportlosen zu liefern, bei denen die Größe von aufeinander folgenden Transportlosen gemäß eines festen Faktors wächst. Zwei Koordinierungsmechanismen, nämlich sofortige und verzögerte Lieferungen, werden verwendet, um den Zeitpunkt der Lieferungen festzulegen. Für dieses Szenario werden mathematische Modelle und die zugehörigen Lösungsverfahren entwickelt. Die Modelle werden dann mittels numerischer Studien veranschaulicht und die Performance der vorgestellten Modelle mit der Situation verglichen, in der gleich große Transportlose zum Käufer geliefert werden. Es werden dabei der Einfluss der Produktionsrate, der Transportkosten und das Verhältnis der Lagerhaltungskosten der Lieferanten zu denen des Käufers auf die Anzahl an Lieferungen und die Gesamtkosten untersucht. Die Ergebnisse deuten darauf hin, dass keines der Modelle (sofortige Lieferung mit gleich großen Transportlosen, sofortige Lieferung mit geometrisch ansteigenden Transportlosen, verzögerte Lieferung mit gleich großen Transportlosen und verzögerte Lieferung mit geometrisch ansteigenden Transportlosen) die jeweils anderen Modelle in allen Szenarien dominiert. Daher muss in der Produktionsplanung sorgfältig abgewogen werden, welcher Koordinationsmechanismus für die vorliegende Planungssituation verwendet werden soll. Die vorgestellten Modelle bieten dafür Entscheidungsunterstützung.

Abstract

This cumulative dissertation consists of five papers, one working paper and four papers published in scientific journals. All five papers deal with lot sizing problems, albeit with different foci: Four papers investigate the Economic Lot Scheduling Problem (ELSP), while the fifth paper studies a variant of the Joint Economic Lot Size (JELS) problem. The structure of this dissertation reflects these two research streams by grouping the first four papers in Part A and by assigning the fifth paper to Part B. Part A studies lot sizing and machine scheduling decisions with a special focus on the ELSP. The ELSP considers a situation where several products have to be produced on a single facility. The objective in this case usually is to generate a cost-minimal production schedule that is free from overlaps and that satisfies the costumers' demand without interruptions. Paper 1 first presents the results of a content analysis to find key themes discussed in research on the ELSP and to identify areas for future research. The remaining four papers develop mathematical models and propose suitable solution methodologies. Paper 2 extends the ELSP to take account of energy consumption during production, proposes solution methodologies and investigates how energy consumption influences the scheduling of products on the machine. Papers 3 and 4 extend the classical ELSP to take account of batch shipments, with Paper 3 employing a heuristic solution approach and Paper 4 adopting an analytical one (dynamic programming). Part B of this dissertation studies lot sizing decisions in a supply chain context. JELS models, in general, study lot sizing decisions in supply chains and aim on deriving lot sizing and transportation policies that optimize the performance of the entire supply chain, instead of focusing on the individual positions of the supply chain members. Paper 5 extends the JELS problem to the case where multiple vendors deliver a product to a single buyer in geometrically increasing batch shipments.

Paper 1 applied a content analysis to the literature on the ELSP including various problem variants and extensions to identify publication patterns, main topics and research trends in this area. The analysis of the sampled articles is carried out for nine different categories: I) type of problem, II) scheduling policy, III) solution methodology, IV) specific assumptions, V) flow pattern, VI) structural properties of the ELSP, VII) scheduling objectives, VIII) extended coverage, and IX) other topics. Based on the results of the content analysis, the work concludes with future research opportunities and provides the basis for Papers 2 to 4 of this dissertation.

Paper 2 studies the ELSP for a situation where the production system incurs energy costs during start-up and shutdown of the machine as well as during idle and production phases. In a first step, the paper proposes energy cost functions for different machine operating states that are then integrated into the Common-Cycle-Approach and into the Basic-Period-Approach. For both scheduling policies, two-stage optimization procedures are developed. The first stage of the solution procedures focuses on the machine occupancy time and determines whether a machine should be in the production or setup mode. In the second stage of the solution procedures, the machine idle time is considered, and the decision is made whether to leave the machine in the idle operation mode or to switch it off. In numerical studies, Paper 2 shows that production schedules significantly change when energy aspects are taken into account. The results also indicate that the developed solution procedure of the Basic-Period-Approach tries to avoid tool changes, and that the corresponding energy costs are reduced, as compared to the original solution procedure.

Paper 3 extends the classical ELSP by taking account of both equal-sized and unequal-sized batch shipments and it compares their performance to the complete lot shipment policy. The objective function is modified to account for setup cost, inventory holding cost and transportation cost. To derive the production policies, the independent solution, the Common-Cycle-Approach, and a heuristic Basic-Period-Approach with two assigning procedures are adapted to the new problem. The numerical studies show that splitting up a lot into equal-sized or unequal-sized batches in the ELSP context can significantly reduce the total cost of the production system.

Paper 4 extends Paper 3 by investigating equal-sized and geometrically increasing batch shipments in the context of an analytical Basic-Period-Approach. First, the mathematical formulation of an analytical Basic-Period-Approach is modified taking equal-sized and geometrically increasing batch shipments into account. Secondly, the solution procedure of the analytical Basic-Period-Approach is adjusted to the planning situation. Subsequently, the developed model and its solution procedure are compared with alternative approaches proposed in Paper 3 to gain insights into the relative advantage of the new approach. It can be shown that the relative performance disadvantage of the analytical Basic-Period-Approach as compared to the other solution procedures can be improved by permitting equal-sized or geometrically increasing batch shipments.

Paper 5 addresses a so-called Joint Economic Lot Size (JELS) model with a single buyer who sources a single product from multiple homogeneous vendors. The existing literature on the JELS problem is extended by assuming that the vendors have the opportunity to deliver their lots in geometrically increasing batch shipments, where subsequent batch shipments increase in size according to a fixed factor. Two coordination mechanism, namely immediate and delayed deliveries, are used to specify the timing of deliveries. For this scenario, mathematical models and associated solution methods are developed. The models are then illustrated in numerical experiments, and the performance of the proposed models is compared to the situation where batches are shipped in equal sizes to the buyer. The influence of the production rate, the transportation cost, and the relation of the inventory holding cost of the vendors to those of the buyer on the number of shipments as well as the total system cost are investigated. The results indicate that none of the models (immediate delivery with equal-sized batch shipments, immediate delivery with geometrically increasing batch shipments, delayed delivery with equal-sized batch shipments and delayed delivery with geometrically increasing batch shipments) dominates the respective other models in all possible scenarios. Hence, production planers have to evaluate carefully which coordination mechanism to use for the planning situation at hand. The proposed models support this evaluation.

Outline

Table of Contents

List of Abbreviations

Introduction:

ELSP	Economic lot scheduling problem
JELS	Joint economic lot size

Paper 1:

ABC	Artificial bee colony
ACA	Ant colony algorithm
ACO	Ant colony optimization
B&B	Branch & bound
BP	Basic period
BPA	Basic-period-approach
CA	Content analysis
CC	Common cycle
CCA	Common-cycle-approach
CS	Cuckoo search
CLSP	Capacitated lot sizing problem
DLSP	Discrete lotsizing and scheduling problem
DP	Dynamic programming
EA	Evolutionary algorithm
EBP	Extended basic period
EBPA	Extended-basic-period-approach
ELDSP	Economic lot and delivery scheduling problem
ELS	Equal lot size
ELSDP	Economic lot scheduling and delivering problem
ELISP	Economic lot and inspection scheduling problem
ELSSP	Economic lot and supply scheduling problem
ELSP	Economic lot scheduling problem
ELSPR	Economic lot scheduling problem with returns
EMQ	Economic manufacture quantity
EOQ	Economic order quantity
EPQ	Economic production quantity

FC	Fundamental cycle
FS-ELSP	Flow shop-economic lot scheduling problem
GA	Genetic algorithm
GLSP	General lotsizing and scheduling problem
GT-ELSP	Group technology-economic lot scheduling problem
GSS	Golden section search
HGA	Hybrid genetic algorithm
ILP	Integer linear programming
ILS	Iterated local search
LB	Lower bound
LP	Linear programming
MILP	Mixed integer linear programming
MINLP	Mixed integer nonlinear programming
MTO	Make-to-order
MTS	Make-to-stock
PLSP	Proportional lotsizing and scheduling problem
POT	Power-of-two
POW2	Power-of-two
PSO	Partical swarm optimization
SA	Simulated annealing
SELSP	Stochastic economic lot scheduling problem
SPT	Shortest processing time
TS	Tabu search
TVLS	Time varying lot size
TVLSA	Time-varying-lot-size-approach
UB	Upper bound
WIP	Work in process
ZSR	Zero switch rule

Paper 2:

ELSP	Economic lot scheduling problem
EUR	Euro
h	Hour
kW	Kilowatt
kWh	Kilowatt hour

Paper 3:

BPA	Basic-period-approach
CCA	Common-cycle-approach
EBPA	Extended-basic-period-approach
ELSP	Economic lot scheduling problem
IS	Independent solution
MCCA	Modified common-cycle-approach of Hanssmann
MHH	Modified heuristic of Haessler and Hogue
MIS	Modified independent solution

Paper 4:

BPA	Basic-period-approach
ELSP	Economic lot scheduling problem
mBPAB	Modified basic-period-approach of Bomberger
mBPAHH	Modified basic-period-approach of Haessler and Hogue
mCCAH	Modified common-cycle-approach of Hanssmann
USD	US-Dollar

Paper 5:

DD	Delayed delivery
ID	Immediate delivery
JELS	Joint economic lot size

List of Figures

Introduction

Paper 1

Paper 2

Paper 3

Paper 4

Paper 5

List of Tables

Introduction

Paper 1

Paper 2

Paper 3

Paper 4

Paper 5

Introduction

This cumulative dissertation consists of five papers, one working paper and four papers published in scientific journals (see Table 1 for an overview). All five papers deal with lot sizing problems, albeit with different foci: Four papers investigate the Economic Lot Scheduling Problem (ELSP), while the fifth paper studies a variant of the Joint Economic Lot Size (JELS) problem. The structure of this dissertation reflects these two research streams by grouping the first four papers in Part A and by assigning the fifth paper to Part B. The main contribution of Part A is the extension of the classical ELSP that was defined by Elmaghraby (1978) as "the problem of accommodating cyclical production patterns when several products are made on a single facility" to account for batch shipments and energy consumption during production, while Part B extends the JELS model that was described by Glock (2012b) as a lot size model that focuses "on coordinated inventory replenishment decisions between buyer and vendor and their impact on the performance of the supply chain" to the case where multiple vendors deliver products to a single buyer in geometrically increasing batch shipments. The content of the five papers is summarized in Figure 1.

In Part A of this dissertation, Paper 1 first presents the results of a content analysis to find key themes discussed in research on the ELSP and to identify areas for future research. The remaining four papers develop mathematical models and propose suitable solution methodologies. Paper 2 extends the ELSP to take account of energy consumption during production, proposes solution methodologies and investigates how energy consumption influences the scheduling of products on the machine. Papers 3 and 4 extend the classical ELSP to take account of batch shipments, with Paper 3 employing a heuristic solution approach and Paper 4 adopting an analytical one (dynamic programming). Finally, Paper 5 in Part B, extends the JELS problem to the case where multiple vendors deliver a product to a single buyer in geometrically increasing batch shipments. In the following, the five papers are summarized briefly, and the research gaps that are addressed in each paper are highlighted.

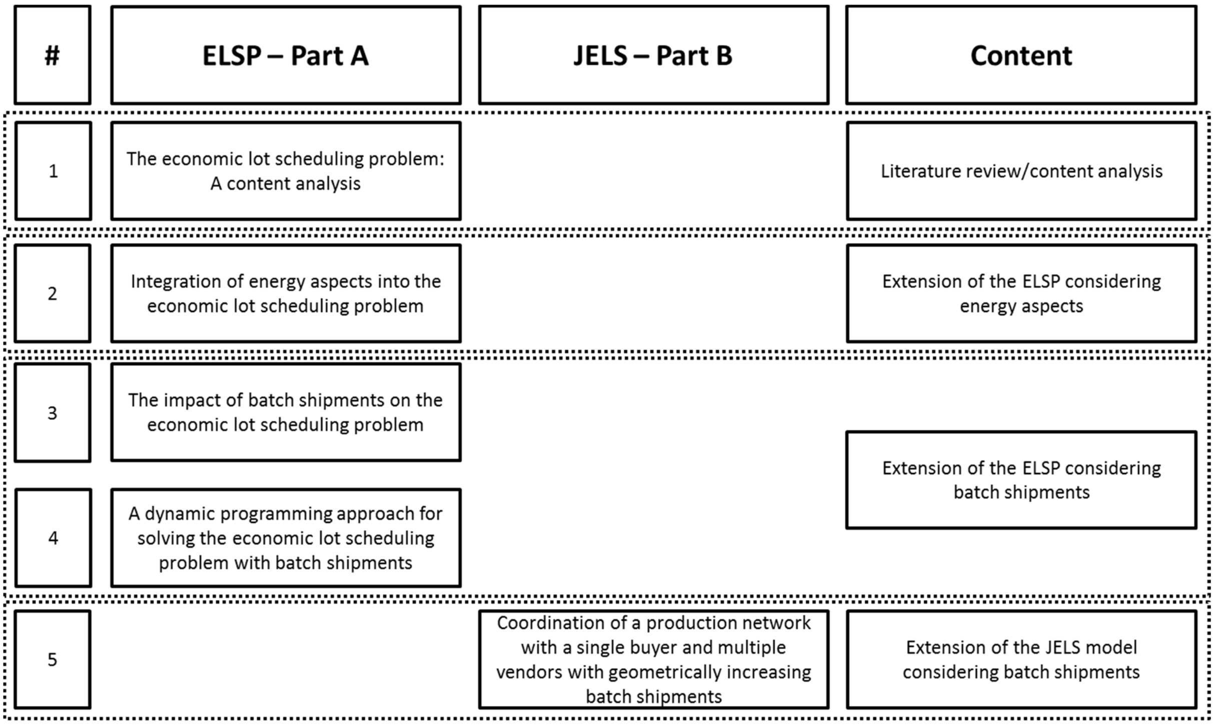

Figure 1: *Classification of the papers included in this cumulative dissertation*

<u>Part A: An investigation of the economic lot scheduling problem with batch shipments and energy considerations</u>

Part A studies lot sizing and machine scheduling decisions with a special focus on the ELSP. The ELSP considers a situation where several products have to be produced on a single facility. The objective in this case usually is to generate a cost-minimal production schedule that is free from overlaps and that satisfies the costumers' demand without interruptions. Paper 1 first analyzes the literature on the ELSP including various problem variants and extensions. To identify publication patterns, main topics and research trends in this area, a content analysis is applied to 228 papers published on the ELSP. Based on the works of Chan et al. (2013) and Santander-Mercado and Jubiz-Diaz (2016), a conceptual framework is first developed deductively and later updated inductively with relevant recording units by analyzing all words, abbreviations and symbols of the final sample. The analysis is carried out for nine different categories: I) type of problem, II) scheduling policy, III) solution methodology, IV) specific assumptions, V) flow pattern, VI) structural properties of the ELSP, VII) scheduling objectives, VIII) extended coverage, and IX) other topics. The results show that the majority of publications on the ELSP have a focus on the development of solution methodologies, and that two scheduling policies, namely the Basic-Period-Approach and the Common-Cycle-Approach, have been especially popular.

The work concludes with suggestions for future research. Two promising research opportunities are the study of energy consumption in production in the context of the ELSP as well as numerical studies that compare the performance of the various solution methodologies that have been proposed in the past. The first research gap is investigated in the second paper contained in this dissertation.

Paper 2 studies the ELSP for a situation where the production system incurs energy costs during start-up and shutdown of the machine as well as during idle and production phases. The consumption of energy and the influence of energy cost on production control has frequently been analyzed both for lot sizing and machine scheduling decision in the past, for example in the works of Collier and Omek (1983), Yildirim and Nezami (2014), Mouzon et al. (2007), or Liu (2016). In light of the fact that the industrial sector is one of the major energy consumers (U.S. Energy Information Administration, 2016), it is surprising that both the literature review of Biel and Glock (2016) and the content analysis in Paper 1 showed that energy aspects have been neglected in the ELSP that combines these two decision problems so far. To close this research gap, the classical ELSP is extended in Paper 2 to take account of energy consumption during production and the consequent energy cost. In a first step, the paper proposes energy cost functions for different machine operating states that are then integrated into the Common-Cycle-Approach originally proposed by Hanssmann (1962) and into the Basic-Period-Approach proposed by Haessler and Hogue (1976). For both scheduling policies, two-stage optimization procedures are developed afterwards. The first stage of the solution procedures focuses on the machine occupancy time and determines whether a machine should be in the production or setup mode. In the second stage of the solution procedures, the machine idle time is considered, and the decision is made whether to leave the machine in the idle operation mode or to switch it off. The results of numerical studies show that production schedules significantly change when energy aspects are taken into account. The results also indicate that the developed solution procedure of the Basic-Period-Approach tries to avoid tool changes, and that the corresponding energy costs are reduced, as compared to the original solution procedure. The models proposed in Paper 2 consequently support production planners in determining both lot sizes and production sequences and in deciding on the operating modes of the production equipment. The proposed models help to lower cost and to increase the energy efficiency of the manufacturing system.

Paper 3 extends the classical ELSP by taking account of both equal-sized and unequal-sized batch shipments. The lot sizing literature has frequently addressed the question of how production quantities (lots) should be delivered from one stage of the production system to the next. Shipping so-called batches (i.e., partial lots) from one stage to the next enables the production system to initiate the consumption of a lot while the production process is still in progress, which reduces inventory in the system and hence inventory carrying cost, albeit at the expense of higher transportation cost. One of the first authors to investigate batch shipments in a lot sizing context was Szendrovits (1975). In his model, a lot may be split up into batches of equal sizes, where the first batch can be shipped to the subsequent stage directly after its completion. Another way to split a lot up into batches was proposed by Goyal (1977), who suggested that subsequent batches should be of unequal sizes. In his model, subsequent batches increase or decrease according to a geometric series depending on the ratio of the stage's production rate to its demand rate, which has been shown to lead to lower total cost as compared to the equal-sized batch shipment policy. In the work of Goyal and Nebebe (2000), the two batch shipment policies were combined in a supply chain context. In this case, the first shipments increase in size by a fixed factor, and the last shipments are of equal sizes. Although the literature has shown that batch shipments can significantly reduce inventory holding cost, there are surprisingly only a few papers that investigate the role of batch shipments in the context of the ELSP. One paper is the one of Buscher (2000), who modified the Common-Cycle-Approach of Hanssmann to take account of equal-sized batch shipments subject to two additional assumptions: I) the planning horizon is finite, and II) the number of batch shipments is equal for all products. Another paper that investigates batch shipments in the ELSP is the one of Ho et al. (2015), who considered the equal-sized batch shipment policy under stochastic demand. Paper 3 extends the existing literature by integrating unequal-sized batch shipments into the ELSP, and it compares their performance to the complete lot and the equal-sized batch shipment policies. The objective functions are modified to account for setup cost, inventory holding cost and transportation cost. To derive the production policies, the independent solution, the Common-Cycle-Approach of Hanssmann (1962), and the Basic-Period-Approach of Haessler and Hogue (1976) with two assigning procedures are adapted to the new problem. The numerical studies show that splitting up a lot into equal-sized or unequal-sized batches in the ELSP context can significantly reduce the total cost of the production system.

Paper 4 extends Paper 3 by investigating equal-sized and geometrically increasing batch shipments in the context of the Basic-Period-Approach proposed by Bomberger (1966). First, the mathematical formulation of the Basic-Period-Approach is modified taking equal-sized and geometrically increasing batch shipments into account. Secondly, the solution procedure of Bomberger is adjusted to the new planning situation. Subsequently, the developed model is compared with alternative approaches proposed in Paper 3, namely the modified Common-Cycle-Approach of Hanssmann and the modified Basic-Period-Approach of Haessler and Hogue to gain insights into the relative advantage of the new model. Two interesting results are obtained. First, it can be shown that the relative performance disadvantage of Bomberger's approach as compared to the other solution procedures can be improved by permitting equal-sized or geometrically increasing batch shipments. Secondly, by reformulating the solution procedure of Bomberger's approach, the results reported in the literature for the classical ELSP (e.g., in Elmaghraby, 1978; Chatfield, 2007) can be improved.

<u>Part B: An investigation of production and transportation policies for multi-actors, multi-stage production systems</u>

Part B of this dissertation studies lot sizing decisions in a supply chain context. Paper 5 addresses a so-called Joint Economic Lot Size (JELS) model with multiple vendors and a single buyer and investigates the impact of geometrically increasing batch shipments on the performance of the supply chain. JELS models, in general, study lot sizing decisions in supply chains and aim on deriving lot sizing and transportation policies that optimize the performance of the entire supply chain, instead of focusing on the individual positions of the supply chain members (Glock 2012b). Earlier research on the JELS problem has shown that especially supply chains with multiple vendors have only infrequently been investigated, and that especially the scheduling of deliveries from the vendors to the buyer requires further investigations (Glock 2012b). Paper 5 addresses this research gap and considers a situation where a single buyer sources a single product from multiple homogeneous vendors. The existing literature on the JELS problem is extended by assuming that the vendors have the opportunity to deliver their lots in geometrically increasing batch shipments, where subsequent batch shipments increase in size according to a fixed factor. Two coordination mechanism proposed earlier by Glock (2012a), namely immediate and delayed deliveries, are used to specify the timing of deliveries. For this scenario, mathematical models and associated solution methods are developed. The models are then illustrated in numerical experiments, and the performance of the proposed models is compared to the situation where batches are shipped in equal sizes to the buyer. The influence of the production rate, the transportation cost, and the relation of the inventory holding cost of the vendors to those of the buyer on the number of shipments as well as the total system cost are investigated. The results indicate that none of the models (immediate delivery with equal-sized batch shipments, immediate delivery with geometrically increasing batch shipments, delayed delivery with equal-sized batch shipments and delayed delivery with geometrically increasing batch shipments) dominates the respective other models in all possible scenarios. Hence, production planers have to evaluate carefully which coordination mechanism to use for the planning situation at hand. The proposed models support this evaluation.

Table 1: *List of papers included in this cumulative dissertation*

Focus	Authors	Title	Journal
An investigation of the economic lot scheduling problem with batch shipments and energy considerations	F.G. Beck, C.H. Glock	The economic lot scheduling problem: A content analysis	Working paper
	F.G. Beck, K. Biel, C.H. Glock	Integration of energy aspects into the economic lot scheduling problem	International Journal of Production Economics
	F.G. Beck, C.H. Glock	The impact of batch shipments on the economic lot scheduling problem	Computers & Industrial Engineering
	F.G. Beck, C.H. Glock	A dynamic programming approach for solving the economic lot scheduling problem with batch shipments	International Journal of Operational Research
An investigation of production and transportation policies for multi-actors, multi-stage production systems	F.G. Beck, C.H. Glock, T. Kim	Coordination of a production network with a single buyer and multiple vendors with geometrically increasing batch shipments	International Journal of Production Economics

References

Biel, K., Glock, C.H., 2016. Systematic literature review of decision support models for energy-efficient production planning. Computers & Industrial Engineering, 101, 243-259.

Bomberger, E., 1966. A dynamic programming approach to a lot size scheduling problem. Management Science, 12(11), 778-784.

Buscher, U., 2000. Lossplitting in der Lossequenzplanung (Lot splitting in the economic lot scheduling problem). WISU – Das Wirtschaftsstudium, 29(2), 197-204.

Chan, H.K., Chung, S.H., Lim, M.K., 2013. Recent research trend of economic-lot scheduling problems. Journal of Manufacturing Technology Management, 24(3), 465-482.

Chatfield, D.C., 2007. The economic lot scheduling problem: a pure genetic search approach. Computers & Operations Research, 34(10), 2865-2881.

Collier, P.I., Ornek, A., 1983. A mathematical model for energy and in-process inventory assessment in a manufacturing system. Applied Energy, 13(4), 265-280.

Elmaghraby, S.E., 1978. The economic lot scheduling problem (ELSP): review and extensions. Management Science, 24(6), 587-598.

Glock, C.H., 2012a. Coordination of a production network with a single buyer and multiple vendors. International Journal of Production Economics, 135(2), 771-780.

Glock, C.H., 2012b. The joint economic lot size: a review. International Journal of Production Economics, 135(2), 671-686.

Goyal, S.K., 1977. Determination of optimum production quantity for a two-stage production system. Operation Research Quarterly, 28(4), 865-870.

Goyal, S.K., Nebebe, F., 2000. Determination of economic production-shipment policy for a single-vendor-single-buyer system. European Journal of Operational Research, 121(1), 175-178.

Haessler, R.W., Hogue, S.L, 1976. Note—A note on the single-machine multi-product lot scheduling problem. Management Science, 22(8), 909-912.

Hanssmann, F., 1962. Operations research in production and inventory control (1st ed.). John Wiley & Sons.

Ho, W. T., Tseng, Y. C., Hsiao, Y. C., 2015. The economic lot-size scheduling problem with equally sized batch shipment policy and stochastic demands. European Journal of Industrial Engineering, 9(2), 195-221.

Liu, C.-H., 2016. Mathematical programming formulations for single-machine scheduling problems while considering renewable energy uncertainty. International Journal of Production Research, 54(4), 1122-1133.

Mouzon, G., Yildirim, M.B., Twomey, J., 2007. Operational methods for minimizing of energy consumption of manufacturing equipment. International Journal of Production Research, 45(18-19), 4247-4271.

Santander-Mercado, A., Jubiz-Diaz, M., 2016. The economic lot scheduling problem: a survey. International Journal of Production Research, 54(16), 4973-4992.

Szendrovits, A.Z., 1975. Manufacturing cycle time determination for a multi-stage economic production quantity model. Management Science, 22(3), 298-308.

U.S. Energy Information Administration, 2016. International Energy Outlook 2016.

Yildirim, M.B., Nezami, F.G., 2014. Integrated maintenance and production planning with energy consumption and minimal repair. The International Journal of Advanced Manufacturing Technology, 74(9-12), 1419-1430.

Part A An investigation of the economic lot scheduling problem with batch shipments and energy considerations

Paper 1 The economic lot scheduling problem: A content analysis

Authors: Fabian G. Beck, Christoph H. Glock

Type of publication: Journal article

Publication details: Working paper

Abstract

The paper at hand addresses the Economic Lot Scheduling Problem (ELSP), which is concerned with finding a feasible and cost-minimal production schedule for multiple items produced in lots on a single machine. The ELSP started to attract the attention of researchers in the 1950s, where the focus was primarily on the development of simple heuristics for solving the problem. Over the subsequent decades, the ELSP has frequently been addressed in the literature, with the subject of research being the development of new scheduling policies or solution procedures or extensions of the scope of the basic ELSP. To date, a large number of journal articles has been published on the ELSP and its model variants.

To identify key research themes, publication patterns and opportunities for future research, the paper at hand applies a content analysis to a sample of 228 papers published on the ELSP. The results of the content analysis indicate that prior research on the ELSP had a strong focus on the development of solution methodologies, and that several topics that are directly connected to lot sizing and scheduling have not attracted much attention in research on the ELSP, such as, for example, energy cost and sustainability.

Keywords:

Economic lot scheduling problem; ELSP; Content analysis; Scheduling; Lot sizing; Literature review

1 Introduction

The Economic Lot Scheduling Problem (ELSP[1]) addresses a situation where a company produces multiple items on a single machine. In its most basic version, the ELSP encompasses two planning problems: the planning of lot sizes for all items and the scheduling of production. The objective of the ELSP is to generate a cost-minimal production schedule that is free from overlaps and that simultaneously satisfies customer demand. To minimize the cost of producing the items, both problems have to be solved simultaneously. Solving the ELSP is challenging, however, as the general version of the problem is NP-hard in the strong sense (see Gallego and Shaw, 1997).

The ELSP is typical for different industrial processes, such as in the woven fiberglass industry (Taylor et al., 1997), in metal forming, molding and stamping or in weaving production lines for carpets (Giri et al., 2003). Due to its practical relevance, the ELSP has been addressed in a large number of publications in the past. Our systematic search of the literature on the ELSP and its extensions that will be explained in more detail in the following sections identified 228 articles published in this area. Increasing publication numbers have also inspired several literature reviews on the ELSP, with the first one being the review of Elmaghraby (1978). After Elmaghraby (1978) three other literature reviews have been published on the ELSP (Vidal-Carreras et al., 2008; Chan et al., 2013; Santander-Mercado and Jubiz-Diaz, 2016) and another one on the related Stochastic Economic Lot Scheduling Problem (Winands et al., 2011).

For research streams with a very high number of publications, it is difficult to give an overview of the state-of-knowledge of the entire domain and to synthesize all research findings in a single literature review. As a consequence, existing reviews of the ELSP focused on specific topics to reduce the number of papers that need to be surveyed. Chan et al. (2013), for example, limited their literature search to the years 1997-2012 to discuss key research streams that emerged during this time span. The authors analyzed more than 100 publications on the ELSP in their review. Santander-Mercado and Jubiz-Diaz (2016) surveyed 126 papers on the ELSP. The authors differentiated between the single- and the multi-facility case and then assigned works contained in their literature sample to four different categories of scheduling policies and three

[1] All abbreviations used in this paper are summarized in the list of abbreviations (see page XVII and XVIII).

solution methodologies. A more detailed overview of related literature reviews is presented in Section 2.2.

The paper at hand adopts a different approach and focuses on the entire ELSP domain without restricting the analysis to a specific sub-topic of this area or to a certain time span. To facilitate analyzing this comprehensive research stream, this paper applies a content analysis (CA) to research published on the ELSP. A CA is a method for identifying publication patterns and trends, and it is especially useful for analyzing large data samples. One major advantage of a CA, in contrast to classical literature reviews, is that the analysis is not based on the opinion of the researchers conducting the CA, but that it instead relies on an evaluation criterion that equates the frequency of occurrence of a recording unit with the importance researchers assign to the corresponding topic. CAs have their origin in social science research, but have become more and more popular in operations and industrial engineering research in recent years as they I) are able to handle large sets of data and II) support a statistical evaluation of the research topic (e.g., Abedinnia et al., 2017; Grosse et al., 2017). The CA at hand contributes to the state-of-knowledge of the ELSP by answering the following research questions:

1. How can research on the ELSP be structured and classified?
2. Which topics related to the ELSP have enjoyed the highest popularity in the past?
3. Which opportunities for future research exist in this area?

The remainder of this paper is organized as follows: The next section introduces the classical ELSP as well as the most popular scheduling policies and solution methodologies and gives an overview of existing literature reviews of the ELSP. Section 3 outlines both the methodology used for generating the literature sample as well as the methodology of the CA applied in this paper. Section 4 presents a classification scheme for the ELSP that is later used for analyzing the literature sample. Section 5 presents the descriptive and the quantitative results of the study, and Section 6 concludes the paper.

2 The economic lot scheduling problem

Section 2.1 gives an overview of the classical ELSP and the different types of scheduling policies and solution methodologies that have been proposed for this problem in the past. Section 2.2 then discusses related literature reviews of the ELSP.

2.1 The ELSP

Rogers (1958) is assumed to be the first author to simultaneously investigate the problem of sizing lots and scheduling the production of several items on a single facility, such that the year 1958 has often been considered as the starting point of research on the ELSP (see Gallego and Joneja, 1994). The assumptions underlying the classical ELSP can be summarized as follows (see Bomberger, 1966):

- Two or more products are produced on a single machine.
- Only one product can be produced by the machine at a time.
- The planning horizon is infinite.
- All parameters are deterministic, known and constant over time.
- Setup cost and setup time are independent of the production sequence.
- Shortages are not allowed.
- Inventory holding cost is directly proportional to the inventory level.

The ELSP is NP-hard in the strong sense for several different problem settings. The so-called independent solution, which is used as a lower bound to the problem, can be easily calculated by minimizing the objective function of every product individually. The independent solution usually leads to an infeasible production schedule with overlaps in the production of items over time.

To calculate a feasible solution for the ELSP, researchers have often assumed that the production cycle is finite and repetitive. In the following, we use the term scheduling policy to refer to approaches that make assumptions on the structure of the production schedule in the ELSP to facilitate solving the problem. Existing scheduling policies can be assigned to one of the following three classes (Chan et al., 2013):

1. Common-Cycle-Approaches that assume a common cycle time for all products.
2. (Extended) Basic-Period-Approaches that permit different cycle times for all products, but that assume that the cycle time of a product has to be an integer multiple of a basic period.
3. Time-Varying-Lot-Size-Approaches that permit different cycle times and lot sizes that may vary during the total cycle for all products.

Santander-Mercado and Jubiz-Diaz (2016) added a fourth scheduling policy to their classification scheme they referred to as the no cycle approach. This approach has not attracted much attention in the past, however, as can be seen in Appendix A.

We further use the term solution procedure to refer to methods employed for determining the production schedule (i.e., calculating lot sizes, production sequences etc.) for the above scheduling policies. According to Elmaghraby (1978) and Santander-Mercado and Jubiz-Diaz (2016), solution procedures for the ELSP can be assigned to one of the following three classes:

1. Exact methods that optimally solve a restricted version of the original problem.
2. Heuristic methods that solve the original version of the problem and that usually do not obtain an optimal solution.
3. Meta-heuristic methods that solve the original version of the problem.

2.2 Literature reviews of the ELSP

The first literature review of the ELSP is the one of Elmaghraby (1978), who identified 25 articles dealing with this problem. Elmaghraby divided the existing literature into two main solution approaches: I) analytical approaches and II) heuristic approaches. He also identified two scheduling policies that had been discussed in the literature at that time: I) the Common-Cycle-Approach, and II) the (Extended) Basic-Period-Approach. The author then assigned the papers contained in his literature sample to these two categories and discussed their solution procedures.

Vidal-Carreras et al. (2008) proposed another literature review of the ELSP and classified earlier publications along six dimensions that represent main assumptions of the classical ELSP and their eventual relaxation, namely I) production rates (fixed vs. variable), II) setup costs and times (independent of/dependent on the production sequence), III) demand rates (static, dynamic, deterministic, stochastic), IV) demand fulfillment (backorders/lost sales permitted/not permitted), V) production capacity (insufficient capacity, capacitated system), and VI) item and demand characteristics (make-to-order, make-to-stock, imperfect quality).

Another review of the ELSP was contributed by Chan et al. (2013), who restricted their analysis to papers published between 1997 and 2012, leading to a literature sample of more than 100 papers. The authors extended Elmaghraby's (1978) review to account for a third scheduling policy, namely Time-Varying-Lot-Size-Approaches, and classified the literature sample accordingly. In addition, they identified five key research themes covered in the ELSP literature: I) non-uniform production rate, II) flow shop, multi-machine, or multi-factory, III) with returns, IV) stochastic problems, and V) sequence-dependent setups.

The most recent review of the ELSP was published by Santander-Mercado and Jubiz-Diaz (2016), who identified 126 papers dealing with the ELSP. The authors first differentiated between works that consider a single facility, and works that investigate the multi-facility case. For both groups, two main classification schemes were proposed. The first one classifies the literature according to four different scheduling policies: I) the Common-Cycle-Approach, II) the (Extended) Basic-Period-Approach, III) the Time-Varying-Lot-Size-Approach, and IV) the no cycle approach. The second classification scheme considers the solution methodology, namely: I) exact methods, II) heuristic methods, and III) meta-heuristic methods.

The work at hand differs from existing literature reviews in the field both in terms of scope and methodology. First, we investigate the entire domain of the ELSP that includes various problem variants and extensions that have been proposed over the years. This leads to a much larger literature sample analyzed in this paper as compared to earlier literature reviews. Secondly, we apply a content analysis to the literature sample that enables us to identify key topics that have been discussed in the literature beyond those included in existing classification schemes.

3 Methodology of the CA

This section outlines the methodology applied in the paper at hand. Section 3.1 first describes the objectives of content analyses and defines the CA methodology used in this paper. Section 3.2 then explains the methodology employed for generating the literature sample that is analyzed using the CA methodology in a later section of this paper.

3.1 Characteristics of content analyses

The literature contains various definitions of the CA. Neuendorf (2002), for example, defined the CA as a systematic, objective, and quantitative analysis of message characteristics. Weber (1990) stated that the CA is a research method that uses several procedures to make valid inferences from text, whereas Seuring and Gold (2012) noted that a "content analysis represents an effective tool for analyzing a sample of research documents in a systematic way". According to Neuendorf (2002), the CA has its origins in World War II, where it was used to analyze large data obtained from propaganda. The CA can be used to identify patterns in large data sets using objective criteria, and it is not restricted to a particular type of information, but can instead be applied to different kinds of media, such as newspapers, pictures, speeches or videos.

According to Neuendorf (2002), there are four major types of content analyses, namely descriptive, inferential, psychometric, and predictive CAs. An inferential CA is used when researchers wish to investigate the implied meaning of the data. The psychometric CA, in turn, analyses messages of individuals to provide a clinical diagnosis or to measure a psychological trait or state of the individual. Predictive CAs try to forecast the responses of the receiver or audience to the messages at hand. This paper applies a descriptive CA to research on the ELSP, with the objective to identify key research themes that have been studied in this field of research. Conclusions derived from the analysis are only valid for the content under study. The methodology of the CA applied in this paper is based on the work of Seuring and Gold (2012), and it can be summarized as follows (for a graphical illustration of the methodology, see Figure 1): After formulating the research questions, we systematically generate the literature sample that is analyzed in the CA (see Section 3.2). Based on the sample obtained in the literature search and considering the reviews of Chan et al. (2013) and Santander-Mercado and Jubiz-Diaz (2016), we then develop a framework with categories, subgroups, terms and recording units that is used for evaluating the literature sample (see Section 4). Finally, the hits obtained for the recording units in the literature sample are analyzed to answer the research questions formulated in Section 1.

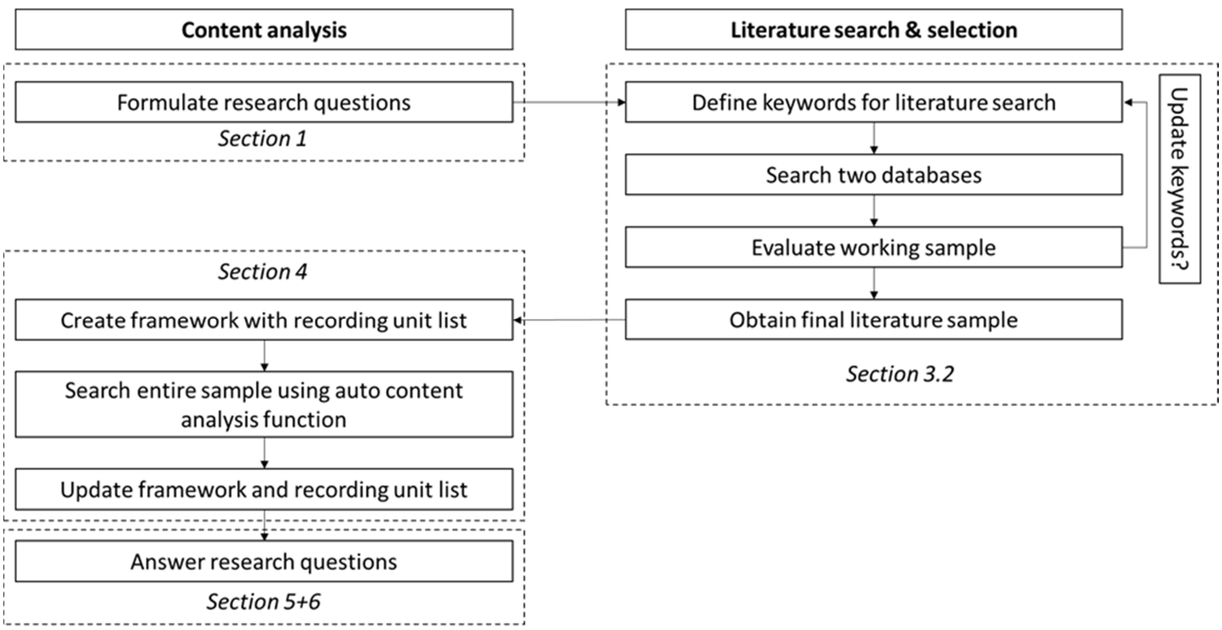

Figure 1: *The methodology of the CA applied in this paper*

3.2 Generation of the literature sample

To generate the literature sample for the CA, this paper employed a systematic literature search and selection methodology. This methodology has been recommended especially for systematic literature reviews and meta-analyses (see, e.g., Rhoades, 2011; Grosse et al., 2015), and it aims on making the generation of the literature sample transparent and reproducible to the reader.

First, we selected two scholarly databases to identify articles that are relevant for the work at hand, namely Ebsco Host and Scopus. Based on the reviews of Chan et al. (2013) and Santander-Mercado and Jubiz-Diaz (2016), we selected only the keywords "ELSP" and "economic lot scheduling problem" for our literature search to ensure that we search the literature as broadly as possible. Using these two keywords, the two databases were searched for articles that contain at least one of the two keywords in their title, abstract or list of keywords (date of the keyword search in the databases: June 26, 2017). The search led to 155 hits in the database Ebsco Host and to 299 hits in the database Scopus. This initial sample was checked for relevance by both authors of this paper. The following selection criteria were applied:

- The language was limited to English.
- Duplicate articles were eliminated.
- Only peer-reviewed academic journals were considered.
- Literature reviews were excluded from the sample.
- Comments that correct minor errors in earlier works were excluded.

In addition, only articles that address lot sizing and scheduling for two or more items were included in the sample to ensure that only works investigating the core problem of the classical ELSP were considered for this analysis. Hence, we excluded all papers dealing with single-item economic production quantity or economic order quantity models from further analysis, for example. To ensure a broad scope of our investigation, we included various model variants of the ELSP, such as the ELSP in a multi-stage supply chain context or the Economic Lot and Supply Scheduling Problem (ELSSP) that combines the Economic Lot Scheduling Problem and the Vehicle Routing Problem.

We then applied the inclusion and exclusion criteria defined above to the 454 papers obtained in the database search. Four papers were excluded from the initial sample since their language was not English, and another 131 papers were eliminated as they were duplicates. In addition, five literature reviews were excluded from the sample, together with 15 articles that considered only a single item. 71 papers investigated topics not relevant for the work at hand, and were therefore excluded as well. The final literature sample consists of 228 articles with references provided in Appendix B. A descriptive analysis of this sample is presented in Section 5.1.

4 Conceptual framework for the ELSP

After the final literature sample has been established, the content has to be brought into a standardized form that can be analyzed by computer software (this step is frequently referred to as coding). EPA (2002) differentiates between coding of manifest content and latent coding. Manifest coding “refers to visible surface content, such as the frequency of words and phrases”, while latent content “refers to the underlying meaning or context of the entire text”. We use a manifest coding procedure for our CA, and as recording units, we choose different words and abbreviations that are counted in our final sample. Considering the number of hits obtained for the recording units, it is then possible to draw conclusions on the relative importance of the coding units and associated topics.

To categorize and evaluate research on the ELSP, we developed a conceptual framework in a two-step approach. Based on Section 2.1 and the reviews of Chan et al. (2013) and Santander-Mercado and Jubiz-Diaz (2016), a conceptual framework with corresponding content categories (main topics), subgroups (of the main categories), terms (general terms and more specific terms), and recording units was first obtained deductively. After completing the analysis of the sampled papers, the framework was updated inductively (see Figure 1).

In the inductive step, the entire literature sample was subjected to an automatic content analysis using the software MAXQDA 12. During the automatic content analysis, all words, abbreviations and (mathematical) symbols were counted, leading to more than 55,000 results. All words with more than 30 hits were carefully examined to identify further recording units that could be added to the initial conceptual framework. Based on the automatic evaluation of recording units, a few categories and subgroups were added to the conceptual framework. Our final conceptual framework for the ELSP consists of the following nine categories:

1. Type of problem: contains recording units related to general characteristics of the problem. In our framework, the type of problem can be dynamic, static, stochastic and/or deterministic.
2. Scheduling policy: defines general patterns that are superimposed on the production sequence. This category considers, for example, the Common-Cycle-

Approach, the Basic-Period-Approach, and the Time-Varying-Lot-Size-Approach.

3. Solution methodology: classifies the mathematical solution approach used for solving the problem at hand. We distinguish between exact methods, heuristic methods, meta-heuristic methods, artificial intelligence, and simulation.

4. Specific assumptions: summarizes assumptions made in developing the ELSP model, e.g. assumptions on the production rate or on the number of products considered.

5. Flow pattern: considers the number of stages, the number of machines per stage, and eventual constraints on the routing of the products through the production system.

6. Structural properties of the ELSP: considers recording units dealing with the boundaries of the problem, i.e. lower and upper bounds, and general mathematical investigations, i.e. complexity and feasibility.

7. Scheduling objectives: refers to the objectives of the ELSP model, e.g. cost or makespan.

8. Extended coverage: considers extensions of the ELSP, e.g. the Group Technology-Economic Lot Scheduling Problem (GT-ELSP) or the Economic Lot and Delivery Scheduling Problem (ELDSP).

9. Other topics: contains subgroups that cannot be clearly assigned to the eight categories above, but that also consider topics related to the ELSP, such as the applicability of ELSP models in practice or sustainability topics.

The main categories and the corresponding subgroups are summarized in Table 1.

Table 1: *Main categories and their corresponding subgroups*

Category	Subgroups
1. Type of problem	dynamic, static, stochastic, deterministic
2. Scheduling policy	Common-Cycle-Approach, Basic-Period-Approach, Time-Varying-Lot-Size-Approach, No cycle approach, 2^x-policy
3. Solution methodology	exact methods, heuristic methods, meta-heuristic methods, artificial intelligence, simulation

4. Specific assumptions	planning horizon, production rate, setup, demand, number of products, shortages, zero switch rule, sequence-dependency, product problems, machine problems
5. Flow pattern	single machine, multi-machine, schedule
6. Structural properties of the ELSP	feasibility, complexity, bounds, theory
7. Scheduling objectives	cost, workload, profit, inventory, makespan
8. Extended coverage	ELSP and its extensions
9. Other topics	green, inventory, practical application, deliveries, basic lot sizing models

After all categories, subgroups, terms and recording units had been defined, the software MAXQDA 12 was used to count the number of hits of every recording unit in the final literature sample. In addition, we counted the number of papers that contain the recording unit under consideration. A comprehensive list of results can be found in Appendix A.

The following aspects were considered both in the counting of recording units as well as in the presentation of results in Appendix A:

- We considered different spellings in the word count, e.g. we searched for both "basic period" and "basic-period". To simplify presentation, only a single spelling is shown in Table A..
- We also considered different spellings for British and American English, e.g. "neighbor" and "neighbour".
- To take account of abbreviations, we used a function provided by the software MAXQDA 12 that ensures that only complete words are counted. Abbreviations were considered as completed words if they had a blank space or a punctuation mark both in front and at the back of them. All abbreviations with this special adjustment are written in capital letters in Table A.1.
- To obtain the hits for some recording units, it was necessary to calculate the hits taking account of the hits obtained for other recording units. For example, to obtain the recording unit hits for "finite horizon", we have to subtract the number of hits for "infinite horizon" from the number of hits for "finite horizon", as the software MAXQDA 12 would count a hit for the recording unit "finite horizon"

both if “finite horizon” or “infinite horizon” was found in a paper (we marked these cases with footnotes in Appendix A).

5 Findings of the study

This section presents the findings of our study. Section 5.1 first analyses the literature sample consisting of 228 articles descriptively (the list of our sample can be found in Appendix B), and Section 5.2 then presents the results of our data analysis.

5.1 Descriptive analysis of the sample

Figure 2 shows the number of sampled articles published per year. The first article contained in our sample was published in 1958 and the last one in 2017. As can be seen, publication numbers on the ELSP exhibited an increasing trend over the years, which underlines the on-going significance of the ELSP and the popularity it enjoys in the research community.

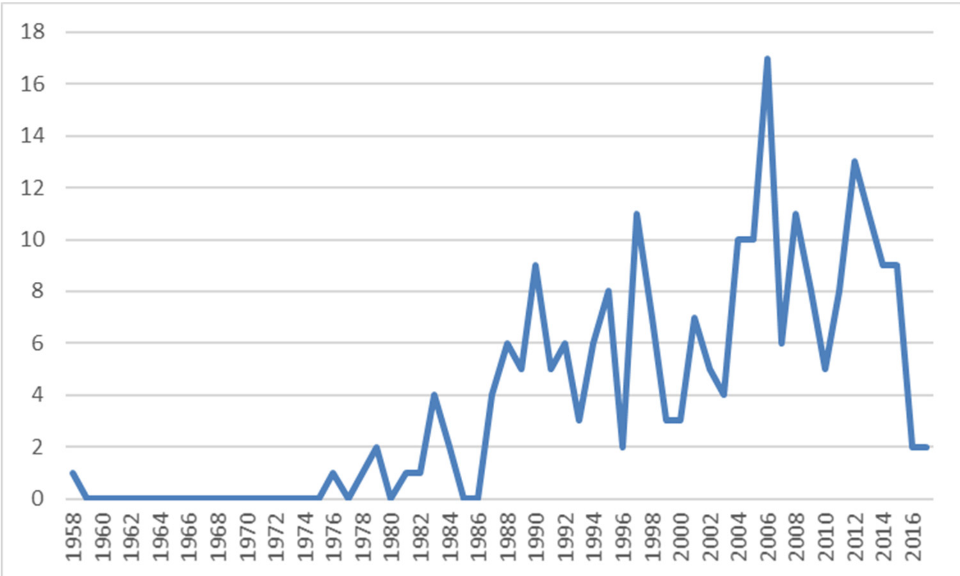

Figure 2: *Number of sampled articles per year*

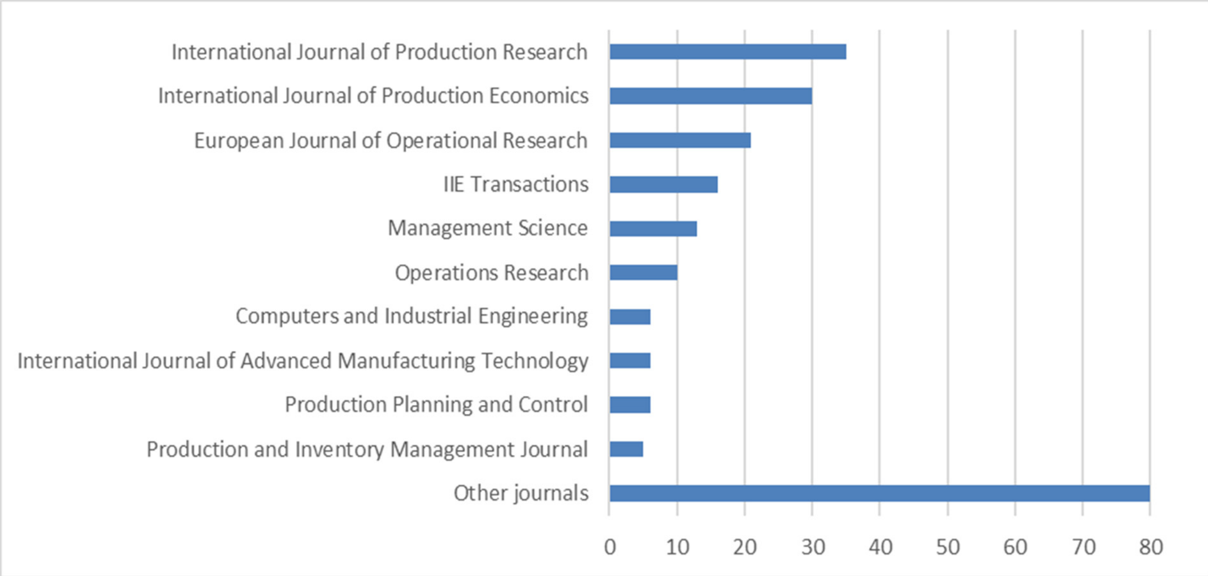

Figure 3: *Number of sampled articles per journal*

Figure 3 shows the journals that published the highest number of articles contained in our sample. As can be seen, more than 50% of the sampled articles were published in only five journals, namely the *International Journal of Production Research* (35 articles), the *International Journal of Production Economics* (30 articles), the *European Journal of Operational Research* (21 articles), *IIE Transactions* (16 articles), and *Management Science* (13 articles).

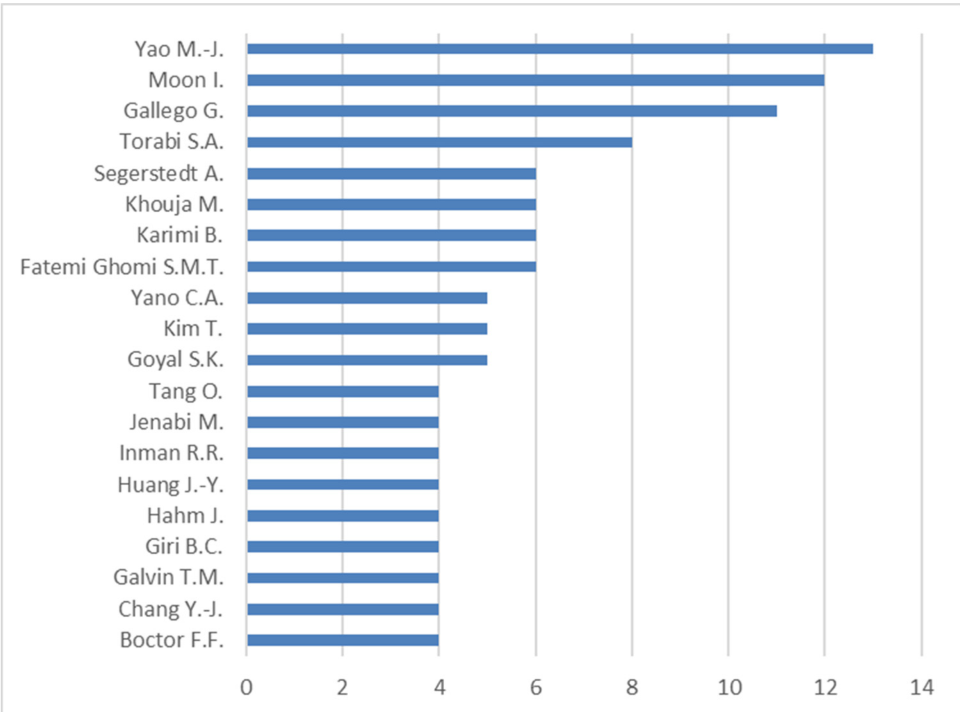

Figure 4: *Authors who published at least four articles contained in our sample*

Our analysis further shows that 316 authors contributed to research on the ELSP and its extensions. Figure 4 summarizes all authors who published at least four articles contained in our sample. The authors who published the highest number of sampled papers are Yao (13 articles), Moon (12 articles), Gallego (11 articles), and Torabi (8 articles).

5.2 Results of the CA

This section presents the results of our CA (see Appendix A for the comprehensive list of recording units, terms, subgroups and categories). Section 5.2.1 first gives an overview of the overall results of the CA, and Section 5.2.2 then describes all nine content categories and their corresponding subgroups in detail.

5.2.1 Overview of the results of the CA

5.2.1.1 Percentage distribution of the categories

Figure 5 shows the percentage distribution of the recording unit hits for the nine categories, which can be seen as an indicator of the relative importance of the different categories. When interpreting the results, one has to keep in mind that the different categories contain a different number of recording units, such that a higher number of recording units in a particular category increases the probability of recording unit hits.[2] It can be seen that more than 40% of the recording unit hits belong to two categories: I) "specific assumptions" (28%) and II) "solution methodology" (17%). This result highlights the importance researchers have attributed to the development of solution procedures for the ELSP in the past. Given that the ELSP is NP-hard in the strong sense, developing better solution procedures for the ELSP and its extensions can contribute to lowering cost and improving the performance of the company. A relatively small number of recording unit hits was obtained for the categories "type of problem" (3%) and "flow pattern" (4%).

[2] The number of recording units contained in a category could, however, be seen as an indicator of the relative importance of that category as well.

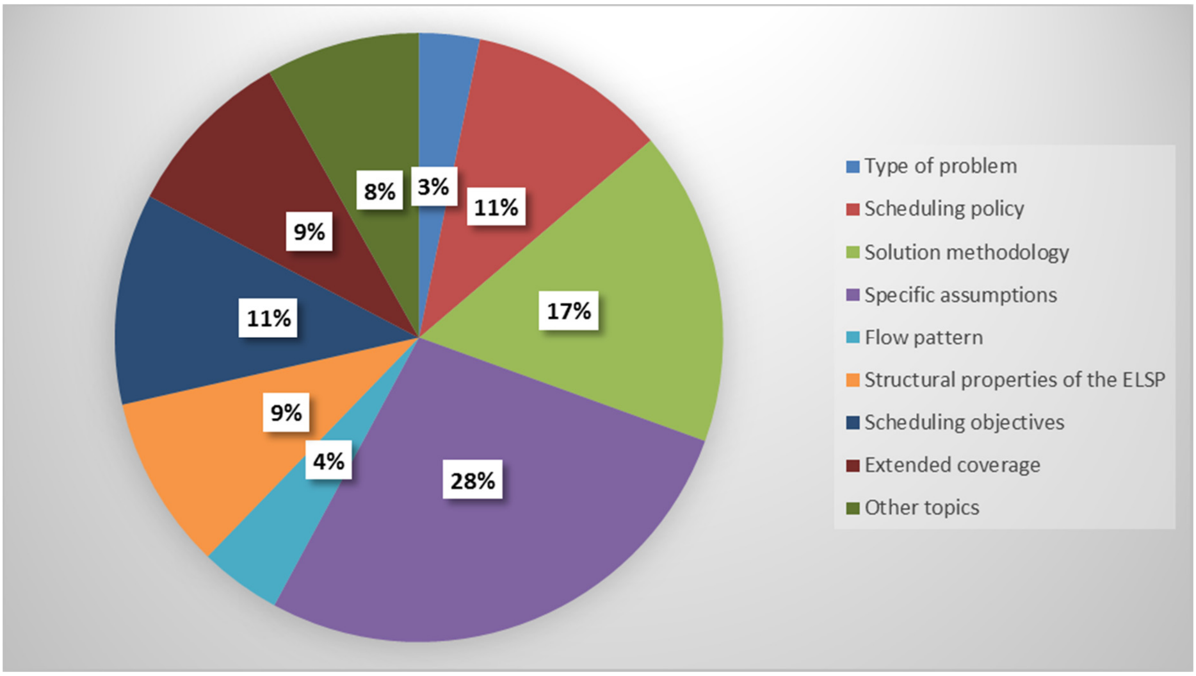

Figure 5: *Percentage distribution of the recording unit hits obtained for the different categories*[3]

5.2.1.2 Top 10 recording unit hits

Table 2 presents the ten recording units that received the highest number of hits in our final sample. The recording units that received the most attention in the literature can roughly be divided into three groups. The first group of recording units ("basic period", i.e. Basic-Period-Approach, and "common cycle", i.e. Common-Cycle-Approach) denotes two scheduling policies. The high number of hits obtained for these two recording units illustrate the high importance researchers attach to scheduling policies for solving the ELSP, and it also shows that the focus of prior research was more on scheduling policies that can still be solved relatively easily, while the more complex Time-Varying-Lot-Size-Approach has attracted less attention. The second group of recording units ("holding cost", "total cost", "setup" and "production rate") deal with general characteristics of the production process, and they refer to different types of cost that play a role in the ELSP and options for the production manager to interfere in the production process (e.g., in case production rates are variable). The third group of recording

[3] To avoid that certain recording units are counted more than once, all recording units that are contained in other recording units and that have not been subtracted from the number of hits yet, such as "unrelated parallel machine" that is already contained in the recording unit "parallel machine", are not considered for this calculation. All recording units where this criterion applies are highlighted with a star in Table A.1.

units (“heuristic”, “lower bound”, “genetic algorithm” and “feasibility”) finally refers to solution procedures and characteristics of the optimization problem and the solution, which shows that solving the ELSP plays an important role in the literature, which may again be a consequence of its complexity.

Table 2: *Top 10 recording units by number of hits obtained in the final sample*[4]

#	Recording unit(s)	Hits	Category
1	setup[5]	4476	Specific assumptions
2	heuristic	3396	Solution methodology
3	basic period, BP, BPA	1837	Scheduling policy
4	common cycle, CC, CCA	1831	Scheduling policy
5	lower bound, LB	1757	Structural properties of the ELSP
6	genetic algorithm, GA	1494	Solution methodology
7	holding cost	1374	Scheduling objectives
8	production rate	1209	Specific assumptions
9	feasibility	1185	Structural properties of the ELSP
10	total cost	1158	Scheduling objectives

5.2.1.3 Top 10 recording unit hits per paper

In this sub-section, we evaluate recording unit hits per paper by dividing the total number of hits obtained for a particular recording unit by the number of papers in which the recording unit was used. This relative measure takes account of the fact that papers mentioning a recording unit very frequently could distort the interpretation of the absolute number of recording unit hits. Table ***3*** presents the top ten recording unit hits per paper. Interestingly, the recording units “setup” and “genetic algorithm” were considered in a large number of papers and also frequently used in these papers. Hence, these recording units and the corresponding topics seem to be of great importance for research on the ELSP. The other recording units, namely “coproduction”, “artificial bee colony algorithm”, “particle swarm optimization”, “fuzzy”, “Economic Lot and Inspection Scheduling Problem” (ELISP), “remanufacturing”, “Economic Lot Scheduling Problem with returns” (ELSPR), and “Economic Lot and Supply Scheduling Problem” (ELSSP), were only used in a small number of papers, which implies that the corresponding

[4] We excluded the recording units “Economic Lot Scheduling Problem” and “ELSP” from this analysis, as it is not surprising that these recording units led to an enormous number of hits in our final sample given the topic of the paper.
[5] We subtract the number of recording unit hits for “setup cost” and excluded the recording units “setup cost” and “setup time” from this analysis.

topics have been extensively discussed in these papers, but that their importance outside of some key papers in the field may not be that high.

Table 3: *Top 10 of recording unit hits per document for the final sample*[6]

#	Recording unit(s)	Hits	Number of documents	Hits per document	Category
1	coproduction	151	2	75.50	Specific assumptions
2	artificial bee colony algorithm, ABC	263	8	32.88	Solution methodology
3	particle swarm optimization, PSO	183	7	26.14	Solution methodology
4	fuzzy	213	9	23.67	Type of problem
5	Economic Lot and Inspection Scheduling Problem, ELISP	47	2	23.50	Extended coverage
6	remanufacturing	268	12	22.33	Specific assumptions
7	Economic Lot Scheduling Problem with returns, ELSPR	171	8	21.38	Extended coverage
8	genetic algorithm, GA	1494	71	21.04	Solution methodology
9	Economic Lot and Supply Scheduling Problem, ELSSP	102	5	20.40	Extended coverage
10	setup	4476	224	19.98	Specific assumptions

5.2.2 Findings for the categories

5.2.2.1 Type of problem

This category consists of four subgroups related to general characteristics of the ELSP. The results are shown in Figure 6. As can be seen, uncertainty plays an important role in our sample since nearly half of the sampled papers contain at least one recording unit belonging to the subgroup "stochastic".[7] The subgroup that received the second highest number of hits is "dynamic", while "static" and "deterministic" received much fewer hits. These results confirm the findings of Abedinnia et al. (2017) in the area of machine scheduling, who also observed a strong emphasis on stochastic scheduling problems in their area. We note, however, that these results may not be fully reflective of the content of the models published on the ELSP. Instead, we noticed a trend in the literature on the ELSP to highlight assumptions deviating from the

[6] We again excluded the recording units "Economic Lot Scheduling Problem" and "ELSP".
[7] Uncertainty in the ELSP has also inspired a dedicated review on this topic, see Winands et al. (2011).

classical ELSP in some detail in the paper, while assumptions that match the classical ELSP were often not explained at all, or only briefly. The fact that the classical ELSP is a deterministic and static model may therefore have induced researchers to discuss especially those assumptions deviating from this standard setting in some detail in their publications, thus driving up the hits for the subgroups "stochastic" and "dynamic".

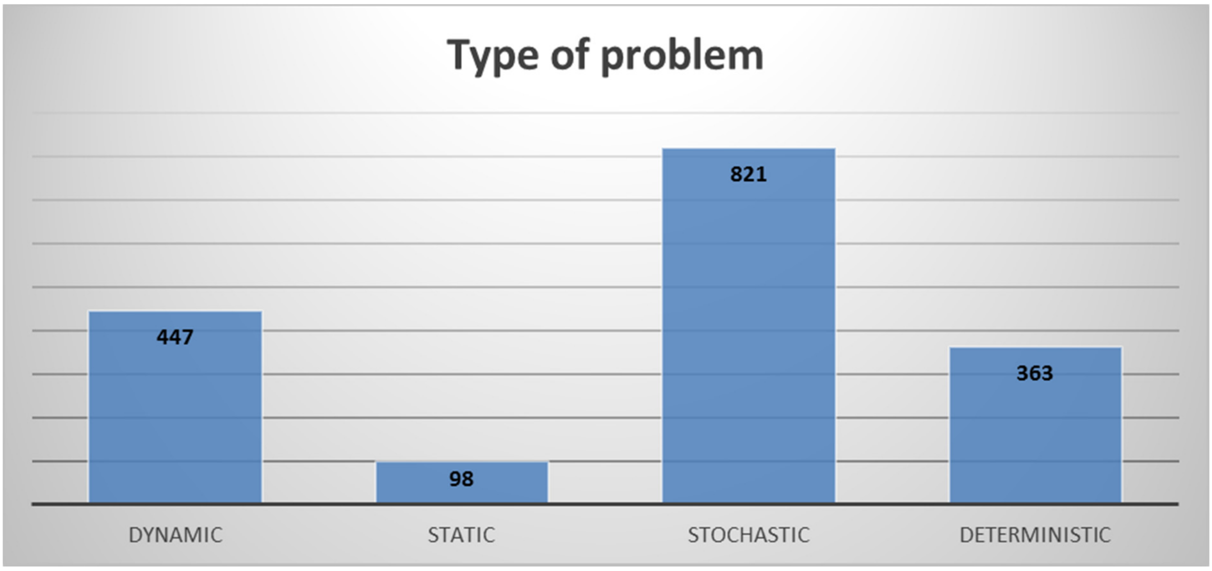

Figure 6: *Subgroups of the category "type of problem" and their number of recording unit hits*

5.2.2.2 Scheduling policy

The category "scheduling policy" is divided into five subgroups, i.e. into four scheduling policies and one special characteristic of a scheduling policy, and the results obtained for the four scheduling policies are shown in Figure 7. The two subgroups that received the highest number of hits are "Common-Cycle-Approach" and "Basic-Period-Approach", which confirms the results of Chan et al. (2013). The popularity of these two scheduling policies can be explained by the fact that they are both easy to apply due to a relatively low computational complexity, and that they lead to machine schedules that can easily be implemented in practice. The Common-Cycle-Approach has also often been considered as an upper bound for the ELSP, such that it has frequently served as a benchmark scheduling policy for evaluating new scheduling approaches.

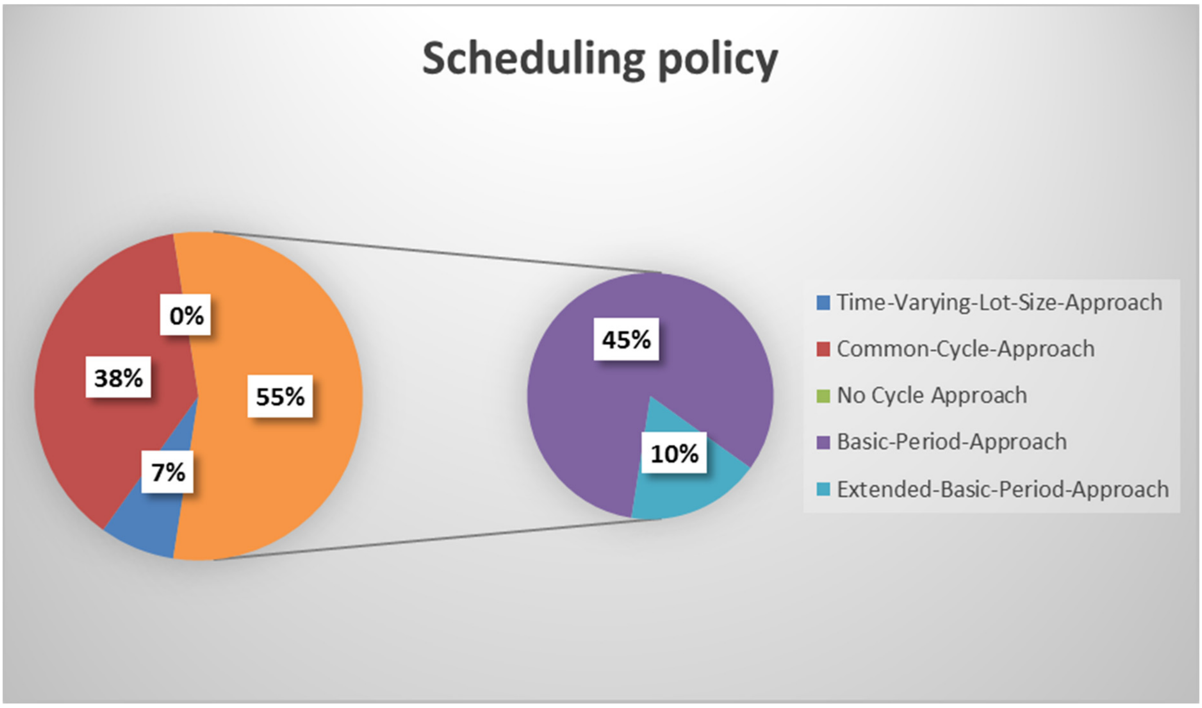

Figure 7: *Subgroups of the category "scheduling policy" and their shares in the recording unit count*

The fourth subgroup "no cycle approach" mentioned by Santander-Mercado and Jubiz-Diaz (2016) obtained zero hits in our sample. This number of hits could indicate that this approach is not be of high relevance for the ELSP.

Finally, the number of hits for the fifth subgroup "2^x-policy" (822 hits) highlights the importance researchers attributed to this special scheduling policy that is a variant of the Basic-Period-Approaches. 2^x-policies assume that the multipliers used in the Basic-Period-Approach are limited to powers of two, which makes it easier to find a solution to the problem.

5.2.2.3 Solution methodology

The category "solution methodology" is divided into five subgroups, and the results for this category are shown in Figure 8. Given that the ELSP is NP-hard in the strong sense, it is not surprising that especially "heuristic methods" and "meta-heuristic methods" were used for solving the problem. Only 15% of the recording unit hits belong to the subgroup "exact methods". Exact methods are commonly used in the ELSP to solve restricted versions of the original problem.

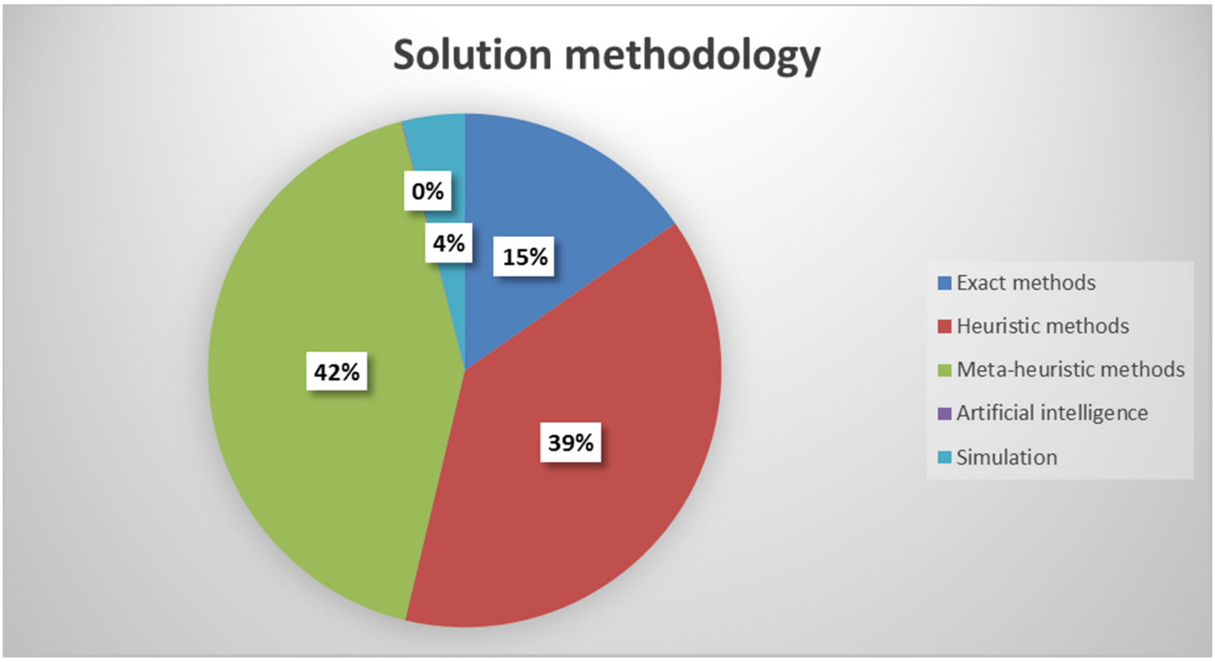

Figure 8: *Subgroups of the category "solution methodology" and their shares in the recording unit count*[8]

Figure 9 gives an overview of the recording unit hits obtained for the subgroup "meta-heuristic methods". As can be seen, "genetic algorithms" dominates this subgroup with 41% of the recording unit hits. Other meta-heuristics that received a significant amount of attention, all with at least 10% of the recording unit hits, are "tabu search", "simulated annealing" and "hybrid genetic algorithms".

[8] To avoid that certain recording units are counted more than once, all recording units that are contained in other recording units and that have not been subtracted from the number of hits yet, such as "integer linear programming" that is already contained in the recording unit "linear programming", are not considered for this calculation. All recording units where this criterion applies are highlighted with a star in Table A.1.

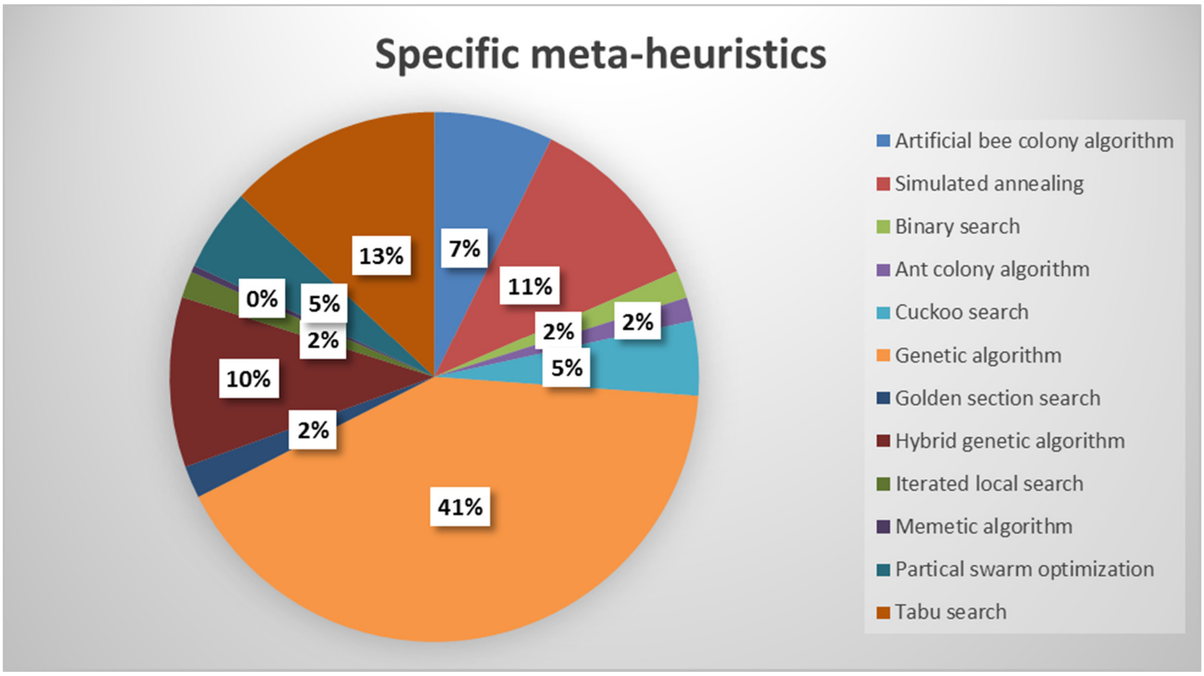

Figure 9: *Percentage distribution of the recording unit hits for specific meta-heuristic methods*

To gain insights into how the popularity of the different solution procedures changed over time, the final literature sample was divided into two groups, with the first group containing all papers that were published between the years 1958 and 1997, and the second group covering the years between 1998 and 2017. The two periods were chosen in such a way that the first publication using a genetic algorithm to solve the ELSP (Khouja et al., 1998) marks the starting point of the second group. The percentage distribution of the recording unit hits for the two groups are shown in Figure 10. For both groups, it can first be seen that the subgroups “artificial intelligence” and “simulation” obtained a small number of recording unit hits. With respect to the other three solution methodologies, a significant difference between the percentage distributions of the two groups was obtained. In the group covering the years 1958 to 1997, more than 80% of the recording unit hits belong to the subgroups “exact methods” and “heuristic methods”, while more than 80% of the recording unit hits obtained for the years 1998 to 2017 belong to the subgroups “heuristic methods” and “meta-heuristic methods”. This result points towards a trend to employ more meta-heuristics and heuristics for solving the ELSP, instead of developing exact solution methods.

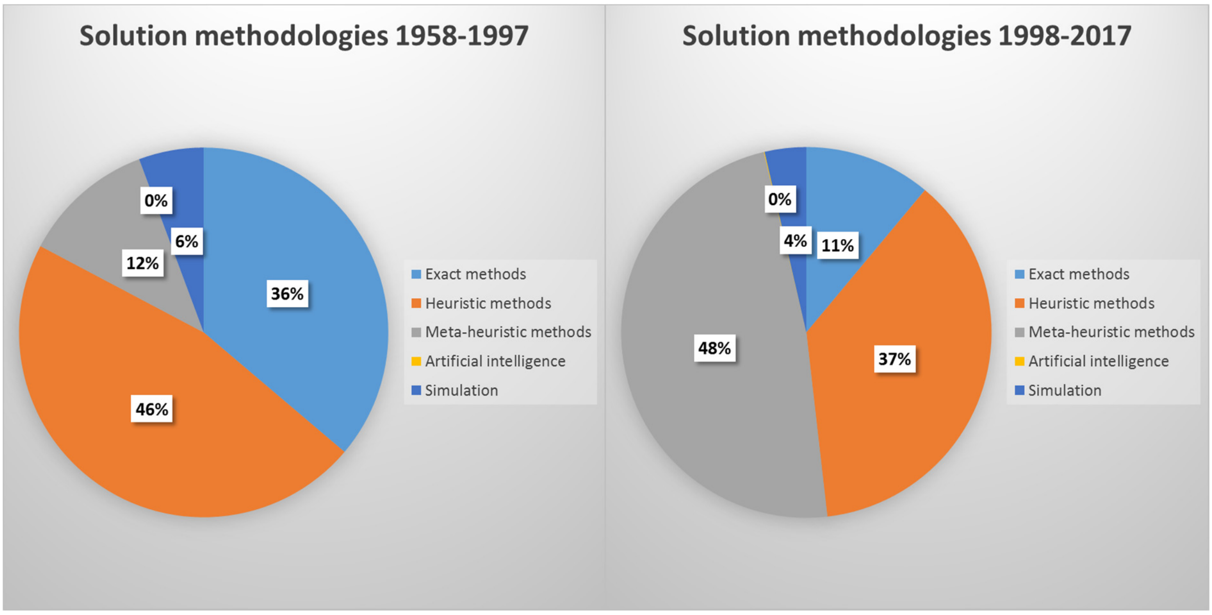

Figure 10: *Percentage distribution of the recording unit hits for the solution methodologies over time*[9]

5.2.2.4 Specific assumptions

The category "specific assumptions" consists of ten subgroups. The first subgroup considers the "planning horizon". The number of recording unit hits for "finite horizon" (82 hits) and "infinite horizon" (74 hits) are approximately the same, but "infinite horizon" (43 to 29) was mentioned in more papers. It is noticeable that this assumption that is important both for the development and the solution of ELSP models has often not been explicitly mentioned in the sampled papers.

Given that a lot sizing and scheduling problem is analyzed in the paper at hand where attributes of the production process play an important role, it is not surprising that a high number of hits was obtained for the recording units "production rate", "setup" and "demand rate". An interesting result for the subgroup "setup" is that in a relatively large number of papers, products were grouped in so-called product families. Changing from one product to another one from the same product family would be associated with a minor setup in this case, while shifting to another product family would entail a costlier and more time-consuming large setup. Product families

[9] To avoid that certain recording units are counted more than once, all recording units that are contained in other recording units and that have not been subtracted from the number of hits yet, such as "integer linear programming" that is already contained in the recording unit "linear programming", are not considered for this calculation. All recording units where this criterion applies are highlighted with a star in Table A.1.

are thus directly connected to the other subgroup “sequence-dependency”, where sequence-dependent setup cost and setup time have been considered.

The hits obtained with respect to the “production rate” can be divided into two groups, namely “fixed production rate” and “variable production rate” (see Figure 11). As can be seen, fixed and variable production rates have received roughly the same number of hits.

Figure 12 illustrates the numbers of hits obtained for different attributes of the “demand rate”. As can be seen, especially the cases of a constant, a random and a stochastic demand rate have attracted some attention.

With respect to the “number of products” considered, most of the papers explicitly referred to a “multi item” (843 hits in 180 papers) problem; “two product” (445 hits in 115 papers) and “three product” (132 hits in 64 papers) problems have also been mentioned a couple of times. For the subgroup “shortages”, the expressions “shortage” (392 hits), “backlog” (258 hits) and “backorder” (584 hits) received most hits in our sample. The number of hits shows that this subgroup also plays an important role in this research area.

The two subgroups “product problems” and “machine problems” refer to the deterioration of products or production equipment. For the subgroup “product problems”, we differentiate between three main problems mentioned in the literature: I) imperfect quality of the products (including the recording units “defective items” (77 hits), “imperfect quality” (36 hits), “failure rate” (8 hits), and “non-conforming item” (34 hits)), II) deteriorating items (including the recording units “deteriorating items” (238 hits) and “shelf life” (273 hits)), and III) remanufacturing of the products (including the recording items “remanufacturing” (268 hits), “reproduction” (62 hits) and “restoration” (64 hits)). Considering the subgroup “machine problems”, we assigned the recording units to two groups: I) disruption and breakdown of the machine, and II) repair and inspection of the machine.

One surprising result is that the subgroup “zero switch rule”, which defines that a new production cycle has to start if and only if the product’s inventory level reaches zero, was only mentioned in 57 papers, although it is an important assumption of the ELSP.

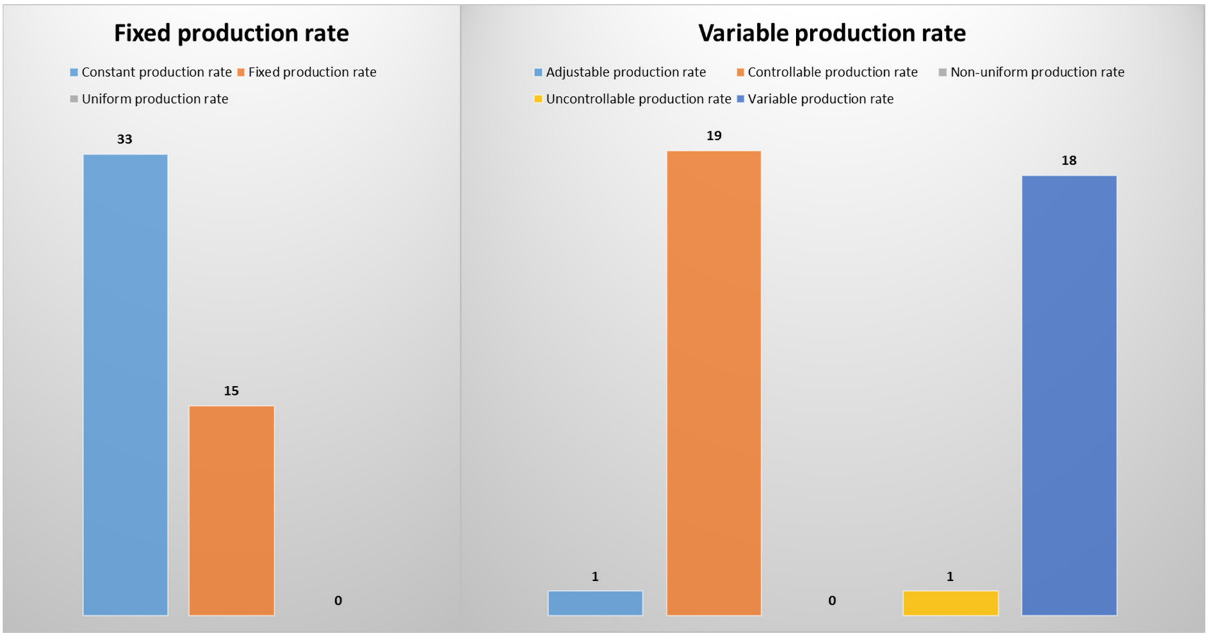

Figure 11: *Number of recording unit hits for different assumptions on the production rates*

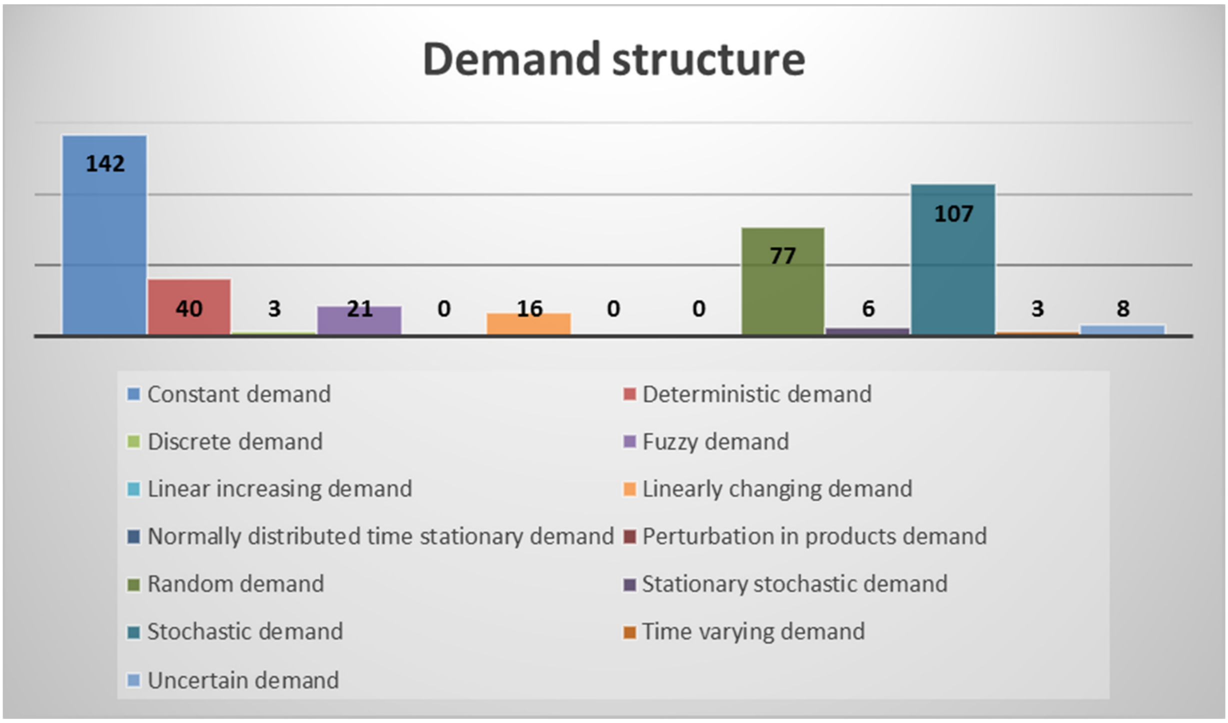

Figure 12: *Number of recording unit hits for different types of demand structures*

5.2.2.5 Flow pattern

The group “flow pattern” consists of three subgroups. Interestingly, although some works referred to the “multi-machine” case (325 hits), the “single-machine” case has received the highest attention in this group (993 hits). With respect to the subgroup “multi-machine”, especially “flow shop” and “job shop” scheduling, “parallel machines” and special production systems with “identical machines” have received the most attention.

5.2.2.6 Structural properties of the ELSP

This category consists of four subgroups that are illustrated in more detail in Figure 13. We first note that all four subgroups have received a high number of hits, which indicates that all four topics are of high importance to the ELSP. Apart from this, it is not surprising to see that “feasibility” (1185 hits) received an especially high number of hits, given that generating a feasible production schedule is one of the main objectives of the ELSP. The recording unit “lower bound” received the highest number of hits in this category (1757 hits). The popularity of this recording unit may reflect the practice to compare new solution procedures or scheduling policies against some kind of benchmark solution, which in the case of the ELSP has frequently been the independent solution. General mathematical terminology, reflected in the subgroups “complexity” and “theory”, have also received some attention.

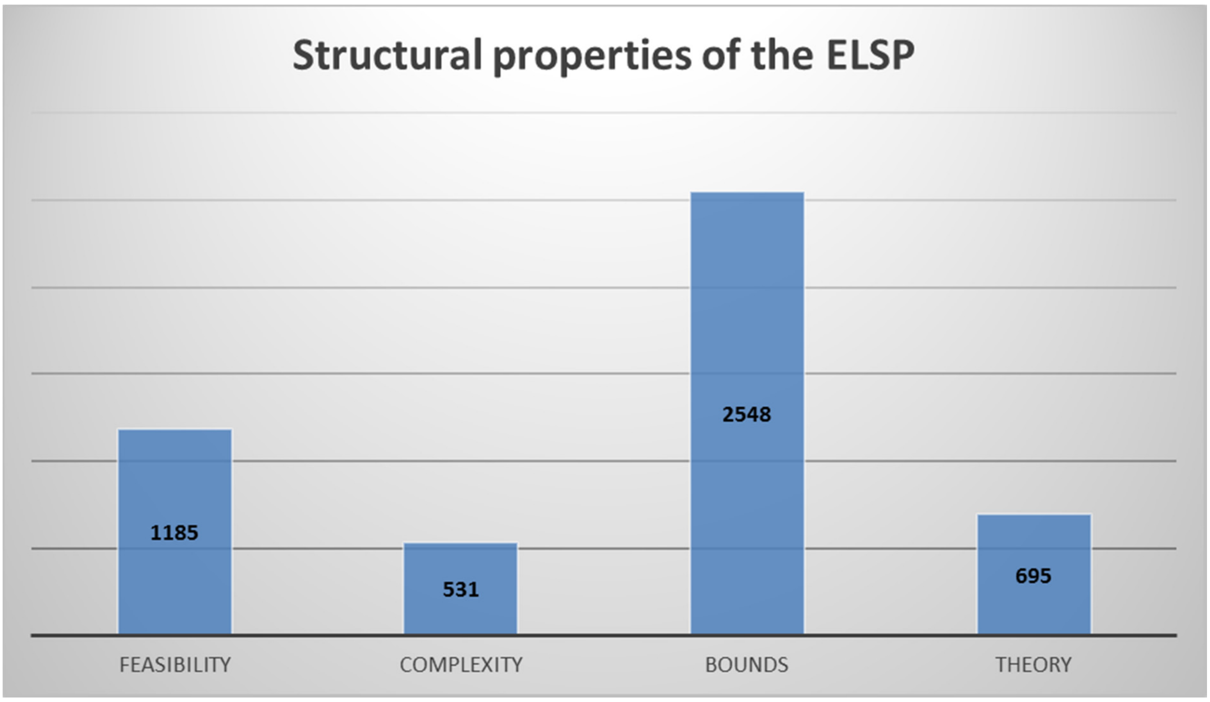

Figure 13: *Subgroups of the category “structural properties of the ELSP” and their respective recording unit hits*

5.2.2.7 *Scheduling objectives*

To compare our results for the category "scheduling objectives" with those obtained by Santander-Mercado and Jubiz-Diaz (2016), we considered the same five subgroups. Our CA confirms the results of Santander-Mercado and Jubiz-Diaz (2016) that the most important objective in research on the ELSP has been cost.

Figure 14 shows the number of hits that were obtained for different cost terms. As can be seen, prior research had a strong focus on total cost. Since total costs are usually made up of setup (changeover) and holding costs, it is not surprising that these recording units also received a substantial number of hits.

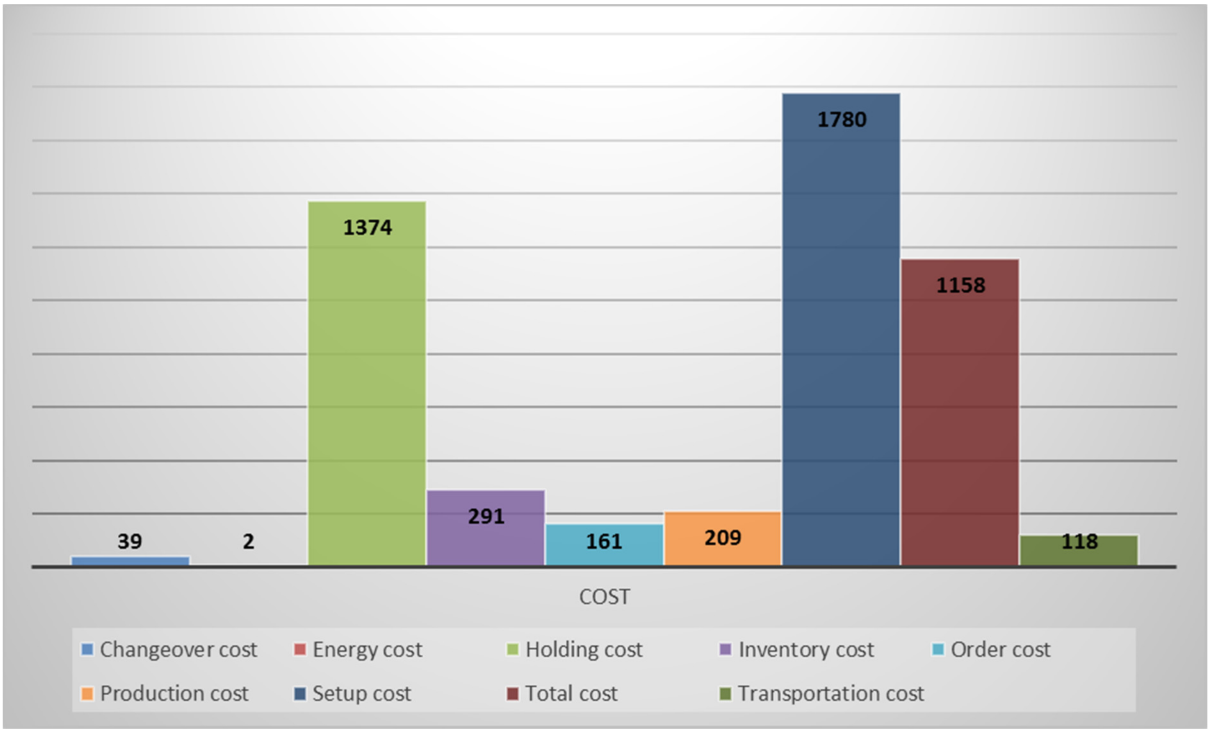

Figure 14: *Number of hits for different cost terms in our sample*

5.2.2.8 *Extended coverage*

The category "extended coverage" consists of a single subgroup only. Figure 15 illustrates the percentage distribution of recording unit hits for different extensions of the ELSP. As could already be seen in Section 5.2.2.1, stochastic problems play an important role in our sample. Hence, it is not surprising that the Stochastic Economic Lot Scheduling Problem (SELSP with 27% of the recording unit count) received the highest number of hits followed by the Economic Lot and Delivery Scheduling Problem (ELDSP) and the Economic Lot Scheduling Problem with returns (ELSPR). For the Economic Lot Scheduling and Delivering Problem (ELSDP), the Proportional Lotsizing and Scheduling Problem (PLSP) and the Economic Lot Scheduling Problem with reworks, we obtained less than five hits for each recording unit.

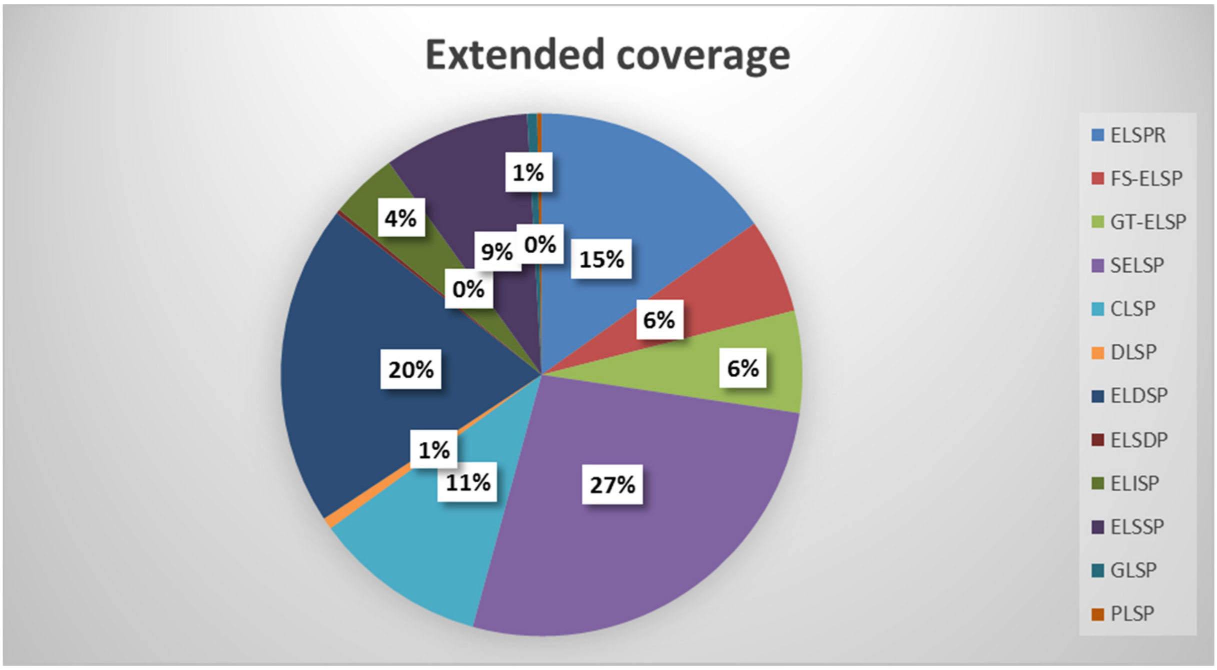

Figure 15: *Category "extended coverage" and their shares in the recording unit count*[10]

5.2.2.9 Other topics

The category "other topics" consists of five subgroups, and their shares in the recording unit count are shown in Figure 16. One interesting result is that green topics appear not to have played a major role in the ELSP, as we only received relatively few recording unit hits for "green" (11 hits) and "waste/wastage" (35 hits). This is surprising, as the scheduling of products and the sizing of lots may influence energy consumption, the use of (virgin) raw materials and the emergence of inventories that all have been shown to impact the environment.

Apart from this, it is not surprising to see that the subgroup "inventory" received a substantial number of hits. Given that the ELSP determines lot sizes that lead to inventory build-up in the company, inventory is directly connected to the research topic at hand. Recording units such as "base-stock", "safety-stock" and "make-to-order" have also been considered quite frequently in our sample.

The subgroup "practical application" highlights types of products that have been considered in the literature. Most of the hits were obtained for the recording unit "food" (252 hits), which,

[10] We again excluded the recording units "Economic Lot Scheduling Problem" and "ELSP".

given that it is a perishable good, may also explain the relatively high number of hits received for “shelf life”. Another recording unit that received some attention is “stamping”, which may have been influenced by the data set published by Bomberger (1966) that has established itself as a standard-data set in this area that is based on a real-world stamping problem.

In the subgroup “deliveries”, most hits were obtained for the recording units “delivery” (826 hits), “shipment” (328 hits), and “batch size” (261 hits). Since the classical lot sizing literature has shown that inventory holding cost can be reduced significantly by shipping partial lots (so-called batches) to the subsequent stages, it is not surprising that batch shipments have received some attention in the ELSP literature as well, particularly in its extensions, such as the ELDSP.

The subgroup “basic lot sizing models” gives a brief overview of recording unit hits related to EOQ, EPQ, and EMQ models. Even though works that propose such models were excluded from our sample, they are still related to the ELSP, which is why some authors referred to models from this area or differentiated the ELSP from related models in the field.

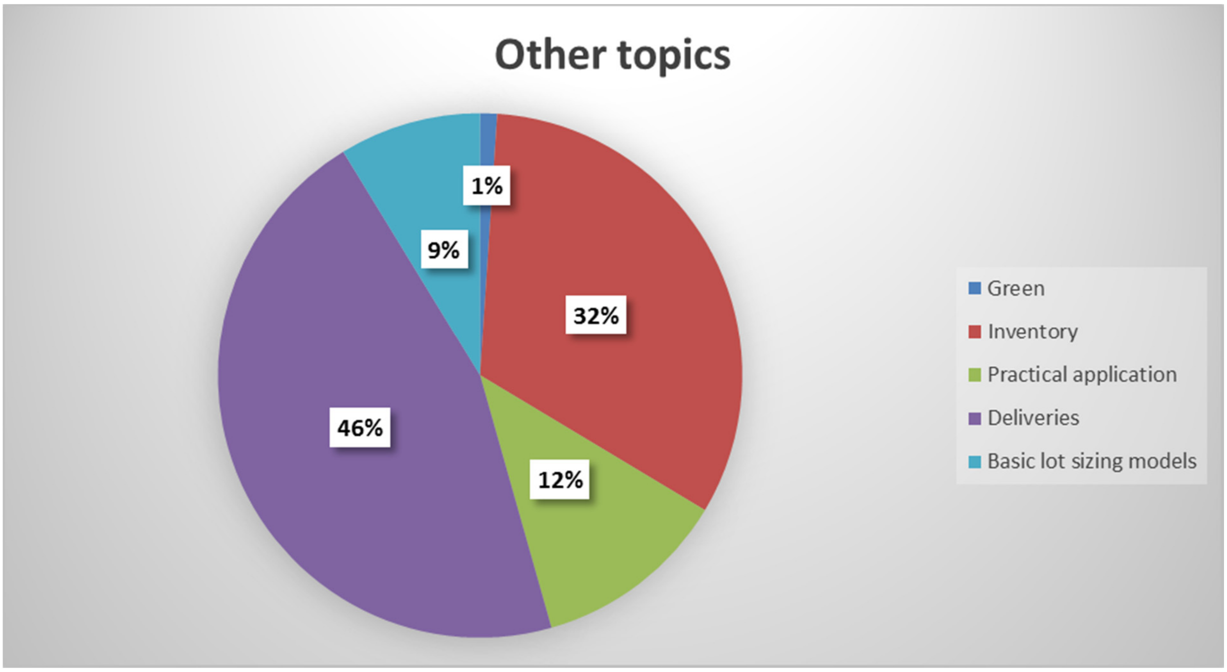

Figure 16: *Subgroups of the category “other topics” and their shares in the recording unit count*

6 Discussion and conclusion

The paper at hand investigated the Economic Lot Scheduling Problem (ELSP) and its extensions and applied a content analysis (CA) to a systematically generated sample of 228 papers.

Based on earlier literature reviews and our analysis of the literature, we developed a conceptual framework in a combined deductive and inductive approach to identify key research topics investigated in the context of the ELSP. Our framework consists of 214 recording units, 42 subgroups and nine categories. Our analysis identified two categories that received more than 40% of the recording unit hits, which highlights the importance these recording units and the associated topics enjoy in the literature: “specific assumptions” and “solution methodology”. In the category “specific assumptions”, the recording units “setup” and “production rate” achieved the highest number of recording unit hits, while for the category “solution methodology”, the recording units “heuristic” and “genetic algorithm” were dominant.

The CA conducted in this paper has limitations. First, the final literature sample was generated by searching two scholarly databases using only two keywords. In addition, only works that appeared in peer-reviewed academic journals were considered relevant. Using other databases or keywords and considering also so-called grey literature (e.g., conference proceedings, books, theses) may have led to a different literature sample and consequently different results. In addition, it was not possible to evaluate all potentially interesting recording units. For example, the abbreviation “IS” has frequently been used for “independent solution”; however, as the word “is” is also often used as a predicate, an automatic count of “is” was not possible with the software MAXQDA 12. In addition, even though a combined deductive and inductive approach was used to generate the conceptual framework, important recording units related to the ELSP could have been missed.

Based on our analysis of the literature sample, we identified the following opportunities for future research:

- As compared to other scheduling policies, we found that the Extended-Basic-Period-Approach, originally formulated by Elmaghraby (1977), did not receive much attention in the literature so far. Compared to the Basic-Period-Approach of Bomberger (1966), the Extended-Basic-Period-Approach uses a less restrictive feasibility constraint, such that it often leads to better results. Hence, applying and extending the Extended-Basic-Period-Approach to the various topics that are currently being studied in the context of the ELSP (such as the energy-aware ELSP) may be promising.
- Our study showed that simulation has not played an important role in the ELSP so far. Prior research has instead often used standard data sets for evaluating the performance

of new scheduling policies or solution approaches, such as the data set proposed by Bomberger (1966). Conducting more extensive numerical experiments or applying simulation to complex manufacturing systems subject to ELSP-type problems may hence contribute to deriving further interesting insights.

- Another interesting opportunity for future research could be the integration of energy efficiency criteria into ELSP models. In light of increasing energy prices and a higher customer demand for products produced in an energy-efficient way, it is surprising to see that energy aspects have thus far not played a major role in the ELSP. Even though a few papers have appeared on this topic recently, further research on energy-efficiency in an ELSP context may be interesting.
- It is striking that above and beyond energy consumption, sustainability topics have not attracted much attention in the ELSP literature in the past, even though sustainability topics have frequently been addressed in the field of lot sizing. Examples include the production of waste and its subsequent recycling or the generation of greenhouse gasses. Integrating these aspects into ELSP models may contribute to lower the environmental impact of production.
- In addition, recording units that received a relatively low number of hits could indicate interesting opportunities for future research. Examples are the “multi-machine” case, e.g. the FS-ELSP, transportation (shipment) cost between two or more stages, or the application of artificial intelligence to the ELSP.

Another opportunity for future research might be to analyze the development of different recording unit hits over time to gain insights into how the relative relevance of different research topics has developed over time and to identify emergent research trends.

References

Abedinnia, H., Glock, C.H., Schneider, M.D., 2017. Machine scheduling in production: a content analysis. Applied Mathematical Modelling, 50, 279-299.

Bomberger, E., 1966. A dynamic programming approach to a lot size scheduling problem. Management Science, 12(11), 778-784.

Chan, H.K., Chung, S.H., Lim, M.K., 2013. Recent research trend of economic-lot scheduling problems. Journal of Manufacturing Technology Management, 24(3), 465-482.

Community culture and the environment: A guide to understanding a sense of place, 2002. U.S. EPA (EPA 842-B-01-003), Office of Water, Washington, DC.

Elmaghraby, S.E., 1977. An extended basic period approach of the ELSP. Report No. 115 Graduate Program in Operations Research, North Carolina University at Raleigh, NC.

Elmaghraby, S.E., 1978. The economic lot scheduling problem (ELSP): review and extensions. Management Science, 24(6), 587-598.

Gallego, G., Joneja, D., 1994. Economic lot scheduling problem with raw material considerations. Operations Research, 42(1), 92-101.

Gallego, G., Shaw, D.X., 1997. Complexity of the ELSP with general cyclic schedules. IIE Transactions, 29(2), 109-113.

Giri, B.C., Moon, I., Yun, W.Y., 2003. Scheduling economic lot sizes in deteriorating production systems. Naval Research Logistics (NRL), 50(6), 650-661.

Grosse, E.H., Glock, C.H., Neumann, W.P., 2015. Human factors in order picking system design: a content analysis. IFAC-PapersOnLine, 48(3), 320-325.

Grosse, E.H., Glock, C.H., Neumann, W.P., 2017. Human factors in order picking: a content analysis of the literature. International Journal of Production Research, 55(5), 1260-1276.

Khouja, M., Michalewicz, Z., Wilmot, M., 1998. The use of genetic algorithms to solve the economic lot size scheduling problem. European Journal of Operational Research, 110(3), 509-524.

Neuendorf, K.A., 2002. The content analysis guidebook. Thousand Oaks: Sage.

Rhoades, E.A., 2011. Literature reviews. Volta Review, 111(3), 353-368.

Rogers, J., 1958. A computational approach to the economic lot scheduling problem. Management Science, 4(3), 264-291.

Santander-Mercado, A., Jubiz-Diaz, M., 2016. The economic lot scheduling problem: a survey. International Journal of Production Research, 54(16), 4973-4992.

Seuring, S., Gold, S., 2012. Conducting content-analysis based literature reviews in supply chain management. Supply Chain Management: An International Journal, 17(5), 544-555.

Taylor, G.D., Taha, H.A., Chowning, K.M., 1997. A heuristic model for the sequence-dependent lot scheduling problem. Production Planning and Control, 8(3), 213-225.

Vidal-Carreras, P.I., Garcia-Sabater, J.P., Marin-Garcia, J.A., Garcia-Sabater, J.J., 2008. ELSP variants: a review. 2nd International Conference on Industrial Engineering and Industrial Management, 1215-1225.

Weber, R.P., 1990. Basic content analysis. Newbury Park, CA: Sage.

Winands, E.M.M., Adan, I.J.B.F., van Houtum, G.J., 2011. The stochastic economic lot scheduling problem: A survey. European Journal of Operational Research, 210(1), 1-9.

Appendix

Appendix A

Table A.1: *Results of the content analysis*

Category	Subgroup	Terms	Recording units	Number of hits in the sample	Number of papers	Hits per paper
1. Type of problem	Dynamic		dynamic[1]	447	92	4.86
	Static		static	98	39	2.51
	Stochastic		fuzzy	213	9	23.67
			probabilistic	26	12	2.17
			stochastic	582	82	7.10
	Deterministic		deterministic	363	122	2.98
2. Scheduling policy	Common-cycle-approach		common cycle, CC, CCA	1831	164	11.16
	Basic-period-approach	General terms	basic period[2], BP, BPA	1837	138	13.31
			fundamental cycle, fundamental period, FC	363	36	10.08
		Extended-Basic-Period-Approach	extended basic, EBP, EBPA	468	71	6.59

[1] The recording unit hits for “dynamic programming” have been subtracted from the recording unit hits for “dynamic”.
[2] The recording unit hits for “extended basic” have been subtracted from the recording unit hits for “basic period”.

	Time-varying-lot-size-approach		time varying lot, TVLSA, TVLS	359	87	4.13
	No cycle approach		no cycle approach	0	0	0.00
	2^x-policy		power-of-two, POT, POW2	822	80	10.28
3. Solution methodology	Exact methods	General terms	branch and bound, B&B	58	29	2.00
			dynamic programming, DP	289	111	2.60
			enumeration	128	47	2.72
			exact algorithm	8	5	1.60
			exact method	14	5	2.80
			linear programming, LP	369	99	3.73
			marginal analysis	44	17	2.59
			optimum	348	95	3.66
		More detailed	integer linear programming, ILP*	52	19	2.74
			lagrange, lagrangian	118	40	2.95
			mixed integer linear programming, MILP*	99	18	5.50
	Heuristic methods	General terms	dispatch rule, dispatching rule	1	1	1.00
			heuristic[3]	3396	200	16.98
			priority rule	55	10	5.50
			g-group heuristic*	17	13	1.31

[3] The recording unit hits for “meta-heuristic” have been subtracted from the recording unit hits for “heuristic”.

		Specific heu-ristics	Johnson’s algorithm	2	1	2.00
			mixed integer nonlinear programming, MINLP*	87	13	6.69
			pt heuristic*	18	7	2.57
			two-group heuristic*	20	11	1.82
	Meta-heuristic methods	General terms	evolutionary algorithm, evolution, EA	224	59	3.80
			greedy algorithm	65	22	2.95
			local search	106	31	3.42
			meta-heuristic	76	24	3.17
			neighborhood	152	36	4.22
		Specific meta-heuristics	artificial bee colony algorithm, ABC	263	8	32.88
			simulated annealing, SA	403	41	9.83
			ant colony algorithm, ACO, ACA	52	14	3.71
			binary search	61	6	10.17
			cuckoo search, CS	165	16	10.31
			genetic algorithm, GA	1494	71	21.04
			golden section search, GSS	71	9	7.89
			hybrid genetic algorithm, HGA*	376	36	10.44
			iterated local search, ILS*	58	3	19.33
			memetic algorithm	15	8	1.88
			partical swarm optimization, PSO	183	7	26.14
			tabu search, taboo, TS	470	66	7.12
			artificial intelligence	3	3	1.00

	Artificial intelligence		artificial neural network	3	3	1.00
	Simulation		simulation	353	64	5.52
4. Specific assumptions	Planning horizon		finite horizon[4]	82	29	2.83
			fixed horizon	1	1	1.00
			infinite horizon	74	43	1.72
	Production rate	General term	production rate	1209	198	6.11
		More detailed	adjustable production rate*	1	1	1.00
			bottleneck	55	20	2.75
			constant production rate*	33	28	1.18
			controllable production rate[5]*	19	9	2.11
			fixed production rate*	15	7	2.14
			learning	130	27	4.81
			non-uniform production rate*	0	0	0.00
			uncontrollable production rate*	1	1	1.00
			uniform production rate*	0	0	0.00
			variable production rate*	18	11	1.64
	Setup	General terms	setup[6]	4476	224	19.98
			changeover[7]	273	37	7.38

[4] The recording unit hits for “infinite horizon” have been subtracted from the recording unit hits for “finite horizon”.
[5] The recording unit hits for “uncontrollable production rate” have been subtracted from the recording unit hits for “controllable production rate”.
[6] The recording unit hits for “setup cost” have been subtracted from the recording unit hits for “setup” since they belong to the category “scheduling objectives”.
[7] The recording unit hits for “changeover cost” have been subtracted from the recording unit hits for “changeover” since they belong to the category “scheduling objectives”.

		More detailed	setup time*	1751	206	8.50
			changeover time*	42	14	3.00
			family, families	695	54	12.87
			group technology	54	15	3.60
	Demand	General term	demand rate	797	192	4.15
		More detailed	constant demand	142	82	1.73
			deterministic demand	40	31	1.29
			discrete demand	3	2	1.50
			fuzzy demand	21	5	4.20
			linear increasing demand	0	0	0.00
			linearly changing demand	16	2	8.00
			normally distributed time stationary demand	0	0	0.00
			perturbation in products demand	0	0	0.00
			random demand	77	36	2.14
			stationary stochastic demand	6	6	1.00
			stochastic demand	107	34	3.15
			time varying demand	3	2	1.50
			uncertain demand	8	7	1.14
	Number of products		two product, two item	445	115	3.87
			three product, three item	132	64	2.06
			multi item, multi product, n-item, n-product	843	180	4.68
	Shortages	General terms	runout	114	14	8.14

			shortage	392	91	4.31
			stock out	307	61	5.03
		More detailed	backlog	258	62	4.16
			backorder	584	86	6.79
			lost order, order lost	0	0	0.00
			lost sale	165	31	5.32
	Zero switch rule		zero switch, ZSR, zero inventory	183	57	3.21
	Sequence-de-pendency		sequence dependent, sequence dependency	315	69	4.57
	Product prob-lems		coproduction	151	2	75.50
			defective item	77	11	7.00
			deterioration, deteriorating item	238	24	9.92
			failure rate	8	6	1.33
			imperfect quality	36	14	2.57
			non-conforming item	34	9	3.78
			remanufacturing	268	12	22.33
			reorder	127	31	4.10
			reproduction	62	21	2.95
			restoration	64	10	6.40
			rework	387	26	14.88
			shelf life	273	22	12.41
			breakdown	75	12	6.25

	Machine prob-lems		disruption	112	17	6.59
			in control	54	16	3.38
			inspection	283	20	14.15
			machine availability	4	3	1.33
			machine reliability	0	0	0.00
			maintenance	90	34	2.65
			out of control	72	15	4.80
			performance decay	47	3	15.67
			process restoration	11	6	1.83
			repair	166	20	8.30
			stability	56	18	3.11
			unreliable machine	0	0	0.00
5. Flow pattern	Single machine		single machine, one facility, one machine, single facility, 1 machine, 1 facility, single stage	993	198	5.02
	Multi-machine	General terms	multi-facility, multi-machine, multi-factory, n-machine, n-facilities, m-machine, m-facilities, two-stage, multi-stage	325	85	3.82
			flow shop	225	33	6.82
			job shop	137	26	5.27
			open shop	4	4	1.00
			identical machine[8]	34	20	1.70

[8] The recording unit hits for "non-identical machine" have been subtracted from the recording unit hits for "identical machine".

		Parallel machines	non-identical machine	4	3	1.33
			parallel machine	122	21	5.81
			unrelated machine	3	3	1.00
			unrelated parallel machine*	32	5	6.40
	Schedule		cyclic schedule[9]	378	94	4.04
			no cyclic schedule	2	2	1.00
			repetitive schedule	15	10	1.50
			rotation schedule	84	16	5.25
6. Structural properties of the ELSP	Feasibility	General term	feasibility	1185	177	6.69
		More detailed	capacity feasibility*	11	7	1.57
			schedule feasibility*	396	112	3.54
	Complexity		complexity	168	99	1.70
			np-complete	38	22	1.73
			np-hard	233	107	2.18
			polynomial	92	40	2.30
	Bounds		lower bound, LB	1757	156	11.26
			upper bound, UB	650	110	5.91
			independent solution	141	50	2.82
	Theory		theorem	328	51	6.43
			lemma	299	40	7.48
			corollary	68	23	2.96

[9] The recording unit hits for "no cyclic schedule" have been subtracted from the recording unit hits for "cyclic schedule".

7. Scheduling objectives	Cost		changeover cost	39	12	3.25
			energy cost	2	2	1.00
			holding cost	1374	213	6.45
			inventory cost	291	93	3.13
			order cost	161	42	3.83
			production cost	209	56	3.73
			setup cost	1780	219	8.13
			total cost	1158	180	6.43
			transportation cost	118	29	4.07
	Workload		work in process, work in progress, WIP	238	37	6.43
			workload	163	15	10.87
	Profit		profit	214	29	7.38
	Inventory		amount of inventory	11	10	1.10
	Makespan		completion time	85	30	2.83
			flow time	50	9	5.56
			makespan	85	14	6.07
			shortest processing time, SPT	55	9	6.11
8. Extended coverage	ELSP and its extensions		economic lot scheduling problem with returns, ELSPR*	171	8	21.38
			economic lot scheduling problem with reworks*	0	0	0.00
			flow shop ELSP, FS-ELSP*	66	4	16.50

			group technology-economic lot scheduling problem, GT-ELSP*	71	9	7.89
			capacitated lot sizing problem, CLSP	121	19	6.37
			discrete lotsizing and scheduling problem, DLSP	8	3	2.67
			economic lot and delivery scheduling problem, ELDSP	224	26	8.62
			economic lot scheduling and delivering problem, ELSDP	3	1	3.00
			economic lot and inspection scheduling problem, ELISP	47	2	23.50
			economic lot and supply scheduling problem, ELSSP	102	5	20.40
			economic lot scheduling problem, ELSP	4391	210	20.91
			general lotsizing and scheduling problem, GLSP	7	2	3.50
			proportional lotsizing and scheduling problem, PLSP	3	2	1.50
			stochastic economic lot scheduling problem, SELSP*	303	24	12.63
9. Other topics	Green		emission	0	0	0.00
			green	11	6	1.83
			sustainable, sustainability	0	0	0.00
			waste, wastage	35	23	1.52
	Inventory		base-stock	328	19	17.26

			buffer	95	30	3.17
			intermediate storage	1	1	1.00
			make-to-order, MTO	178	16	11.13
			make-to-stock, MTS	149	18	8.28
			order up to level	49	10	4.90
			procurement	74	25	2.96
			safety-stock	415	38	10.92
			storage	142	37	3.84
	Practical application		chemical	91	33	2.76
			fashion	41	31	1.32
			food	252	36	7.00
			pharmaceutical	42	10	4.20
			plastic	38	26	1.46
			stamping	64	24	2.67
	Deliveries		batch size	261	61	4.28
			delivery	826	64	12.91
			equal-lot[10], ELS	91	33	2.76
			lead time	178	36	4.94
			lot-for-lot	4	2	2.00
			resource constraint	16	13	1.23
			routing	103	19	5.42

[10] The recording unit hits for "unequal-lot" have been subtracted from the recording unit hits for "equal-lot".

			shipment	328	28	11.71
			shipping	104	21	4.95
			unequal-lot	4	4	1.00
			vehicle	92	13	7.08
	Basic lot sizing models		economic order quantity, EOQ	181	68	2.66
			economic production quantity, EPQ	148	38	3.89
			economic manufacture quantity, EMQ	55	10	5.50

Appendix B

Aardal, K., Larsson, T., 1990. A benders decomposition based heuristic for the hierarchical production planning problem. European Journal of Operational Research, 45(1), 4-14.

Adelman, D., Barz, C., 2014a. A price-directed heuristic for the economic lot scheduling problem. IIE Transactions, 46(12), 1343-1356.

Adelman, D., Barz, C., 2014b. A unifying approximate dynamic programming model for the economic lot scheduling problem. Mathematics of Operations Research, 39(2), 374-402.

Adhisatya, F.Y., Ai, T.J., Gong, D.C., 2015. Economic lot scheduling problem with two imperfect key modules. Procedia Manufacturing, 4, 30-37.

Adil, S.H., Azhar Ali, S.S., Hussaan, A., Raza, K., 2013a. Solution of economic lot scheduling problem: a hybrid meta-heuristic approach. Life Science Journal, 10(3), 303-309.

Adil, S.H., Azhar Ali, S.S., Hussaan, A., Raza, K., 2013b. Hybridization of multiple intelligent schemes to solve economic lot scheduling problem using basic period approach. Life Science Journal, 10(2), 2992-3005.

Akrami, B., Karimi, B., Moattar Hosseini, S.M., 2006. Two metaheuristic methods for the common cycle economic lot sizing and scheduling in flexible flow shops with limited intermediate buffers: the finite horizon case. Applied Mathematics and Computation, 183(1), 634-645.

Alle, A., Pinto, J.M., Papageorgiou, L.G., 2004. The economic lot scheduling problem under performance decay. Industrial and Engineering Chemistry Research, 43(20), 6463-6475.

Allen, S.J., 1990. Production rate planning for two products sharing a single process facility: a real-world case study. Production & Inventory Management Journal, 31(3), 24-29.

Anderson, E.J., 1990. Testing feasibility in a lot scheduling problem. Operations Research, 38(6), 1079-1088.

Axsäter, S., 1982. Alternative dynamic programming approaches to obtain upper bounds for the economic lot scheduling problem. Engineering Costs & Production Economics, 6(1-4), 17-23.

Axsäter, S., 1984. Lower bounds for the economic lot scheduling problem using aggregation. European Journal of Operational Research, 17(2), 201-206.

Axsäter, S., 1987. An extension of the extended basic period approach for economic lot scheduling problems. Journal of Optimization Theory and Applications, 52(2), 179-189.

Bae, H., Moon, I., Yun, W., 2014. Economic lot and supply scheduling problem: a time-varying lot sizes approach. International Journal of Production Research, 52(8), 2422-2435.

Bae, H., Moon, I., Yun, W., 2016. A time-varying lot sizes approach for the economic lot scheduling problem with returns. International Journal of Production Research, 54(11), 3380-3396.

Banerjee, A., 2009. Simultaneous determination of multiproduct batch and full truckload shipment schedules. International Journal of Production Economics, 118(1), 111-117.

Banerjee, A., Pyreddy, V.R., Kim, S.L., 1996. Investment policy for multiple product setup reduction under budgetary and capacity constraints. International Journal of Production Economics, 45(1-3), 321-327.

Beck, F.G., Glock, C.H., 2016. The impact of batch shipments on the economic lot scheduling problem. Computers & Industrial Engineering, 96, 126-139.

Ben-Daya, M., Hariga, M., 2000. Economic lot scheduling problem with imperfect production processes. Journal of the Operational Research Society, 51(7), 875-881.

Boctor, F.F., 1988. A more efficient formulation of the single machine, static demand, lot scheduling problem. Engineering Costs and Production Economics, 14(1), 3-10.

Boctor, F.F., Poulin, P., 2005. Heuristics for the N-product, M-stage, economic lot sizing and scheduling problem with dynamic demand. International Journal of Production Research, 43(13), 2809-2828.

Bollapragada, R., Croce, F.D., Ghirardi, M., 2011. Discrete-time, economic lot scheduling problem on multiple, non-identical production lines. European Journal of Operational Research, 215(1), 89-96.

Bollapragada, R., Rao, U., 1999. Single-stage resource allocation and economic lot scheduling on multiple, nonidentical production lines. Management Science, 45(6), 889-904.

Bourland, K.E., Yano, C.A., 1994. The strategic use of capacity slack in the economic lot scheduling problem with random demand. Management Science, 40(12), 1690-1704.

Bourland, K.E., Yano, C.A., 1997. A comparison of solution approaches for the fixed-sequence economic lot scheduling problem. IIE Transactions, 29(2), 103-108.

Brander, P., Forsberg, R., 2006. Determination of safety stocks for cyclic schedules with stochastic demands. International Journal of Production Economics, 104(2), 271-295.

Brander, P., Levén, E., Segerstedt, A., 2005. Lot sizes in a capacity constrained facility - a simulation study of stationary stochastic demand. International Journal of Production Economics, 93-94(SPEC.ISS.), 375-386.

Brander, P., Segerstedt, A., 2009. Economic lot scheduling problems incorporating a cost of using the production facility. International Journal of Production Research, 47(13), 3611-3624.

Bulut, O., Tasgetiren, M.F., 2014. An artificial bee colony algorithm for the economic lot scheduling problem. International Journal of Production Research, 52(4), 1150-1170.

Cárdenas-Barrón, L.E., 2012. A note on models for a family of products with shelf life, and production and shortage costs in emerging markets. International Journal of Industrial Engineering Computations, 3(2), 277-280.

Carreno, J.J., 1990. Economic lot scheduling for multiple products on parallel identical processors. Management Science, 36(3), 348-358.

Casas-Liza, J., Pinto, J.M., Papageorgiou, L.G., 2005. Mixed integer optimization for cyclic scheduling of multiproduct plants under exponential performance decay. Chemical Engineering Research and Design, 83(10 A), 1208-1217.

Chaieb Memmi, I., Hammani Laaroussi, S., 2013. A new approach for solving capacitated lot sizing and scheduling problem with sequence and period-dependent setup costs. Journal of Industrial Engineering and Management, 6(4), 1027-1054.

Chan, H.K., Chung, S.H., Chan, T.M., 2012. Combining genetic approach and integer programming to solve multi-facility economic lot-scheduling problem. Journal of Intelligent Manufacturing, 23(6), 2397-2405.

Chandrasekaran, C., Rajendran, C., Chetty, O.V.K., Hanumanna, D., 2009. A two-phase metaheuristic approach for solving Economic Lot Scheduling Problems. International Journal of Operational Research, 4(3), 296-322.

Chandrasekaran, C., Rajendran, C., Chetty, O.V.K., Hanumanna, D., 2007. Metaheuristics for solving economic lot scheduling problems (ELSP) using time-varying lot-sizes approach. European Journal of Industrial Engineering, 1(2), 152-181.

Chang, P.-T., Yao, M.-J., Huang, S.-F., Chen, C.-T., 2006. A genetic algorithm for solving a fuzzy economic lot-size scheduling problem. International Journal of Production Economics, 102(2), 265-288.

Chang, Y.J., Yao, M.J., 2009. A genetic algorithm for solving the economic lot scheduling problem with reworks. Journal of the Chinese Institute of Industrial Engineers, 26(5), 411-425.

Chang, Y.J., Yao, M.J., 2011. New heuristics for solving the economic lot scheduling problem with reworks. Journal of Industrial and Management Optimization, 7(1), 229-251.

Chatfield, D.C., 2007. The economic lot scheduling problem: a pure genetic search approach. Computers & Operations Research, 34(10), 2865-2881.

Cheng, F., Yan, H., Yang, J., 1998. Production scheduling of continuous flow lines: multiple products with setup times and costs. Production and Operations Management, 7(4), 387-400.

Chung, S.H., Chan, H.K., 2012. A two-level genetic algorithm to determine production frequencies for economic lot scheduling problem. IEEE Transactions on Industrial Electronics, 59(1), 611-619.

Clausen, J., Ju, S., 2006. A hybrid algorithm for solving the economic lot and delivery scheduling problem in the common cycle case. European Journal of Operational Research, 175(2), 1141-1150.

Co, H.C., Sharafali, M., Goh, M., 2001. Scheduling part-families under FMS: To mix or not to mix? International Transactions in Operational Research, 8(2), 139-153.

Cooke, D.L., Rohleder, T.R., 2006. Inventory evaluation and product slate management in large-scale continuous process industries. Journal of Operations Management, 24(3), 235-249.

Cooke, D.L., Rohleder, T.R., Silver, E.A., 2004. Finding effective schedules for the economic lot scheduling problem: a simple mixed integer programming approach. International Journal of Production Research, 42(1), 21-36.

Cunha Neto, E., Ferreira Filho, V.J.M., Arruda, E.F., 2015. Stochastic economic lot sizing and scheduling problem with pitch interval, reorder points and flexible sequence. International Journal of Production Research, 53(19), 5948-5961.

Davis, S.G., 1990. Scheduling economic lot size production runs. Management Science, 36(8), 985-998.

Davis, S.G., 1995. An improved algorithm for solving the economic lot size problem (ELSP). International Journal of Production Research, 33(4), 1007-1026.

Ditt, S., Kuhn, H., 1997. An improved algorithm for solving the economic lot size problem (ELSP): A note. International Journal of Production Research, 35(6), 1785-1787.

Dobson, G., 1987. The economic lot-scheduling problem: Achieving feasibility using time-varying lot sizes. Operations Research, 35(5), 764-771.

Dousthaghi, S., Tavakkoli-Moghaddam, R., 2012. An economic lot and delivery scheduling problem with the fuzzy shelf life in a flexible job shop with unrelated parallel machines. International Journal of Industrial Engineering Computations, 3(4), 663-680.

Dousthaghi, S., Tavakkoli-Moghaddam, R., Makui, A., 2013. Solving the economic lot and delivery scheduling problem in a flexible job shop with unrelated parallel machines and a shelf life by a proposed hybrid PSO. International Journal of Advanced Manufacturing Technology, 68(5-8), 1401-1416.

El-Najdawi, M.K., 1992. A compact heuristic for common cycle lot-size scheduling in multi-stage, multi-product production processes. International Journal of Production Economics, 27(1), 29-41.

El-Najdawi, M.K., 1995. An experimental evaluation of single-stage lot-size scheduling heuristics in a multi-stage, multi-product production environment. Journal of Business Logistics, 16(2), 175-195.

Elhafsi, M., Bai, S.X., 1997. The common cycle economic lot scheduling problem with backorders: Benefits of controllable production rates. Journal of Global Optimization, 10(3), 283-303.

Eynan, A., 2003. The benefits of flexible production rates in the economic lot scheduling problem. IIE Transactions (Institute of Industrial Engineers), 35(11), 1057-1064.

Faaland, B.H., Schmitt, T.G., Arreola-Risa, A., 2004. Economic lot scheduling with lost sales and setup times. IIE Transactions, 36(7), 629-640.

Federgruen, A., Katalan, Z., 1996. The stochastic economic lot scheduling problem: Cyclical base-stock policies with idle times. Management Science, 42(6), 783-796.

Federgruen, A., Katalan, Z., 1998. Determining production schedules under base-stock policies in single facility multi-item production systems. Operations Research, 46(6), 883-898.

Federgruen, A., Katalan, Z., 1999. The impact of adding a make-to-order item to a make-to-stock production system. Management Science, 45(7), 980-994.

Fitz-Gerald, J.M., Billington, E.J., 1981. Manufacturing sequences for the economic lot scheduling problem. Bulletin of the Australian Mathematical Society, 24(3), 405-431.

Fransoo, J.C., Sridharan, V., Bertrand, J.W.M., 1995. A hierarchical approach for capacity coordination in multiple products single-machine production systems with stationary stochastic demands. European Journal of Operational Research, 86(1), 57-72.

Fujita, S., 1978. The application of marginal analysis to the economic lot scheduling problem. AIIE Transactions, 10(4), 354-361.

Gallego, G., 1990a. An extension to the class of easy economic lot scheduling problems. IIE Transactions (Institute of Industrial Engineers), 22(2), 189-190.

Gallego, G., 1990b. Scheduling the production of several items with random demands in a single facility. Management Science, 36(12), 1579-1592.

Gallego, G., 1993. Reduced production rates in the economic lot scheduling problem. International Journal of Production Research, 31(5), 1035-1046.

Gallego, G., 1994. When is a base stock policy optimal in recovering disrupted cyclic schedules? Naval Research Logistics (NRL), 41(3), 317-333.

Gallego, G., Joneja, D., 1994. Economic lot scheduling problem with raw material considerations. Operations Research, 42(1), 92-101.

Gallego, G., Moon, I., 1992. The effect of externalizing setups in the economic lot scheduling problem. Operations Research, 40(3), 614-619.

Gallego, G., Moon, I., 1995. Strategic investment to reduce setup times in the economic lot scheduling problem. Naval Research Logistics (NRL), 42(5), 773-790.

Gallego, G., Roundy, R., 1992. The economic lot scheduling problem with finite backorder costs. Naval Research Logistics (NRL), 39(5), 729-739.

Gallego, G., Shaw, D.X., 1997. Complexity of the ELSP with general cyclic schedules. IIE Transactions, 29(2), 109-113.

Galvin, T.M., 1987a. Economic lot scheduling problem with sequence-dependent setup costs. Production & Inventory Management, 28(1), 96-105.

Galvin, T.M., 1987b. Simulating the economic lot scheduling problem: part I. model description. Production & Inventory Management Journal, 28(4), 32-38.

Galvin, T.M., 1988. Simulating the economic lot scheduling problem: part II. experimental results. Production & Inventory Management Journal, 29(1), 11-15.

Galvin, T.M., 1989. Increasing production efficiency through restricted sourcing and sequence-dependent setup costs. Production and Inventory Management Journal, 30(1), 58-62.

Garn, W., Aitken, J., 2015. Agile factorial production for a single manufacturing line with multiple products. European Journal of Operational Research, 245(3), 754-766.

Geng, P.C., Vickson, R.G., 1988. Two heuristics for the economic lot scheduling problem: an experimental study. Naval Research Logistics (NRL), 35(4), 605-617.

Gim, B., Han, M.H., 1997. Economic scheduling of products with N components on a single machine. European Journal of Operational Research, 96(3), 570-577.

Giri, B.C., Moon, I., 2004a. Note on an economic lot scheduling problem under budgetary and capacity constraints. International Journal of Production Economics, 91(3), 229-234.

Giri, B.C., Moon, I., 2004b. Accounting for idle capacity cost in the scheduling of economic lot sizes. International Journal of Production Research, 42(4), 677-691.

Giri, B.C., Moon, I., Yun, W.Y., 2003. Scheduling Economic Lot Sizes in Deteriorating Production Systems. Naval Research Logistics, 50(6), 650-661.

Glass, C.A., 1992. Feasibility of scheduling lot sizes of three products on one machine. Management Science, 38(10), 1482-1494.

Glass, C.A., 1994. Feasibility of scheduling lot sizes of two frequencies on one machine. European Journal of Operational Research, 75(2), 354-364.

Goncalves, J.F., Leachman, R.C., 1998. A hybrid heuristic and linear programming approach to multi-product machine scheduling. European Journal of Operational Research, 110(3), 548-563.

Goncalves, J.F., Leachman, R.C., Gascon, A., Xiong, Z.K., 1994. Heuristic scheduling policy for multi-item, multi-machine production systems with time-varying, stochastic demands. Management Science, 40(11), 1455-1468.

Goncalves, J.F., Sousa, P.S.A., 2011. A genetic algorithm for lot sizing and scheduling under capacity constraints and allowing backorders. International Journal of Production Research, 49(9), 2683-2703.

Goyal, S.K., Deshmukh, S.G., 1997. Integrated procurement- production system in a just-in-time environment-modelling and analysis. Production Planning and Control, 8(1), 31-36.

Graves, S.C., 1979. On the deterministic demand multi-product single-machine lot scheduling problem. Management Science, 25(3), 276-280.

Grznar, J., Riggle, C., 1997. An optimal algorithm for the basic period approach to the economic lot scheduling problem. Omega, 25(3), 355-364.

Gupta, D., 1992. On the economic lot scheduling problem with backlogging: The common cycle approach. Operations Research Letters, 12(2), 101-109.

Haessler, R.W., 1979. An improved extended basic period procedure for solving the economic lot scheduling problem. AIIE Transactions, 11(4), 336-340.

Hahm, J., Yano, C.A., 1995a. Economic lot and delivery scheduling problem: powers of two policies. Transportation Science, 29(3), 222-241.

Hahm, J., Yano, C.A., 1995b. The economic lot and delivery scheduling problem: models for nested schedules. IIE Transactions (Institute of Industrial Engineers), 27(2), 126-139.

Hahm, J., Yano, C.A., 1995c. The economic lot and delivery scheduling problem: the common cycle case. IIE Transactions (Institute of Industrial Engineers), 27(2), 113-125.

Haji, R., Mansuri, M., 1995. Optimum common cycle for scheduling a single-machine multiproduct system with a budgetary constraint. Production Planning and Control, 6(2), 151-156.

Haksoz, C., Pinedo, M., 2011. Economic lot scheduling with resources in parallel. International Journal of Production Research, 49(9), 2625-2641.

Hanson, B.B., Hodgson, T.J., Kay, M.G., King, R.E., Thoney-Barletta, K.A., 2015. On the economic lot scheduling problem: stock-out prevention and system feasibility. International Journal of Production Research, 53(16), 4903-4916.

Hennet, J.-C., 2001. A common cycle approach to lot-scheduling in multistage manufacturing systems. Production Planning & Control, 12(4), 362-371.

Ho, W.T., Tseng, Y.C., Hsiao, Y.C., 2015. The economic lot-size scheduling problem with equally sized batch shipment policy and stochastic demands. European Journal of Industrial Engineering, 9(2), 195-221.

Hockstra, D.J., 1997. Simple policies in the economic lot scheduling problem with zero setup costs. Production Planning & Control, 8(1), 25-30.

Holmbom, M., Segerstedt, A., van der Sluis, E., 2013. A solution procedure for economic lot scheduling problems even in high utilisation facilities. International Journal of Production Research, 51(12), 3765-3777.

Hoque, M.A., Kingsman, B.G., 2006. Synchronization in common cycle lot size scheduling for a multi-product serial supply chain. International Journal of Production Economics, 103(1), 316-331.

Horng, S.-C., 2015. Combining artificial bee colony with ordinal optimization for stochastic economic lot scheduling problem. IEEE Transactions on Systems, Man & Cybernetics. Systems, 45(3), 373-384.

Hsu, W.L., 1983. On the general feasibility test of scheduling lot sizes for several products on one machine. Management Science, 29(1), 93-105.

Huang, J.Y., Yao, M.J., 2006. An improved heuristic for solving the economic lot scheduling problem in flow shops under power-of-two policy. Journal of the Chinese Institute of Industrial Engineers, 23(2), 127-140.

Huang, J.Y., Yao, M.J., 2007. A new algorithm for determining production schedules when solving the multi-product economic lot sizing problem in flow shops. International Journal of Advanced Manufacturing Technology, 33(5-6), 560-578.

Huang, J.Y., Yao, M.J., 2008. A genetic algorithm for solving the economic lot scheduling problem in flow shops. International Journal of Production Research, 46(14), 3737-3761.

Huang, J.Y., Yao, M.J., 2013. On the optimal lot-sizing and scheduling problem in serial-type supply chain system using a time-varying lot-sizing policy. International Journal of Production Research, 51(3), 735-750.

Hwang, H., Moon, D.H., 1991. A production inventory model for producing two-products at a single facility with deteriorating raw materials. Computers & Industrial Engineering, 20(1), 141-147.

Inman, R.R., Jones, P.C., 1993. Decomposition for scheduling flexible manufacturing systems. Operations Research, 41(3), 608-617.

Inman, R.R., Jones, P.C., Gallego, G.M., 1991. Economic lot scheduling of fully loaded processes with external setups. Naval Research Logistics (NRL), 38(5), 699-713.

Jenabi, M., Fatemi Ghomi, S.M.T., Torabi, S.A., Karimi, B., 2007. Two hybrid meta-heuristics for the finite horizon ELSP in flexible flow lines with unrelated parallel machines. Applied Mathematics and Computation, 186(1), 230-245.

Jenabi, M., Fatemi Ghomi, S.M.T., Torabi, S.A., 2008. Finite horizon economic lot and delivery scheduling problem: flexible flow lines with unrelated parallel machines and sequence dependent setups. International Journal of Engineering, Transactions A: Basics, 21(2), 143-158.

Jensen, M.T., Khouja, M., 2004. An optimal polynomial time algorithm for the common cycle economic lot and delivery scheduling problem. European Journal of Operational Research, 156(2), 305-311.

Jodlbauer, H., 2006. An approach for integrated scheduling and lot-sizing. European Journal of Operational Research, 172(2), 386-400.

Jones, P.C., Inman, R.R., 1989. When is the economic lot scheduling problem easy? IIE Transactions (Institute of Industrial Engineers), 21(1), 11-20.

Kamath B, N., Bhattacharya, S., 2007. Lead time minimization of a multi-product, single-processor system: a comparison of cyclic policies. International Journal of Production Economics, 106(1), 28-40.

Karalli, S.M., Flowers, A.D., 2006. The multiple-family ELSP with safety stocks. Operations Research, 54(3), 523-531.

Kayvanfar, V., Zandieh, M., 2012. The economic lot scheduling problem with deteriorating items and shortage: an imperialist competitive algorithm. International Journal of Advanced Manufacturing Technology, 62(5-8), 759-773.

Kelle, P., Clendenen, G., Dardeau, P., 1994. Economic lot scheduling heuristic for random demands. International Journal of Production Economics, 35(1-3), 337-342.

Khouja, M., 1997. The scheduling of economic lot sizes on volume flexible production systems. International Journal of Production Economics, 48(1), 73-86.

Khouja, M., 1999. A note on deliberately slowing down output in a family production context. International Journal of Production Research, 37(17), 4067-4077.

Khouja, M., 2000. The economic lot and delivery scheduling problem: common cycle, rework, and variable production rate. IIE Transactions (Institute of Industrial Engineers), 32(8), 715-725.

Khouja, M., Michalewicz, Z., Vijayaragavan, P., 1998. Evolutionary algorithm for economic lot and delivery scheduling problem. Fundamenta Informaticae, 35(1-4), 113-123.

Khouja, M., Michalewicz, Z., Wilmot, M., 1998. The use of genetic algorithms to solve the economic lot size scheduling problem. European Journal of Operational Research, 110(3), 509-524.

Khoury, B.N., Abboud, N.E., Tannous, M.M., 2001. The common cycle approach to the ELSP problem with insufficient capacity. International Journal of Production Economics, 73(2), 189-199.

Kia, H., Ghodsypour, S.H., Davoudpour, H., 2017. A hybrid imperialist competitive algorithm for solving economic lot and delivery scheduling problem in a four-stage supply chain. Advances in Mechanical Engineering, 9(2), 1-20.

Kim, D., Mabert, V.A., Pinto, P.A., 1993. Integrative cycle scheduling approach for a capacitated flexible assembly system. Decision Sciences, 24(1), 126-147.

Kim, T., Goyal, S.K., 2010. Design of flexible capacity plan for multiple items in a single production facility. International Journal of Procurement Management, 3(2), 167-180.

Kim, T., Goyal, S.K., 2011. Composite sourcing policy for multiple products on a single production facility. International Journal of Logistics Systems and Management, 8(1), 86-100.

Kim, T., Goyal, S.K., 2012. Composite sourcing policy considering raw-material consumption. International Journal of Logistics Systems and Management, 11(4), 535-549.

Kim, T., Hong, Y., Chang, S.Y., 2006a. Joint economic procurement-production-delivery policy for multiple items in a single-manufacturer, multiple-retailer system. International Journal of Production Economics, 103(1), 199-208.

Kim, T., Hong, Y., Chang, S.Y., 2006b. Joint economic production-shipment policy in a single-manufacturer- multiple-item system. International Journal of Industrial Engineering : Theory Applications and Practice, 13(4), 357-363.

Kuhn, H., Liske, T., 2011. Simultaneous supply and production planning. International Journal of Production Research, 49(13), 3795-3813.

Kuhn, H., Liske, T., 2014. An exact algorithm for solving the economic lot and supply scheduling problem using a power-of-two policy. Computers & Operations Research, 51, 30-40.

Kuo, H.W., Inman, R.R., 1990. A practical heuristic for the group technology economic lot scheduling problem. International Journal of Production Research, 28(4), 709-722.

Larraneta, J., Onieva, L., 1988. The economic lot-scheduling problem: a simple approach. Journal of the Operational Research Society, 39(4), 373-379.

Lee, C.Y., Danusaputro, S., 1989. Economic lot scheduling for two-product problem. IIE Transactions (Institute of Industrial Engineers), 21(2), 162-169.

Levén, E., Segerstedt, A., 2007. A scheduling policy for adjusting economic lot quantities to a feasible solution. European Journal of Operational Research, 179(2), 414-423.

Liberopoulos, G., Pandelis, D., Hatzikonstantinou, O., 2013. The stochastic economic lot sizing problem for non-stop multi-grade production with sequence-restricted setup changeovers. Annals of Operations Research, 209(1), 179-205.

Lin, G.C., Kroll, D.E., Lin, C.J., 2006. Determining a common production cycle time for an economic lot scheduling problem with deteriorating items. European Journal of Operational Research, 173(2), 669-682.

Lin, G.C., Wu, A., Gong, D.-C., Huang, B., Ma, W.-N., 2014. On a multi-product lot scheduling problem subject to an imperfect process with standby modules. International Journal of Production Research, 52(8), 2243-2257.

Liu, J., Wu, L., Zhou, Z., 2008. A time-varying lot size method for the economic lot scheduling problem with shelf life considerations. European Journal of Industrial Engineering, 2(3), 337-355.

Löhndorf, N., Minner, S., 2013. Simulation optimization for the stochastic economic lot scheduling problem. IIE Transactions, 45(7), 796-810.

Löhndorf, N., Riel, M., Minner, S., 2014. Simulation optimization for the stochastic economic lot scheduling problem with sequence-dependent setup times. International Journal of Production Economics, 157(1), 170-176.

Ma, W.N., Gong, D.C., Lin, G.C., 2010. An optimal common production cycle time for imperfect production processes with scrap. Mathematical and Computer Modelling, 52(5-6), 724-737.

Markowitz, D.M., Reiman, M.I., Wein, L.M., 2000. The stochastic economic lot scheduling problem: Heavy traffic analysis of dynamic cyclic policies. Operations Research, 48(1), 136-154.

Matthews, J.P., 1988. The optimality of the 'zero-switch' rule for a class of economic lot-scheduling problems. Journal of the Operational Research Society, 39(12), 1155-1161.

Maxwell, W.L., Singh, H., 1983. The effect of restricting cycle times in the economic lot scheduling problem. IIE Transactions (Institute of Industrial Engineers), 15(3), 235-241.

Mohammadi, M., Musa, S.N., Bahreininejad, A., 2014. Optimization of mixed integer nonlinear economic lot scheduling problem with multiple setups and shelf life using metaheuristic algorithms. Advances in Engineering Software, 78, 41-51.

Mohammadi, M., Musa, S.N., Bahreininejad, A., 2015. Optimization of economic lot scheduling problem with backordering and shelf-life considerations using calibrated metaheuristic algorithms. Applied Mathematics and Computation, 251, 404-422.

Mokhlesian, M., Fatemi Ghomi, S.M.T., Jolai, F., 2010. Economic lot scheduling problem with consideration of money time value. International Journal of Industrial Engineering Computations, 1(2), 121-138.

Monahan, J.P., 1976. Production scheduling of perishable products. International Journal of Production Research, 14(6), 689-697.

Moon, D.H., Christy, D.P., 1998. Determination of optimal production rates on a single facility with dependent mold lifespan. International Journal of Production Economics, 54(1), 29-40.

Moon, I., Gallego, G., Simchi-Levi, D., 1991. Controllable production rates in a family production context. International Journal of Production Research, 29(12), 2459-2470.

Moon, I., Giri, B.C., Choi, K., 2002. Economic lot scheduling problem with imperfect production processes and setup times. Journal of the Operational Research Society, 53(6), 620-629.

Moon, I., Hahm, J., Lee, C., 1998. The effect of the stabilization period on the economic lot scheduling problem. IIE Transactions (Institute of Industrial Engineers), 30(11), 1009-1017.

Moon, I., Silver, E.A., Choi, S., 2002. Hybrid genetic algorithm for the economic lot-scheduling problem. International Journal of Production Research, 40(4), 809-824.

Moon, I.K., Cha, B.C., Bae, H.C., 2006. Hybrid genetic algorithm for group technology economic lot scheduling problem. International Journal of Production Research, 44(21), 4551-4568.

Narro Lopez, M.A., Kingsman, B.G., 1991. The economic lot scheduling problem: theory and practice. International Journal of Production Economics, 23(1-3), 147-164.

Nilsson, K., Segerstedt, A., 2008. Corrections of costs to feasible solutions of economic lot scheduling problems. Computers & Industrial Engineering, 54(1), 155-168.

Öner, S., Bilgiç, T., 2008. Economic lot scheduling with uncontrolled co-production. European Journal of Operational Research, 188(3), 793-810.

Osman, H., Demirli, K., 2012. Economic lot and delivery scheduling problem for multi-stage supply chains. International Journal of Production Economics, 136(2), 275-286.

Ouenniche, J., Bertrand, J.W.M., 2001. The finite horizon economic lot sizing problem in job shops: the multiple cycle approach. International Journal of Production Economics, 74(1-3), 49-61.

Ouenniche, J., Boctor, F.F., 2001a. The multi-product, economic lot-sizing problem in flow shops: the powers-of-two heuristic. Computers & Operations Research, 28(12), 1165-1182.

Ouenniche, J., Boctor, F.F., 2001b. The two-group heuristic to solve the multi-product, economic lot sizing and scheduling problem in flow shops. European Journal of Operational Research, 129(3), 539-554.

Park, K.S., Yun, D.K., 1984. A stepwise partial enumeration algorithm for the economic lot scheduling problem. IIE Transactions (Institute of Industrial Engineers), 16(4), 363-370.

Pasandideh, S.H.R., Niaki, S.T.A., Nobil, A.H., Cárdenas-Barrón, L.E., 2015. A multiproduct single machine economic production quantity model for an imperfect production system under warehouse construction cost. International Journal of Production Economics, 169, 203-214.

Pesenti, R., Ukovich, W., 2003. Economic lot scheduling on multiple production lines with resource constraints. International Journal of Production Economics, 81/82, 469.

Philipoom, P.R., Rees, L.P., Taylor, B.W., 1989. Solving the economic lot-scheduling problem using the method of prime subperiods. Decision Sciences, 20(4), 794-809.

Prasetyo, H., 2013. Comment on "multiple cycle economic lot and delivery-scheduling problem in a two-echelon supply chain". International Journal of Advanced Manufacturing Technology, 64(9-12), 1541-1543.

Prasetyo, H., Lee, S.H., 2013. Comment on "simultaneous determination of multiproduct batch and full truckload shipment schedules". International Journal of Production Economics, 144(2), 405-408.

Rahman, H.F., Sarker, R., Essam, D., 2015. A genetic algorithm for permutation flow shop scheduling under make to stock production system. Computers & Industrial Engineering, 90, 12-24.

Rajaram, K., Karmarkar, U.S., 2004. Campaign planning and scheduling for multiproduct batch operations with applications to the food-processing Industry. Manufacturing & Service Operations Management, 6(3), 253-269.

Ramani, S., Narayanan, N., 1992. Single facility, multi-item lot sizing under just-in-time and "cyclic scheduling for improvement". International Journal of Production Economics, 26(1-3), 333-339.

Raza, A.S., Akgunduz, A., 2008. A comparative study of heuristic algorithms on Economic Lot Scheduling Problem. Computers & Industrial Engineering, 55(1), 94-109.

Raza, S.A., Akgunduz, A., Chen, M.Y., 2006. A tabu search algorithm for solving economic lot scheduling problem. Journal of Heuristics, 12(6), 413-426.

Rogers, J., 1958. A computational approach to the economic lot scheduling problem. Management Science, 4(3), 264-291.

Roundy, R., 1989. Rounding off to powers of two in continuous relaxations of capacitated lot sizing problems. Management Science, 35(12), 1433-1442.

Ruiz-Torres, A.J., Santiago, P.I., Chung, W., 2010. The campaign and lot size scheduling problem: a modification of the economic lot scheduling problem for the pharmaceutical industry. International Journal of Logistics Systems and Management, 7(2), 184-197.

Salvietti, L., Smith, N.R., 2008. A profit-maximizing economic lot scheduling problem with price optimization. European Journal of Operational Research, 184(3), 900-914.

Salvietti, L., Smith, N.R., Cárdenas-Barrón, L.E., 2014. A stochastic profit-maximising economic lot scheduling problem with price optimisation. European Journal of Industrial Engineering, 8(2), 193-221.

Sarin, S.C., Kalir, A.A., Chen, M., 2008. A single-lot, unified cost-based flow shop lot-streaming problem. International Journal of Production Economics, 113(1), 413-424.

Schweitzer, P.J., Silver, E.A., 1983. Mathematical pitfalls in the one machine multiproduct economic lot scheduling problem. Operations Research, 31(2), 401-405.

Shirodkar, V.A., Madhusudanan Pillai, V., Sridharan, R., 2011. On the feasibility of sequence-dependent economic lot scheduling problem. International Journal of Production Research, 49(10), 2925-2939.

Shirodkar, V.A., Sridharan, R., Madhusudanan Pillai, V., 2011. Effective allocation of idle time in the group technology economic lot scheduling problem. International Journal of Production Research, 49(24), 7493-7513.

Smith-Daniels, V.L., Ritzman, L.P., 1988. A model for lot sizing and sequencing in process industries. International Journal of Production Research, 26(4), 647-674.

Smits, S.R., Wagner, M., De Kok, T.G., 2004. Determination of an order-up-to policy in the stochastic economic lot scheduling model. International Journal of Production Economics, 90(3), 377-389.

Soman, C.A., Van Donk, D.P., Gaalman, G.J.C., 2006. Comparison of dynamic scheduling policies for hybrid make-to-order and make-to-stock production systems with stochastic demand. International Journal of Production Economics, 104(2), 441-453.

Soman, C.A., Van Donk, D.P., Gaalman, G.J.C., 2004. A basic period approach to the economic lot scheduling problem with shelf life considerations. International Journal of Production Research, 42(8), 1677-1689.

Sox, C.R., Muckstadt, J.A., 1997. Optimization-based planning for the stochastic lot-scheduling problem. IIE Transactions (Institute of Industrial Engineers), 29(5), 349-357.

Sule, D.R., 1983. Effect of learning and forgetting on economic lot size scheduling problem. International Journal of Production Research, 21(5), 771-786.

Sun, H., Huang, H.C., Jaruphongsa, W., 2009. A genetic algorithm for the economic lot scheduling problem under the extended basic period and power-of-two policy. CIRP Journal of Manufacturing Science and Technology, 2(1), 29-34.

Sun, H., Huang, H.C., Jaruphongsa, W., 2010. The economic lot scheduling problem under extended basic period and power-of-two policy. Optimization Letters, 4(2), 157-172.

Sun, H., Jaruphongsa, W., Huang, H.C., 2009. Genetic algorithms for the multiple-machine economic lot scheduling problem. International Journal of Advanced Manufacturing Technology, 43(11-12), 1251-1260.

Taj, S., Nedeltcheva, G.N., Pfeil, G., Roumaya, M., 2012. A spread-sheet model for efficient production and scheduling of a manufacturing line/cell. International Journal of Production Research, 50(4), 1141-1154.

Tang, L.C., Lee, L.H., 2005. A simple recovery strategy for economic lot scheduling problem: a two-product case. International Journal of Production Economics, 98(1), 97-107.

Tang, O., Teunter, R., 2006. Economic lot scheduling problem with returns. Production and Operations Management, 15(4), 488-497.

Taylor, G.D., Taha, H.A., Chowning, K.M., 1997. A heuristic model for the sequence-dependent lot scheduling problem. Production Planning and Control, 8(3), 213-225.

Teunter, R., Kaparis, K., Tang, O., 2008. Multi-product economic lot scheduling problem with separate production lines for manufacturing and remanufacturing. European Journal of Operational Research, 191(3), 1241-1253.

Teunter, R., Tang, O., Kaparis, K., 2009. Heuristics for the economic lot scheduling problem with returns. International Journal of Production Economics, 118(1), 323-330.

Torabi, S.A., Fatemi Ghomi, S.M.T., Karimi, B., 2005a. The finite horizon economic lot scheduling in flexible flow lines. International Journal of Engineering, Transactions B: Applications, 18(3), 273-284.

Torabi, S.A., Fatemi Ghomi, S.M.T., Karimi, B., 2005b. The economic lot and delivery scheduling problem in flexible job shops. Amirkabir (Journal of Science and Technology), 16(62 B), 83-91.

Torabi, S.A., Fatemi Ghomi, S.M.T., Karimi, B., 2006. A hybrid genetic algorithm for the finite horizon economic lot and delivery scheduling in supply chains. European Journal of Operational Research, 173(1), 173-189.

Torabi, S.A., Jenabi, M., 2008. A basic period approach for solving the economic lot and delivery scheduling In flexible flow lines. International Journal of Engineering, Transactions A: Basics, 21(4), 361-374.

Torabi, S.A., Jenabi, M., 2009. Multiple cycle economic lot and delivery-scheduling problem in a two-echelon supply chain. International Journal of Advanced Manufacturing Technology, 43(7-8), 785-798.

Torabi, S.A., Karimi, B., Fatemi Ghomi, S.M.T., 2005. The common cycle economic lot scheduling in flexible job shops: the finite horizon case. International Journal of Production Economics, 97(1), 52-65.

Van den Broecke, F., Van Landeghem, H., Aghezzaf, E.-H., 2005. An application of cyclical master production scheduling in a multi-stage, multi-product environment. Production Planning & Control, 16(8), 796-809.

Van den Broecke, F., Van Landeghem, H., Aghezzaf, E.-H., 2008. Implementing a near-optimal solution for the multi-stage, multi-product capacitated lot-sizing problem by rolling out a cyclical production plan. International Journal of Production Economics, 112(1), 121-137.

Vidal-Carreras, P.I., Garcia-Sabater, J.P., Coronado-Hernandez, J.R., 2012. Economic lot scheduling with deliberated and controlled coproduction. European Journal of Operational Research, 219(2), 396-404.

Viswanathan, S., Goyal, S.K., 2002. On ’Manufacturing batch size and ordering policy for products with shelf lives’. International Journal of Production Research, 40(8), 1965-1970.

Wagner, B.J., Davis, D.J., 2002. A search heuristic for the sequence-dependent economic lot scheduling problem. European Journal of Operational Research, 141(1), 133-146.

Wagner, B.J., Davis, D.J., Kher, H.V., 2002. The production of several items in a single facility with linearly changing demand rates. Decision Sciences, 33(3), 317-346.

Wagner, M., Smits, S.R., 2004. A local search algorithm for the optimization of the stochastic economic lot scheduling problem. International Journal of Production Economics, 90(3), 391-402.

Wang, J., Li, X., Zhu, X., 2012. Intelligent dynamic control of stochastic economic lot scheduling by agent-based reinforcement learning. International Journal of Production Research, 50(16), 4381-4395.

Yan, C., Liao, Y., Banerjee, A., 2013. Multi-product lot scheduling with backordering and shelf-life constraints. Omega, 41(3), 510-516.

Yang, H., Low, V., Zhang, C., Zheng, L., Miao, L., 2017. Behaviour perception-based disruption models for the parallel machine capacitated lot-sizing and scheduling problem. International Journal of Production Research, 55(11), 3058-3072.

Yao, M.J., 2005. The economic lot scheduling problem without capacity constraints. Annals of Operations Research, 133(1-4), 193-205.

Yao, M.J., Chen, S.C., Chang, Y.J., 2012. A common cycle approach for solving the economic lot and inspection scheduling problem. Journal of Industrial and Management Optimization, 8(1), 141-162.

Yao, M.J., Chen, S.C., Chang, Y.J., Tseng, T.Y., 2012. Solving the economic lot and inspection scheduling problem using the extended basic period approach under power-of-two policy. Journal of the Chinese Institute of Industrial Engineers, 29(1), 43-60.

Yao, M.J., Elmaghraby, S.E., 2001. The economic lot scheduling problem under power-of-two policy. Computers and Mathematics with Applications, 41(10-11), 1379-1393.

Yao, M.J., Elmaghraby, S.E., Chen, I.C., 2003. On the feasibility testing of the economic lot scheduling problem using the extended basic period approach. Journal of the Chinese Institute of Industrial Engineers, 20(5), 435-448.

Yao, M.J., Huang, J.X., 2005. Solving the economic lot scheduling problem with deteriorating items using genetic algorithms. Journal of Food Engineering, 70(3), 309-322.

Zanoni, S., Segerstedt, A., Tang, O., Mazzoldi, L., 2012. Multi-product economic lot scheduling problem with manufacturing and remanufacturing using a basic period policy. Computers & Industrial Engineering, 62(4), 1025-1033.

Zipkin, P.H., 1991. Computing optimal lot sizes in the economic lot scheduling problem. Operations Research, 39(1), 56-63.

Zoller, K., 1990. Coordination of replenishment activities. International Journal of Production Research, 28(6), 1123-1136.

Paper 2 Integration of energy aspects into the economic lot scheduling problem[1]

Authors: Fabian G. Beck, Konstantin Biel, Christoph H. Glock

Type of publication: Journal article

Publication details: *International Journal of Production Economics* (VHB-JOURQUAL3: B), 209, 399-410, 2019.

Abstract

Due to the role the manufacturing sector plays in the depletion of resources and the generation of greenhouse gas emissions, the consumption of energy has more and more often made it onto research agendas in the area of production planning over the last decade. The work at hand integrates energy aspects into the well-known Economic Lot Scheduling Problem (ELSP) by taking account of the cost arising from the product-dependent energy usage of the production facility during machine startups and shutdowns as well as during tool change, idle, and production phases. To determine a cyclic production schedule that minimizes the sum of tool change, inventory holding, and energy usage costs, we use the Common-Cycle-Approach of Hanssmann (1962) and the Basic-Period-Approach of Haessler and Hogue (1976) and adjust them accordingly. Using the data sets of Bomberger and Eilon, we show that considering energy cost in the ELSP affects the resulting cyclic production schedule and significantly reduces the company's energy cost.

Keywords:

Economic lot scheduling problem; ELSP; Energy usage; Energy efficiency; Production planning; Basic-period-approach

[1] Reprinted and slightly modified from *International Journal of Production Economics*, 209, Beck, F.G., Biel, K., Glock, C.H., Integration of energy aspects into the economic lot scheduling problem, 399-410, 2019, https://doi.org/10.1016/j.ijpe.2018.02.012, with permission from Elsevier.

1 Introduction

In 2012, the industrial sector used 54 percent of the total delivered energy[2] worldwide. Even though the delivered energy usage of the transportation, the commercial, and the residential sectors is expected to grow faster than that of the industrial sector, the industrial sector is expected to remain the largest delivered energy consumer at least until 2040. Within the industrial sector, manufacturing activities were responsible for two thirds of the energy usage in 2012, and this share is expected to increase slightly until 2040 (U.S. Energy Information Administration, 2016b).

In light of this development, and driven by steadily rising energy prices and an increasing public awareness of the impact energy-related CO_2 emissions have on the environment, researchers and practitioners are constantly looking for measures to make production processes more energy efficient (U.S. Energy Information Administration, 2016a). Typically, these measures can be assigned to one of two categories. The first category embraces measures that aim at changing the design and operation of machines and tools from an engineering perspective. This may include lightweight construction, material selection optimization, and module integration (Zhou et al., 2016). In contrast, measures belonging to the second category primarily focus on reducing energy usage by changing the way the machine is utilized, which in most cases results in an adjustment of the production schedule. From a managerial point of view, production planners may lower energy usage by reducing the idle times of machines, coordinating operating times of machines, and synchronizing jobs to be processed with machine tools (Zeng et al., 2009). The paper at hand investigates measures that belong to the second category as these measures are generally not associated with large investments as opposed to purchasing more energy-efficient production equipment. Thus, practitioners usually concentrate on energy management first before investing in advanced machinery (Bi and Wang, 2012).

Before energy efficiency measures made their way on the agendas of researchers and practitioners, manufacturing companies controlled their production processes primarily based on production-related goals such as the minimization of setup and inventory holding costs or the minimization of flow time or tardiness. Nowadays, manufacturing companies increasingly gear their production processes towards energy-related criteria, in addition to production-related

[2] Delivered energy, also referred to as net energy, represents the amount of energy delivered to the site of use. It does not comprise conversion losses incurred over the course of the energy supply process (U.S. Energy Information Administration, 2016b).

goals. To this end, recent decision support tools explicitly consider the energy usage attached to different machine operating modes and thus facilitate tracking the true cost associated with production scheduling. This, in turn, enables companies to schedule production in a more energy- and cost-efficient way. On the one hand, energy-aware production models enable companies to compute production schedules that minimize energy usage and energy cost to mitigate the increasing pressure from rising energy prices. On the other hand, these models are able to consider CO_2 emission constraints that make it possible to compute optimal production schedules that comply with environmental law.

As a result of the developments described above, numerous scientific articles have recently been published that integrated various energy efficiency criteria into traditional production planning problems. Biel and Glock (2016c) provided a comprehensive overview of this stream of research. This review identified a few articles on energy-efficient lot sizing (e.g., Biel and Glock, 2016a; Zanoni et al., 2014) and a larger number of articles on energy-efficient machine scheduling (e.g., Artigues et al., 2013; Luo et al., 2013; Mouzon et al., 2007). However, it is striking that the well-known Economic Lot Scheduling Problem (ELSP) has not been studied in the context of energy efficiency so far. The paper at hand intends to close this research gap by incorporating energy considerations into two popular approaches for solving the ELSP. Hence, its contribution is threefold. First, it integrates the product-dependent energy usage of the production facility during machine startups and shutdowns as well as during tool change, idle, and production phases into the ELSP. Secondly, it adjusts the Common-Cycle-Approach of Hanssmann (1962) and the Basic-Period-Approach of Haessler and Hogue (1976) to account for energy usage cost, in addition to tool change and inventory holding costs, when computing a cyclic production schedule for the ELSP. Lastly, using the data sets of Bomberger and Eilon, the paper shows that considering energy cost in the ELSP affects the resulting cyclic production schedule, and that it may significantly reduce the company's energy cost.

The remainder of the article is organized as follows: Section 2 reviews the literature on the ELSP as well as on related energy-efficient production planning models. Section 3 provides a detailed problem description. Section 4 describes the energy-consuming machine operating modes and extends the traditional ELSP as well as the corresponding solution approaches to consider energy usage cost. Section 5 then examines the impact of the energy considerations on the cyclic production schedule in a numerical study, and Section 6 concludes the paper.

2 Literature review

The literature relevant to this article mainly originates from two streams of research: I) research on the ELSP and II) research on the integration of energy aspects into decision support models for production planning. Both research streams will be reviewed briefly in the following.

2.1 Recent research on the ELSP

The ELSP dates back to Rogers (1958), who was the first author to analyze lot sizing for multiple items that need to be produced on the same facility. Since this seminal paper, a plethora of works has been published on the ELSP. As suggested by Elmaghraby (1978) and Santander-Mercado and Jubiz-Diaz (2016), research on the ELSP can be differentiated according to two important model attributes: the scheduling policy and the solution procedure. With respect to scheduling policies, the literature on the ELSP usually differentiates between three approaches: I) the Common-Cycle-Approach (e.g., Hanssmann, 1962), II) the (Extended) Basic-Period-Approach (e.g., Bomberger, 1966; Doll and Whybark, 1973; Haessler and Hogue, 1976), and III) the Time-Varying-Lot-Size-Approach (e.g., Dobson, 1987; Zipkin, 1991; Moon et al., 2002). With respect to solution procedures, Santander-Mercado and Jubiz-Diaz (2016) differentiated between exact methods, which solve a restricted/simplified version of the original problem, and heuristic or meta-heuristic methods, which try to find a good solution to the original problem.

In a recent survey, Chan et al. (2013) showed that the ELSP still attracts a lot of attention among researchers. Further, they investigated different research trends of the ELSP between 1997 and 2012 and identified five main topics: I) non-uniform production rates, II) flow shop, multi-machine, or multi-factory, III) with returns, IV) stochastic problems, and V) sequence-dependent setups.

Works that belong to the first research stream modified the assumption of the classical ELSP that the production rate is uniform. Elhafsi and Bai (1997), for example, assumed that the production rates of the products can be adjusted anytime during the production run, varied at the beginning of each production run, and during each production run to reduce the production cost. Ben-Daya and Hariga (2000) considered the case where a facility starts producing imperfect items after a random point in time. This results in a situation where the production rate is essentially reduced after some time, such that the production rate becomes non-uniform.

According to Chan et al. (2013), the second research stream of the ELSP considers the cases of flow shops, multiple machines or multiple factories. Haksöz and Pinedo (2011), for example, investigated the multiple-parallel-machine ELSP with different speeds of the machines. The objective of their work was to minimize the total cost per unit time by assigning products to a certain number of machines. They investigated three different models, which were finally solved by heuristics. These models include the following cases: I) the cycles of all machines have to be cyclical and of the same length, II) allowing different cycle lengths for every machine in contrast to I), and III) relaxing the assumption that the schedule has to be cyclical.

A third stream of research analyzes environmental aspects in the context of the ELSP. Chowdhury and Sarker (2001), for example, presented a modified ELSP with product shelf life. Since the shelf life constraint influences the storage of the products in terms of duration and quantity, the authors paid special attention to the production rate and the cycle time and their adjustments. Tang and Teunter (2006) modified the ELSP to consider remanufacturing of returned items and used the Common-Cycle-Approach for finding a feasible schedule. As remanufacturing of returned products/cars is of great interest in the automotive sector, remanufacturing of water pumps for diesel engines was investigated in a case study in this paper.

In a fourth research stream, Chan et al. (2013) mentioned stochastic problems as one of the main research trends of the ELSP and pointed out that especially stochastic demand is in the focus of current research. An overview of the so called Stochastic Economic Lot Scheduling Problem was presented by Sox et al. (1999) and Winands et al. (2011).

Works that belong to the fifth research stream relax another basic assumption of the ELSP: Setup times are assumed to be no longer independent of the production sequence. Brander and Forsberg (2005), for example, considered a disassembly process on a single facility with sequence-dependent setups.

2.2 Research on energy-aware production planning

As indicated in Section 1, numerous articles were published in recent years that integrated energy aspects into traditional production planning models. In the vast majority of these articles, energy aspects were taken account of in the form of energy usage, energy cost, or energy-related CO_2 emissions. Since the ELSP may be considered as a hybrid of the economic lot sizing and the single-machine scheduling problems (Rogers, 1958), and since no energy-aware ELSP has been developed thus far, research on energy-aware lot sizing and energy-aware machine scheduling is most closely related to the work at hand. The work of Zanoni et al. (2014) was among the first to take account of energy cost in calculating an optimal lot size in a two-stage production system in addition to inventory holding and setup costs. The authors considered energy usage of the machines during production, setup, and idle times, and showed in a numerical analysis that considering energy cost clearly impacts the lot sizing decision and is an essential step towards capturing the true cost of manufacturing. Biel and Glock (2016a) extended the model of Zanoni et al. (2014) by incorporating a waste heat recovery system into the energy supply system of the production environment. Biel and Glock (2016b), in turn, transferred this approach to a serial multi-stage production system and added an electrical energy storage system to better exploit the benefits of the waste heat recovery system. Both models highlighted the importance of integrating production planning and energy usage decisions to foster energy efficiency in manufacturing. In addition to energy usage, Bazan et al. (2015) also took energy-related CO_2 emissions from production and transportation processes in two single-vendor (manufacturer) single-buyer (retailer) production-inventory systems into consideration. The authors showed that the goals of minimizing system cost and minimizing energy-related CO_2 emissions may be conflicting.

Compared with research on energy-aware lot sizing, research on energy-aware machine scheduling is far more advanced. One of the first and most influential works is the one of Mouzon et al. (2007), who proposed a mixed-integer linear program that schedules jobs on a single machine such that energy usage and total completion time are minimized simultaneously. In recent years, this concept was transferred to numerous machine scheduling problems, the most prominent being the parallel machine scheduling problem (e.g., Artigues et al., 2013; Ding et al., 2015; Liu, 2014) and various flow shop (e.g., Fang et al., 2011; Luo et al., 2013; Sharma et al., 2015) and job shop problems (e.g., Liu et al., 2014; May et al., 2015; Moon and Park, 2014). Besides the type of machine scheduling problem, there are two integral characteristics to classify energy-aware machine scheduling models: I) the objectives or constraints specifying the integration of energy aspects into the model, and II) the definition of the machine operating modes whose energy usage impacts the production planning decision. With respect to the first characteristic, energy-aware machine scheduling problems typically aim at minimizing or at least restricting either energy usage (e.g., Liu et al., 2014; May et al., 2015), energy cost (e.g., Artigues et al., 2013; Ding et al., 2015; Moon and Park, 2014), or energy-related CO_2 emissions (e.g., Fang et al., 2011; Liu, 2014; Sharma et al., 2015). With respect to the second characteristic, these models take account of the energy usage during production (e.g., Fang et al., 2011;

Moon and Park, 2014), during idle times (e.g., Liu et al., 2014; Luo et al., 2013), during setup (e.g., Mouzon et al., 2007; Sharma et al., 2015), and/or during tool change (e.g., May et al., 2015; Wang et al., 2015).

2.3 Summary

Despite a plethora of research on energy-aware production planning, prior research has, to the best of the authors' knowledge, made no attempt so far to integrate energy aspects into the Economic Lot Scheduling Problem. Hence, the paper at hand contributes to closing this research gap at the interface of energy-aware lot sizing and energy-aware machine scheduling by proposing an energy-aware ELSP. The classical formulation of the ELSP constitutes the starting point of our investigation. The following sections extend the classical ELSP to take account of energy usage during machine setup, tool change, idle times, and production. The objective of the model will be the minimization of the sum of inventory holding cost, setup cost, and energy-related cost.

3 Problem description and terminology

The paper at hand extends the classical ELSP for the single-machine-multi-product case with the typical cost elements, i.e. inventory holding and setup costs, by additionally considering energy cost resulting from different machine operating modes (the machine operating modes are described in more detail in Section 4.1). For the machine in question, we assume that each operating mode is associated with an individual power requirement and consequently energy cost. We integrate these machine operating modes into the classical ELSP in the following, and use results obtained in prior research to assess the power requirement of each machine operating mode. For solving the modified ELSP, two popular scheduling policies and their corresponding solution procedures, namely the Common-Cycle-Approach of Hanssmann and the Basic-Period-Approach of Haessler and Hogue, are adjusted to match the new problem settings.

In addition to what has already been stated, we assume the following hereafter:

- only one product can be produced at a time on the machine;
- the planning horizon is infinite;
- all parameters are constant over time;
- product shortages are not allowed;

- tool change cost and time for a tool change are required for producing each item, and they are known and independent of the production sequence;
- the production facility has already gone through the startup phase at the beginning of the planning horizon.

Energy-related assumptions will be presented along with the modeling of energy usage in Section 4.1.

Throughout the paper, the following terminology is used:

Indices:

i	Product with $i = 1,2, \dots, N$
o	Basic period with $o = 1,2, \dots, K$

Parameters:

A_i	Tool change cost per production lot for product i [EUR]
e	Energy usage charge [EUR/kWh]
f_i	Multiplication factor representing the relation of the power required during the tool change of the machine for product i to the idle power of the machine [−]
g	Multiplication factor representing the relation of the power required during the shutdown and startup of the machine to the idle power of the machine [−]
h_i	Inventory holding cost per unit per unit of time for product i [EUR/(item·h)]
l^{BE}	Break-even duration [h]
l^{sdsu}	Sum of the durations of the shutdown and startup phases [h]
l^{sr}	Step range [h]
N	Number of products [−]
p_i	Production rate for product i [items/h]
PR^{idle}	Power required to keep the machine in the idle operating mode [kW]
PR^{off}	Power required when the machine is turned off [kW]
PR_i^{prc}	Power required by the machine when processing product i [kW]
PR^{sdsu}	Power required during the shutdown and startup phases [kW]
PR_i^{tlch}	Power required by the machine when being retooled before processing product i [kW]

r_i	Demand rate for product i [items/h]
s_i	Tool change time for product i [h]
v_i	Specific energy required by the machine to process one item of product i [kWh/item]
W	Idle power of the machine [kW]

Decision variables:

FC	Length of the basic period [h]
K	Number of basic periods within the total cycle with $K = \max\{k_1, k_2, \ldots, k_N\}$[−]
k_i	Integer multiplier; product i is produced every k_i basic periods [−]
T	Length of the common cycle [h]
T_i	Cycle time of product i [h]
T^{idle}	Idle time of the machine at the end of the total cycle in case of the Common-Cycle-Approach [h]
T_o^{idle}	Idle time of the machine at the end of basic period o in case of the Basic-Period-Approach [h]
T^{occ}	Machine occupancy time within the total cycle in case of the Common-Cycle-Approach [h]
T_o^{occ}	Machine occupancy time within basic period o in case of the Basic-Period-Approach [h]
TC	Total average cost with $TC = TC^{occ} + TC^{idle}$ [EUR/h]
TC^{idle}	Average cost when the machine is idle (i.e., when neither the tool is changed nor the machine is processing) [EUR/h]
TC_o^{idle}	Average cost of basic period o when the machine is idle (i.e., when neither the tool is changed nor the machine is processing) [EUR/h]
TC^{occ}	Average cost when the machine is occupied (i.e., when either the tool is changed or the machine is processing) [EUR/h]
TC_i^{occ}	Average cost when the machine is occupied with product i [EUR/h]
u_i	Auxiliary variable for product i [−]
$Y_{i,o}$	Binary variable which equals 1 if product i is produced in basic period o, and 0 otherwise [−]

Definitions:

$\lceil x \rceil$	Ceiling function, which gives the smallest integer greater than or equal to x
$\lfloor x \rfloor$	Floor function, which gives the largest integer less than or equal to x

To ensure that the problem remains feasible, the necessary condition for the so-called net utilization has to hold for both problem variants investigated in the following. The condition can be formulated as follows (e.g. Elmaghraby, 1978; Beck and Glock, 2016):

$$\sum_{i=1}^{N} \frac{r_i}{p_i} < 1 \tag{1}$$

Note that condition (1) implies that $r_i < p_i, \forall i$.

4 Model description

The model description is divided into two parts. Section 4.1 describes the different machine operating modes and how their power requirements can be modeled. Subsequently, Section 4.2 integrates the machine operating modes and their respective power requirements into two popular approaches for solving the ELSP.

4.1 Modeling of machine operating modes

As indicated in Section 2, existing approaches for energy-aware production planning differ in the way they model machine operating modes and their respective power requirements. Since the ELSP can be broken down into startup/shutdown, tool change, idle, and processing phases, we need to integrate the power requirements of these machine operating modes into the two considered scheduling policies and the corresponding modified solution procedures.

4.1.1 Relevant transitions between machine operating modes

Figure 1 illustrates the transitions between the machine operating modes relevant to the ELSP (cf. Wang et al., 2015) along with the power requirements of the considered machine operating modes which are explained in detail in Section 4.1.2. Before a machine can process a lot, it first needs to go through a general startup phase (transition a), followed by a product-specific tool change phase (transition b). Then, the machine is "ready to produce" and the production of a lot can be initiated (transition c). If the subsequent lot is scheduled immediately after the previous lot has been finished, the tool(s) of the machine need(s) to be changed immediately (transition d). However, if there is some idle time scheduled on the machine before the next lot is due, the machine can be either switched to the idle operating mode (transition e) or shut down (transition g) and ultimately be switched off (transition h) completely. If the machine is switched to the idle operating mode, only a product-specific tool change is required before the machine can process the next lot (transition f). However, if the machine is shut down and switched off, it needs to go through the general startup phase again (transition a) before the tool(s) can be changed (transition b) and the machine can process the next lot (transition c). Lastly, we note that Figure 1 only features transitions that are used in regular operations. Other transitions, such as a direct transition from the startup to the shutdown operating mode, are technologically feasible, but irrational from an economic point of view in regular operations.

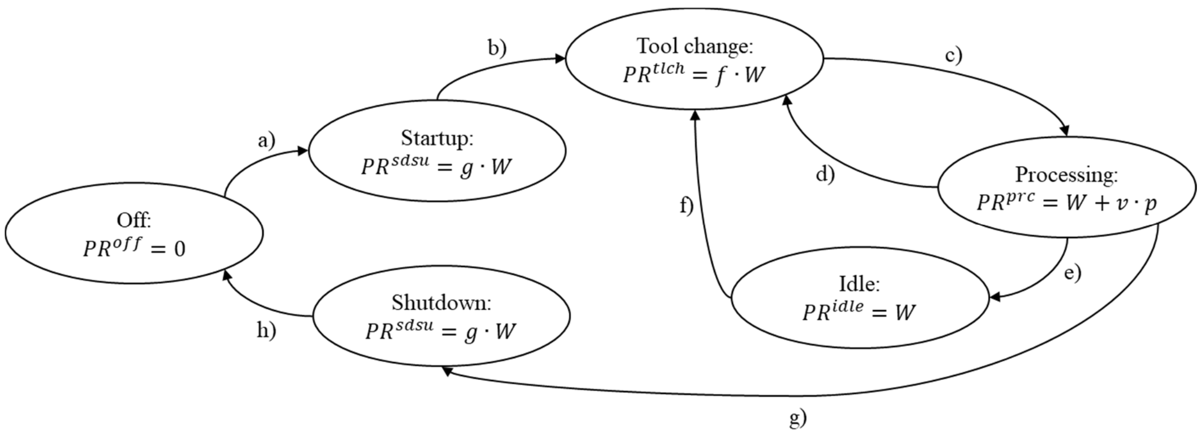

Figure 1: *Overview of machine operating modes and associated transitions and power requirements*

4.1.2 Power requirements of machine operating modes

We assume that the production facility is run using electric power. The amount of electric power required, PR, varies with the machine operating modes (see Figure 1). When the machine is turned off, no energy is used ($PR^{off} = 0$). However, as in Wang et al. (2015), we assume that a constant amount of power W is required to keep the machine in the idle operating mode where the machine is “ready to produce” after a product-specific tool change. According to Gutowski et al. (2006), the power required by the machine when processing product i, PR_i^{prc}, comprises a constant part and a variable part. While the constant part resembles the power required to keep the machine in a “ready-to-produce” mode, the variable part corresponds to the specific electrical energy required to perform an operation:

$$PR_i^{prc} = W + v_i \cdot p_i \tag{2}$$

The power required by the machine when being retooled before processing product i, PR_i^{tlch}, can be assumed constant and generally higher than the idle power. According to Zanoni et al. (2014), it can be modeled as

$$PR_i^{tlch} = f_i \cdot W, \tag{3}$$

where $f_i > 1, \forall i$. The power required during the shutdown and startup phases, PR^{sdsu}, can also be assumed constant and higher than the idle power:

$$PR^{sdsu} = g \cdot W, \tag{4}$$

where $g > 1$. Additionally, Zhou et al. (2016) argue that in most cases $PR^{sdsu} > \max_{i \in \{1,2,\ldots,N\}} PR_i^{tlch}$ holds.

4.1.3 Machine shutdown/startup policy

After finishing a lot, the question arises whether the machine should be switched to the idle operating mode or whether it should be shut down and started up again before the next lot is scheduled. To answer this question, we use the concept of break-even duration introduced by Mouzon et al. (2007). According to this concept, the machine should be shut down and switched off between processing two successive lots if the idle time between these lots is longer than the shutdown and startup time and if the energy required to shut the machine down and start it up again is lower than the energy required to keep the machine in the idle operating mode. Since we assume that the amount of power required during startup and shutdown operating mode equals $g \cdot W$, the break-even duration, l^{BE}, simply equals the sum of the durations of the shutdown and startup phases, l^{sdsu}, multiplied by g, where $g > 1$:

$$l^{BE} = \max\left\{\frac{l^{sdsu} \cdot g \cdot W}{W}, l^{sdsu}\right\} = g \cdot l^{sdsu} \tag{5}$$

4.2 Integration of energy aspect into two solution approaches

To integrate energy usage into the ELSP, we adjust the Common-Cycle-Approach of Hanssmann (1962) (cf. Section 4.2.1) and the Basic-Period-Approach of Haessler and Hogue (1976) (cf. Section 4.2.2) accordingly. To this end, we employ a two-stage optimization procedure (cf. Figure 2). In case of the Common-Cycle-Approach, the first stage determines the optimal common cycle, T, solely focusing on tool change, inventory holding, and energy cost arising in times when the machine is occupied (i.e., when either the tool is changed or the machine is processing). In case of the Basic-Period-Approach, the first stage determines the length of the basic period, FC, and the k-vector, which contains the number of basic periods between production runs for all N products, again merely focusing on tool change, inventory holding, and energy cost arising in times when the machine is occupied. In both solution approaches, the second stage determines whether to shut down and restart the machine or to leave it in the idle operating mode when the machine is idle (i.e., when neither the tool is changed nor the machine is processing) based on the break-even duration (cf. Section 4.1.3).

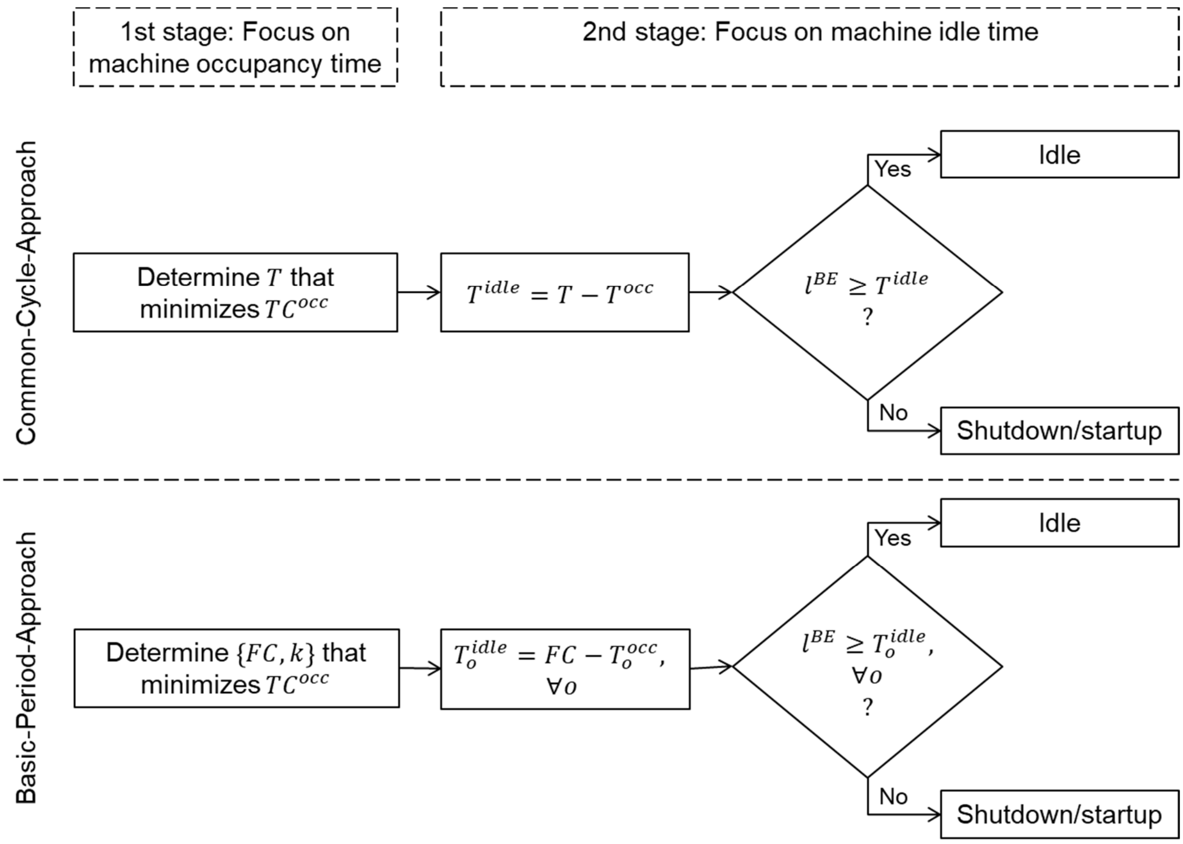

Figure 2: *Two-stage optimization procedure of adjusted Common-Cycle-Approach and Basic-Period-Approach*

4.2.1 Common-cycle-approach

The Common-Cycle-Approach, presented by Hanssmann in 1962, determines only one (common) cycle for all products and is therefore an easy and fast way to compute a feasible schedule. As the assumption of an equal consumption time for one lot of each product is very restrictive, the solution obtained by the Common-Cycle-Approach is often considered as an upper bound for the ELSP.

The total average cost associated with the traditional Common-Cycle-Approach only features tool change and inventory holding costs. On the first stage of the two-stage optimization procedure, we extend this total cost function by the energy cost to derive the average cost when the machine is occupied:

$$TC^{OCC}(T) = \frac{1}{T} \cdot \sum_{i=1}^{N} \left(\underbrace{A_i}_{Tool\ change\ cost} + \underbrace{\frac{h_i \cdot r_i}{2} \cdot T^2 \cdot \left(1 - \frac{r_i}{p_i}\right)}_{Inventory\ holding\ cost} + \underbrace{\left(s_i \cdot f_i \cdot W + (W + v_i \cdot p_i) \cdot \frac{r_i}{p_i} \cdot T\right) \cdot e}_{Energy\ cost} \right) \tag{6}$$

As in the case that does not consider energy cost, product-specific tool change cost is incurred every time the production of a new product is initiated. Furthermore, inventory holding cost accrue for products kept in stock. Besides these two classical cost components of the ELSP, Eq. (6) features the energy cost that results from the energy usage in the tool change and processing operating modes (cf. Section 4.1) multiplied by the energy usage charge, e.

To determine the optimal length of the common consumption cycle, we formulate the first-order condition of Eq. (6) with respect to T:

$$\frac{dTC^{OCC}(T)}{dT} = \sum_{i=1}^{N} \left(-\frac{A_i}{T^2} + \frac{h_i \cdot r_i}{2} \cdot \left(1 - \frac{r_i}{p_i}\right) - (s_i \cdot f_i \cdot W) \cdot \frac{e}{T^2} \right) = 0 \tag{7}$$

Solving Eq. (7) for T yields

$$T_{opt} = \sqrt{\frac{2 \cdot \sum_{i=1}^{N} (A_i + (s_i \cdot f_i \cdot W) \cdot e)}{\sum_{i=1}^{N} \left(h_i \cdot r_i \cdot \left(1 - \frac{r_i}{p_i}\right) \right)}} \tag{8}$$

However, the optimal total cycle time obtained using Eq. (8) is only feasible if it is greater than or equal to the machine occupancy time of all N products within the total cycle, i.e. the sum of the tool change and the processing times (c.f. Elmaghraby, 1978):

$$T_{opt} \geq \sum_{i=1}^{N} s_i + \sum_{i=1}^{N} \frac{r_i}{p_i} \cdot T_{opt} \tag{9}$$

Treating Eq. (9) as an equation and rearranging it, the minimum total cycle length can be determined by

$$T_{min} = \frac{\sum_{i=1}^{N} s_i}{1 - \sum_{i=1}^{N} \frac{r_i}{p_i}} \tag{10}$$

To guarantee the feasibility of the total cycle length and to conclude the first stage of the two-stage optimization procedure, we set the total cycle time to the larger value of the two values obtained from Eqs. (8) and (10):

$$T = \max\{T_{opt}, T_{min}\} \tag{11}$$

Based on this total cycle time, the second stage of the two-stage optimization procedure determines whether to shut down and restart the machine or to leave it in the idle operating mode when the machine is idle (i.e., when neither the tool is changed nor the machine is processing). To this end, the idle time at the end of the total cycle needs to be calculated by subtracting the machine occupancy time, i.e. the sum of tool change and processing time, from the length of the total cycle:

$$T^{idle} = T - \left(\sum_{i=1}^{N} s_i + \sum_{i=1}^{N} \frac{r_i}{p_i} \cdot T \right) \tag{12}$$

The comparison of the idle time, T^{idle}, and the break-even duration, l^{BE}, (cf. Section 4.1.3) then determines in which machine operating mode to switch when the machine is idle:

(1) If $l^{BE} \geq T^{idle}$, the machine is switched to the idle operating mode and the additional costs can be calculated as $TC^{idle} = W \cdot T^{idle} \cdot \frac{e}{T}$,

(2) else, the machine is shut down and started up again, and the additional costs can be determined by $TC^{idle} = l^{sdsu} \cdot g \cdot W \cdot \frac{e}{T}$.

Finally, the total average costs, TC, can be calculated by adding TC^{occ} with the T-value obtained from Eq. (11) and TC^{idle}.

4.2.2 Basic-period-approach

In case the Basic-Period-Approach is used, the optimization procedure is again split into two stages (cf. Figure 2). The average cost when the machine is occupied corresponds to:

$$TC^{occ}(k_1, k_2, \dots, k_N, FC) = \frac{1}{FC} \cdot \sum_{i=1}^{N} \left(\underbrace{\frac{A_i}{k_i}}_{Tool\ change\ cost} + \underbrace{\frac{h_i \cdot r_i}{2} \cdot k_i \cdot FC^2 \cdot \left(1 - \frac{r_i}{p_i}\right)}_{Inventory\ holding\ cost} + \underbrace{\left(s_i \cdot f_i \cdot W + (W + v_i \cdot p_i) \cdot \frac{r_i}{p_i} \cdot k_i \cdot FC\right) \cdot \frac{e}{k_i}}_{Energy\ cost} \right) \tag{13}$$

The solution procedure for the proposed model will be divided into three steps in the following (cf. Beck and Glock, 2016).

<u>Step 1:</u>

First, the so-called *independent solution* is calculated as a lower bound. Here, all products are considered separately and their cost functions are minimized individually:

$$TC_i^{OCC}(T_i) = \frac{A_i}{T_i} + \frac{h_i \cdot r_i}{2} \cdot T_i \cdot \left(1 - \frac{r_i}{p_i}\right) + \left(s_i \cdot f_i \cdot W + (W + v_i \cdot p_i) \cdot \frac{r_i}{p_i} \cdot T_i\right) \cdot \frac{e}{T_i} \tag{14}$$

The first-order condition of Eq. (14) with respect to T_i can be written as follows:

$$\frac{dTC_i^{OCC}(T_i)}{dT_i} = -\frac{A_i}{T_i^2} + \frac{h_i \cdot r_i}{2} \cdot \left(1 - \frac{r_i}{p_i}\right) - (s_i \cdot f_i \cdot W) \cdot \frac{e}{T_i^2} = 0 \tag{15}$$

Solving Eq. (15) for T_i leads to

$$T_{i,opt} = \sqrt{\frac{2 \cdot (A_i + s_i \cdot f_i \cdot W \cdot e)}{h_i \cdot r_i \cdot \left(1 - \frac{r_i}{p_i}\right)}} \tag{16}$$

Inserting the individual optimal cycle length derived from Eq. (16) into Eq. (14) and summing up TC_i^{occ} over all N products leads to the total average cost of the independent solution. The preliminary length of the basic period for Step 2 equals

$$FC = \min\{T_{1,opt}, T_{2,opt}, \dots, T_{N,opt}\} \tag{17}$$

Step 2:

To carry out the procedure which assigns the products to the basic period, subsequently referred to as assigning procedure, in Step 3, we determine the maximum permissible integer multipliers for each product i and a starting value for the length of the basic period. Since the cycle time of each product has to be an integer multiple of the basic period – with the multipliers being restricted to powers of two as in Haessler and Hogue (1976), i.e. $k_i \in \{1,2,4,8,\dots\}, \forall i$ – the product-related cost functions need to be modified using the relation $T_i = k_i \cdot FC$:

$$\begin{aligned} TC_i^{OCC}(k_i, FC) = {} & \frac{A_i}{k_i \cdot FC} + \frac{h_i \cdot r_i}{2} \cdot k_i \cdot FC \cdot \left(1 - \frac{r_i}{p_i}\right) \\ & + \left(s_i \cdot f_i \cdot W + (W + v_i \cdot p_i) \cdot \frac{r_i}{p_i} \cdot k_i \cdot FC\right) \cdot \frac{e}{k_i \cdot FC} \end{aligned} \tag{18}$$

The following procedure is repeated until all k_i-values stay constant for two consecutive calculations. First, we calculate an auxiliary value for all N products using the results obtained in Step 1. In the following repetitions, we need updated values for the length of the basic period FC:

$$u_i = \frac{T_{i,opt}}{FC} \tag{19}$$

Three cases may arise for all products:

(1) If $u_i < 1$, set $k_i = 1$,

(2) else if $u_i \in \{1,2,4,8,\dots\}$, set $k_i = u_i$,

(3) else choose the next lower k_i^- and the next higher k_i^+ integer, with $k_i^- < u_i < k_i^+$ and $k_i^-, k_i^+ \in \{1,2,4,8,\ldots\}$. Using these values, the optimal k_i-values for all N products using Eq. (18) can be calculated in the following way:

1. If $TC_i^{occ}(k_i^-, FC) < TC_i^{occ}(k_i^+, FC)$, set $k_i = k_i^-$,
2. else if $TC_i^{occ}(k_i^+, FC) \leq TC_i^{occ}(k_i^-, FC)$, set $k_i = k_i^+$.

In order to update the length of the basic period FC, the sum of the product-related average cost functions across all N products is minimized with respect to FC:

$$TC^{OCC}(FC) = \sum_{i=1}^{N} TC_i^{OCC}(k_i, FC) \tag{20}$$

The first order condition of Eq. (20) with respect to FC equals

$$\frac{dTC^{OCC}(FC)}{dFC} = \sum_{i=1}^{N}\left(\frac{h_i \cdot r_i}{2} \cdot k_i \cdot \left(1 - \frac{r_i}{p_i}\right)\right) - \frac{1}{FC^2} \cdot \sum_{i=1}^{N}(A_i + s_i \cdot f_i \cdot W \cdot e) \cdot \frac{1}{k_i} = 0 \tag{21}$$

Solving Eq. (21) for FC yields

$$FC_{opt} = \sqrt{\frac{\sum_{i=1}^{N}(A_i + s_i \cdot f_i \cdot W \cdot e) \cdot \frac{1}{k_i}}{\sum_{i=1}^{N}\left(\frac{h_i \cdot r_i}{2} \cdot k_i \cdot \left(1 - \frac{r_i}{p_i}\right)\right)}} \tag{22}$$

Since the length of the recalculated basic period might not be sufficiently long to include the required tool changes, Eilon (1962) (chap. 14) and Haessler (1971) presented a necessary condition for feasibility:

$$FC_{check} \geq \frac{\sum_{i=1}^{N} \frac{s_i}{k_i}}{1 - \sum_{i=1}^{N} \frac{r_i}{p_i}} \tag{23}$$

Treating Eq. (23) as an equation, the minimum length of the basic period can be determined by

$$FC_{min} = \frac{\sum_{i=1}^{N} \frac{s_i}{k_i}}{1 - \sum_{i=1}^{N} \frac{r_i}{p_i}} \tag{24}$$

Finally, the updated value for the length of the basic period is set to $FC = max\{FC_{opt}, FC_{min}\}$ and the auxiliary variables u_i can be recalculated using Eq. (19).

As already pointed out above, the results of Step 2 determine the starting values for the assigning procedure in Step 3. In Step 3, the value for the length of the basic period of Step 2 will be referred to as FC_{start} and the k-vector with $k = (k_1, k_2, \dots, k_N)$ of Step 2 will be referred to as the vector of the maximum permissible k_i-values for each product i.

Step 3:

We use the assigning procedure I as presented in Beck and Glock (2016). As in Beck and Glock (2016), we also propose an evaluation of all possible k-vectors between the vector of the maximum permissible k_i-values for each product i of Step 2 and the unit vector. In the following, we refer to these possible k-vectors as the set of candidate solutions.

To derive a feasible solution, the three conditions mentioned in Haessler and Hogue (1976) and Beck and Glock (2016) need to hold. Since the assigning procedure used here is identical to the one presented by Beck and Glock (2016), we refer the reader to their paper for further information. After all products have been assigned to the basic periods, the following test for feasibility, which is the modified condition (III) of Beck and Glock (2016), has to be made:

$$FC_{test} = \left\{ \frac{\sum_{i=1}^{N} Y_{i,o} \cdot s_i}{1 - \sum_{i=1}^{N} \frac{Y_{i,o} \cdot r_i \cdot k_i}{p_i}} | o = 1,2, \dots, K \right\} \tag{25}$$

If conditions (I) and (II) of Beck and Glock (2016) are not violated and $min(FC_{test}) \geq 0$ and $max(FC_{test}) \leq FC$ hold, a feasible solution has been found.

In addition, in case a feasible solution has been found, the idle time of every basic period needs to be calculated:

$$T_o^{idle} = FC - \sum_{i=1}^{N} Y_{i,o} \cdot \left(s_i + \frac{r_i \cdot k_i \cdot FC}{p_i} \right), \forall o \tag{26}$$

As in case of the Common-Cycle-Approach (cf. Section 4.2.1), the comparison of the idle time, T_o^{idle}, and the break-even duration, l^{BE}, (cf. Section 4.1.3) then determines for each basic period in which machine operating mode to switch when the machine is idle (cf. Figure 2):

(1) If $l^{BE} \geq T_o^{idle}$, the machine is switched to the idle operating mode and the additional costs can be calculated as $TC_o^{idle} = W \cdot T_o^{idle} \cdot \frac{e}{K \cdot FC}$ with $K = max\{k_1, k_2, \dots, k_N\}$,

(2) else, the machine is shut down and started up again, and the additional costs can be determined by $TC_o^{idle} = l^{sdsu} \cdot g \cdot W \cdot \frac{e}{K \cdot FC}$ with $K = max\{k_1, k_2, \dots, k_N\}$.

Finally, the total average costs, TC, can be calculated by adding TC^{occ} and TC^{idle} with $TC^{idle} = \sum_{o=1}^{K} TC_o^{idle}$.

As already pointed out in Beck and Glock (2016), it is possible to calculate an upper bound for the length of the basic period FC_{max} for every k-vector. All values greater than FC_{max} lead to higher total average cost because Eq. (20) is convex in FC for a given k-vector (cf. Appendix A). Therefore, the cost function Eq. (20) has to be solved for FC using the current best cost value for a feasible solution TC_{ub}. This leads to

$$FC_{max} = \frac{-\left(\sum_{i=1}^{N}\left((W + v_i \cdot p_i) \cdot \frac{r_i}{p_i} \cdot e\right) - TC_{ub}\right)}{2\left(\sum_{i=1}^{N} \frac{k_i \cdot r_i}{2} \cdot \left(1 - \frac{r_i}{p_i}\right) \cdot h_i\right)} + \frac{\sqrt{\left(\sum_{i=1}^{N}\left((W + v_i \cdot p_i) \cdot \frac{r_i}{p_i} \cdot e\right) - TC_{ub}\right)^2 - 4 \cdot \left(\sum_{i=1}^{N} \frac{k_i \cdot r_i}{2} \cdot \left(1 - \frac{r_i}{p_i}\right) \cdot h_i\right) \cdot \left(\sum_{i=1}^{N} \frac{A_i + s_i \cdot f_i \cdot W \cdot e}{k_i}\right)}}{2\left(\sum_{i=1}^{N} \frac{k_i \cdot r_i}{2} \cdot \left(1 - \frac{r_i}{p_i}\right) \cdot h_i\right)} \quad (27)$$

Step 3 can be summarized as follows:

3.1 Set $k_i = 1, \forall i$ and determine TC_{ub}, i.e. the solution of the Common-Cycle-Approach (cf. Section 4.2.1).

3.2 Take one k-vector from the set of candidate solutions and calculate FC_{max} using Eq. (27). If $FC_{max} < FC_{start}$ or $FC_{max} \in \mathbb{C}$ with $Im(FC_{max}) \neq 0$, consider another k-vector from the set of candidate solutions that has not yet been investigated and go back to the beginning of Step 3.2, else go to Step 3.3.

3.3 Set $FC = FC_{start}$ and start using the assigning procedure described in Beck and Glock (2016) until all products have been assigned to the basic periods. If a feasible solution with TC^{occ} has been found, TC^{idle} needs to be added (cf. second optimization stage in Figure 2) to derive the TC. In case $TC < TC_{ub}$, set $TC_{ub} = TC$. If $FC + l^{sr} \leq FC_{max}$, set $FC = FC + l^{sr}$ and restart the assigning procedure, else if there is still another k-vector from the set of candidate solutions that has not been considered so far, go back to Step 3.2, else stop.

5 Numerical study

In order to investigate how the consideration of energy usage impacts the production schedules for the ELSP, we conducted a numerical study. Hence, the major goals of this numerical study are to assess I) how the production schedules derived from the Common-Cycle-Approach and the Basic-Period-Approach differ depending on whether or not energy usage is considered in the optimization and II) to what extent the total cost change as a result. To this end, we used the data set of Bomberger (1966) corrected by Chatfield (2007), which is traditionally used to evaluate solution approaches of the ELSP.

Table 1 summarizes the Bomberger data set, adjusted to a working day of 8 h and extended by product-specific energy usage data. The idle power of the machine, W, is set to 50 kW. A shutdown and startup phase is assumed to last 2 h, while the multiplication factor for the power required during the shutdown and startup of the machine, g, is set to 4.0. The energy usage charge, e, is assumed to equal 0.15 EUR/kWh.

For both the Common-Cycle-Approach and the Basic-Period-Approach, we calculated two production schedules. The first one was calculated without considering energy usage. Afterwards, energy cost associated with the resulting production schedule were added to the tool change and inventory holding costs. The second production schedule was calculated considering energy usage as described in Section 4.2 using the step range for the Basic-Period-Approach $l^{sr} = 0.01$. This procedure enabled us to directly compare the computed production schedules as well as the attached total cost.

Table 1: *Bomberger data set including product-specific energy usage data*

Pro- duct i	p_i [items/h]	r_i [items/h]	$10^{-5}h_i$ [EUR/(item·h)]	A_i [EUR]	s_i [h]	f_i [-]	v_i [kWh/item]
1	3,750.00	50.00	0.033854	15	1.00	1.25	0.005
2	1,000.00	50.00	0.924479	20	1.00	1.5	0.04
3	1,187.50	100.00	0.664063	30	2.00	2.5	0.01
4	937.50	200.00	0.520833	10	1.00	1.25	0.025
5	250.00	10.00	14.505208	110	4.00	3	0.065
6	750.00	10.00	1.393229	50	2.00	2.5	0.03
7	300.00	3.00	7.812500	310	8.00	1.5	0.02
8	162.50	42.50	30.729167	130	4.00	2.75	0.05
9	250.00	42.50	4.687500	200	6.00	1.75	0.07
10	1,875.00	50.00	0.208333	5	1.00	2	0.015

Table 2 shows the results of the modified Common-Cycle-Approach. As expected, the total average cost associated with the ELSP considering energy usage dominates the total average cost associated with the classical ELSP without energy considerations. In addition, one can see that the lengths of the fundamental cycles significantly differ from each other. Hence, considering power requirements clearly impacts the production schedules and the total cost.

Table 2: *Comparison of solutions derived from the Common-Cycle-Approach based on the Bomberger data set*

	Classical ELSP	**ELSP with energy**
T [h]	342.03	422.26
Upper bound of the total average costs [EUR/h]	15.55	15.38

The same holds true for the solutions derived from the Basic-Period-Approach (cf. Table 3). The results differ significantly in several ways: The consideration of energy usage in the optimization results in a reduction of total costs per hour by 1.7 percent. Among others, this can be traced back to the different k-vectors, which lead to different schedules. By definition, the total average cost associated with Step 3 is higher than the total average cost of the independent solution. However, the comparison of the total average cost of Step 3 of the classical ELSP and of the ELSP considering energy cost with the corresponding lower bounds of the total average cost shows that our procedure leads to a good result: While the total average cost of Step 3 of the classical ELSP exceeds the corresponding lower bound by 1.8 percent, the total average cost of Step 3 of the ELSP considering energy cost only lies 1.4 percent above the respective lower bound. Furthermore, the two relations of the total idle time and total tool change time to the total cycle length illustrate that the procedure considering energy usage tries to avoid tool changes and to reduce associated energy cost.

Table 3: *Comparison of the solutions derived from the Basic-Period-Approach based on the Bomberger data set*

		Classical ELSP	**ELSP with energy**
Step 1	FC of the independent solution [h]	156.23	209.92
	Lower bound of the total average cost [EUR/h]	14.10	13.92
Step 2	FC_{start} [h]	162.38	237.93
	k	{8,2,2,1,2,4,8,1,4,2}	{8,2,2,1,2,4,8,1,2,2}
Step 3	FC [h]	187.40	253.96
	k	{8,2,2,1,2,4,8,1,2,2}	{2,1,2,1,2,4,8,1,2,2}
	Total average cost [EUR/h]	14.35	14.11
	Number of times the machine is shut down and started up per total cycle	3	3
	Relation of the total idle time to the total cycle length	0.045	0.060
	Relation of the total tool change time to the total cycle length	0.073	0.057

Figure 3 visualizes the differences between the production schedules of one total cycle derived from the Basic-Period-Approach even better. Even though, in both cases the Basic-Period-Approach recommends splitting the production schedule across eight basic periods and shutting down and starting up the machine three times, the number of production runs of products 1 and 2 changes from 8 and 2 for the classical ELSP to 2 and 1 for the ELSP with energy, respectively.

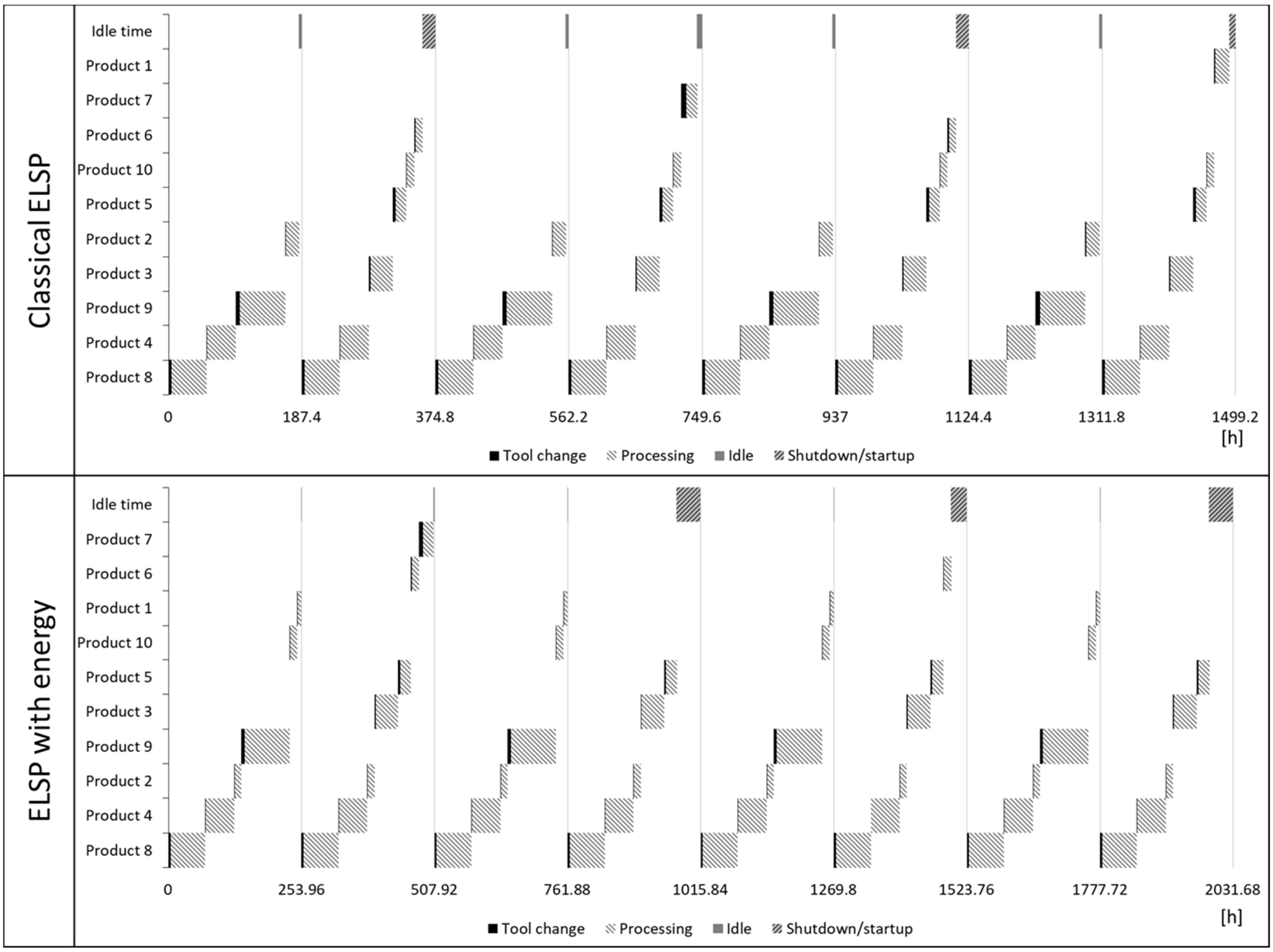

Figure 3: *Comparison of production schedules derived from the Basic-Period-Approach based on the Bomberger data set*

Another numerical example using the modified Eilon (1962) (chap. 14) data set can be found in Appendix B. The results also clearly support the implications derived from the Bomberger data set.

6 Conclusions and managerial implications

The paper at hand extended the classical Economic Lot Scheduling Problem to the case where energy cost resulting from energy usage in different machine operating modes is taken into consideration. Energy cost plays an important role for tracking the true cost associated with production scheduling. Hence, our model helps to derive production schedules which feature a more realistic representation of the total cost and support reaching environmental goals often specified by legislation.

After integrating energy cost into the objective functions of both the Common-Cycle-Approach of Hanssmann and the Basic-Period-Approach of Haessler and Hogue, the corresponding solution procedures were modified according to a two-stage optimization procedure. The first stage computes an optimal production schedule by minimizing the sum of tool change, inventory holding, and energy costs arising in times when the machine is occupied (i.e., when either the tool is changed or the machine is processing). The second stage determines whether to shut down and restart the machine or to leave it in the idle operating mode when the machine is idle (i.e., when neither the tool is changed nor the machine is processing) based on the break-even duration.

From the numerical analysis using the modified data sets of Bomberger and Eilon, several managerial implications regarding the ELSP with energy cost can be derived:

- Considering energy cost associated with the energy usage during processing, tool change, shutdown and startup, and idling may lead to production schedules and lot sizes which differ significantly from the production schedules and lot sizes calculated without considering energy cost.
- Depending on the production process, energy cost may contribute a considerable share to the total production cost. Hence, to capture the true cost associated with a production schedule, production planning models need to reproduce energy usage in different machine operating modes as realistically as possible. Then, the consideration of energy cost in production scheduling may result in substantial cost reductions.
- Production planners should not only focus on the times when the tool of a machine is changed or the machine is processing when determining a production schedule. They should instead also be aware of the energy used in times a machine is idle and control the machine operating modes accordingly.

In a next step, it may be of interest to take account of time-varying energy usage charges many manufacturing companies face. Furthermore, the integration of energy cost into other approaches commonly used to solve the ELSP, such as the Time-Varying-Lot-Sizes-Approach, may further underscore how production scheduling is impacted by the consideration of energy usage.

References

Artigues, C., Lopez, P., Haït, A., 2013. The energy scheduling problem: industrial case-study and constraint propagation techniques. International Journal of Production Economics, 143(1), 13-23.

Bazan, E., Jaber, M.Y., Zanoni, S., 2015. Supply chain models with greenhouse gases emissions, energy usage and different coordination decisions. Applied Mathematical Modelling, 39(17), 5131-5151.

Beck, F.G., Glock, C.H., 2016. The impact of batch shipments on the economic lot scheduling problem. Computers & Industrial Engineering, 96, 126-139.

Ben-Daya, M., Hariga, M., 2000. Economic lot scheduling problem with imperfect production processes. The Journal of Operational Research Society, 51(7), 875-881.

Bi, Z.M., Wang, L., 2012. Optimization of machining processes from the perspective of energy consumption: a case study. Journal of Manufacturing Systems, 31(4), 420-428.

Biel, K., Glock, C.H., 2016a. On the use of waste heat in a two-stage production system with controllable production rates. International Journal of Production Economics, 181, 174-190.

Biel, K., Glock, C.H., 2016b. Prerequisites of efficient decentralized waste heat recovery and energy storage in production planning. Journal of Business Economics, 87(1), 41-72.

Biel, K., Glock, C.H., 2016c. Systematic literature review of decision support models for energy-efficient production planning. Computers & Industrial Engineering, 101, 243-259.

Bomberger, E., 1966. A dynamic programming approach to a lot size scheduling problem. Management Science, 12(11), 778-784.

Brander, P., Forsberg, R., 2005. Cyclic lot scheduling with sequence-dependent set-ups: a heuristic for disassembly processes. International Journal of Production Research, 43(2), 295-310.

Chan, H.K., Chung, S.H., Lim, M.K., 2013. Recent research trend of economic-lot scheduling problems. Journal of Manufacturing Technology Management, 24(3), 465-482.

Chatfield, D.C., 2007. The economic lot scheduling problem: a pure genetic search approach. Computers & Operations Research, 34(10), 2865-2881.

Chowdhury, M.R., Sarker, B.R., 2001. Manufacturing batch size and ordering policy for products with shelf lives. International Journal of Production Research, 39(7), 1405-1426.

Ding, J.Y., Song, S., Zhang, R., Chiong, R., Wu, C., 2015. Parallel machine scheduling under time-of-use electricity prices: New models and optimization approaches. IEEE Transactions on Automation Science and Engineering, 13(2), 1138-1154.

Dobson, G., 1987. The economic lot-scheduling problem: achieving feasibility using time-varying lot sizes. Operations Research, 35(5), 764-771.

Doll, C.L., Whybark, D.C., 1973. An iterative procedure for the single-machine multi-product lot scheduling problem. Management Science, 20(1), 50-55.

Eilon, S., 1962. Elements of production planning and control. New York: MacMillan.

Elhafsi, M., Bai, S.X., 1997. The common cycle economic lot scheduling problem with backorders: benefits of controllable production rates. Journal of Global Optimization, 10(3), 283-303.

Elmaghraby, S.E., 1978. The economic lot scheduling problem (ELSP): review and extensions. Management Science, 24(6), 587-598.

Fang, K., Uhan, N., Zhao, F., Sutherland, J.W., 2011. A new approach to scheduling in manufacturing for power consumption and carbon footprint reduction. Journal of Manufacturing Systems, 30(4), 234-240.

Gutowski, T., Dahmus, J., Thiriez, A., 2006. Electrical energy requirements for manufacturing processes. In: 13th CIRP International Conference on Life Cycle Engineering, 31, 623-638.

Haessler, R.W., 1971. A note on scheduling a multi-product single-machine system for an infinite planning period. Management Science, 18(4), 240-241.

Haessler, R.W., Hogue, S.L., 1976. Note—a note on the single-machine multi-product lot Scheduling problem. Management Science, 22(8), 909-912.

Haksöz, C., Pinedo, M., 2011. Economic lot scheduling with resources in parallel. International Journal of Production Research, 49(9), 2625-2641.

Hanssmann, F., 1962. Operations Research in Production and Inventory Control (1st ed.). John Wiley & Sons.

Liu, C.H., 2014. Approximate trade-off between minimisation of total weighted tardiness and minimisation of carbon dioxide (CO2) emissions in bi-criteria batch scheduling problem. International Journal of Computer Integrated Manufacturing, 27(8), 759-771.

Liu, Y., Dong, H., Lohse, N., Petrovic, S., Gindy, N., 2014. An investigation into minimising total energy consumption and total weighted tardiness in job shops. Journal of Cleaner Production, 65, 87-96.

Luo, H., Du, B., Huang, G.Q., Chen, H., Li, X., 2013. Hybrid flow shop scheduling considering machine electricity consumption cost. International Journal of Production Economics, 146(2), 423-439.

May, G., Stahl, B., Taisch, M., Prabhu, V., 2015. Multi-objective genetic algorithm for energy-efficient job shop scheduling. International Journal of Production Research, 53(23), 7071-7089.

Moon, I., Silver, E.A., Choi, S., 2002. Hybrid genetic algorithm for the economic lot-scheduling problem. International Journal of Production Research, 40(4), 809-824.

Moon, J.Y., Park, J., 2014. Smart production scheduling with time-dependent and machine-dependent electricity cost by considering distributed energy resources and energy storage. International Journal of Production Research, 52(13), 3922-3939.

Mouzon, G., Yildirim, M.B., Twomey, J., 2007. Operational methods for minimization of energy consumption of manufacturing equipment. International Journal of Production Research, 45(18-19), 4247-4271.

Rogers, J., 1958. A computational approach to the economic lot scheduling problem. Management Science, 4(3), 264-291.

Santander-Mercado, A., Jubiz-Diaz, M., 2016. The economic lot scheduling problem: a survey. International Journal of Production Research, 54(16), 4973-4992.

Sharma, A., Zhao, F., Sutherland, J.W., 2015. Econological scheduling of a manufacturing enterprise operating under a time-of-use electricity tariff. Journal of Cleaner Production, 108, 256-270.

Sox, C.R., Jackson, P.L., Bowman, A., Muckstadt, J.A., 1999. A review of the stochastic lot scheduling problem. International Journal of Production Economics, 62(3), 181-200.

Tang, O., Teunter, R., 2006. Economic lot scheduling problem with returns. Production and Operations Management, 15(4), 488-497.

U.S. Energy Information Administration, 2016a. Annual Energy Outlook 2016 with Projections to 2040.

U.S. Energy Information Administration, 2016b. International Energy Outlook 2016.

Wang, S., Lu, X., Li, X.X., Li, W.D., 2015. A systematic approach of process planning and scheduling optimization for sustainable machining. Journal of Cleaner Production, 87, 914-929.

Winands, E.M.M., Adan, I.J.B.F., van Houtum, G.J., 2011. The stochastic economic lot scheduling problem: a survey. European Journal of Operational Research, 210(1), 1-9.

Zanoni, S., Bettoni, L., Glock, C.H., 2014. Energy implications in a two-stage production system with controllable production rates. International Journal of Production Economics, 149, 164-171.

Zeng, L.-L., Zou, F.-X., Xu, X.-H., Gao, Z., 2009. Dynamic scheduling of multi-task for hybrid flow-shop based on energy consumption. In: Information and Automation, 2009. ICIA '09. International Conference on Information and Automation, 478-482.

Zhou, L., Li, J., Li, F., Meng, Q., Li, J., Xu, X., 2016. Energy consumption model and energy efficiency of machine tools: a comprehensive literature review. Journal of Cleaner Production, 112(5), 3721-3734.

Zipkin, P.H., 1991. Computing optimal lot sizes in the economic lot scheduling problem. Operations Research, 39(1), 56-63.

Appendix

Appendix A

The second partial derivative of Eq. (6) with respect to T can be calculated as follows:

$$\frac{d^2TC^{OCC}(T)}{dT^2} = \sum_{i=1}^{N} \left(\frac{2 \cdot A_i}{T^3} + 2 \cdot (s_i \cdot f_i \cdot W) \cdot \frac{e}{T^3} \right) > 0 \tag{A.1}$$

Thus, Eq. (6) is convex in T.

The second partial derivative of Eq. (14) with respect to T_i can be calculated as follows:

$$\frac{d^2TC_i^{OCC}(T_i)}{dT_i^2} = \frac{2 \cdot A_i}{T_i^3} + 2 \cdot (s_i \cdot f_i \cdot W) \cdot \frac{e}{T_i^3} > 0 \tag{A.2}$$

Thus, Eq. (14) is convex in T_i.

The second partial derivative of Eq. (20) with respect to FC can be calculated as follows:

$$\frac{d^2TC^{OCC}(FC)}{dFC^2} = \frac{2}{FC^3} \cdot \sum_{i=1}^{N} (A_i + s_i \cdot f_i \cdot W \cdot e) \cdot \frac{1}{k_i} > 0 \tag{A.3}$$

Thus, for a given vector k, Eq. (20) is convex in FC.

Appendix B

Table B.1 summarizes the Eilon data set, modified by Haessler and Hogue, adjusted to a working day of 8 h, and extended by product-specific energy usage data. The idle power of the machine, W, is set to 100 kW. A shutdown and startup phase is assumed to last 5 h, while the multiplication factor for the power required during the shutdown and startup of the machine, g, is set to 8.0. The energy usage charge, e, is assumed to equal 0.15 EUR/kWh.

Table B.1: *Eilon data set modified by Haessler and Hogue including product-specific energy usage data*

Product i	p_i [items/h]	r_i [items/h]	$10^{-5}h_i$ [EUR/(item·h)]	A_i [EUR]	s_i [h]	f_i [-]	v_i [kWh/item]
1	16.625	2.5	5.7625	3,000	32	1.25	0.5
2	37.5	3	3.9	1,800	19.2	2.5	0.75
3	33.25	3.75	8.1375	3,600	38.4	3	0.95
4	18.25	4.5	14.75	1,500	16	2.75	0.8
5	66.5	5	14.875	6,000	32	1.75	0.6
6	46.625	6.25	10.5875	30,000	64	3	0.4

Table B.2 presents the results of the modified Common-Cycle-Approach. Once again, the total average cost associated with the ELSP considering energy usage dominates the total average cost associated with the classical ELSP without energy considerations. As in case of the Bomberger data set, the fundamental cycle of the ELSP considering energy usage is significantly longer than the fundamental cycle of the classical ELSP.

Table B.2: *Comparison of solutions derived from the Common-Cycle-Approach based on the Eilon data set*

	Classical ELSP	**ELSP with energy**
T [h]	6370.93	6867.11
Upper bound of the total average costs [EUR/h]	30.08	30.03

Table B.3 shows the corresponding results when using the Basic-Period-Approach. Again, the total average cost of the independent solution with energy dominates the corresponding total average cost of the classical ELSP. The k-vector of Step 2 indicates that only for product 1 the maximum permissible integer multipliers differ from each other. Hence, it may be possible that the next step leads to worse results because $k_1 = 4$ is not possible in case of the ELSP under consideration of energy usage, although it would be good to use a k-value of $k_1 = 4$ for product 1. As in case of the Bomberger data set, the total average cost when considering energy usage is lower than the total average cost when disregarding energy usage. However, in case of the

Eilon data set, the difference is not as significant. In addition, as in case of the Bomberger data set, the solution procedure for the ELSP with energy tries to avoid tool changes and the associated energy cost. This follows from the relation of the total tool change time to the total cycle length which is lower in the case of the ELSP under consideration of energy usage compared to the classical ELSP.

Table B.3: *Comparison of the solutions derived from the Basic-Period-Approach based on the Eilon data set*

		Classical ELSP	ELSP with energy
Step 1	FC of the independent solution [h]	2449.28	2939.14
	Lower bound of the total average cost [EUR/h]	28.54	28.46
Step 2	FC_{start} [h]	2388.99	2748.40
	k	{4,2,2,1,2,4}	{2,2,2,1,2,4}
Step 3	FC [h]	2388.99	2790.32
	k	{4,2,2,1,2,4}	{2,2,2,1,2,4}
	Total average cost [EUR/h]	28.92	28.83
	Number of times the machine is shut down and started up per total cycle	4	4
	Relation of the total idle time to the total cycle length	0.166	0.168
	Relation of the total tool change time to the total cycle length	0.035	0.033

Figure B.1 illustrates the corresponding production schedules of one total cycle. Overall, the results of the Eilon data set proof that the energy usage clearly impacts the optimal production schedule of the ELSP derived from both the Common-Cycle-Approach and the Basic-Period-Approach and thus reinforce the results of the Bomberger data set.

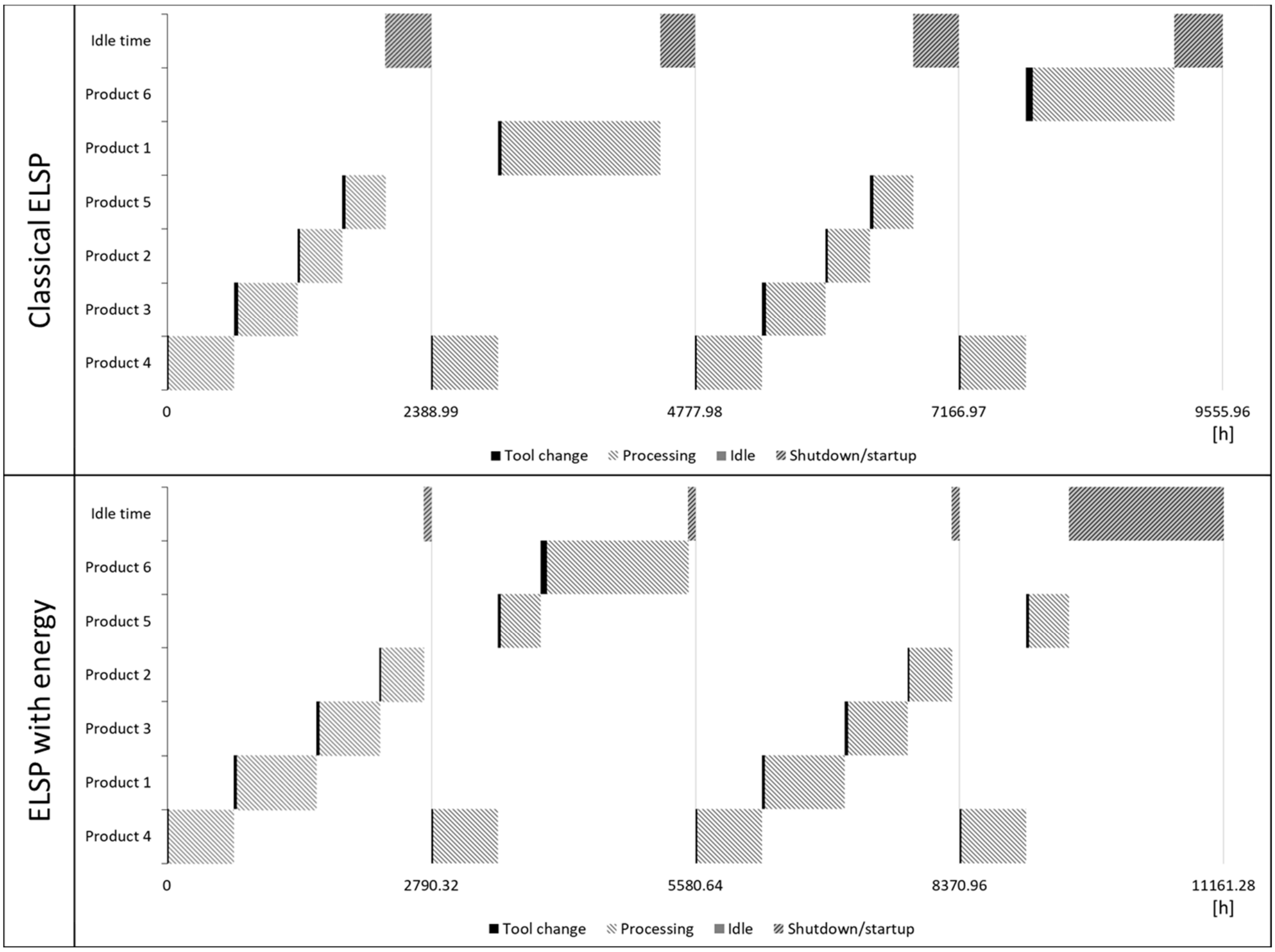

Figure B.1: *Comparison of production schedules derived from the Basic-Period-Approach based on the Eilon data set*

Paper 3 The impact of batch shipments on the economic lot scheduling problem[1]

Authors: Fabian G. Beck, Christoph H. Glock

Type of publication: Journal article

Publication details: *Computers & Industrial Engineering* (VHB-JOURQUAL3: B), 96, 126-139, 2016.

Abstract

This paper studies the Economic Lot Scheduling Problem (ELSP) for the single-machine-multi-product case. In contrast to earlier research, it assumes that a lot may be split up into batches, which are then shipped individually to the consuming stage. Integrating batch shipments into the ELSP helps to reduce cycle times at the expense of higher transportation costs. This paper selects two popular approaches used in the literature to solve the ELSP, namely the Common-Cycle-Approach of Hanssmann and the Basic-Period-Approach of Haessler and Hogue, extends them to include both equal- and unequal-sized batch shipments, and suggests a solution procedure for each approach. Both model extensions are illustrated using the modified Bomberger and Eilon data sets, and they are compared to the special case where only complete lots are transferred to the next stage. Finally, ideas for future research are presented.

Keywords:

Economic lot scheduling problem; ELSP; Equal-sized batch shipments; Unequal-sized batch shipments; Basic-period-approach; Haessler and Hogue

---

[1] Reprinted and slightly modified from *Computers & Industrial Engineering,* 96, Beck, F.G., Glock, C.H., The impact of batch shipments on the Economic Lot Scheduling Problem, 126-139, 2016, https://doi.org/10.1016/j.cie.2016.02.021, with permission from Elsevier.

1 Introduction

The Economic Lot Scheduling Problem (ELSP) refers to a situation where several items are produced on the same facility on a repetitive basis (Rogers, 1958). Products produced on the facility are shipped to a consuming stage, which could represent the end customer(s) or other production stages processing the items. In the ELSP, the production planner has to define lot sizes in such a way that all products can be produced on the machine without overlaps, and in addition it has to be made sure that the items can be consumed on the subsequent stage without interruption. The ELSP occurs in many different industries, such as the chemical process industry (Carstensen, 1999) or in metal stamping applications (Bomberger, 1966). The ELSP is, in general, NP-hard (see Gallego & Shaw, 1997; Hsu, 1983), which is why heuristics are usually employed to solve this problem.

As will be shown in the next section, prior research on the ELSP has concentrated on developing heuristics that help finding a solution that comes close to the independent solution, which is usually treated as a theoretical lower bound to the problem. Chan, Chung, and Lim (2013) showed that the ELSP is still a very popular research field. In their review of 100 articles published during the years 1997–2012, they showed that the most popular solution approaches for the ELSP in recent years have been the Common-Cycle-Approach (CCA), which was used in 41% of the reviewed articles, and the Basic-Period-Approach (BPA), which was used in 28% of the sampled papers. In addition to works that aim on improving the solution of the classical ELSP, other authors introduced extensions of the classical ELSP by assuming that production rates are variable (Eynan, 2003), by studying multiple machines (Sun, Jaruphongsa, & Huang, 2009), or by considering sequence-dependent setup times and/or cost (Brander & Forsberg, 2005).

In our survey of the literature, we noticed that prior research consistently made the assumption that products produced in the ELSP can be consumed directly after their completion, which implies that each unit of a product is shipped individually from the producing to the consuming stage. It is clear that shipping individual items between both stages is economical only in case both stages are located in close geographic proximity and in case shipment costs are low. If shipment costs are high, production managers may decide to forward several items together in a single shipment to reduce the overall number of shipments and the corresponding transportation costs. Shipping complete lots, which is a mode of transportation that has very frequently been discussed in the literature on lot sizing, obviously helps to reduce the number of shipments and the associated costs; it may, however, result in high inventory carrying costs in case lot sizes are large.

To balance inventory carrying and transportation costs, the literature suggests splitting up a lot into several so-called batches and to make a shipment from the first to the second stage whenever a batch has been completed (e.g., Goyal, 1977; Hill, 1997; Szendrovits, 1975). Lot sizing models with batch shipments are a generalization of the two extreme cases described above, as high shipment costs would lead to a single batch shipment, i.e. a situation where only complete lots are shipped between both stages, whereas low transportation cost would lead to a high number of batch shipments and ultimately a situation where individual items are shipped. Interestingly, both the shipping of complete lots as well as the option to split up lots into batches have thus far not been investigated thoroughly in the context of the ELSP, despite the large number of works that have been published on the sizing of batches in inventory models (Glock, 2012; Glock, Grosse, & Ries, 2014).

The use of batches may influence the ELSP in several ways. Especially if compared to the case where only complete lots are shipped, batch shipments may help to reduce inventory holding costs due to an earlier start of the consumption phase. In addition, permitting batch shipments may lead to larger lots and less frequent setups, which reduces setup times and which may facilitate finding a feasible solution to the problem.

In light of the benefits the use of batch shipments may offer, the objective of the work at hand is to introduce two modified heuristic solution procedures for the ELSP based on Haessler and Hogue's (1976) approach, with both procedures permitting batch shipments. The contributions of this paper can be summarized as follows:

(I) The paper extends the ELSP to the cases of complete lots as well as equal- and unequal-sized batch shipments.

(II) The paper develops modified solution procedures for the new planning situations.

(III) The paper modifies the assignment procedure proposed by Haessler and Hogue (1976).

The solution procedures will be evaluated against the Common-Cycle-Approach of Hanssmann (1962), which will be modified in this paper to account for batch shipments as well.

The remainder of this paper is structured as follows: The next two sections review related literature and outline the definitions and assumptions used in this paper. Section 4 presents the models proposed in this paper, and Section 5 contains a numerical example to illustrate our solution procedures. Section 6 concludes the article.

2 Literature review

Solution procedures for the ELSP can roughly be divided into three categories with different assumptions, namely Common-Cycle-Approaches, Basic-Period-Approaches, and Time-Varying-Lot-Sizes-Approaches. Elmaghraby (1978) further differentiated solution procedures into two main categories: analytical approaches, which find an optimum solution of a restricted version of the original ELSP, and heuristics, which try to solve the original problem as good as possible.

The Common-Cycle-Approach (CCA) was introduced by Hanssmann (1962) as an easy way to solve the ELSP. The CCA assumes an identical cycle time for each product, which leads to exactly one setup per product per cycle. The aim of the CCA is to determine the length of the total cycle, which has to be long enough to accommodate the production of each product. Due to its restrictive assumptions, the solution obtained for the CCA is usually considered as an upper bound to the problem.

Dobson (1987) used the ideas of Maxwell (1964) and Delaporte and Thomas (1977) and developed the so-called Time-Varying-Lot-Sizes-Approach. The basic idea of this approach is to relax two restrictive assumptions that were made in earlier approaches. First, the lot sizes of every product are allowed to vary during the planning horizon. Secondly, the production times of every product are no longer limited to values smaller than the length of a basic period. One major drawback of the Time-Varying-Lot-Sizes-Approach is that calculating lot sizes may be time-consuming (Carstensen, 1999).

The focus of our literature review is on the Basic-Period-Approach (BPA) in the following, which is the approach that will be extended in this paper. To avoid the shortcomings of the CCA, Bomberger (1966) introduced the BPA, which permits different cycle times for different products. To reduce the complexity of the problem, the cycle time of each product was restricted to an integer multiple of a basic period (which is sometimes also referred to as the "fundamental cycle"). Bomberger solved the ELSP using a dynamic programming approach based on the functional equation approach of Bellman (1957). Doll and Whybark (1973) presented a heuristic for the BPA that relies on an iterative procedure for determining the production frequencies and the length of the basic period. One drawback of Doll and Whybark's heuristic is that it does not guarantee that a feasible solution is found. Therefore, Haessler and Hogue (1976) extended the solution procedure of Doll and Whybark to make sure that a feasible solution is found. In addition, Haessler and Hogue restricted the integer multipliers used for calculating individual cycle times to powers of two to reduce the number of solutions that have to be tested for optimality. The heuristic of Geng and Vickson (1988) modified the calculation of the integer multipliers suggested by Doll and Whybark (1973) and Haessler and Hogue (1976). Their approach is to choose the integer multiple next to the value determined by the quotient of the individual optimal cycle time of the independent solution and the minimum of these individual optimal cycle times, again limited to powers of two. In addition, the assignment of the products to the basic periods was also modified in their paper to minimize idle times within the basic periods.

Elmaghraby (1978) presented the so-called Extended-Basic-Period-Approach (EBPA), which assumed that two consecutive periods with the same cycle length exist. In case of two separate periods, it is possible that each of them has different constraints. Thus, for the EBPA, it is not necessary that all products are produced in a single period, which makes it easier to find a feasible solution to the problem. An improvement of the EBPA of Elmaghraby (1978) was proposed by Haessler (1979).

Khouja, Michalewicz, and Wilmot (1998) used a genetic algorithm for solving the ELSP. The problem was formulated again as a Basic-Period-Approach. A genetic algorithm was also used by Chatfield (2007) and Sun, Huang, and Jaruphongsa (2009) to solve the ELSP, but in contrast to Khouja et al. (1998), their works were based on the EBPA.

One of the few works that considered batch shipments in the ELSP is the one of Buscher (2000), who integrated equal-sized batch shipments into the CCA of Hanssmann (1962). In contrast to Hanssmann, Buscher made the assumption that the planning horizon is finite. In addition, he assumed that the number of batch shipments has to be equal for all products, and suggested a solution procedure that calculates the number of lots and the number of batch shipments per lot. Ho, Tseng, and Hsiao (2015) also took equal-sized batch shipments into account and assumed in addition that the demand is stochastic. Another related paper is the one of Ho, Lai, and Huang (2014), who studied a single-supplier multiple-retailer supply chain where the supplier produces multiple products on a single facility. The authors assumed that a joint cycle is implemented in the system, and that each product is produced exactly once in each replenishment cycle. The authors then used a combination of equal- and unequal sized batch shipments for transporting products from the supplier to the retailers.

Even though batch shipments have not received much attention in the ELSP literature, there is a significant amount of research on batch shipments in other inventory models. Research on batch shipments usually differentiates between three alternative batch shipment policies: (I) equal-sized batch shipments, where subsequent shipments from a lot have the same size (e.g. Szendrovits, 1975); (II) unequal-sized batch shipments, where the sizes of subsequent shipments from a lot vary, e.g. according to a geometric series (e.g. Goyal, 1977; Hill, 1997); and (III) a combination of both (e.g. Goyal & Nebebe, 2000). Prior research has frequently shown that using batch shipments is a suitable measure for reducing the cost of an inventory system. As a result, batch shipments have been investigated in many different areas, including (company-internal) production systems (e.g. Bogaschewsky, Buscher, & Lindner, 2001) and supply chains (e.g. Kim & Glock, 2013).

Our review of the literature shows that only three works studied batch shipments in the ELSP or one of its extensions. Clearly, using batch shipments helps to reduce cycle inventory, which is why their use in the ELSP deserves more attention than it received in the past. This paper contributes to closing this research gap by extending two popular heuristics for solving the ELSP to take account of batch shipments. In contrast to the works of Buscher (2000), Ho et al. (2015) and Ho et al. (2014), we use a Basic-Period-Approach for coordinating the system and employ both equal- and unequal-sized batch shipments for transporting products from the producing to the consuming stage.

3 Problem description

This paper modifies the classical ELSP to take account of batch shipments that are transported from the producing to the consuming stage.

The following assumptions are made in developing the proposed models:

1. N products are produced on a single machine.
2. All parameters are deterministic and constant over time.
3. The planning horizon is infinite.

4. The machine can only produce one item at a time.
5. Inventory holding costs are directly proportional to inventory levels.
6. Shortages are not allowed.
7. Setup times and setup costs are independent of the production sequence.
8. The lot size and the number of batches are constant over time.

To make sure that the products can be consumed without interruption at the consuming stage, prior research implemented a so-called zero switch rule, which specifies that the production of a product has to start at the point in time when the inventory of that product reaches zero (Maxwell, 1964). In the case considered here, i.e. in the case where batch shipments are used, production has to start earlier to permit an uninterrupted consumption. Therefore, we use a modified zero switch rule, which stipulates that the production of the first batch has to be finished at the point in time when the inventory of this product reaches zero.

The following notation will be used in this paper:

Variables:

FC	Length of the fundamental cycle
k_i	Integer multiplier; product i is produced every k_i fundamental cycles
q_i	Lot size of product i in a production cycle
m_i	Number of batch shipments of a lot of product i
T	Total cycle time
T_i	Cycle time of product i

Parameters:

p_i	Production rate of product i in units per unit of time
pt_i	Total production time per lot of product i
r_i	Demand rate of product i in units per unit of time
rt_i	Consumption time per lot of product i
T_{pp}	Length of the planning period in units of time
B_i	Total demand of product i in units in the planning period, where $B_i = T_{pp}r_i$
h_i	Inventory holding costs per unit per unit of time for product i
l_{sr}	Length of the step range
s_i	Setup costs per setup for product i

st_i	Setup time per setup for product i
tc_i	Transportation costs per batch shipment for product i
N	Number of products
N_{sr}	Number of step ranges
o	Index of the fundamental cycle
u_i	Auxiliary variable for product i
w	Number of fundamental cycles within the total cycle
Y	Binary variable
z_i	Number of production cycles of product i within the total cycle

Definitions:

C	Total average costs per unit of time
C_i	Average costs per unit of time of product i
$\lceil ... \rceil$	Ceiling function
$\lfloor ... \rfloor$	Floor function

In the scenario considered here, the production capacity of the facility has to be large enough to ensure that demand can be satisfied without interruption. Therefore, it is necessary to evaluate the net utilization of the facility to make sure that the production capacity is large enough and that the problem is feasible. A feasible ELSP thus has to satisfy the following condition:

$$\sum_{i=1}^{N} \frac{r_i}{p_i} < 1 \tag{1}$$

Several articles stated that $\sum_{i=1}^{N} \frac{r_i}{p_i} \leq 1$ has to hold (e.g. Elmaghraby, 1978; Hanssmann, 1962); however, we note that in case where $\sum_{i=1}^{N} \frac{r_i}{p_i} = 1$, there is no time left for performing setups, which is why we use the condition specified above. Despite the general feasibility of the ELSP, the net utilization constraint also influences the solution complexity of the ELSP. In general, the higher the net utilization, the more difficult it is to find a feasible solution (Doll & Whybark, 1973).

The aim of the heuristics presented in the following is to minimize the total average costs per unit of time for a feasible solution.

4 The ELSP with batch shipments

4.1 The inventory structure

This paper extends the classical ELSP to the case where batch shipments are transported from the producing to the consuming stage. Using batch shipments is appropriate in case shipment costs arise for forwarding products from the first to the second stage; in this case, it is not efficient to ship each unit of each product individually to the consuming stage, as was assumed in prior research on the ELSP so far. This paper considers two different batch shipment policies that have received much attention in the past. First, Section 4.2 discusses the case of equal-sized batch shipments where all batches have the same size, and then Section 4.3 discusses the case of unequal-sized batch shipments where subsequence shipments increase in size according to a constant factor.

Figures 1-4 illustrate four different shipment policies considered in this paper for a single product and two consecutive lots. We assume that the production of product i starts at time t_0. In case of the classical ELSP (Figure 1), each individual item is sent to the next stage directly after its completion. In this case, consumption can start immediately after initializing production. In case complete lots (equal- or unequal-sized batches) are shipped (cf. Figures 2-4), in contrast, consumption can start not before the first lot (batch) has been completed, i.e. at time t_1. In case batch shipments are used, the first batch is consumed at time t_2, and the first lot is consumed completely at time t_3. If unequal-sized batches are shipped, then the first batch can be consumed earlier than in the case of an equal-sized batch shipment policy due to its smaller size. To ensure that consuming the products is possible without interruption, it is necessary to start production of the second lot at time t_4. As can be seen for the classical ELSP, a new production cycle starts at the end of the last consumption cycle; in case batch shipments or complete lots are shipped, both cycles overlap. The length of the consumption cycles is the same for all shipment policies if the lot sizes (here: q_i) are identical.

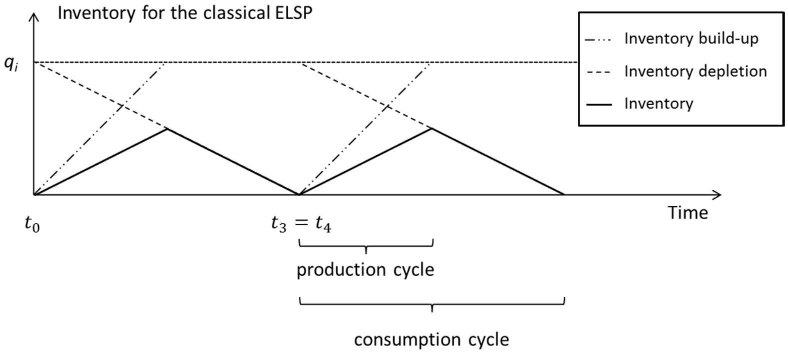

Figure 1: *Inventory-time plots for two lots of one product for the classical ELSP*

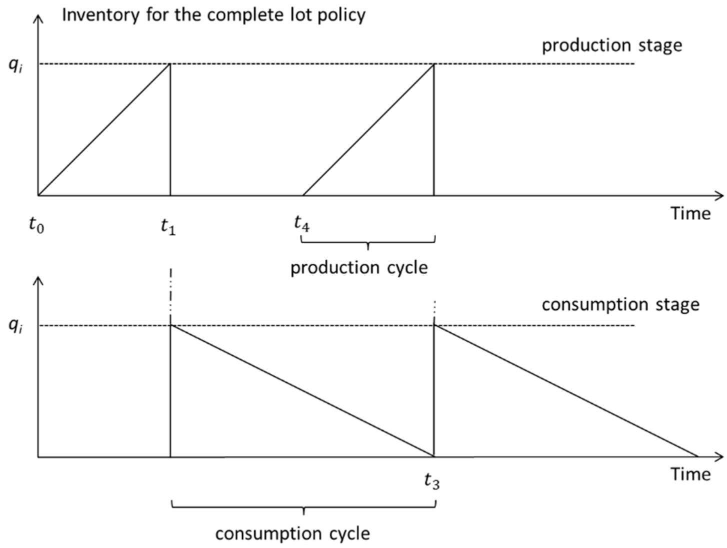

Figure 2: *Inventory-time plots for two lots of one product for the complete lot policy*

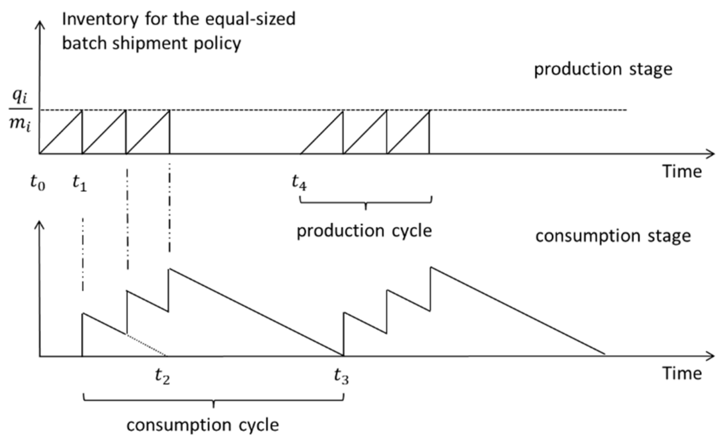

Figure 3: *Inventory-time plots for two lots of one product for the ELSP with equal-sized batch shipments*

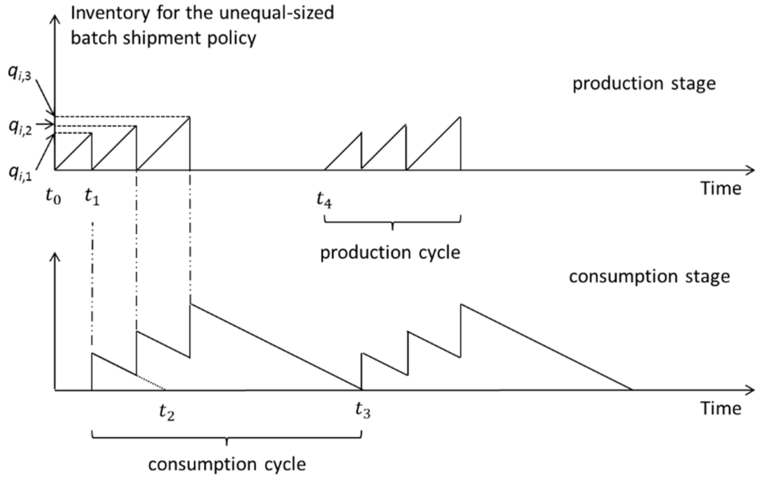

Figure 4: *Inventory-time plots for two lots of one product for the ELSP with unequal-sized batch shipments*

4.2 Equal-sized batch shipments

4.2.1 Modified independent solution (MIS)

First, we consider the case where equal-sized batch shipments are transported from the producing to the consuming stage. In a first step, we calculate a lower bound for the problem at hand. As is common in the literature on the ELSP, we use the so-called "independent solution", i.e. the solution where the objective function of each product is minimized individually without considering possible overlaps in the production schedule, as the lower bound. Minimizing the objective functions of the products individually obviously leads to the lowest possible total cost for the problem at hand; the independent solution results in an infeasible production schedule in many cases, however.

In case of an equal-sized batch shipment policy, we use the total cost function developed by Szendrovits (1975), who studied the case of equal-sized batch shipments in a two-stage inventory model with a single product. The costs per unit of time of product i can thus be calculated by summing up inventory carrying, setup and transportation costs:

$$C_i(q_i, m_i) = \frac{q_i}{2m_i}\left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1)\left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] h_i B_i + (s_i + m_i tc_i)\frac{B_i}{q_i} \tag{2}$$

using the relations $q_i = T_i r_i$ and $B_i = T_{pp} r_i$ with $T_{pp} = 1$, Eq. (2) can be rewritten as follows:

$$C_i(T_i, m_i) = \frac{T_i r_i^2}{2m_i}\left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1)\left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] h_i + (s_i + m_i tc_i)\frac{1}{T_i} \tag{3}$$

The first-order condition of Eq. (3) with respect to T_i can be calculated as:

$$\frac{dC_i(T_i, m_i)}{dT_i} = \frac{r_i^2}{2m_i}\left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1)\left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] h_i - (s_i + m_i tc_i)\frac{1}{T_i^2} = 0 \tag{4}$$

Solving Eq. (4) for T_i yields

$$T_{i,opt}(m_i) = \sqrt{\frac{2m_i(s_i + m_i tc_i)}{r_i^2\left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1)\left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] h_i}} \tag{5}$$

The first-order condition of Eq. (3) with respect to m_i can be formulated as follows:

$$\frac{dC_i(T_i, m_i)}{dm_i} = \frac{T_i r_i^2}{2m_i}\left(\frac{1}{r_i} - \frac{1}{p_i}\right) h_i - \frac{T_i r_i^2}{2m_i^2}\left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1)\left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] h_i + \frac{tc_i}{T_i} = 0 \tag{6}$$

Solving Eq. (6) for m_i yields

$$m_{i,opt}(T_i) = T_i r_i \sqrt{\frac{h_i}{tc_i p_i}} \tag{7}$$

Inserting Eq. (5) into Eq. (7) leads to a closed-form expression for m_i that can be used to calculate an optimal real-valued solution for all m_i:

$$m_{i,opt}\left(T_{i,opt}\right) = \sqrt{\frac{2s_i r_i}{tc_i(p_i - r_i)}} \tag{8}$$

Since the solution for $m_{i,opt}$ obtained using Eq. (8) has to be a natural number > 0, the following procedure can be used for rounding $m_{i,opt}$:

1. If $m_{i,opt} < 1$, set $m_i = 1$,
2. else if $m_{i,opt} \in \mathbb{N}$, set $m_i = m_{i,opt}$,
3. else calculate $\lfloor m_{i,opt} \rfloor$ and $\lceil m_{i,opt} \rceil$. Using these two values, compute $T_{i,opt,floor}\left(\lfloor m_{i,opt} \rfloor\right)$ and $T_{i,opt,ceiling}\left(\lceil m_{i,opt} \rceil\right)$ using Eq. (5). Using these two values,

we can now calculate the optimal combination of T_i- and m_i-values for all N products using Eq. (3) as follows:

(1) If $C_i\left(T_{i,opt,floor}, \lfloor m_{i,opt} \rfloor\right) < C_i\left(T_{i,opt,ceiling}, \lceil m_{i,opt} \rceil\right)$, set $T_i = T_{i,opt,floor}$ and $m_i = \lfloor m_{i,opt} \rfloor$,

(2) else set $T_i = T_{i,opt,ceiling}$ and $m_i = \lceil m_{i,opt} \rceil$.

The lower bound of the ELSP, $C_{lower\ bound}$, which is the cost of the modified independent solution, can now be calculated by summing up $C_i(T_i, m_i)$ given in Eq. (3) over all N.

4.2.2 Modified common-cycle-approach of Hanssmann (MCCA)

One of the earliest and easiest approaches for solving the ELSP is the CCA of Hanssmann. As pointed out in the previous section, the CCA has often been used as an upper bound for the ELSP in the literature. The total cost per unit of time for the case of equal-sized batch shipments can be written as follows:

$$C = \sum_{i=1}^{N} C_i(T_i, m_i) \tag{9}$$

with the individual cost function of item i per unit of time, $C_i(T_i, m_i)$, as in Eq. (3).

Each product i is produced exactly once in each cycle, such that $T_i = T, \forall i$. The objective function (9) can now be written as follows:

$$\begin{aligned} C &= \sum_{i=1}^{N} \left[\frac{T r_i^2}{2m_i} \left(\left(\frac{1}{r_i} + \frac{1}{p_i} \right) + (m_i - 1) \left(\frac{1}{r_i} - \frac{1}{p_i} \right) \right) h_i + (s_i + m_i tc_i) \frac{1}{T} \right] \\ &= T \sum_{i=1}^{N} \left[\frac{r_i^2}{2m_i} \left(\left(\frac{1}{r_i} + \frac{1}{p_i} \right) + (m_i - 1) \left(\frac{1}{r_i} - \frac{1}{p_i} \right) \right) h_i \right] \\ &\quad + \frac{1}{T} \sum_{i=1}^{N} (s_i + m_i tc_i) \end{aligned} \tag{10}$$

The first-order condition of Eq. (10) with respect to T can be written as follows:

$$\frac{dC}{dT} = \sum_{i=1}^{N} \left[\frac{r_i^2}{2m_i} \left(\left(\frac{1}{r_i} + \frac{1}{p_i} \right) + (m_i - 1) \left(\frac{1}{r_i} - \frac{1}{p_i} \right) \right) h_i \right] - \frac{1}{T^2} \sum_{i=1}^{N} (s_i + m_i tc_i) = 0 \tag{11}$$

Solving Eq. (11) for T yields

$$T_{opt}(m_i) = \sqrt{\frac{2\sum_{i=1}^{N}(s_i + m_i tc_i)}{\sum_{i=1}^{N} \left[\frac{r_i^2}{m_i} \left(\left(\frac{1}{r_i} + \frac{1}{p_i} \right) + (m_i - 1) \left(\frac{1}{r_i} - \frac{1}{p_i} \right) \right) h_i \right]}} \tag{12}$$

As can be easily shown, the optimal shipment frequencies in the CCA are the same as those given in Eq. (7). Inserting $m_{i,opt}$ from Eq. (7) with $T_i = T, \forall i$ into Eq. (12) leads to:

$$T_{opt}(m_{i,opt}) = \sqrt{\frac{2\sum_{i=1}^{N} s_i}{\sum_{i=1}^{N} r_i h_i \left(1 - \frac{r_i}{p_i}\right)}} \tag{13}$$

After calculating the total cycle time using Eq. (13), the solution has to be tested for feasibility. To ensure that the production plan is feasible, the gross utilization of the facility has to be smaller than or equal to the total cycle time (see Elmaghraby, 1978):

$$T_{opt} \geq \sum_{i=1}^{N} st_i + \sum_{i=1}^{N} \frac{r_i}{p_i} T_{opt} \tag{14}$$

Treating (14) as an equation and rearranging the resulting expression, the minimum total cycle time can be calculated as

$$T_{min} = \frac{\sum_{i=1}^{N} st_i}{1 - \sum_{i=1}^{N} \frac{r_i}{p_i}} \tag{15}$$

The feasible solution for the total cycle time in the MCCA can now be determined as follows:

$$T_{total\ cycle\ time} = max\{T_{opt}, T_{min}\} \tag{16}$$

Using this result, the optimal shipment frequency for product i can be calculated as

$$m_{i,opt}(T_{total\ cycle\ time}) = T_{total\ cycle\ time} r_i \sqrt{\frac{h_i}{p_i tc_i}} \tag{17}$$

Since the value obtained with Eq. (17) has to be a natural number > 0, the procedure proposed in Section 4.2.1 can be used for rounding m_i to a natural number, keeping in mind that in the MCCA, $T_i = T, \forall i$.

To calculate an optimal solution for the MCCA, it might be necessary to repeat the above procedure and to update $T_{total\ cycle\ time}$ with Eqs. (12), (15) and (16) and m_i for all N products until a stable solution has been found. As we are searching only for an upper bound, we run the above procedure only once without any repetitions. After solutions for T and all m_i have been found, an upper bound to the ELSP can be calculated using Eq. (10).

4.2.3 Modified heuristic of Haessler and Hogue (MHH)

This section modifies the heuristic of Haessler and Hogue (1976). As was already explained in the literature review, the heuristic of Haessler and Hogue is an extension of Doll and Whybark's (1973) heuristic. As the original procedure of Haessler and Hogue, the solution procedure proposed here will also be divided into four steps:

Step 1:

To initialize the procedure, we calculate the modified independent solution (see Section 4.2.1) to determine the shipment frequencies for all products as well as the preliminary fundamental cycle time FC:

$$FC = min\{T_1, \dots, T_N\} \tag{18}$$

The preliminary fundamental cycle time will be updated in later steps of the heuristic.

Step 2:

Secondly, the individual cost function (3) of item i is written as a function of FC using the relation $T_i = k_i FC$:

$$C_i(k_i, FC) = \frac{k_i FC r_i^2}{2m_i}\left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1)\left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] h_i + (s_i + m_i tc_i)\frac{1}{k_i FC} \tag{19}$$

The values of the k-vector with $k = (k_1, k_2, \dots, k_N)$ are limited to powers of two, i.e. $k_i \in \{1,2,4,8,\dots\}, \forall i$. To determine a preliminary k-vector, an auxiliary variable u_i is calculated for all N products by dividing the individual optimal cycle length calculated in Step 1 through the preliminary fundamental cycle length also determined in Step 1:

$$u_i = \frac{T_{i,opt}}{FC} \tag{20}$$

Once the FC has been updated in a later step of the procedure, u_i is recalculated using the updated FC-value.

The following three cases may occur for all products:

1. If $u_i < 1$, set $k_i = 1$,
2. else if $u_i \in \{1,2,4,8,\dots\}$, set $k_i = u_i$,
3. else determine the next lower k_i^- and the next higher k_i^+ integer, such that $k_i^- < u_i < k_i^+$ and $k_i^-, k_i^+ \in \{1,2,4,8,\dots\}$. Using these values, we can now calculate the optimal k_i-values for all N products using Eq. (19) as follows:

   (1) If $C_i(k_i^-, FC) < C_i(k_i^+, FC)$, set $k_i = k_i^-$,

   (2) else if $C_i(k_i^+, FC) \leq C_i(k_i^-, FC)$, set $k_i = k_i^+$.

Using the new k-vector, we compute a new FC-value. For this purpose, we need the total average cost function per unit of time, which can be obtained by summing up Eq. (19) over all N products, i.e.

$$C(FC) = \sum_{i=1}^{N} C_i(k_i, FC) \tag{21}$$

The first-order condition of $C(FC)$ with respect to FC equals:

$$\frac{dC(FC)}{dFC} = \sum_{i=1}^{N} \frac{k_i r_i^2}{2m_i}\left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1)\left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] h_i - \frac{1}{FC^2}\sum_{i=1}^{N}(s_i + m_i tc_i)\frac{1}{k_i} = 0 \tag{22}$$

Solving Eq. (22) for FC yields

$$FC_{opt} = \sqrt{\frac{\sum_{i=1}^{N}(s_i + m_i tc_i)\frac{1}{k_i}}{\sum_{i=1}^{N}\frac{k_i r_i^2}{2m_i}\left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1)\left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] h_i}} \tag{23}$$

A necessary condition for feasibility was proposed by Eilon (1962, chap. 14) and Haessler (1971) to ensure that the fundamental cycle time calculated in Eq. (23) is long enough to permit the required setups:

$$FC_{check} \geq \frac{\sum_{i=1}^{N} \frac{st_i}{k_i}}{1 - \sum_{i=1}^{N} \frac{r_i}{p_i}} \tag{24}$$

Treating (24) as an equation, the minimum FC-value can be calculated as

$$FC_{min} = \frac{\sum_{i=1}^{N} \frac{st_i}{k_i}}{1 - \sum_{i=1}^{N} \frac{r_i}{p_i}} \tag{25}$$

The value for the fundamental cycle is updated in case FC_{min} exceeds FC_{opt} from (23):

$$FC = max\{FC_{opt}, FC_{min}\} \tag{26}$$

The new FC-value from (26) is used to calculate new values for u_i using Eq. (20) and to update all k_i-values. Afterward, a new FC-value has to be determined according to the procedure described above. The procedure terminates if all k_i-values are identical for two consecutive calculations. The outcome of Step 2 is a k-vector with the maximum permissible integer multipliers for each product i and a value for the fundamental cycle that is used as the starting value for the scheduling procedure in Step 3 of this heuristic (we refer to the FC-value obtained in this step as FC_{start} in the following). The final k_i-values for each product i are limited to the interval between 1 and the maximum k_i-values determined in Step 2, and they are calculated in the subsequent steps of this procedure.

Step 3:

The production plan generated by the MHH consists of w fundamental cycles (an example is presented in Figure 5); the production plan is repeated once all w fundamental cycles have been completed. In Step 3 of the MHH, we assign all N products to the different fundamental cycles of the production plan. The number of fundamental cycles within the total cycle, w, depends on the realized k-vector, and it can be calculated as follows:

$$w = max\{k_1, \ldots, k_N\} \tag{27}$$

In the total production cycle, which consists of w fundamental cycles, product i is produced z_i times:

$$z_i = \frac{w}{k_i} \text{ for all } i = 1, \dots, N \tag{28}$$

In the following, we use a binary variable to indicate whether product i is produced in a particular fundamental cycle:

$$Y_{io} = \begin{cases} 1, & \text{if product } i \text{ is produced in the oth fundamental cycle} \\ 0, & \text{else} \end{cases}$$

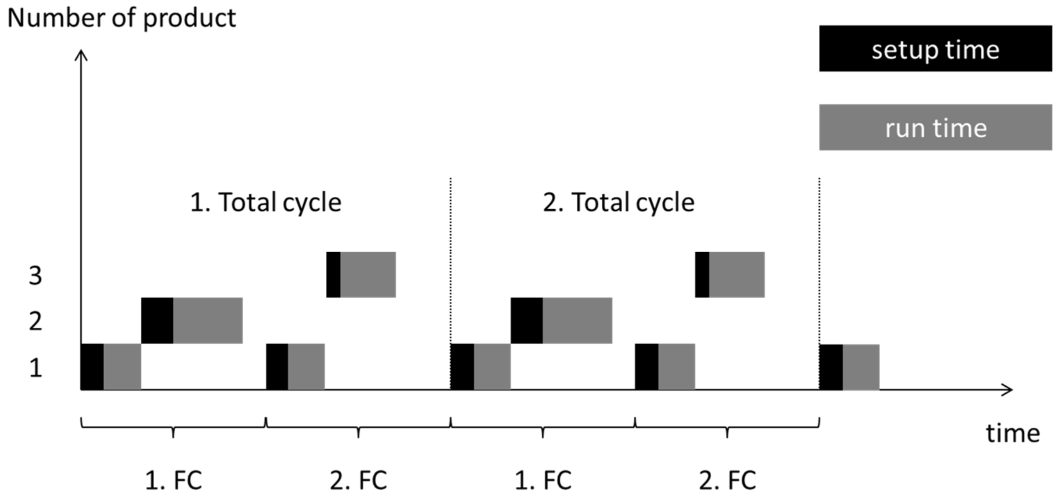

Figure 5: *Production plan for three products and two fundamental cycles*

We now try to find a feasible production schedule by evaluating different k-vectors between the unit vector and the vector of maximum permissible k_i-values for each product i determined in Step 2. All possible k-vectors between the unit vector and the vector of maximum permissible k_i-values for each product i with $k_i \in \{1,2,4,8,\dots\}$ are referred to as the set of candidate solutions in the following. For each vector in the set of candidate solutions, we have to

- determine how the N products should be assigned to the w fundamental cycles;
- verify whether a feasible solution can be found for the respective k-vector;
- update the length of the fundamental cycle FC if necessary.

To ensure that a production plan is feasible, the following three conditions, which were already presented by Haessler and Hogue (1976), have to be satisfied:

(I) $\sum_{o=1}^{w} Y_{io} = z_i$ for all $i = 1, \dots, N$.

(II) $Y_{io} = Y_{io+k_i}$ for all $i = 1, \dots, N$ with $k_i \neq w$ as well as $o = 1, \dots, w - k_i$.

(III) $\sum_{i=1}^{N} Y_{io} \left(st_i + \frac{r_i k_i FC}{p_i} \right) \leq FC$ for all $o = 1, \dots, w$.

Condition (I) guarantees that the number of setups of all products within the total cycle is correct. Constraint (II) ensures that all products are classified correctly with respect to the fundamental cycles based on their multipliers. Condition (III) finally makes sure that the length of the fundamental cycle is sufficient for permitting both setting up and producing all products assigned to the oth fundamental cycle with $o = 1, \dots, w$.

In the following, we propose two different solution procedures for finding a feasible production schedule. The costs obtained with Procedure I are always lower than or equal to those derived with Procedure II, but the runtime of Procedure II is shorter.

Procedure I: This procedure modifies and concretizes the algorithm of Haessler and Hogue (1976). In their 1976 paper, Haessler and Hogue recommended in the third step of their procedure somewhat vaguely to (p. 910) "systematically increase […] the fundamental cycle time until a feasible solution is found" in case the solution should be infeasible. We concretize this procedure as described in the following.

First, we calculate an upper bound on the average costs per unit of time using the MCCA (see Section 4.2.2) with $k_i = 1, \forall i$ and the m-vector with $m = (m_1, m_2, \dots, m_N)$ calculated in Step 1. After checking the length of the total cycle T using conditions (14)-(16), a first upper bound can be calculated using Eq. (10).

To assign the products to the different fundamental cycles, the total production times, which equal the sum of setup and run times, have to be calculated for all N products:

$$pt_i = st_i + \frac{k_i FC r_i}{p_i} \tag{29}$$

For a given k-vector, the products are now sorted according to their k_i-values in ascending order. If the k_i-values of two or more products are equal, the products are sorted in descending order of their production time, and the product with the longest total production time is selected first. Products are assigned to the fundamental cycles as follows: Starting from $o = 1$ and choosing the first fundamental cycle that still has enough time for producing product i, the product is assigned to this cycle with the earliest possible starting time. In case none of the fundamental cycles has enough free time available for producing product i, the product is assigned to the fundamental cycle with the longest free time left or to the fundamental cycle that violates the cycle length the least. After all N products have been assigned to the fundamental cycles, the following calculation, which is a rearrangement of condition (III) formulated above, has to be made:

$$FC_{test} = \left\{ \frac{\sum_{i=1}^{N} Y_{io} st_i}{1 - \sum_{i=1}^{N} \frac{Y_{io} r_i k_i}{p_i}} | o = 1, \dots, w \right\} \tag{30}$$

It is now necessary to differentiate between two cases:

(1) If $min(FC_{test}) \geq 0$ and $max(FC_{test}) \leq FC_{start}$, a feasible solution has been found and the corresponding costs can be calculated using Eq. (21),

(2) else modify the fundamental cycle time.

If a modification of the fundamental cycle time is necessary, we proceed as follows: Since the cost function (21) is convex in FC for given vectors m and k (see Appendix B), it is possible to calculate a maximum fundamental cycle time FC_{max} for every k-vector using the current upper bound C_{ub} (see Figure 6). Solving $C(FC_{max})$ defined above for FC_{max}, we get:

$$FC_{max} = \frac{C_{ub} + \sqrt{(C_{ub})^2 - 4(\sum_{i=1}^{N} \frac{k_i r_i^2}{2m_i} \left[\left(\frac{1}{r_i} + \frac{1}{p_i} \right) + (m_i - 1) \left(\frac{1}{r_i} - \frac{1}{p_i} \right) \right] h_i)(\sum_{i=1}^{N} (s_i + m_i tc_i) \frac{1}{k_i})}}{2(\sum_{i=1}^{N} \frac{k_i r_i^2}{2m_i} \left[\left(\frac{1}{r_i} + \frac{1}{p_i} \right) + (m_i - 1) \left(\frac{1}{r_i} - \frac{1}{p_i} \right) \right] h_i)} \tag{31}$$

For a given k-vector, all parameters in Eq. (31) are fixed values except the length of the fundamental cycle FC, and also the value of the current upper bound, C_{ub}, is known.

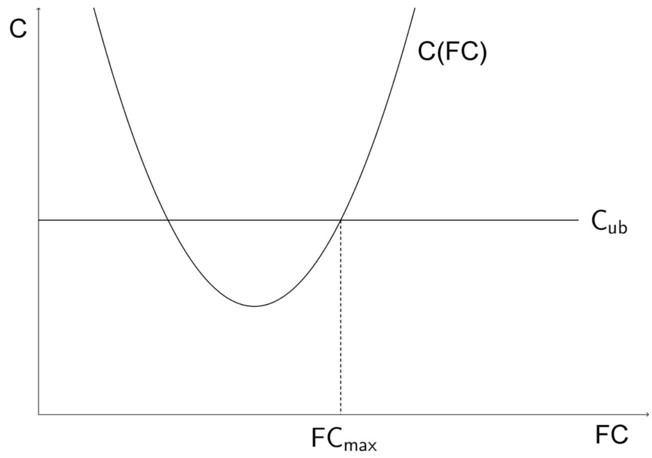

Figure 6: *Graphical illustration of the calculation of* FC_{max}

Comparing FC_{max} to FC_{start} calculated in Step 2, two cases may arise:

(1) If $FC_{max} > FC_{start}$, it might be possible to find a feasible and better solution than C_{ub},

(2) else if $FC_{max} \leq FC_{start}$ or $FC_{max} \in \mathbb{C}$ with $Im\ (FC_{max}) \neq 0$, no better solution can be found for the given k-vector.

In case (1), we introduce two possible step ranges for increasing the length of the fundamental cycle:

1. For the given k-vector, we define a maximum number of alternative FC-values that can be evaluated, N_{sr}, which leads to the following step range:

$$l_{sr} = \frac{FC_{max} - FC_{start}}{N_{sr}}$$

2. We determine a fixed step range for the given k-vector, e.g. $l_{sr} = 0.01$.

Procedure I described here has to be repeated for all k-vectors in the set of candidate solutions. It can be summarized as follows:

I.1 Set $k_i = 1, \forall i$ and calculate C_{ub}.

I.2 Take one k-vector from the set of candidate solutions and determine FC_{max} according to Eq. (31).

I.3 If $FC_{max} > FC_{start}$, set $FC = FC_{start}$ and assign the products to the fundamental cycles and go to Step I.4, else consider a k-vector from the set of candidate solutions that has not yet been evaluated and go back to Step I.2.

I.4 If a feasible solution with costs C_{new} has been found and $C_{new} \geq C_{ub}$, go back to Step I.2, else if a feasible solution with costs C_{new} has been found and $C_{new} < C_{ub}$, set $C_{ub} = C_{new}$ and go back to Step I.2, else if $FC + l_{sr} \leq FC_{max}$, set $FC = FC + l_{sr}$ and assign the products again to the fundamental cycles, else consider another k-vector from the set of candidate solutions that has not yet been evaluated and go back to Step I.2.

Select the solution with the lowest costs and go to Step 4.

Procedure II: Procedure II proposes an easy and fast way to find a feasible solution. Instead of a systematic increase of the length of the fundamental cycle FC combined with a new assignment for each change in FC, we assign the products only once for every k-vector from the set of candidate solutions.

As in Procedure I, the total production times, pt_i, of all products, the number of fundamental cycles within the total cycle, w, the number of production cycles for every product within the total cycle, z_i, the binary variable, Y_{io}, and the three conditions for feasibility of Haessler and Hogue (1976) are needed. In addition, the following calculation has to be made after all products have been assigned:

$$FC_{test} = \left\{ \frac{\sum_{i=1}^{N} Y_{io} st_i}{1 - \sum_{i=1}^{N} \frac{Y_{io} r_i k_i}{p_i}} | o = 1, \dots, w \right\} \tag{30}$$

Procedure II can be summarized as follows:

II.1 Starting with FC_{start}, as long as the set of candidate solutions is not empty, select a k-vector from the set of candidate solutions and assign all products to the fundamental cycles using the algorithm described in Procedure I and go to Step II.2, else stop.

II.2 If $min(FC_{test}) < 0$, no feasible solution can be found for the current k-vector and go back to Step II.1, else go to Step II.3.

II.3 If $max(FC_{test}) \leq FC_{start}$, a feasible solution is found for $FC = FC_{start}$ and the corresponding costs can be calculated using Eq. (21) and go back to Step II.1, else go to Step II.4.

II.4 Set $FC = max(FC_{test})$ and calculate the corresponding costs using Eq. (21) and go back to Step II.1.

Select the solution with the lowest costs and go to Step 4.

Step 4:

This step tries to improve the number of batch shipments of all N products calculated in Step 1 in order to decrease the total average costs per unit of time. First, the length of the basic period FC and the N individual integer multiples k_i calculated in Step 3 are used to compute the lot sizes for all N products. Varying the number of batch shipments for fixed values of FC and all k_i is possible because the variation does not influence the production schedule gained in Step 3, and it does not violate the modified zero switch rule; it may, however, reduce the total cost of the solution. The lot sizes for all N products can be calculated as follows:

$$q_i = k_i FC r_i \tag{32}$$

Using the relation $q_i = T_i r_i$, Eq. (7) can be written as

$$m_{i,new} = q_i \sqrt{\frac{h_i}{tc_i p_i}} \tag{33}$$

Since $m_{i,new}$ obtained with Eq. (33) has to be a natural number > 0, $m_{i,new}$ has to be rounded to a natural number using the following procedure:

1. If $m_{i,new} < 1$, set $m_i = 1$,
2. else if $m_{i,new} \in \mathbb{N}$, set $m_i = m_{i,new}$,
3. else calculate $\lfloor m_{i,new} \rfloor$ and $\lceil m_{i,new} \rceil$. Using these two values, we can now determine the final m_i-values for all N products using Eq. (19):

   (1) If $C_i(\lfloor m_{i,new} \rfloor) < C_i(\lceil m_{i,new} \rceil)$, set $m_i = \lfloor m_{i,new} \rfloor$,

   (2) else set $m_i = \lceil m_{i,new} \rceil$.

4.3 Unequal-sized batch shipments

4.3.1 Modified independent solution (MIS)

We next study the case where unequal-sized batch shipments are used for transporting the finished product from the producing to the consuming stage, with subsequent batches increasing in size according to a geometric series. In this case, we can again use the modified independent solution as a lower bound to the problem. The costs per unit of time of product i can be calculated according to Goyal (1977), who studied a two-stage inventory model with unequal-sized batch shipments and a single product. The total costs per unit of time can thus be calculated by summing up inventory carrying, setup and transportation costs:

$$C_i(q_i, m_i) = \frac{q_i}{2}\left(\frac{1}{r_i} + \frac{1}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2} h_i B_i + (s_i + m_i tc_i)\frac{B_i}{q_i} \tag{34}$$

where

$$A(m_i) = \sum_{j=1}^{m_i}\left(\frac{p_i}{r_i}\right)^{j-1} = \frac{\left(\frac{p_i}{r_i}\right)^{m_i} - 1}{\left(\frac{p_i}{r_i}\right) - 1} \tag{35}$$

and

$$D(m_i) = \frac{\left(\frac{p_i}{r_i}\right)^{2m_i} - 1}{\left(\frac{p_i}{r_i}\right)^2 - 1} \tag{36}$$

Using the relations $q_i = T_i r_i$ and $B_i = T_{pp} r_i$ with $T_{pp} = 1$, expression (34) can be written as follows:

$$C_i(T_i, m_i) = \frac{T_i r_i}{2}\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2} h_i + (s_i + m_i tc_i)\frac{1}{T_i} \tag{37}$$

The first-order condition of Eq. (37) with respect to T_i can be calculated as follows:

$$\frac{dC_i(T_i, m_i)}{dT_i} = \frac{r_i}{2}\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2} h_i - (s_i + m_i tc_i)\frac{1}{T_i^2} = 0 \tag{38}$$

Solving Eq. (38) for T_i yields

$$T_{i,opt}(m_i) = \sqrt{\frac{2(s_i + m_i tc_i)}{r_i\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2}h_i}} \tag{39}$$

Inserting Eq. (39) into Eq. (37) leads to the following cost function

$$C_i\left(T_{i,opt}, m_i\right) = \frac{1}{A(m_i)}\sqrt{2(s_i + m_i tc_i) r_i \left(1 + \frac{r_i}{p_i}\right) D(m_i) h_i} \tag{40}$$

Bogaschewsky et al. (2001) showed that (34) is convex in m_i. Therefore, the number of batch shipments for product i can be calculated by successively increasing m_i starting with $m_i = 1$ until the cost function (40) increases. The following solution procedure has to be repeated for all N products:

Step 1: Set $m_i = 1$ and $C_i^* = +\infty$.

Step 2: Calculate $C_i\left(T_{i,opt}, m_i\right)$ according to (40).

Step 3: If $C_i\left(T_{i,opt}, m_i\right) < C_i^*$, set $C_i^* = C_i\left(T_{i,opt}, m_i\right)$, $m_i = m_i + 1$ and go to Step 2, else set $m_i = m_i - 1$ and determine $T_{i,opt}(m_i)$ from (39).

The lower bound of the ELSP, $C_{lower\ bound}$, which is the cost of the MIS, can now be calculated by summing up $C_i\left(T_{i,opt}(m_i), m_i\right)$ given in Eq. (37) over all N products.

4.3.2 Modified common-cycle-approach of Hanssmann (MCCA)

The total average costs per unit of time for the MCCA can be determined by summing up the average costs per unit of time given in Eq. (37), $C_i(T_i, m_i)$, over all N products, i.e.

$$C = \sum_{i=1}^{N} C_i(T_i, m_i) \tag{41}$$

Applying the same procedure to (41) as in Section 4.2.2, the optimal total cycle time can be calculated as follows:

$$T_{opt} = \sqrt{\frac{2\sum_{i=1}^{N}(s_i + m_i tc_i)}{\sum_{i=1}^{N} r_i\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2}h_i}} \tag{42}$$

We now adopt the m_i-values for all N products calculated in the MIS in Section 4.3.1, which enables us to calculate T_{opt} (cf. also the following procedure). After the test for feasibility as in Section 4.2.2, the total cycle time can also be determined by Eq. (16).

To reduce the total cost per unit of time, we again use the enumeration scheme as in Section 4.3.1 for determining the optimal number of batch shipments. The following solution procedure has to be repeated for all N products:

Step 1: Set $m_i = 1$ and $C_i^* = +\infty$.

Step 2: Calculate $C_i(T_{total\ cycle\ time}, m_i)$ according to Eq. (37).

Step 3: If $C_i(T_{total\ cycle\ time}, m_i) < C_i^*$, set $C_i^* = C_i(T_{total\ cycle\ time}, m_i)$, $m_i = m_i + 1$ and go to Step 2, else set $m_i = m_i - 1$.

To calculate an optimal solution for the MCCA, it might be necessary to repeat the above procedure and to update $T_{total\ cycle\ time}$ with Eq. (16) and m_i for all N products until a stable solution has been found. As we are searching only for an upper bound, we run the above procedure only once without any repetitions. After solutions for the total cycle time T and all m_i have been found, an upper bound to the ELSP can be calculated by using Eq. (41) and setting $T_i = T, \forall i$.

4.3.3 Modified heuristic of Haessler and Hogue (MHH)

The procedure proposed in this section is similar to the one described in Section 4.2.3 and is also divided into four steps:

Step 1:

In the first step, we calculate the modified independent solution (see Section 4.3.1) to determine the shipment frequencies for all products as well as the preliminary fundamental cycle length using Eq. (18).

Step 2:

Step 2 proposed here is almost identical to Step 2 described in Section 4.2.3. For this reason, we introduce only the equations that have changed as a result of the different batch shipment policy employed. Instead of Eq. (19), the following expression is used:

$$C_i(k_i, FC) = \frac{k_i FC r_i}{2}\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2}h_i + (s_i + m_i tc_i)\frac{1}{k_i FC} \quad (43)$$

Eq. (23) has to be replaced by

$$FC_{opt} = \sqrt{\frac{\sum_{i=1}^{N}(s_i + m_i tc_i)\frac{1}{k_i}}{\sum_{i=1}^{N}\frac{k_i r_i}{2}\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2}h_i}} \quad (44)$$

Step 3:

The solution procedures used in Step 3 are also quite similar to those presented in Section 4.2.3. Changes that are necessary due to the different batch shipment policy employed will be outlined in the following.

Procedure I: Eq. (21) is replaced by

$$C = FC\sum_{i=1}^{N}\frac{k_i r_i}{2}\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2}h_i + \frac{1}{FC}\sum_{i=1}^{N}(s_i + m_i tc_i)\frac{1}{k_i} \quad (45)$$

Also the maximum fundamental cycle length has to be calculated differently:

$$FC_{max} = \frac{C_{ub} + \sqrt{(C_{ub})^2 - 4\left(\sum_{i=1}^{N}\frac{k_i r_i}{2}\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2}h_i\right)\left(\sum_{i=1}^{N}(s_i + m_i tc_i)\frac{1}{k_i}\right)}}{2\left(\sum_{i=1}^{N}\frac{k_i r_i}{2}\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2}h_i\right)} \quad (46)$$

Procedure II: Procedure II uses the same cost function required than in Procedure I, Eq. (45). Apart from this, Procedure II is identical to the one proposed in Section 4.2.3.

Step 4:

As in Section 4.2.3, this step tries to improve the number of batch shipments of all N products calculated in Step 1 in order to reduce the total average costs per unit of time. The lot sizes for all N products can be calculated as follows:

$$q_i = k_i FC r_i \quad (32)$$

Using the same relation $q_i = T_i r_i$ as in Section 4.2.3, Eq. (37) can be written as

$$C_i(m_i) = \frac{q_i}{2}\left(1 + \frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2} h_i + (s_i + m_i tc_i)\frac{r_i}{q_i} \tag{47}$$

As Eq. (47) is convex in m_i (see Bogaschewsky et al., 2001), the number of batches can be derived by successively increasing m_i from $m_i = 1$ until the cost function (47) increases for the first time. The following solution procedure has to be applied to all N products:

Step 1: Set $m_i = 1$ and $C_i^* = +\infty$.

Step 2: Calculate $C_i(m_i)$ according to Eq. (47).

Step 3: If $C_i(m_i) < C_i^*$, set $C_i^* = C_i(m_i)$, $m_i = m_i + 1$ and go to Step 2, else set $m_i = m_i - 1$.

5 Numerical examples

To illustrate and compare the developed ELSP models with batch shipments and their solution procedures, the modified Bomberger (1966) problem, which has often been used as a standard data set for the ELSP, is considered and extended to include transportation cost rates tc_i for all products (see Table 1). In addition, we consider the case where only complete lots are shipped to the next stage. For the case where only complete lots are shipped, the solution procedure proposed in Section 4.2.3 with $m_i = 1, \forall i$ can be used. Mathematica 8.1 was used for calculating results, which are summarized in Tables 2-5. In the following, we present the results of the four steps of the solution procedures developed above for illustration purposes.

Table 2 contains results for the independent solution (IS) and the upper bounds for the modified Bomberger problem. Obviously, the lower and the upper bound of the unequal-sized batch shipment policy dominate the results of the equal-sized batch shipment policy and the policy where only complete lots are shipped. As can be seen, a tendency could be observed that for products 5, 8 and 9, more batch shipments per lot are used than for the other products, which use only a single batch. One possible explanation is that the products' inventory holding costs are relatively high and the transportation costs per delivery are relatively low as compared to the other products, which induced this result.

Table 1: *Data for the modified Bomberger problem*

Product	**Production rate (items/day)**	**Demand rate (items/day)**	**Holding cost (dollar/day)**	**Setup cost (dollar/day)**	**Transportation cost (dollar/ shipment)**	**Setup time (days)**
i	p_i	r_i	$10^{-4}h_i$	s_i	tc_i	st_i
1	30,000	400	0.02708	15	1	0.25
2	8000	400	0.73958	20	6	0.25
3	9500	800	0.53125	30	4	0.5
4	7500	1600	0.41667	10	10	0.25
5	2000	80	11.60417	110	2	1
6	6000	80	1.11458	50	5	0.5
7	2400	24	6.25	310	8	2
8	1300	340	24.58333	130	4	1
9	2000	340	3.75	200	6	1.5
10	15,000	400	0.16667	5	7	0.25

Table 2: *Independent solutions and upper bounds for the modified Bomberger problem*

	Classical ELSP (without batch shipments)	**Equal-sized batch shipments**	**Unequal-sized batch shipments**	**Complete lot**
Independent solution				
FC	19.53	20.63	21.07	15.94
m	-	(1,1,1,1,2, 1,1,5,4,1)	(1,1,2,1,2, 1,1,3,3,1)	(1,1,1,1,1, 1,1,1,1,1)
Lower bound of the costs/day	31.62	35.90	34.32	38.83
Common-Cycle Approach of Hanssmann				
FC	63.78	63.78	63.78	63.78
m	-	(1,1,2,2,3, 1,1,14,4,1)	(1,1,2,2,2, 1,1,5,3,1)	(1,1,1,1,1, 1,1,1,1,1)
Upper bound of the costs/day	44.50	48.83	46.42	62.16

***Table 3:** Determining FC and the k-vector for the modified Bomberger problem (Step 2)*

	Classical ESLP (without batch shipments)	**Equal-sized batch shipments**	**Unequal-sized batch shipments**	**Complete lot**
FC	33.49	32.42	30.03	32.42
k	(4,1,1,1,2,4,8,1,2,1)	(4,1,1,1,2,4,8,1,2,2)	(8,1,2,1,2,4,8,1,2,2)	(4,1,1,1,2,4,8,1,2,2)

The results for the MHH of Step 2 can be seen in Table 3. The most interesting point is that the k-vector of the maximum permissible integer multipliers for each product i is equal for the complete lot and the equal-sized batch shipment policy; for the classical ELSP and the unequal-sized batch shipment policy, the k-vectors are different. As a consequence, it is possible that the following steps lead to worse results because a value, for example, of $k_1 = 8$ is only possible for the unequal-sized batch shipment policy. This means that this step might sometimes be too restrictive because values which are higher than the calculated k_i's are not possible in the following steps, even though they might lead to better results.

***Table 4:** Results for the modified Bomberger problem (Step 3)*

	Classical ESLP (without batch shipments)		**Equal-sized batch shipments**		**Unequal-sized batch shipments**		**Complete lot**	
	Procedure I	Procedure II	Procedure I	Procedure II	Procedure I	Procedure II	Procedure I	Procedure II
FC	47.03	47.03	49.55	47.03	37.49	37.49	47.03	47.03
k	(2,1,1,1, 2,4,4,1,1, 1)	(2,1,1,1, 2,4,4,1,1, 1)	(2,1,1,1, 1,4,4,1,1, 2)	(2,1,1,1, 2,4,4,1,1, 1)	(1,1,2,1, 2,4,4,1,2, 1)	(1,1,2,1, 2,4,4,1,2, 1)	(2,1,1,1, 2,4,4,1,1, 1)	(2,1,1,1, 2,4,4,1,1, 1)
Cost/ day	38.07	38.07	42.98	43.16	38.07	38.07	51.58	51.58

The difference in the lengths of the FCs (see Table 4) can be explained by the k-vector. Usually, the lower the values of the elements of the k-vector, the higher the length of FC, because more products have to be produced in the same FC. For the classical ELSP, the complete lot and the equal-sized batch shipment policies, the results are identical for Procedure II. Only the costs per day differ from each other because of the different inventory policies. For the modified Bomberger problem, it does not matter if Procedure I or II is used for calculating the results of the classical ELSP, the complete lot and the unequal-sized batch shipment policies. Only in case of the equal-sized batch shipment policy, the results differ from each other. Once again, the unequal-sized batch shipment policy dominates the results of the equal-sized batch shipment policy and the policy where only complete lots are shipped.

Table 5: *Results for the modified Bomberger problem (Step 4)*

	Equal-sized batch shipments		**Unequal-sized batch shipments**	
	Procedure I	Procedure II	Procedure I	Procedure II
m	(1,1,2,2,2,1,1,12,3,1)	(1,1,1,2,4,1,1,11,3,1)	(1,1,2,2,2,1,1,4,3,1)	(1,1,2,2,2,1,1,4,3,1)
Cost/ day	42.09	42.34	37.70	37.70

Table 5 shows that it is possible to improve the results obtained in Step 3 by recalculating the number of batches derived in Step 1. In all cases, improving the results obtained in Step 3 is possible. As was already shown above, products 5, 8 and 9 have the highest numbers of batches. In comparison to the solutions obtained for the MCCA (see Step 1), the results are significantly better, such that the modified heuristic of Haessler and Hogue proposed in this paper should be used instead of the MCCA in practice.

Another numerical example (the modified Eilon (1962, chap. 14) problem) can be found in Appendix A.

6 Conclusion

The paper at hand extended the classical Economic Lot Scheduling Problem to the case where either complete lots or batch shipments are transported from the producing to the consuming stage. Batch shipments are especially beneficial in cases where the distance between the producing and the consuming stages is too large to be ignored, and where transportation costs arise for each shipment. In such situations, the procedures developed in this paper can be used to calculate lot sizes and shipment quantities that help to reduce total costs as compared to the classical ELSP. Three different shipment policies were considered in this paper, namely the transfer of complete lots, the transfer of equal-sized batch shipments, and the transfer of unequal-sized batch shipments.

After developing mathematical models for the three shipment policies, solution procedures were proposed. For coordinating the production system, this paper extended the Basic-Period-Approach of Haessler and Hogue (1976) as well as Hanssmann's Common-Cycle-Approach. For the equal- and unequal-sized batch shipment policies, two alternative solution procedures for the BPA were presented. The results of the paper indicate that batch shipments lead to significant cost savings as compared to complete lots.

A comparison between equal- and unequal-sized batch shipments shows that the unequal-sized policy always leads to lower total costs than the policy that uses equal-sizes batches. In practice, it might, however, be difficult to implement a shipment policy with varying batch sizes, such that a practitioner might prefer equal-sized batches. To reduce implementation problems, future research could restrict batch sizes to integer multiples of a basic lot size (which could equal a container or box, for example), which would make it easier to standardize shipments between both stages. Future research could also try to improve the solution procedures presented in this work, for example by using more loops and/or combining the four steps suggested in this paper in a single step for planning the schedule, the lot sizes and the number of batch shipments simultaneously. Future research could also further investigate the influence of learning and forgetting effects on the ELSP, as was already done by Sule (1983). If learning and forgetting are considered, the setup times and/or the production times could increase or decrease over time, which leads to time-varying lot-sizes. This suggests that the Time-Varying-Lot-Sizes-Approach of Dobson (1987) could be used to solve this problem. Improving the solution by relaxing the power-of-two-assumption made in this paper could also be interesting, even though this would increase the solution complexity.

References

Bellman, R., 1957. Dynamic programming. Princeton, New Jersey: Princton University Press.

Bogaschewsky, R.W., Buscher, U.D., Lindner, G., 2001. Optimizing multi-stage production with constant lot size and varying number of unequal sized batches. Omega, 29(2), 183-191.

Bomberger, E., 1966. A dynamic programming approach to a lot size scheduling problem. Management Science, 12(11), 778-784.

Brander, P., Forsberg, R., 2005. Cyclic lot scheduling with sequence-dependent set-ups: a heuristic for disassembly processes. International Journal of Production Research, 43(2), 295-310.

Buscher, U., 2000. Lossplitting in der Lossequenzplanung (Lot splitting in the economic lot scheduling problem). WISU – Das Wirtschaftsstudium, 29(2), 197-204.

Carstensen, P., 1999. Das Economic Lot Scheduling Problem – Überblick und LP-basiertes Verfahren (The economic lot scheduling problem – survey and LP-based method). OR Spektrum, 21(4), 429-460.

Chan, H.K., Chung, S.H., Lim, M.K., 2013. Recent research trend of economic-lot scheduling problems. Journal of Manufacturing Technology Management, 24(3), 465-482.

Chatfield, D.C., 2007. The economic lot scheduling problem: a pure genetic search approach. Computers & Operations Research, 34(10), 2865-2881.

Delaporte, C.M., Thomas, L.J., 1977. Lot size and sequencing for n products on one facility. Management Science, 23(10), 1070-1079.

Dobson, G., 1987. The economic lot-scheduling problem: achieving feasibility using time-varying lot sizes. Operations Research, 35(5), 764-771.

Doll, C.L., Whybark, D.C., 1973. An iterative procedure for the single-machine multi-product lot scheduling problem. Management Science, 20(1), 50-55.

Eilon, S., 1962. Elements of production planning and control. New York: MacMillan.

Elmaghraby, S.E., 1978. The economic lot scheduling problem (ELSP): review and extensions. Management Science, 24(6), 587-598.

Eynan, A., 2003. The benefits of flexible production rates in the economic lot scheduling problem. IIE Transactions, 35(11), 1057-1064.

Gallego, G., Shaw, D.X., 1997. Complexity of the ELSP with general cyclic schedules. IIE Transactions, 29(2), 109-113.

Geng, P., Vickson, R., 1988. Two heuristics for the economic lot scheduling problem: an experimental study. Naval Research Logistics (NRL), 35(4), 605-617.

Glock, C.H., 2012. The joint economic lot size: a review. International Journal of Production Economics, 135(2), 671-686.

Glock, C.H., Grosse, E.H., Ries, J.M., 2014. The lot sizing problem: a tertiary study. International Journal of Production Economics, 155, 39-51.

Goyal, S.K., 1977. Determination of optimum production quantity for a two-stage production system. Operation Research Quarterly, 28(4), 865-870.

Goyal, S.K., Nebebe, F., 2000. Determination of economic production-shipment policy for a single-vendor-single-buyer system. European Journal of Operational Research, 121(1), 175-178.

Haessler, R.W., 1971. A note on scheduling a multi-product single-machine system for an infinite planning period. Management Science, 18(4), 240-241.

Haessler, R.W., 1979. An improved extended basic period procedure for solving the economic lot scheduling problem. AIIE Transaction, 11(4), 336-340.

Haessler, R.W., Hogue, S., 1976. Note—a note on the single-machine multi-product lot scheduling problem. Management Science, 22(8), 909-912.

Hanssmann, F., 1962. Operations research in production and inventory control (1st ed.). John Wiley & Sons.

Hill, R.M., 1997. The single-vendor single-buyer integrated production-inventory model with generalised policy. European Journal of Operational Research, 97(3), 493-499.

Ho, W.-T., Lai, S.F., Huang, Y.K., 2014. An optimal mixed batch shipment policy for multiple items in a single-supplier multiple-retailer integrated system. Journal of Optimization Theory and Applications, 160(2), 636-658.

Ho, W.-T., Tseng, Y.-C., Hsiao, Y.-C., 2015. The economic lot-size scheduling problem with equally sized batch shipment policy and stochastic demands. European Journal of Industrial Engineering, 9(2), 195-221.

Hsu, W.L., 1983. On the general feasibility test of scheduling lot sizes for several products on one machine. Management Science, 29(1), 93-105.

Khouja, M., Michalewicz, Z., Wilmot, M., 1998. The use of genetic algorithm to solve the economic lot size scheduling problem. European Journal of Operational Research, 110(3), 509-524.

Kim, T., Glock, C.H., 2013. A multi-stage joint economic lot size model with lead time penalty costs. Computers & Industrial Engineering, 66(1), 133-146.

Maxwell, W.L., 1964. The scheduling of economic lot sizes. Naval Research Logistics Quarterly, 11(2), 89-124.

Rogers, J., 1958. A computational approach to the economic lot scheduling problem. Management Science, 4(3), 264-291.

Sule, D.R., 1983. Effect of learning and forgetting on economic lot size scheduling problem. International Journal of Production Research, 21(5), 771-786.

Sun, H., Huang, H.-C., Jaruphongsa, W., 2009. A genetic algorithm for the economic lot scheduling problem under the extended basic period and power-of-two policy. CIRP Journal of Manufacturing Science and Technology, 2(1), 29-34.

Sun, H., Jaruphongsa, W., Huang, H.-C., 2009. Genetic algorithms for the multiple-machine economic lot scheduling problem. International Journal of Advanced Manufacturing Technology, 43(11-12), 1251-1260.

Szendrovits, A.Z., 1975. Manufacturing cycle time determination for a multi-stage economic production quantity model. Management Science, 22(3), 298-308.

Appendix

Appendix A

Appendix A illustrates the solution procedures developed in this paper using the modified Eilon data sets for the ELSP. As for the Bomberger data set, we again added transportation costs tc_i for all products (see Table A.1). Appendix A is structured similarly than the numerical examples presented in Section 5. The results of the four steps of the solution procedures are summarized in Table A.2-A.5.

Table A.1: *Data for the modified Eilon problem*

Product	Production rate (items/day)	Demand rate (items/day)	Holding cost (dollar/day)	Setup cost (dollar/day)	Transportation cost (dollar/ shipment)	Setup time (days)
i	p_i	r_i	h_i	s_i	tc_i	st_i
1	133	20	0.00461	3000	10	4
2	200	24	0.00312	1800	12	2.4
3	266	30	0.00651	3600	6	4.8
4	146	36	0.0118	1500	4	2
5	532	40	0.0119	6000	2	4
6	373	50	0.00847	30,000	10	8

Table A.2: *Independent solutions and upper bounds for the modified Eilon problem (Step 1)*

	Classical ELSP (without batch shipments)	Equal-sized batch shipments	Unequal-sized batch shipments	Complete lot
Independent solution				
FC	96.82	96.90	97.37	75.37
m	-	(10,6,12,16,22,30)	(4,3,4,5,4,5)	(1,1,1,1,1,1)
Lower bound of the costs/day	324.41	329.88	325.28	370.37
Common-Cycle Approach of Hanssmann				
FC	252.09	252.09	252.49	218.26
m	-	(9,7,15,40,33,18)	(4,3,4,6,4,4)	(1,1,1,1,1,1)
Upper bound of the costs/day	364.15	369.62	364.93	421.00

Table A.3: *Determining FC and the k-vector for the modified Eilon problem (Step 2)*

	Classical ESLP (without batch shipments)	**Equal-sized batch shipments**	**Unequal-sized batch shipments**	**Complete lot**
FC	94.51	94.50	94.73	82.91
k	(4,2,2,1,2,4)	(4,2,2,1,2,4)	(4,2,2,1,2,4)	(4,2,2,1,2,4)

Table A.4: *Results for the modified Eilon problem (Step 3)*

	Classical ESLP (without batch shipments)		**Equal-sized batch shipments**		**Unequal-sized batch shipments**		**Complete lot**	
	Procedure I	Procedure II	Procedure I	Procedure II	Procedure I	Procedure II	Procedure I	Procedure II
FC	96.01	130.32	96.02	130.32	96.02	130.32	96.02	155.12
k	(4,2,2,1,2,4)	(4,2,2,1,1,4)	(4,2,2,1,2,4)	(4,2,2,1,1,4)	(4,2,2,1,2,4)	(4,2,2,1,1,4)	(4,2,2,1,2,4)	(1,2,1,1,1,2)
Cost/day	326.97	338.29	332.50	344.08	327.85	339.01	377.19	386.97

Table A.5: *Results for the modified Eilon problem (Step 4)*

	Equal-sized batch shipments		**Unequal-sized batch shipments**	
	Procedure I	Procedure II	Procedure I	Procedure II
m	(14,5,12,16,26,29)	(19,7,16,21,17,39)	(4,3,4,5,4,5)	(4,3,4,5,4,5)
Cost/day	332.44	343.75	327.85	339.01

Appendix B

The total cost function for the equal-sized batch shipment policy is given as follows:

$$C(FC) = FC \sum_{i=1}^{N} \frac{k_i r_i^2}{2m_i} \left[\left(\frac{1}{r_i} + \frac{1}{p_i} \right) + (m_i - 1) \left(\frac{1}{r_i} - \frac{1}{p_i} \right) \right] h_i + \frac{1}{FC} \sum_{i=1}^{N} (s_i + m_i tc_i) \frac{1}{k_i} \tag{21}$$

The second partial derivative of (21) with respect to FC can be calculated as:

$$\frac{d^2 C(FC)}{dFC^2} = \frac{2}{FC^3} \sum_{i=1}^{N} (s_i + m_i tc_i) \frac{1}{k_i} > 0 \tag{48}$$

Thus, for given vectors of m and k, (21) is convex in FC.

The total cost function for the unequal-sized batch shipment policy is given as follows:

$$C(FC) = FC\sum_{i=1}^{N}\frac{k_i r_i}{2}\left(1+\frac{r_i}{p_i}\right)\frac{D(m_i)}{\left(A(m_i)\right)^2}h_i + \frac{1}{FC}\sum_{i=1}^{N}(s_i + m_i tc_i)\frac{1}{k_i} \tag{45}$$

The second partial derivative of (45) with respect to FC can be calculated as:

$$\frac{d^2C(FC)}{dFC^2} = \frac{2}{FC^3}\sum_{i=1}^{N}(s_i + m_i tc_i)\frac{1}{k_i} > 0 \tag{49}$$

Thus, for given vectors of m and k, (45) is convex in FC.

Paper 4 A dynamic programming approach for solving the economic lot scheduling problem with batch shipments[1]

Authors: Fabian G. Beck, Christoph H. Glock

Type of publication: Journal article

Publication details: *International Journal of Operational Research* (VHB-JOURQUAL3: C), In Press.

Abstract

This note investigates the Economic Lot Scheduling Problem (ELSP) with batch shipments. It first modifies an existing formulation of the ELSP to account both for the cases of equal-sized and geometrically increasing batch shipments, and it then adapts the popular dynamic programming approach of Bomberger to the new planning situation. In addition, the paper specifies some steps of Bomberger's solution procedure that had been formulated imprecisely in the original publication of the author. The paper compares the solution approach proposed in this note to the popular methods of Hanssmann as well as Haessler & Hogue in a numerical experiment and highlights the influence of the batch shipments on the relative performance of the solution procedures. Our results show that the proposed modification reduces the performance disadvantage of Bomberger's Basic-Period-Approach, which may be interesting especially for practitioners that are interested in an easy-to-apply procedure for solving the ELSP in practice. Our changes to Bomberger's solution procedure support finding the lowest total cost solution that had not always been obtained in earlier publications.

Keywords:

Economic lot scheduling problem, ELSP, Bomberger's method, Basic-period-approach, BPA, batch shipments, dynamic programming

[1] Reprinted and slightly modified from *International Journal of Operational Research*, Beck, F.G., Glock, C.H., A dynamic programming approach for solving the economic lot scheduling problem with batch shipments, In Press, with permission from Inderscience Publishers. Inderscience retains copyright of the article.

1 Introduction

The Economic Lot Scheduling Problem (ELSP) that studies the production of multiple items on a single machine has attracted the attention of researchers for many years (see Chan et al. (2013) and Santander-Mercado & Jubiz-Diaz (2016) for recent reviews of the ELSP and Glock et al. (2014) for a recent review of the broad domain of lot sizing). In its most basic version, the ELSP aims on constructing a (feasible) production schedule that is free from overlaps (i.e., the machine is not busy with more than a single product at any point in time), satisfies end customer demand, and minimizes the sum of inventory holding and setup costs.

The ELSP has been shown to be NP-hard in the strong sense for several different problem settings (see Gallego & Shaw, 1997), which has led to the advent of various solution methods for generating feasible production schedules. In a recent review of the ELSP, Santander-Mercado and Jubiz-Diaz (2016) categorized the solution methodologies into three groups: I) analytical methods (that derive an exact solution to a relaxed version of the original problem), II) heuristic methods, and III) meta-heuristic methods. The ELSP has frequently been extended in the past, with recent extensions including product returns (Bae et al., 2016), sequence-dependent setup times and costs (Adelman & Barz, 2014), as well as backordering and shelf-life constraints (Chandrasekaran et al., 2009; Yan et al., 2013).

Beck and Glock (2016) extended the ELSP to take account of partial shipments that can be made from a lot (so-called batch shipments). If batch shipments are possible, the company does not have to wait until the entire lot has been completed before shipping items to the subsequent stage (which could be another machine or the end customer); instead, products can be shipped to the next stage while the production of the lot is still in progress. Prior research has shown that batch shipments can help to reduce cycle times and system inventory, at the expense of higher transportation costs (e.g., Szendrovits, 1975; Goyal, 1977). Since batch shipments often lead to larger lot sizes, they can also support generating a feasible production schedule in the ELSP, as larger lots lead to less frequent setups and consequently a smaller fraction of the total production capacity that is consumed by setups.

To solve the ELSP with batch shipments, Beck & Glock (2016) adapted the popular Basic-Period-Approach (BPA) of Haessler & Hogue (1976) to the new planning situation. Haessler & Hogue's (1976) solution procedure for the ELSP is usually classified as a heuristic procedure. The paper at hand adopts a different approach to the problem and modifies a popular analytical method for solving the ELSP, namely Bomberger's (1966) dynamic programming approach, to take account both of equal-sized and geometrically increasing batch shipments that can be made from a lot for each product. Bomberger's (1966) method is often outperformed by other solution procedures, but still enjoys a high popularity in the literature as it I) presents an easy way to calculate a feasible solution for a given length of the basic period (e.g. one week or one month), and II) leads to a low computational effort in most cases as it neglects scheduling issues in finding the length of the basic period and the number of basic periods between two subsequent production runs of all products. The popularity of Bomberger's (1966) approach is also reflected in the numbers of citations it has received over the years (see Appendix A for details). For these reasons, this note focuses on the approach of Bomberger in the following. As will be shown, using batch shipments as proposed in this paper also reduces an eventual performance disadvantage of Bomberger's method.

The remainder of this paper is structured as follows: The next section outlines the assumptions made in developing the proposed model. Section 3 introduces the modified model and the corresponding solution procedures both for the cases of equal-sized and geometrically increasing batch shipments. Section 4 modifies the data set introduced by Bomberger (1966) to suit the new planning situation and then compares the modified Basic-Period-Approach of Bomberger (mBPAB) developed in the paper at hand to the modified Basic-Period-Approach of Haessler & Hogue (mBPAHH) as well as the modified Common-Cycle-Approach of Hanssmann (mCCAH; for the original Common-Cycle-Approach, see Hanssmann, 1962). The latter two modifications have already been proposed by Beck & Glock (2016). The last section concludes the note.

2 Model assumptions and definitions

This paper studies a scenario where multiple products are produced on a single facility and where products are transported to the subsequent stage in equal-sized or geometrically increasing batch shipments. This scenario has already been studied in Beck & Glock (2016) with the following assumptions made:

1. N products are manufactured on a single facility.
2. All parameters are deterministic and constant over time.
3. The planning horizon is infinite.
4. Only one product can be produced at a time.

5. Shortages are not allowed.
6. Setup cost and setup times are independent of the production sequence.
7. All lot sizes and the number of batches are constant over time.
8. The modified zero switch rule has to hold (see Beck & Glock, 2016).

Note that assumptions #1 to #6 are standard assumptions for the ELSP that have frequently been made in the past (e.g., Bomberger, 1966; Chatfield, 2007).

Throughout the paper, the following terminology is used:

Indices:

i, j	Product with $i, j = 1,2, \dots, N$

Parameters:

h_i	Inventory holding cost per unit per unit of time for product i with $i = 1,2, \dots, N$ [USD/(item·day)]
l^{sr}	Step range [day]
N	Number of products [-]
p_i	Production rate for product i with $i = 1,2, \dots, N$ [items/day]
r_i	Demand rate for product i with $i = 1,2, \dots, N$ [items/day]
s_i	Setup cost per lot for product i with $i = 1,2, \dots, N$ [USD]
st_i	Setup time per lot for product i with $i = 1,2, \dots, N$ [day]
tc_i	Transportation cost per batch shipment for product i with $i = 1,2, \dots, N$ [USD/shipment]

Decision variables:

FC	Length of the basic period [day]
k_i	Integer multiplier; product i is produced every k_i basic periods with $i = 1,2, \dots, N$ [-]
m_i	Number of batch shipments from a lot of product i with $i = 1,2, \dots, N$ [-]
T_i	Cycle time of product i with $i = 1,2, \dots, N$ [day] (implicitly given)

Definitions:

C_i	Average cost for product i with $i = 1,2, \dots, N$ [USD/day]
F	Total average cost [USD/day]
F_j	Minimum average cost to produce the remaining products $j, \dots, N$ [USD/day]
$tocc_{j-1}$	Machine occupancy time, i.e., the sum of setup and production times, for one lot of the first $j-1$ products [day]
tf_j	Remaining idle time for the machine for scheduling products $j, \dots, N$ after the first $j-1$ products have already been scheduled [day]
pt_i	Machine occupancy time per lot of product i with $i = 1,2, \dots, N$ [day]
$\lceil x \rceil$	Ceiling function that returns the smallest integer greater than or equal to x
$\lfloor x \rfloor$	Floor function that returns the largest integer less than or equal to x

Finally, to guarantee that a feasible solution exists, the following net utilization constraint has to hold for the problem at hand (e.g., Elmaghraby, 1978; Beck & Glock, 2016):

$$\sum_{i=1}^{N} \frac{r_i}{p_i} < 1 \tag{1}$$

Note that condition (1) implies that $r_i < p_i, \forall i$.

3 Model development

This section adapts Bomberger's (1966) dynamic programming approach to the situation where batch shipments can be made from a lot. Section 3.1 first introduces assumptions that are valid both for the cases of equal-sized and geometrically increasing batch shipments. Sections 3.2 and 3.3 then discuss the two batch shipment structures considered in this paper.

3.1 General assumptions for Bomberger's BPA

In all Basic-Period-Approaches for the ELSP, the assumption is made that the cycle time of a product is an integer multiple of a so-called basic period:

$$T_i = k_i \cdot FC, \forall i \tag{2}$$

To reduce the computational effort associated with determining optimal values for the k_i-multipliers, some authors restricted their values to powers of two, i.e., $k_i \in \{1,2,4,8 \dots\}, \forall i$ (e.g., Haessler & Hogue, 1976). The paper at hand does not make this assumption and assumes that

$k_i \in \mathbb{N}, \forall i$. The same assumption needs to be made for the length of the basic period, i.e., $FC \in \mathbb{N}$ (see Bomberger, 1966; Elmaghraby, 1978; Chatfield, 2007).

To ensure that a feasible production schedule can be found, Bomberger (1966) required that the sum of setup times and production times for one lot of all N products does not exceed the length of the basic period:

$$\sum_{i=1}^{N} pt_i = \sum_{i=1}^{N} \left(\frac{k_i \cdot FC \cdot r_i}{p_i} + st_i \right) \leq FC \tag{3}$$

Treating the middle and right part of Eq. (3) as an equation and rearranging leads to:

$$FC = \frac{\sum_{i=1}^{N} st_i}{1 - \sum_{i=1}^{N} \frac{k_i \cdot r_i}{p_i}} \tag{4}$$

Bomberger (1966) suggested using a trial-and-error approach to determine the length of the basic period that was not precisely described in his paper. Here, we propose an easy way to calculate boundaries for FC. Eq. (4) can be used to calculate a lower bound for the length of the basic period for the mBPAB. Obviously, the denominator in (4) adopts the largest value if $k_i = 1, \forall i$. In this case, the lower bound is reduced to:

$$FC_{min} = \left\lceil \frac{\sum_{i=1}^{N} st_i}{1 - \sum_{i=1}^{N} \frac{r_i}{p_i}} \right\rceil \tag{5}$$

As an upper bound for the length of the basic period, we choose the maximum of the individual optimal cycle lengths calculated in Step 1 of the procedures proposed by Beck & Glock (2016) in their Section 4.2.3 (equal-sized batch shipments) and Section 4.3.3 (geometrically increasing batch shipments), respectively, i.e.

$$FC_{max} = \lceil \max\{T_1, T_2, \dots, T_N\} \rceil \tag{6}$$

3.2 BPA with equal-sized batch shipments

The average cost that result for product i if equal-sized batches are used can be formulated as in Beck & Glock (2016):

$$C_i(k_i, m_i, FC) = \frac{k_i \cdot FC \cdot r_i^2}{2 \cdot m_i} \cdot \left[\left(\frac{1}{r_i} + \frac{1}{p_i}\right) + (m_i - 1) \cdot \left(\frac{1}{r_i} - \frac{1}{p_i}\right)\right] \cdot h_i + (s_i + m_i \cdot tc_i) \cdot \frac{1}{k_i \cdot FC} \tag{7}$$

The total average cost can be calculated by summing Eq. (7) up over all products:

$$F(k, m, FC) = \min_{k,m}\left(\sum_{i=1}^{N} C_i(k_i, m_i, FC)\right) \tag{8}$$

with $k = (k_1; k_2; \dots; k_N)$ and $m = (m_1; m_2; \dots; m_N)$.

To calculate optimal values for the k_i and m_i for all N products, we adapt Bomberger's (1966) procedure in the following. First, we assume that for a given length of the basic period FC, the products $1, \dots, j-1$ have already been scheduled, resulting in a machine occupancy time $tocc_{j-1}$. The remaining machine idle time available for producing products $j, \dots, N$ can thus be calculated as

$$tf_j = FC - tocc_{j-1} \tag{9}$$

The minimum average cost to produce the remaining products $j, \dots, N$, considering the remaining machine idle time tf_j, can be determined as follows:

$$F_j(tf_j, FC) = \min_{k,m}\left(\sum_{i=j}^{N} C_i(k_i, m_i, FC)\right) \tag{10}$$

subject to

$$\sum_{i=j}^{N}\left(st_i + \frac{r_i}{p_i} \cdot k_i \cdot FC\right) \le tf_j \tag{11}$$

The functional equation of Eq. (10) can be written as

$$\begin{aligned} F_j(tf_j, FC) &= \min_{k_j, m_j}\left(C_j(k_j, m_j, FC) + F_{j+1}(tf_j - pt_j, FC)\right) \\ &= \min_{k_j, m_j}\left(C_j(k_j, m_j, FC) + F_{j+1}\left(tf_j - st_j - \frac{r_j}{p_j} \cdot k_j \cdot FC, FC\right)\right) \end{aligned} \tag{12}$$

with

$$1 \leq k_j \leq k_{j,max} = \left\lfloor \frac{(tf_j - st_j) \cdot p_j}{r_j \cdot FC} \right\rfloor \tag{13}$$

and

$$F_{N+1}(tf_{N+1}, FC) = 0 \tag{14}$$

For a given value of FC, Bomberger's BPA always starts with the last product, i.e., with $j = N$. To be able to calculate Eqs. (9) and (13), the time needed to produce the products $1, \dots, N-1$, i.e., $tocc_{N-1}$, needs to be known. Since $tocc_{N-1}$ is not known in this step of the procedure, Bomberger (1966) suggested using different values of $tocc_{N-1}$ with a certain step range, i.e., $tocc_{N-1} = \Delta T, 2\Delta T, \dots, T$. This may lead to a high computational effort, however, and it is also difficult to initiate since no further information about the step range is given in Bomberger's paper.

For this reason, we propose an easy way to calculate an upper bound $k_{j,max}$ for all N products by assuming that all products $i \in \{1, \dots, N\} \backslash j$ occupy the machine for the shortest possible time. The latter results for $k_i = 1$ for $i \in \{1, \dots, N\} \backslash j$ (see Appendix B for details):

$$k_{j,max} = \left\lfloor \frac{\left[FC \cdot \left(1 - \sum_{i=1}^{j-1} \frac{r_i}{p_i} - \sum_{i=j+1}^{N} \frac{r_i}{p_i}\right) - \sum_{i=1}^{N} st_i\right] \cdot p_j}{r_j \cdot FC} \right\rfloor \tag{15}$$

Using Eqs. (13) and (15), it is possible to directly calculate all feasible k_j-values for all N products for the given FC-value and to determine the corresponding decision tree (see Appendix C for an example).

For given values of k_j and FC, it is then possible to determine the optimal real-valued solution for the number of batch shipments of product j, i.e., m_j (see Beck & Glock, 2016):

$$m_j^* = r_j \cdot k_j \cdot FC \cdot \sqrt{\frac{h_j}{tc_j \cdot p_j}} \tag{16}$$

Since the number of batch shipments of product j is restricted to a natural number > 0, the real value obtained by Eq. (16) may have to be rounded as follows:

1. If $m_j^* < 1$, set $m_j = 1$ and calculate $C_j(k_j, m_j, FC)$,
2. else if $m_j^* \in \mathbb{N}$, set $m_j = m_j^*$ and calculate $C_j(k_j, m_j, FC)$,
3. else determine $\lfloor m_j^* \rfloor$ and $\lceil m_j^* \rceil$ and do a cost comparison using Eq. (7):
   3.1. If $C_j(k_j, \lfloor m_j^* \rfloor, FC) < C_j(k_j, \lceil m_j^* \rceil, FC)$, set $m_j = \lfloor m_j^* \rfloor$ and calculate $C_j(k_j, m_j, FC)$,
   3.2. else set $m_j = \lceil m_j^* \rceil$ and calculate $C_j(k_j, m_j, FC)$.

To exclude those k_i-value combinations that violate the constraints formulated above, Eqs. (3) and (11) can be used. Note that a combination of k_i-values for products $j, \dots, N$ is not feasible if their machine occupancy time is larger than the remaining machine idle time if the first $j-1$ products (that have not yet been scheduled) would be produced with their respective lowest machine occupancy time that results for $k_i = 1, \forall i = 1, \dots j-1$. k_i-value combinations thus can be excluded from further analysis if the following condition is satisfied:

$$\sum_{i=j}^{N} pt_i = \sum_{i=j}^{N} \left(\frac{k_i \cdot FC \cdot r_i}{p_i} + st_i \right) > FC - \sum_{i=1}^{j-1} \left(\frac{FC \cdot r_i}{p_i} + st_i \right) = tf_j \qquad (17)$$

The above procedure has to be repeated for all $FC \in [FC_{min}, FC_{max}]$.

Appendix C illustrates how certain k_i-value combinations can be excluded from further analysis in an example.

3.3 BPA with geometrically increasing batch shipments

The average cost function for the case of geometrically increasing batch shipments for product i can be formulated as in Beck & Glock (2016):

$$C_i(k_i, m_i, FC) = \frac{k_i \cdot FC \cdot r_i}{2} \cdot \left(1 + \frac{r_i}{p_i}\right) \cdot \frac{D(m_i)}{\left(A(m_i)\right)^2} \cdot h_i + (s_i + m_i \cdot tc_i) \cdot \frac{1}{k_i \cdot FC} \qquad (18)$$

with $D(m_i) = \frac{\left(\frac{p_i}{r_i}\right)^{2 \cdot m_i} - 1}{\left(\frac{p_i}{r_i}\right)^2 - 1}$ and $A(m_i) = \frac{\left(\frac{p_i}{r_i}\right)^{m_i} - 1}{\left(\frac{p_i}{r_i}\right) - 1}$

The optimal length of the basic period and the k_j-multipliers can be calculated as described in Sections 3.1. and 3.2. using Eq. (18) instead of Eq. (7). To calculate an optimal solution for the number of batch shipments per product, m_j, we utilize the convexity property of Eq. (18) outlined in Bogaschewsky et al. (2001) for the single-product case. Since it is not possible to derive a closed-form expression for m_j, we use the following iterative solution procedure for given values of k_j and FC:

1. Set $m_j = 1$ and $C_j^* = \infty$.
2. Determine $C_j(k_j, m_j, FC)$ according to Eq. (18).
3. If $C_j(k_j, m_j, FC) < C_j^*$, set $C_j^* = C_j(k_j, m_j, FC)$, $m_j = m_j + 1$ and go to Step 2, else set $m_j = m_j - 1$ and calculate $C_j(k_j, m_j, FC)$ from Eq. (18).

4 Numerical examples

This section compares the dynamic programming approach proposed in this paper to the mBPAHH and the mCCAH proposed in Beck & Glock (2016) using the Bomberger (1966) data set as corrected by Chatfield (2007). Table 1 shows the data set including specific transportation cost for every product taken from Beck and Glock (2016).

Table 1: *Data for the modified Bomberger problem*

Product i	**Production rate** p_i **(items/day)**	**Demand rate** r_i **(items/day)**	**Holding cost** $10^{-4}h_i$ **(USD/day)**	**Setup cost** s_i **(USD/day)**	**Transportation cost** tc_i **(USD/shipment)**	**Setup time** st_i **(days)**
1	30,000	400	0.02708	15	1	0.125
2	8000	400	0.73958	20	6	0.125
3	9500	800	0.53125	30	4	0.25
4	7500	1600	0.41667	10	10	0.125
5	2000	80	11.60417	110	2	0.5
6	6000	80	1.11458	50	5	0.25
7	2400	24	6.25	310	8	1
8	1300	340	24.58333	130	4	0.5
9	2000	340	3.75	200	6	0.75
10	15,000	400	0.16667	5	7	0.125

To calculate the cost of the mBPAHH, we use procedure 1 proposed in Beck and Glock (2016). The step range in this procedure is set to $l^{sr} = 0.01$. In addition, Step 3 of Beck & Glock (2016) is modified to evaluate all FC-values with $FC \in [FC_{start}; FC_{max}]$ for all k-vectors from the set of candidate solutions without terminating the procedure once a feasible solution for a single k-vector has been found, which may lead to better results.

Table 2: *Results for the modified Bomberger problem*

	Classical ELSP (without batch shipments)	**Equal-sized batch shipments**	**Geometrically increasing batch shipments**
Independent solution			
FC	19.53	20.63	21.07
m	-	(1,1,1,1,2,1,1,5,4,1)	(1,1,2,1,2,1,1,3,3,1)
Lower bound of the costs/day	31.62	35.90	34.32
Common-Cycle-Approach of Hanssmann (modified)			
FC	42.75	42.75	43.37
m	-	(1,1,1,1,3,1,1,9,2,1)	(1,1,2,2,2,1,1,5,2,1)
Upper bound of the costs/day	41.17	45.79	43.65
Basic-Period-Approach of Haessler & Hogue (modified)			
FC	23.42	23.66	24.25
k	(8,2,2,1,2,4,8,1,2,2)	(8,2,2,1,2,4,8,1,2,2)	(8,2,2,1,2,4,8,1,2,2)
m	-	(1,1,2,1,2,1,1,6,3,1)	(1,1,2,1,2,1,1,4,3,1)
Costs/day	32.07	36.35	34.73
Basic-Period-Approach of Bomberger (modified)			
FC	39	39	43
k	(1,1,1,1,1,1,3,1,1,1)	(1,1,1,1,1,1,3,1,1,1)	(1,1,1,1,1,1,4,1,1,1)
m	-	(1,1,1,2,2,1,1,9,2,1)	(1,1,2,2,2,1,1,4,2,1)
Costs/day	36.62	41.07	39.09

Table 2 presents the results obtained using three different solution procedures as well as the lower bound. The mBPAHH outperforms the modified procedure proposed in this paper, with the mBPAB outperforming the mCCAH. We note, however, that the relative performance disadvantage of the mBPAB compared to the mBPAHH reported in the literature is reduced if batch shipments are used. For the case where equal- (unequal-) sized batch shipments are used, the performance disadvantage is 12.99% (12.55%), while it is 14.19% in the classical ELSP. Using batch shipments as proposed in this note is therefore beneficial in case companies are interested in utilizing the advantages Bomberger's procedure offers, which is especially a higher flexibility in considering alternative k_i-values than Haessler & Hogue's method (the latter method restricts the k_i-values to powers of two) and the fact that Bomberger's approach can consider given basic periods much easier than Haessler & Hogue's method. Bomberger's method is also associated with a shorter runtime than the approach of Haessler & Hogue.

We finally also point out that our result for the classical ELSP solved with the mBPAB differs from the one reported in the literature. Bomberger (1966) calculated a basic period length $FC = 40$ with the corresponding cost $F = 36.65$, which is also mentioned in Elmaghraby (1978) and Chatfield (2007), while our result is $FC = 39$ with the corresponding cost $F = 36.62$. This difference in results originates from the adjusted solution procedure introduced above that was not formulated precisely in the original publication of Bomberger (1966) and that may lead, as was shown in this numerical example, to unnecessary high cost.

5 Conclusion

This note proposed an analytical method for solving the ELSP with batch shipments. For this purpose, the BPA of Bomberger (1966) was modified to account for two batch shipment policies, namely equal-sized and geometrically increasing batch shipments. In addition, we modified Bomberger's (1996) original solution procedure in two ways to remove imprecision in the corresponding earlier algorithm: I) instead of a trial-and-error approach with interpolation and extrapolation between various FC-values, we determine a maximum and a minimum value for the length of the basic period to state exactly which FC-values need to be evaluated; II) instead of using an arbitrary search over $tocc_j$ to calculate the $k_{i,max}$-values, we present an easy way to directly calculate intervals for all feasible k_i-values for all N products for a given FC-value.

Our modifications make two major contributions to the literature. First, the improvement of Bomberger's solution procedure supports obtaining lower total cost, which was illustrated by comparing the results obtained in this note to those obtained in earlier publications. Secondly, introducing batch shipments into Bomberger's model reduces its performance disadvantage as compared to other solution approaches that was observed frequently in the past, which makes it an interesting solution procedure for the ELSP especially from a practical point of view. For example, a production planner may be interesting in considering a certain given length of the basic period, e.g., one week, in scheduling production, which can easily be inserted into the BPA proposed in this paper, as opposed to Haessler & Hogue, for example. Bomberger's approach is usually also associated with a shorter runtime, which is another advantage in a practical application.

To gain further insights into how the use of batch shipments influences both the total costs of the ELSP as well as the generation of a feasible production schedule, future research could investigate batches in other solution approaches as well, e.g., the Time-Varying-Lot-Size-Approach of Dobson (1987), the Extended-Basic-Period-Approach of Elmaghraby (1977), or the Genetic-Search-Approach of Chatfield (2007). It would also be interesting to study if the results of these approaches differ significantly from the approaches already developed. In addition, dividing a lot into a combination of equal-sized and geometrically increasing batch shipments could also be interesting, which is a shipment structure that leads to the lowest total cost in joint economic lot size models, for example (see, e.g., Hill, 1999).

References

Adelman, D., Barz, C., 2014. A price-directed heuristic for the economic lot scheduling problem. IIE Transactions, 46(12), 1343-1356.

Bae, H., Moon, I., Yun, W., 2016. A time-varying lot sizes approach for the economic lot scheduling problem with returns. International Journal of Production Research, 54(11), 3380-3396.

Beck, F.G., Glock, C.H., 2016. The impact of batch shipments on the economic lot scheduling problem. Computers & Industrial Engineering, 96, 126-139.

Bogaschewsky, R.W., Buscher, U.D., Lindner, G., 2001. Optimizing multi-stage production with constant lot size and varying number of unequal sized batches. Omega, 29(2), 183-191.

Bomberger, E., 1966. A dynamic programming approach to a lot size scheduling problem. Management Science, 12(11), 778-784.

Chan, H.K., Chung, S.H., Lim, M.K., 2013. Recent research trend of economic-lot scheduling problems. Journal of Manufacturing Technology Management, 24(3), 465-482.

Chandrasekaran, C., Rajendran, C., Chetty, O. V. K., Hanumanna, D., 2009. A two-phase metaheuristic approach for solving Economic Lot Scheduling Problems. International Journal of Operational Research, 4(3), 296-322.

Chatfield, D.C., 2007. The economic lot scheduling problem: a pure genetic search approach. Computers & Operations Research, 34(10), 2865-2881.

Dobson, G., 1987. The economic lot-scheduling problem: achieving feasibility using time-varying lot sizes. Operations Research, 35(5), 764-771.

Elmaghraby, S.E., 1977. An extended basic period approach of the ELSP. Report No. 115 Graduate Program in Operations Research, North Carolina University at Raleigh, NC.

Elmaghraby, S.E., 1978. The economic lot scheduling problem (ELSP): review and extensions. Management Science, 24(6), 587-598.

Gallego, G., Shaw, D.X., 1997. Complexity of the ELSP with general cyclic schedules. IIE Transactions, 29(2), 109-113.

Glock, C.H., Grosse, E.H., Ries, J.M., 2014. The lot sizing problem: a tertiary study. International Journal of Production Economics, 155, 39-51.

Goyal, S.K., 1977. Determination of optimum production quantity for a two-stage production system. Operation Research Quarterly, 28(4), 865-870.

Haessler, R.W., Hogue, S., 1976. Note—a note on the single-machine multi-product lot scheduling problem. Management Science, 22(8), 909-912.

Hanssmann, F., 1962. Operations research in production and inventory control (1st ed.). John Wiley & Sons.

Hill, R.M., 1999. The optimal production and shipment policy for the single-vendor single-buyer integrated production-inventory problem. International Journal of Production Research, 37(11), 2463-2475.

Santander-Mercado, A., Jubiz-Diaz, M., 2016. The economic lot scheduling problem: a survey. International Journal of Production Research, 54(16), 4973-4992.

Szendrovits, A.Z., 1975. Manufacturing cycle time determination for a multi-stage economic production quantity model. Management Science, 22(3), 298-308.

Yan, C., Liao, Y., Banerjee, A., 2013. Multi-product lot scheduling with backordering and shelf-life constraints. Omega, 41(3), 510-516.

Appendix

Appendix A

Figure A.1 illustrates the number of citations the work of Bomberger (1966) received over time.

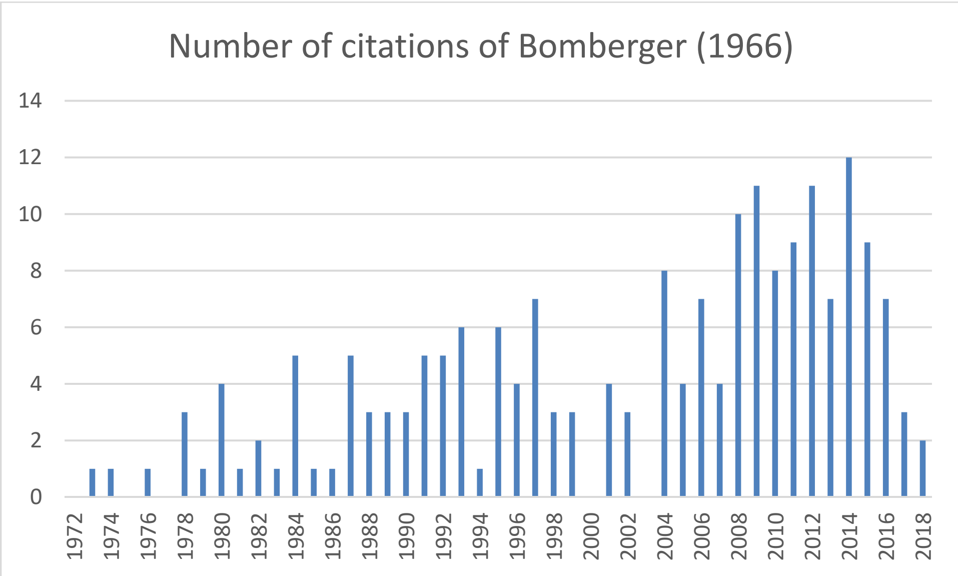

Figure A.1: *Number of citations of Bomberger (1966)*

Appendix B

This appendix derives an upper bound on $k_j, \forall j = 1, \dots, N$. For a given length of the basic period FC, we start with product $j = N$. To determine the maximum feasible value for k_N, we utilize the constraint that all products need to be produced in one basic period (Bomberger, 1966). From Eqs. (3) and (11), we conclude that the minimum occupation time for the remaining products $1, \dots, N-1$ results where the k_i-values of these products equals 1, i.e., $k_i = 1$, $\forall i = 1, \dots, N-1$. The remaining machine idle time can then be calculated by subtracting this minimum occupation time, which can be seen as an actual value for $tocc_{N-1}$, from the length of the basic period FC, i.e., $FC - \sum_{i=1}^{N-1}\left(\frac{FC \cdot r_i}{p_i} + st_i\right)$. Finally, the occupation time of product $j = N$ has to be less than or equal to the remaining machine idle time, and it can be expressed as follows:

$$\frac{FC \cdot r_N \cdot k_N}{p_N} + st_N \leq FC - \sum_{i=1}^{N-1}\left(\frac{FC \cdot r_i}{p_i} + st_i\right) \tag{19}$$

Solving Eq. (19) for k_N yields

$$k_N \leq \frac{\left(FC \cdot \left(1 - \sum_{i=1}^{N-1} \frac{r_i}{p_i}\right) - \sum_{i=1}^{N} st_i\right) \cdot p_N}{r_N \cdot FC} \tag{20}$$

Since k_N has to be a natural number, $k_{N,max}$ can be calculated as follows:

$$k_{N,max} = \left\lfloor \frac{\left(FC \cdot \left(1 - \sum_{i=1}^{N-1} \frac{r_i}{p_i}\right) - \sum_{i=1}^{N} st_i\right) \cdot p_N}{r_N \cdot FC} \right\rfloor \tag{21}$$

For any product j, assuming that all other products are produced at their minimum machine occupancy time, Eq. (19) can be formulated as follows:

$$\frac{FC \cdot r_j \cdot k_j}{p_j} + st_j \leq FC - \sum_{i=1}^{j-1} \left(\frac{FC \cdot r_i}{p_i} + st_i\right) - \sum_{i=j+1}^{N} \left(\frac{FC \cdot r_i}{p_i} + st_i\right) \tag{22}$$

Solving Eq. (22) and taking into account that k_j has to be a natural number leads to Eq. (15).

Appendix C

To illustrate our new calculation procedure to determine the $k_{j,max}$-values and the resulting decision tree of the mBPAB for a given FC-value, we use the Bomberger data set presented in Table 1. The mBPAB with $FC = 39$ then leads to the following maximum values for k_j with $j = 1, \dots, N$ obtained using Eq. (15):

Table C.1: *Maximum k_j-values for the Bomberger data set and $FC = 39$*

j	1	2	3	4	5	6	7	8	9	10
$k_{j,max}$	2	1	1	1	1	2	3	1	1	1

The corresponding decision tree is illustrated in Figure C.1. In addition, we marked branches (k_i-value-combinations) that do not have to be evaluated if Eq. (17) is used (light grey).

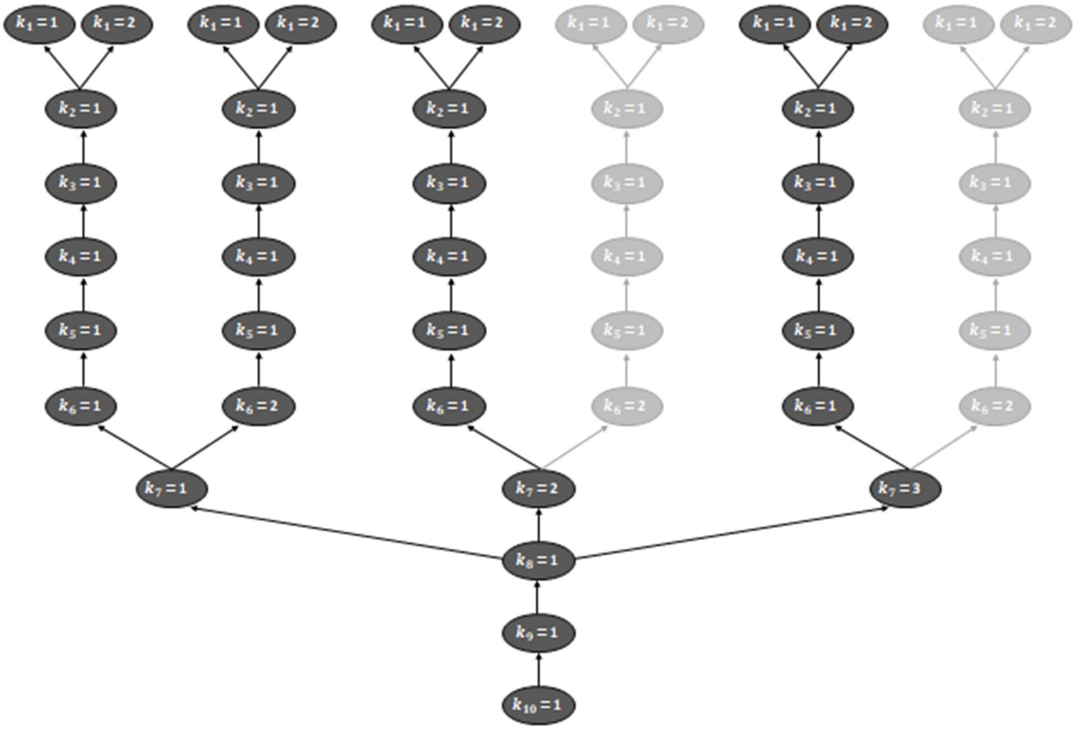

Figure C.1: *Decision tree for the Bomberger data set and* $FC = 39$

Part B An investigation of production and transportation policies for multi-actors, multi-stage production systems

Paper 5 Coordination of a production network with a single buyer and multiple vendors with geometrically increasing batch shipments[1]

Authors: Fabian G. Beck, Christoph H. Glock, Taebok Kim

Type of publication: Journal article

Publication details: *International Journal of Production Economics* (VHB-JOURQUAL3: B), 193, 633-646, 2017

Abstract

This paper considers the joint economic lot size model for the special case where multiple vendors supply a single product to a single buyer. The production lots of each vendor are transferred to the buyer in batches increasing in size according to a fixed factor, where the size of the batches may differ from vendor to vendor. To coordinate the production cycles of the vendors and the consumption cycle of the buyer, we use two different shipment policies, namely one policy where batches are shipped to the buyer directly after their completion, and one policy where shipments are made whenever the inventory at the buyer reaches zero. Mathematical models are proposed for each policy, and solution procedures are suggested. Subsequently, the performance of the policies is evaluated in a numerical experiment, and both policies are compared to the situation where batches of equal sizes are shipped from the vendors to the buyer. The paper concludes with suggestions for further research.

Keywords:

Joint economic lot size; JELS; Integrated inventory; Supply chain management; Supply chain coordination; Geometric batch shipment policy; Multiple vendors; Single buyer

[1] Reprinted and slightly modified from *International Journal of Production Economics*, 193, Beck, F.G., Glock, C.H., Kim, T., Coordination of a production network with a single buyer and multiple vendors with geometrically increasing batch shipments, 633-646, 2017, https://doi.org/10.1016/j.ijpe.2017.07.015, with permission from Elsevier.

1 Introduction

The coordination of supply chains has been a popular research topic for many years. One stream of research has concentrated on developing decision support models that assist practitioners in determining optimal production and distribution policies for supply chain operations. Among these decision support models, so-called Joint Economic Lot Size (JELS) models have received a particularly high attention in recent years.

JELS models consider a situation where one or more vendors supply one or more products to a single or a group of buyers. The objective of JELS models is to determine production and distribution policies that minimize the total costs of the entire supply chain instead of optimizing the individual positions of selected supply chain members.

As was shown in a review of Glock (2012b), publication numbers on JELS models increased almost exponentially in recent years. Popular topics in this area were the determination of shipment policies (e.g., Banerjee, 1986; Hill, 1997; Goyal and Nebebe, 2000), the impact of supply chain coordination on environmental issues (e.g., Jaber et al., 2013; Bazan et al., 2015), or the effect of performance improvement investments on the efficiency of the supply chain (e.g., Woo et al., 2000; Pan and Yang, 2002; Ouyang et al., 2004).

It is interesting to note that prior research on the JELS problem had a one-sided focus on single vendor-single buyer models as well as single vendor-multiple buyer-scenarios, and that situations where a buyer sources a product from multiple vendors have only infrequently been considered. Glock (2012b) pointed out in his literature review that out of the 155 papers included in his literature sample, only seven papers discussed a scenario with more than a single vendor. Clearly, the one-sided focus of prior research on single vendor-settings does not reflect what can be observed in practice, where it is often common to purchase a component from more than a single source (see, for example, Costantino and Pellegrino, 2010). Even though a couple of JELS models that consider more than a single vendor have been published since Glock's (2012b) review appeared, multiple-vendor JELS models are still heavily underrepresented in the literature.

The paper at hand contributes to closing this research gap by studying alternative shipment policies in a multiple vendor-single buyer Joint Economic Lot Size model. More specifically, the paper tries to answer the questions of a) how the production cycles of the vendors and the consumption cycle of the buyer should be coordinated, and b) how items produced at the vendors should be shipped to the buyer to minimize the total costs of the system. To answer these research questions, the paper at hand extends an earlier paper of Glock (2012a), which is one of the first works that studied a multiple vendor-single buyer JELS model. Glock's (2012a) work, however, assumed that lots are transported in equal-sized batches to the buyer, which may lead to a delayed initiation of the consumption process at the buyer and unnecessarily high inventory carrying cost. The paper at hand thus extends the work of Glock (2012a) to the case of a geometrically increasing batch shipment policy, which has been shown to lead to lower total costs in many cases in the past.

The remainder of this paper is structured as follows: The next two sections review related literature and present the definitions and assumptions used in this paper. Sections 4 and 5 present two mathematical models for alternative shipment policies as well as their solution procedures. Section 6 illustrates the proposed models in a numerical experiment, and Section 7 concludes this paper.

2 Literature review

Due to the large number of Joint Economic Lot Size models published in the past, a comprehensive review of the JELS literature is not within the scope of this paper. For a structured literature review of JELS models, the reader may refer to Glock (2012b). In the following, we present a brief overview of JELS models that are most relevant to the work at hand.

Goyal (1977) was one of the first authors who considered a lot size model with vendor-buyer coordination with the vendor manufacturing at an infinite production rate. Goyal (1977) thus implicitly assumed that the vendor is a reseller of the product sourcing it from another vendor. Banerjee (1986) extended the work of Goyal (1977) to account for a finite production rate at the vendor. While Goyal (1977) assumed that the vendor may combine several orders of the buyer in a large production/order quantity, Banerjee (1986) assumed that only complete lots are shipped from the vendor to the buyer, which was later referred to as the lot-for-lot policy. Agrawal and Raju (1996) extended the work of Banerjee (1986) by assuming that a production lot may be split up into equal-sized batches that may be shipped individually from the vendor to the buyer. The advantage of batch shipments is that the vendor does not have to wait until the entire lot has been finished, but instead may ship products earlier, which leads to an earlier start of the consumption process at the buyer and lower inventory levels in the supply chain.

Another option for shipping a lot from the vendor to the buyer is a geometric batch shipment policy. In this case, batches usually increase in size according to a fixed factor, which enables the buyer to initiate consumption earlier than in the case of equal-sized shipments. The first authors to consider a geometric batch shipment policy in a JELS model were Chatterjee and Ravi (1991) and Goyal (1995). Viswanathan (1998) compared the case of equal-sized batches and the geometric batch shipment policy and showed that either policies may lead to lower total system costs, and that the relative advantage of both policies depends on the scenario studied. Goyal and Nebebe (2000) and Goyal (2000) then combined geometrically increasing and equal-sized batch shipments and showed that using both types of batches may lead to another reduction in total costs. The optimal policy for the single vendor-single buyer JELS model was proposed by Hill (1999), who showed that a particular combination of geometrically increasing and equal-sized batch shipments results in the lowest total system costs for this case.

Other researchers developed JELS models with more than a single actor on each stage. Joglekar and Tharthare (1990) were among the first to consider multiple buyers in a Joint Economic Lot Size model. The authors assumed that buyers are homogeneous, i.e. that they have identical performance and cost parameters. This paper was extended by Banerjee and Burton (1994), who assumed that buyers are heterogeneous, i.e. that their performance and cost parameters may differ.

Only a few authors studied the case were a buyer sources a product from multiple vendors. Kim and Goyal (2009) presented one of the first single buyer-multiple vendor JELS models. They studied two distinct delivery policies in their model, namely lumpy deliveries and phased deliveries. In the case of lumpy deliveries, all vendors ship their lots at the same time to the buyer, whereas in the case of phased deliveries, the buyer is supplied from only one vendor whenever its inventory level reaches zero. Phased deliveries obviously lead to more frequent incoming shipments at the buyer. Glock (2011) combined the single buyer-multiple vendor JELS model with a vendor selection problem and assumed that the buyer has the option to select vendors from a pool of pre-selected vendors. After vendors have been selected, the model again aims on minimizing the total costs of the newly established supply chain. Glock and Kim (2014) studied the single buyer-multiple vendor JELS model and assumed that the buyer has the option to assign vendors to groups, where each group then delivers their batches in a consolidated shipment to the buyer to save transportation costs. Glock and Kim (2015) considered CO_2 emissions in a multiple vendor-single buyer JELS model and assumed that emitting greenhouse gases leads to additional costs that need to be considered in coordinating the supply chain. In their model, the authors studied a supply chain where heterogeneous trucks are used to transport the product to the buyer. Ben-Daya and Al-Nassar (2008) proposed a JELS model for a three-layer supply chain and assumed that each stage consists of multiple actors. The authors proposed a so-called non-delayed equal-sized shipments policy, where shipments are delivered to the next stage before the lot has been completed. Another paper investigating a three-layer supply chain is the one of Jaber and Goyal (2008), who considered multiple suppliers, a single vendor and multiple buyers. Seliaman and Ahmad (2009) extended these earlier works on three-layer supply chains by studying a supply chain with n stages and multiple actors on each stage. Lots were shipped from stage to stage using equal-sized batch shipments. Leung (2010) proposed another multi-stage multi-actors JELS model.

The paper that is closest related to the work at hand is the one of Glock (2012a), who also studied a supply chain consisting of a single buyer and multiple vendors. The author assumed that the vendors deliver the product in equal-sized batch shipments to the buyer and developed two different shipment policies to coordinate the system: In the case of immediate deliveries (ID), the vendors deliver their respective batches consecutively, such that a batch is dispatched to the buyer directly after its completion. In the case of delayed deliveries (DD), the vendors deliver batches consecutively again, but only dispatch a shipment when the buyer has depleted its inventory. The shipment policy DD leads to the majority of inventory being kept at the vendors, while the policy ID transfers the majority of inventory to the buyer.

Compared to a scenario where only complete lots are delivered to the buyer, Glock's (2012a) shipment policies always lead to a reduction in total system costs. Limiting batch shipments to equal sizes, however, may be too restrictive. As has been shown by several authors, using a geometric batch shipment policy in a supply chain may lead to another reduction in total system costs (e.g., Hill, 1997, 1999; Glock, 2009). This paper consequently extends the models of Glock (2012a) by assuming that the size of batch shipments increases by a fixed factor. In numerical experiments, we show that this generalization may significantly reduce the total costs of the supply chain.

3 Assumptions and definitions

This paper studies a scenario where multiple vendors supply a single type of product to a single buyer. We assume that a large buyer faces a network of small vendors with low production capacities, which means that more than a single vendor is needed to satisfy the buyer's demand. As will be shown in more detail below, vendors with relatively small production rates may cause overlaps in the delivery cycle, which leads to a complex expression for the inventory carrying cost at the buyer or the vendors, respectively. In the models proposed here, the buyer requests an uninterrupted flow of materials, and therefore the sum of the production rates of the vendors have to exceed the buyer's demand rate. In addition, it is assumed that the vendors are homogeneous with identical problem parameters. Homogeneous vendors can be found in the juice production industry, for example, where a large number of fruit-farmers face a small number of large beverage factories, or in the milk processing industry as explained in Glock (2012a). The aim of the paper is to minimize the total costs of the system studied here.

In addition to what has already been stated, the following assumptions are made:

1. All parameters are deterministic and constant over time.
2. The buyer seeks to minimize its vendor base and maintains relationships only to as many vendors as are necessary to ensure an uninterrupted supply of materials (see also Glock, 2012a).
3. A lot produced at one of the vendors is sent to the buyer in batches that increase in size according to a fixed factor.
4. Shortages are not allowed.
5. The number of batches is equal for all vendors. The production lot size – and consequently the size of individual batch shipments – may differ from vendor to vendor, however.
6. Deliveries are instantaneous, i.e. transportation time is not considered.
7. We consider an infinite horizon for the models. All problem parameters are given on a "per unit of time" basis, where one unit of time could be a day, a week or a month, for example. In addition, we specify a planning period that could extend over multiple units of time and calculate the total costs of the system for the planning period. Thus, if the demand per unit of time (e.g., per day) is d and the planning period is T (e.g., one week), then the demand in the planning period (e.g., demand per week) would be $D = dT$. Explicitly considering the planning period T in the models enables the decision maker to comfortably calculate the decision variables and total costs for any period of interest. If

the decision maker is interested in obtaining only the costs per unit of time, setting $T = 1$ would normalize the total cost function to one unit of time.

8. For the sake of brevity, the male gender is used to refer to individuals that could be male or female.

The following terminology is used throughout the paper:

Definitions:

consumption cycle:	the time needed to consume the order quantity Q at the buyer
production cycle:	the time needed to produce a lot q_i at vendor *i*
$\lceil \dots \rceil$	ceiling function

Indices:

i	vendor with $i = 1,2, \dots, n$
j	batch shipment in a production cycle with $j = 1, \dots, m$

Variables:

β_i	proportion of the total order quantity that is produced by vendor *i* with $\sum_{i=1}^{n} \beta_i = 1$
q_i	production lot size of vendor *i*, where $q_i = \beta_i Q = q_{i,1} \sum_{j=1}^{m} \lambda^{j-1}$
$q_{i,j}$	size of the *j*th batch shipment in a production cycle of vendor *i*, where $q_{i,j} = q_{i,1}\lambda^{j-1}$
Q	buyer's total order quantity with $Q = \sum_{i=1}^{n} q_i$
m	number of batch shipments per production cycle of each vendor
λ	proportional increase in the size of subsequent batch shipments in a production cycle
$ct_{i,j}$	consumption time of the *j*th batch of vendor *i* at the buyer
$pt_{i,j}$	production time of the *j*th batch of vendor *i*
TC^b	total costs of the buyer in the planning period *T*
TC_i^v	total costs of vendor *i* in the planning period *T*
TC^{sys}	total costs of the system in the planning period *T*
TWI	time-weighted inventory

Parameters:

c_O	buyer's ordering cost per order
c_S	vendor's setup cost per setup
c_T	sum of the buyer's handling and the vendor's transportation cost per delivery

c_T^v	vendor's transportation cost per delivery
c_T^b	buyer's handling cost per delivery
T	length of the planning period
d	demand rate in units per unit of time at the buyer
D	total demand at the buyer in the planning period, with $D = Td$
h^b	unit inventory carrying charges per unit of time at the buyer
h^v	unit inventory carrying charges per unit of time at the vendors
n	number of vendors, with $n = \left\lceil \frac{d}{p} \right\rceil$
p	production rate per unit of time of each vendor

The timing of the deliveries of the batch shipments could also be seen as a decision variable; it is, however, fully determined by the implemented delivery structure (see Section 4.1) and the batch sizes and thus indirectly given once the other variables have been determined.

4 Model development

4.1 Delivery structure

As in Glock (2012a), we consider two different delivery structures in this paper. The delivery structures proposed by Glock (2012a) are extended in the following to account for geometrically increasing batch shipments.

Production and delivery are structured as follows: The buyer orders a lot of size Q at time t_0, which is allocated to the vendors in such a way that q_i equals the production lot size of vendor i with $q_i = \beta_i Q = q_{i,1} \sum_{j=0}^{m-1} \lambda^j$. As was already pointed out by Glock (2012a), there are two basic alternatives to ensure an uninterrupted supply of products at the buyer in this case. In both scenarios, the vendors deliver their batches successively to the buyer. In the first scenario, each batch is delivered directly after its completion. In the second scenario, only the respective first batches of each vendor are delivered directly after their completion, and subsequent batches are delivered when the inventory level of the buyer reaches zero. In the following, we refer to both coordination policies as follows:

- Overlapping production cycles with immediate delivery (ID)
- Overlapping production cycles with delayed delivery (DD)

The main difference between these two coordination policies is the stage where the majority of inventory is kept. In case the DD policy is implemented, the majority of inventory is kept at the vendors, such that this policy leads to low costs if keeping inventory at the vendors is cheaper than keeping it at the buyer. In case the ID policy is implemented, the majority of inventory is kept at the buyer, such that this policy should be used in the opposite case. Both coordination policies are illustrated in Figures 1 and 2 for the cases of three vendors, and they will be explained in more detail in the following.

If coordination policy ID is used, vendor 1 initiates production directly after receiving the order of the buyer at time t_0. The first batch of vendor 1, $q_{1,1}$, is finished at time $t_2 = \frac{q_{1,1}}{p}$ and the second batch $q_{1,2}$ at time $t_4 = \frac{q_{1,1}+q_{1,2}}{p} = \frac{q_{1,1}\sum_{j=0}^{1}\lambda^j}{p}$. After its completion, the first batch is directly sent to the buyer. As was already pointed out earlier in this section, vendor 1 is not able to finish the second batch before the first batch that was delivered to the buyer has been used up. To avoid interruptions in the consumption process at the buyer, the second vendor has to start production in such a way that his first batch is finished at time $t_3 = \frac{q_{1,1}}{p} + \frac{q_{1,1}}{d}$. Production of this batch thus has to be initiated at $t_1 = \frac{q_{1,1}}{p} + \frac{q_{1,1}}{d} - \frac{q_{2,1}}{p}$. Figure 1 illustrates a possible scenario where three vendors (only two are shown in the figure for the sake of brevity) are required to satisfy demand at the buyer. As we assumed that $np > d$, a cycle surplus inventory may occur with the delivery of the second batch by the first vendor, and it may occur for every subsequent batch of each vendor. We define the cycle surplus for any delivery $k = 2,\dots,m$ of vendor i as the sum of all deliveries received by the buyer and including the last delivery of vendor i, reduced by the consumption that occurred during this time span (see Figure 1). As a result, the buyer's inventory may increase over time, and his inventory level only reaches zero again after all m batches of all n vendors have been consumed, i.e. at time $t_5 = \frac{q_{1,1}}{p} + \sum_{j=1}^{m}\sum_{i=1}^{n}\frac{q_{i,j}}{d}$.

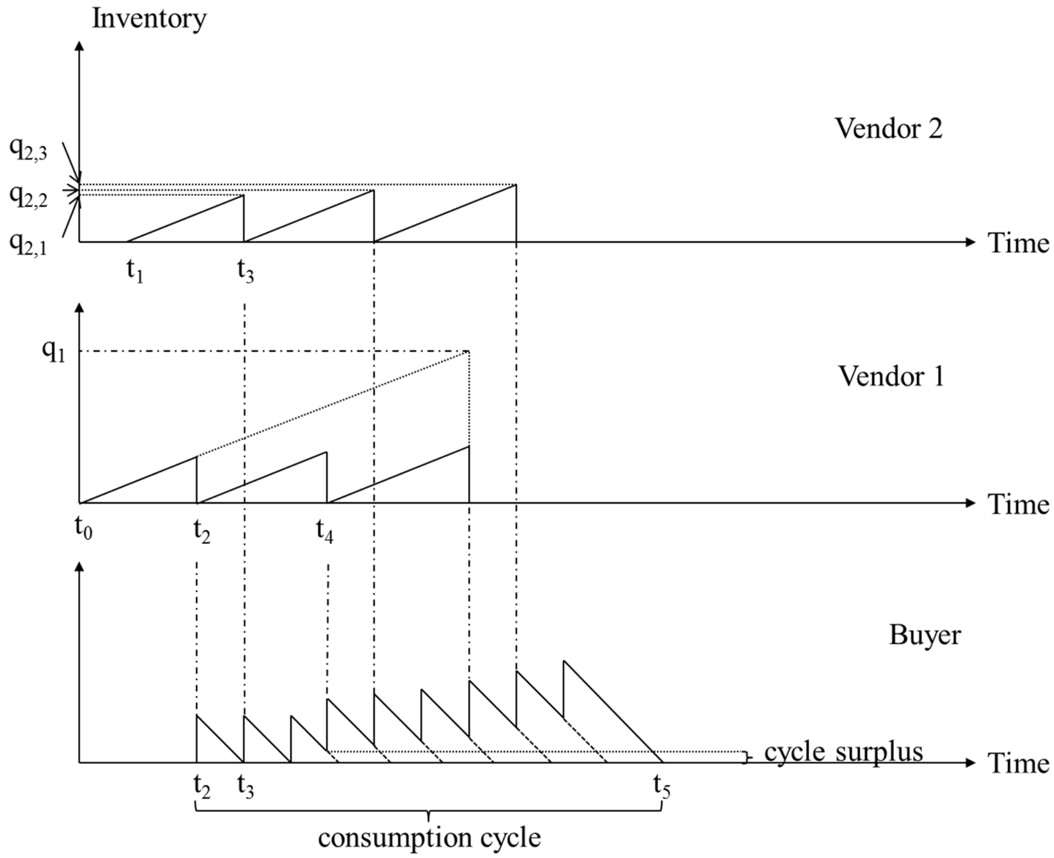

Figure 1: *Inventory-time plots for model ID for the case of three vendors with three batch shipments each (two vendors shown in the figure)*

The second coordination policy, DD, is illustrated in Figure 2 for three vendors each delivering two batches (again, only two vendors are shown in the figure). If policy DD is implemented, the vendors only ship batches to the buyer when the buyer is just about to run out of inventory. As a consequence, the majority of inventory is kept at the vendors in this case. For example, vendor 1 finishes his second batch at time $t_4 = \frac{q_1}{p}$, but keeps it in inventory until the first batch of vendor 3 has been used up by the buyer, i.e. until time $t_6 = \frac{q_{1,1}}{p} + \sum_{i=1}^{n} \frac{q_{i,1}}{d}$.

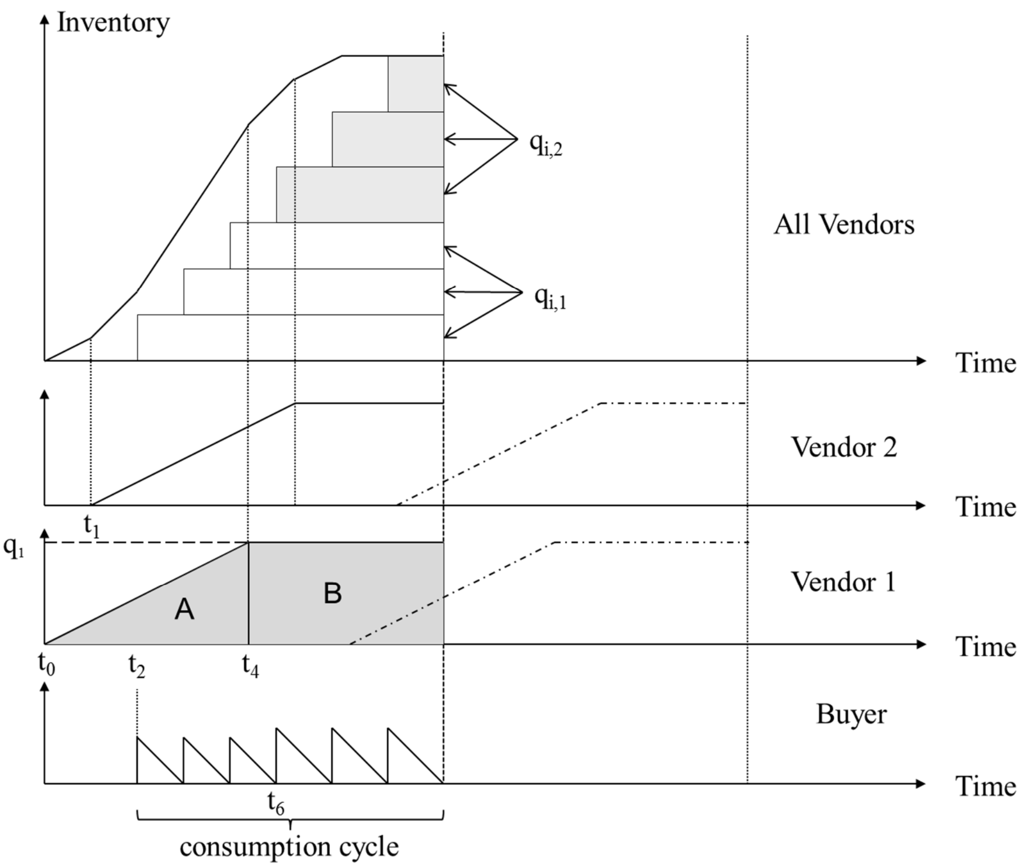

Figure 2: *Cumulative production and consumption quantities for model DD for three vendors with two batch shipments each (two vendors shown in the figure)*

The following sub-sections present mathematical models for both coordination policies.

4.2 Overlapping production cycles with immediate delivery

The inventory patterns of the buyer and the vendors for the first coordination policy (ID) are illustrated in Figure 1. The total costs per lot of vendor i can be calculated by summing up inventory carrying, setup and transportation costs for m batches:

$$TC^{v}_{i,per\ lot} = \sum_{j=1}^{m} \frac{q_{i,1}^{2}\lambda^{2j-2}h^{v}}{2p} + c_S + mc_T^{v} \tag{1}$$

Multiplying Eq. (1) with the number of lots in the planning period, $\frac{D}{Q}$, leads to the total costs of vendor i:

$$TC_i^v = \left(\sum_{j=1}^{m} \frac{q_{i,1}^2 \lambda^{2j-2} h^v}{2p} + c_S + mc_T^v\right)\frac{D}{Q} \tag{2}$$

The timing of deliveries assumed here entails that batches are shipped to the buyer from time to time even though the inventory level at the buyer is still positive. As explained in Section 4.1, a cycle surplus inventory may occur at any delivery $k = 2, \ldots, m$ of vendor j. For the k^{th} batch delivered by vendor j, it can be calculated as follows:

$$\left(\sum_{i=j}^{n} q_{i,1}\lambda^{k-2} + \sum_{i=1}^{j-1} q_{i,1}\lambda^{k-1}\right) - \frac{q_{j,1}\lambda^{k-1} d}{p} \tag{3}$$

To avoid shortages, we restrict the cycle surplus to values equal to or larger than zero. As a result, we can derive an upper bound on λ from Eq. (3):

$$\lambda \leq \frac{\sum_{i=j}^{n} q_{i,1}}{q_{j,1}\frac{d}{p} - \sum_{i=1}^{j-1} q_{i,1}} \tag{4}$$

Regarding the constraint on λ, we note that λ has to be strictly positive by definition to arrive at a feasible solution for the lot sizes. Consequently, the denominator on the right-hand side of Eq. (4) is restricted to positive values. Eq. (4) has to hold for all vendors $j = 1, \ldots, n$, and hence the minimal permissible value for λ for any of the n vendors is the upper bound on λ. Inserting the relation $q_{i,1} = \frac{\beta_i Q}{\sum_{j=1}^{m} \lambda^{j-1}}$ (see Section 3) into Eq. (4), the upper bound on λ can consequently be simplified as follows:

$$\lambda \leq \min\{\frac{\sum_{i=j}^{n} \beta_i}{\beta_j \frac{d}{p} - \sum_{i=1}^{j-1} \beta_i}, \forall j = 1, \ldots, n\} \tag{5}$$

To calculate the inventory of the buyer, we modify the method introduced by Joglekar (1988) and calculate the total inventory during the consumption of the lot size Q reduced by the inventory that is kept at the vendors. Figure 3 is used to illustrate the procedure.

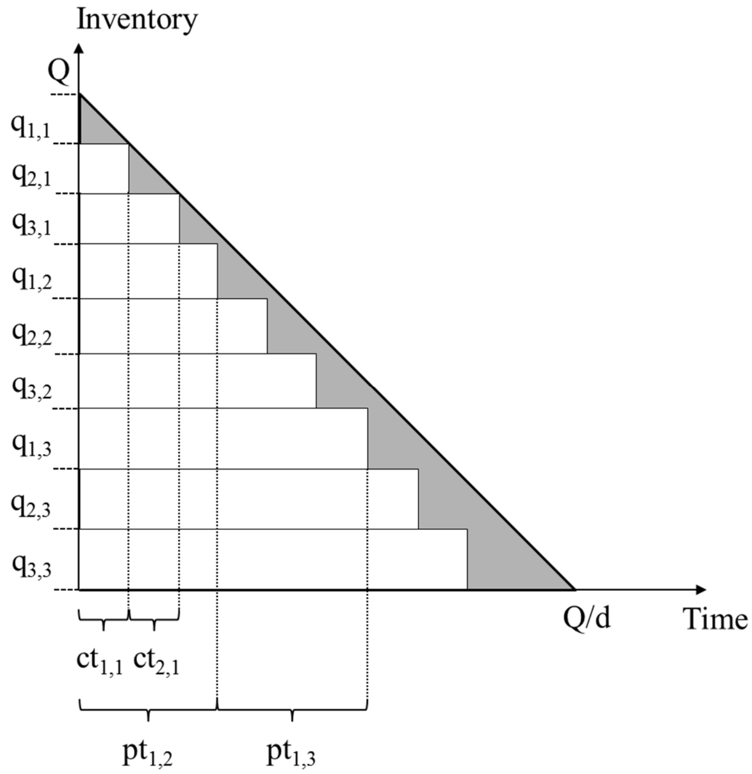

Figure 3: *Inventory-time plot of the buyer for model ID for the case of three vendors*

The inventory per lot that would occur at the buyer if the entire order quantity was delivered at once equals a triangle with height Q and width Q/d, which corresponds to:

$$\frac{Q^2}{2d} \tag{6}$$

This area needs to be reduced by the inventory that is kept at the vendors and the inventory that has not yet been built up in the system due to the phased start of the production processes at the vendors. This inventory corresponds to the white rectangles in Figure 3. In calculating the size of the white rectangles, it has to be kept in mind that all batches of each vendor are delivered directly after their completion.

The area of the first $(n-1)$ white rectangles with solid-line boundary can be computed by multiplying the height of each batch, $q_{i+1,1}$, with the time that is required to consume the first i batches that have been delivered earlier. Summing the resulting expression up over all $(n-1)$ batches leads to:

$$\sum_{i=1}^{n-1}\left(q_{i+1,1}\sum_{j=1}^{i}ct_{j,1}\right)=\sum_{i=1}^{n-1}\left(q_{i+1,1}\sum_{j=1}^{i}\frac{q_{j,1}}{d}\right) \tag{7}$$

For the remaining $n(m-1)$ batches, the corresponding rectangles can be determined by summing up the relevant consumption and production times for the width of the rectangle multiplied with the size of the batch in question:

$$\sum_{i=1}^{n}\sum_{j=2}^{m}\left(\left(\sum_{k=1}^{i-1}ct_{k,1}+\sum_{k=2}^{j}pt_{i,k}\right)q_{i,j}\right)$$
$$=\sum_{i=1}^{n}\sum_{j=2}^{m}\left(\left(\sum_{k=1}^{i-1}\frac{q_{k,1}}{d}+\sum_{k=2}^{j}\frac{q_{i,1}\lambda^{k-1}}{p}\right)q_{i,1}\lambda^{j-1}\right) \tag{8}$$

The time-weighted inventory of the buyer can now be calculated by subtracting Eqs. (7) and (8) from Eq. (6):

$$TWI^{b}=\frac{Q^{2}}{2d}-\sum_{i=1}^{n-1}\left(q_{i+1,1}\sum_{j=1}^{i}\frac{q_{j,1}}{d}\right)-\sum_{i=1}^{n}\sum_{j=2}^{m}\left(\left(\sum_{k=1}^{i-1}\frac{q_{k,1}}{d}+\sum_{k=2}^{j}\frac{q_{i,1}\lambda^{k-1}}{p}\right)q_{i,1}\lambda^{j-1}\right) \tag{9}$$

The total costs of the buyer can now be calculated if the time-weighted inventory of the buyer is multiplied with the unit inventory carrying charges per unit of time, h^{b}, and the number of lots in the planning period, $\frac{D}{Q}$, and if the ordering costs per order and the handling costs per delivery are considered in addition:

$$TC^{b}=\left(\left(\left(\frac{Q^{2}}{2d}-\sum_{i=1}^{n-1}\left(q_{i+1,1}\sum_{j=1}^{i}\frac{q_{j,1}}{d}\right)\right.\right.\right.$$
$$\left.\left.-\sum_{i=1}^{n}\sum_{j=2}^{m}\left(\left(\sum_{k=1}^{i-1}\frac{q_{k,1}}{d}+\sum_{k=2}^{j}\frac{q_{i,1}\lambda^{k-1}}{p}\right)q_{i,1}\lambda^{j-1}\right)\right)h^{b}+c_{O} \tag{10}$$
$$\left.+nmc_{T}^{b}\right)\frac{D}{Q}$$

The total costs of the system equal the sum of all TC_i^v and TC^b:

$$TC^{sys}=\sum_{i=1}^{n}TC_{i}^{v}+TC^{b}$$
$$=\left(\left(\left(\frac{Q^{2}}{2d}-\sum_{i=1}^{n-1}\left(q_{i+1,1}\sum_{j=1}^{i}\frac{q_{j,1}}{d}\right)\right.\right.\right.$$
$$\left.\left.-\sum_{i=1}^{n}\sum_{j=2}^{m}\left(\left(\sum_{k=1}^{i-1}\frac{q_{k,1}}{d}+\sum_{k=2}^{j}\frac{q_{i,1}\lambda^{k-1}}{p}\right)q_{i,1}\lambda^{j-1}\right)\right)h^{b} \tag{11}$$
$$\left.+\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{q_{i,1}^{2}\lambda^{2j-2}}{2p}h^{v}+c_{0}+n(c_{S}+mc_{T})\right)\frac{D}{Q}$$

Note that c_T represents the aggregated handling and transportation cost that occurs at the buyer and the vendors, respectively, i.e. $c_T = c_T^b + c_T^v$.

4.3 Overlapping production cycles with delayed delivery

The inventory pattern of the buyer and the vendors for the second coordination policy DD is illustrated in Figure 2. We use the method introduced by Joglekar (1988) for calculating the time-weighted inventory. For vendor i, the time-weighted inventory can be calculated by adding areas A and B shown in Figure 2 and subtracting the cumulative quantity shipped to the buyer (i.e., the rectangles denoted with $q_{i,j}$ in Figure 2). Area A for vendor i corresponds to:

$$\frac{q_i^2}{2p} \tag{12}$$

The time span between the placement of the order by the buyer and the point in time when the last unit of the order has been consumed by the buyer can be calculated as follows:

$$\frac{Q}{d}+\frac{q_{1,1}}{p} \tag{13}$$

Area B for vendor i is a rectangle with height q_i and width $\left(\frac{Q}{d}+\frac{q_{1,1}}{p}\right)-\left(\frac{q_{1,1}}{p}+\sum_{j=1}^{i-1}\frac{q_{j,1}}{d}-\frac{q_{i,1}}{p}\right)-\frac{q_i}{p}$. The width of the rectangle can be calculated by subtracting the time required to produce the lot at vendor i, i.e. $\frac{q_i}{p}$, and the time between the placement of the order by the buyer and the start of production at vendor i, i.e. $\left(\frac{q_{1,1}}{p}+\sum_{j=1}^{i-1}\frac{q_{j,1}}{d}-\frac{q_{i,1}}{p}\right)$, from Eq. (13). Therefore, the size of Area B amounts to:

$$\left(\left(\frac{Q}{d}+\frac{q_{1,1}}{p}\right)-\left(\frac{q_{1,1}}{p}+\sum_{j=1}^{i-1}\frac{q_{j,1}}{d}-\frac{q_{i,1}}{p}\right)-\frac{q_i}{p}\right)q_i \tag{14}$$

Summing up Eqs. (12) and (14) over all vendors leads to the following expression:

$$\sum_{i=1}^{n}\frac{q_i^2}{2p}+\left(\left(\frac{Q}{d}+\frac{q_{1,1}}{p}\right)-\frac{q_1}{p}\right)q_1+\sum_{i=2}^{n}\left(\frac{Q}{d}-\left(\sum_{j=1}^{i-1}\frac{q_{j,1}}{d}-\frac{q_{i,1}}{p}\right)-\frac{q_i}{p}\right)q_i \tag{15}$$

The cumulative quantity shipped to the buyer has to be subtracted from the total inventory of all vendors, and it can be calculated as the sum of the rectangles denoted with $q_{i,j}$ in Figure 2. Rectangle $k+1$, with $k=0,\dots,m-1$, of vendor i is of height $\lambda^k q_{i,1}$ and width $\left(\frac{Q}{d}-\sum_{l=0}^{k-1}\sum_{x=1}^{n}\frac{\lambda^l q_{x,1}}{d}-\sum_{j=1}^{i-1}\frac{\lambda^k q_{j,1}}{d}\right)$, where the latter is the time span between the dispatch of this batch and the end of the consumption cycle (see, for example, Joglekar (1988) and Glock (2011)):

$$\sum_{k=0}^{m-1}\sum_{i=1}^{n}\left(\left(\frac{Q}{d}-\sum_{l=0}^{k-1}\sum_{x=1}^{n}\frac{\lambda^{l}q_{x,1}}{d}-\sum_{j=1}^{i-1}\frac{\lambda^{k}q_{j,1}}{d}\right)\lambda^{k}q_{i,1}\right) \tag{16}$$

The time-weighted inventory for all vendors thus equals:

$$\begin{aligned} TWI^{v} = {} & \sum_{i=1}^{n}\frac{q_i^2}{2p}+\left(\left(\frac{Q}{d}+\frac{q_{1,1}}{p}\right)-\frac{q_1}{p}\right)q_1+\sum_{i=2}^{n}\left(\frac{Q}{d}-\left(\sum_{j=1}^{i-1}\frac{q_{j,1}}{d}-\frac{q_{i,1}}{p}\right)-\frac{q_i}{p}\right)q_i \\ & -\sum_{k=0}^{m-1}\sum_{i=1}^{n}\left(\left(\frac{Q}{d}-\sum_{l=0}^{k-1}\sum_{x=1}^{n}\frac{\lambda^{l}q_{x,1}}{d}-\sum_{j=1}^{i-1}\frac{\lambda^{k}q_{j,1}}{d}\right)\lambda^{k}q_{i,1}\right) \end{aligned} \tag{17}$$

The total costs of all vendors can now be derived if the time-weighted inventory for all vendors is multiplied with the unit inventory carrying charges per unit of time, h^{v}, and the number of lots in the planning period, $\frac{D}{Q}$, and if the setup costs per setup and the transportation costs per delivery are considered in addition:

$$TC^{v}=\left(TWI^{v}h^{v}+n(c_S+mc_T^{v})\right)\frac{D}{Q} \tag{18}$$

Note that in contrast to model ID, the cycle surplus is always zero in model DD. For model DD, instead of formulating a constraint on the cycle surplus, it is necessary to ensure that the production time for the k^{th} batch, $\forall k=2,\dots,m$, delivered by vendor j, $\forall j=1,\dots,n$, does not exceed the consumption time at the buyer's side for all deliveries received since (and including) the last delivery of vendor j, i.e.

$$\frac{q_{j,1}\lambda^{k-1}}{p}\leq\frac{\left(\sum_{i=j}^{n}q_{i,1}\lambda^{k-2}+\sum_{i=1}^{j-1}q_{i,1}\lambda^{k-1}\right)}{d} \tag{19}$$

Rearranging Eq. (19) leads to the same expression as Eq. (4) and finally to the same upper bound on λ as for model ID, which is given in Eq. (5).

The total costs of the buyer can be calculated as follows:

$$TC^b = \left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{q_{i,1}^2\lambda^{2j-2}}{2d}h^b + c_O + nmc_T^b\right)\frac{D}{Q} \tag{20}$$

The total costs of the system consist of the sum of Eqs. (18) and (20):

$$TC^{sys} = \left(TWI^v h^v + \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{q_{i,1}^2\lambda^{2j-2}}{2d}h^b + c_O + n(c_S + mc_T)\right)\frac{D}{Q} \tag{21}$$

As in Section 4.2, c_T represents the aggregate handling and transportation cost at the buyer and the vendors, i.e. $c_T = c_T^b + c_T^v$.

5 Solution of the models

5.1 Model ID

Using the relation $q_{i,1} = \frac{\beta_i Q}{\sum_{l=1}^{m}\lambda^{l-1}}$, the total system cost function in Eq. (11) can be re-written as

$$TC^{sys} = DQ\left(\left(\frac{1}{2d} - \sum_{i=1}^{n-1}\left(\frac{\beta_{i+1}}{\sum_{l=1}^{m}\lambda^{l-1}}\sum_{j=1}^{i}\frac{\beta_j}{d\sum_{l=1}^{m}\lambda^{l-1}}\right)\right.\right.$$
$$\left. - \sum_{i=1}^{n}\sum_{j=2}^{m}\left(\left(\sum_{k=1}^{i-1}\frac{\beta_k}{d\sum_{l=1}^{m}\lambda^{l-1}} + \sum_{k=2}^{j}\frac{\beta_i\lambda^{k-1}}{p\sum_{l=1}^{m}\lambda^{l-1}}\right)\frac{\beta_i}{\sum_{l=1}^{m}\lambda^{l-1}}\lambda^{j-1}\right)\right)h^b \tag{22}$$
$$\left. + \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\beta_i^2\lambda^{2j-2}}{2(\sum_{l=1}^{m}\lambda^{l-1})^2 p}h^v\right) + \left(c_0 + n(c_S + mc_T)\right)\frac{D}{Q}$$

The second partial derivative of Eq. (22) with respect to Q can be calculated as follows:

$$\frac{d^2TC^{sys}}{dQ^2} = \frac{2D(c_0 + n(c_S + mc_T))}{Q^3} \geq 0 \tag{23}$$

Thus, for given values of β_i, λ and m, Eq. (22) is convex in Q. From the first partial derivative of Eq. (22) in Q, we get:

$$Q_{opt} = \sqrt{\frac{(c_0 + n(c_S + mc_T))}{\alpha}} \tag{24}$$

where

$$\alpha = \left(\left(\frac{1}{2d} - \sum_{i=1}^{n-1} \left(\frac{\beta_{i+1}}{\sum_{l=1}^{m} \lambda^{l-1}} \sum_{j=1}^{i} \frac{\beta_j}{d \sum_{l=1}^{m} \lambda^{l-1}} \right) \right.\right.$$

$$\left. - \sum_{i=1}^{n} \sum_{j=2}^{m} \left(\left(\sum_{k=1}^{i-1} \frac{\beta_k}{d \sum_{l=1}^{m} \lambda^{l-1}} + \sum_{k=2}^{j} \frac{\beta_i \lambda^{k-1}}{p \sum_{l=1}^{m} \lambda^{l-1}} \right) \frac{\beta_i}{\sum_{l=1}^{m} \lambda^{l-1}} \lambda^{j-1} \right) \right) h^b \tag{25}$$

$$\left. + \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\beta_i^2 \lambda^{2j-2}}{2(\sum_{l=1}^{m} \lambda^{l-1})^2 p} h^v \right)$$

Inserting Eq. (24) into Eq. (22) leads to the following total cost function:

$$TC^{sys} = 2D\sqrt{\left(c_0 + n(c_S + mc_T)\right)\alpha} \tag{26}$$

The sum of all β_i has to be equal to 1, and every β_i is defined on a finite interval. Clearly, to ensure that the model remains feasible, each vendor cannot produce more than $\frac{p}{d}$ percent of the total buyer demand, and in addition each vendor has to produce at least $1 - (n-1)\frac{p}{d}$ percent of the total buyer demand to avoid shortages.

Since numerical experiments showed that the objective function (22) is convex in m (one example can be found in Appendix A), and since we are not able to derive a closed-form solution for the optimal number of batch shipments due to the complexity of the objective function, the number of batch shipments was optimized by increasing m stepwise starting with $m = 1$ until an increase in the total costs occurred. In this case, we adopted $m = m - 1$ as the solution to the problem (cf., for a similar solution procedure, Bogaschewsky et al., 2001). Due to the assumption that shipment sizes should be non-decreasing, one could use a simple search algorithm (see, for example, Gill et al., 1981) to optimize λ on the interval $\left[1; \min\{\frac{\sum_{i=j}^{n} \beta_i}{\beta_j \frac{d}{p} - \sum_{i=1}^{j-1} \beta_i}, \forall j = 1, \dots, n\}\right]$, where the right interval boundary is taken from Eq. (5).

The solution procedure applied to model ID can now be summarized as follows:

Step 1: Set $m = 1$ and $TC^* = \infty$.

Step 2: Calculate $\beta_i \epsilon \left]1 - (n-1)\frac{p}{d}; \frac{p}{d}\right[, \forall i = 1, \dots, n$ with $\sum_{i=1}^{n} \beta_i = 1$ and $\lambda \epsilon \left[1; \min\{\frac{\sum_{i=j}^{n} \beta_i}{\beta_j \frac{d}{p} - \sum_{i=1}^{j-1} \beta_i}, \forall j = 1, \dots, n\}\right]$ that minimizes TC^{sys}. Calculate TC^{sys} according to Eq. (26). If $TC^{sys} > TC^*$, go to Step 4, else go to Step 3.

Step 3: Set $TC^* = TC^{sys}, \beta_i^* = \beta_i, \forall i, \lambda^* = \lambda, m = m + 1$ and go to Step 2.

Step 4: $m_{opt} = m - 1,\ \lambda_{opt} = \lambda^*, \beta_{i,opt} = \beta_i^*, \forall i$. Determine Q_{opt} from Eq. (24).

In Step 2, we use the standard-solver Minimize of Wolfram Mathematica 8.1 to calculate an approximate solution to the above problem. Since the values for β_i and λ are defined on finite intervals, Mathematica is able to compute near-optimal solutions.

5.2 Model DD

Inserting $q_{i,1} = \frac{\beta_i Q}{\sum_{j=1}^{m} \lambda^{j-1}}$ and $q_i = \beta_i Q$ into Eq. (21), the total system cost function can be written as

$$TC^{sys} = DQ\left(\left(\sum_{i=1}^{n}\frac{\beta_i^2}{2p} + \left(\left(\frac{1}{d} + \frac{\beta_1}{p\sum_{t=1}^{m}\lambda^{t-1}}\right) - \frac{\beta_1}{p}\right)\beta_1 + \sum_{i=2}^{n}\left(\frac{1}{d} - \left(\sum_{j=1}^{i-1}\frac{\beta_j}{d\sum_{t=1}^{m}\lambda^{t-1}} - \frac{\beta_i}{p\sum_{t=1}^{m}\lambda^{t-1}}\right) - \frac{\beta_i}{p}\right)\beta_i - \sum_{k=0}^{m-1}\sum_{i=1}^{n}\left(\left(\frac{1}{d} - \sum_{l=0}^{k-1}\sum_{x=1}^{n}\frac{\lambda^l\beta_x}{d\sum_{o=1}^{m}\lambda^{o-1}} - \sum_{j=1}^{i-1}\frac{\lambda^k\beta_j}{d\sum_{l=1}^{m}\lambda^{l-1}}\right)\lambda^k\frac{\beta_i}{\sum_{l=1}^{m}\lambda^{l-1}}\right)\right)h^v + \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\beta_i^2\lambda^{2j-2}}{2d\left(\sum_{l=1}^{m}\lambda^{l-1}\right)^2}h^b\right) + \left(c_O + n(c_S + mc_T)\right)\frac{D}{Q} \tag{27}$$

The second partial derivative of this function in Q can be calculated as follows:

$$\frac{d^2TC^{sys}}{dQ^2} = \frac{2D(c_0 + n(c_S + mc_T))}{Q^3} \geq 0 \tag{28}$$

Thus, for given values of β_i, λ and m, Eq. (27) is convex in Q. From the first partial derivative of Eq. (27) in Q, we get

$$Q_{opt} = \sqrt{\frac{(c_0 + n(c_S + mc_T))}{\gamma}} \tag{29}$$

where

$$
\begin{aligned}
\gamma = \Bigg(\Bigg(&\sum_{i=1}^{n}\frac{\beta_i^2}{2p} + \Bigg(\Bigg(\frac{1}{d} + \frac{\beta_1}{p\sum_{t=1}^{m}\lambda^{t-1}}\Bigg) - \frac{\beta_1}{p}\Bigg)\beta_1 \\
&+ \sum_{i=2}^{n}\Bigg(\frac{1}{d} - \Bigg(\sum_{j=1}^{i-1}\frac{\beta_j}{d\sum_{t=1}^{m}\lambda^{t-1}} - \frac{\beta_i}{p\sum_{t=1}^{m}\lambda^{t-1}}\Bigg) - \frac{\beta_i}{p}\Bigg)\beta_i \\
&- \sum_{k=0}^{m-1}\sum_{i=1}^{n}\Bigg(\Bigg(\frac{1}{d} - \sum_{l=0}^{k-1}\sum_{x=1}^{n}\frac{\lambda^l\beta_x}{d\sum_{o=1}^{m}\lambda^{o-1}} \\
&- \sum_{j=1}^{i-1}\frac{\lambda^k\beta_j}{d\sum_{l=1}^{m}\lambda^{l-1}}\Bigg)\lambda^k\frac{\beta_i}{\sum_{l=1}^{m}\lambda^{l-1}}\Bigg)\Bigg)h^v + \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\beta_i^2\lambda^{2j-2}}{2d(\sum_{l=1}^{m}\lambda^{l-1})^2}h^b\Bigg)
\end{aligned}
\tag{30}
$$

Inserting Eq. (29) into Eq. (27) leads to the following total system cost function:

$$TC^{sys} = 2D\sqrt{(c_0 + n(c_S + mc_T))\gamma} \tag{31}$$

β_i is defined on the same interval as in Section 5.1. We assume again that λ is limited to values equal to or larger than 1 and use the same procedure as in Section 5.1 to optimize the objective function. The batch shipment frequency m is again enumerated.

The solution procedure for model DD can be summarized as follows:

Step 1: Set $m = 1$ and $TC^* = \infty$.

Step 2: Calculate $\beta_i \epsilon \left]1 - (n-1)\frac{p}{d}; \frac{p}{d}\right[, \forall i = 1, \dots, n$ with $\sum_{i=1}^{n}\beta_i = 1$ and $\lambda \epsilon \left[1; \min\{\frac{\sum_{i=j}^{n}\beta_i}{\beta_j\frac{d}{p} - \sum_{i=1}^{j-1}\beta_i}, \forall j = 1, \dots, n\}\right]$ that minimizes TC^{sys}. Calculate TC^{sys} according to Eq. (31). If $TC^{sys} > TC^*$, go to Step 4, else go to Step 3.

Step 3: Set $TC^* = TC^{sys}, \beta_i^* = \beta_i, \forall i, \lambda^* = \lambda, m = m + 1$ and go to Step 2.

Step 4: $m_{opt} = m - 1,\ \lambda_{opt} = \lambda^*, \beta_{i,opt} = \beta_i^*, \forall i$. Determine Q_{opt} from Eq. (29).

In Step 2, we use the standard-solver Minimize of Wolfram Mathematica 8.1 to calculate an approximate solution to the above problem. Since the values for β_i and λ are defined on finite intervals, Mathematica is able to compute near-optimal solutions.

6 Numerical examples

To illustrate the behavior of the coordination policy proposed in this paper, a numerical study with the following data set was performed:

Vendors: $p = 200, c_T = 15, c_S = 80, h^v = 2.5$

Buyer: $d = 500, c_O = 50, h^b = 3.5, T = 5, D = 2{,}500$

In this example, because of $n = \left\lceil \frac{d}{p} \right\rceil = 3$, three vendors are needed to guarantee an uninterrupted supply of materials at the buyer's side. The policies proposed in this paper are compared to the policies suggested by Glock (2012a) to assess the impact of using geometrically increasing batch shipments: If we set $\lambda = 1$, subsequent batches have the same size, and our models reduce to those proposed by Glock (2012a). We used Wolfram Mathematica 8.1 for our calculations. The results are summarized in Table 1.

Table 1: *Results of the numerical study*

	ID		**DD**	
	Equal shipments	*Increasing shipments*	*Equal shipments*	*Increasing shipments*
Lot size (Q)	934.07	1,002.77	1,065.46	1,002.77
Lot sizes of the vendors (q_1, q_2, q_3)	(373.63,186.81, 373.63)	(303.83,333.30, 365.64)	(426.19,213.09, 426.19)	(303.83,333.30, 365.64)
Number of batch shipments (m)	8	7	9	7
Proportional factor (λ)	1	1.32	1	1.32
Total costs (TC^{sys})	3,479.40	3,016.66	3,261.5	3,016.66

Table 1 shows that models DD and ID with geometrically increasing batch shipments lead to the lowest total system costs, whereas model ID with equal-sized shipments leads to the highest total system costs for the scenario considered. As can be seen, the models lead to lot sizes between 934.07 and 1065.46. For the number of batch shipments, a similar result was obtained for all models, and 7 to 9 shipments are needed in all cases. Another result is that in all four models studied here, the vendors produce different lot sizes, even though they are homogeneous with respect to their cost and performance parameters. Only for the equal-sized shipments models, vendors 1 and 3 (ID and DD) produce the same lot sizes. Another interesting result of this example is that the results obtained for the models DD and ID with geometrically increasing batch shipments are identical. Permitting geometrically increasing batch shipments obviously gives the system the opportunity to reduce (or even avoid) the cycle surplus at the buyer by increasing the inventory that is kept at the vendors. This is especially beneficial in case the inventory carrying cost are (much) higher at the buyer than at the vendors. As the ratio of the buyer's inventory holding cost to the inventory holding cost of the vendors gets lower, the system decides to keep more and more inventory at the buyer, and the cycle surplus increases again. Permitting geometrically increasing batch shipments and introducing the variable λ into the model obviously gives the system additional flexibility as compared to the models proposed by Glock (2012a).

To gain further insights into the models, we examined the influence of an increase in p ($p \in \{170; 180; 190; \dots; 240\}$) on the total system costs of the different models and on the number of batch shipments (The exact results for the total system costs and the number of batch shipments can be found in Appendix B. In addition, the values for the β_i and λ are also presented in Appendix B.). The values of p were chosen in such a way to ensure that the number of required vendors constantly equals 3. In case n is not changed, a higher value of p leads to more idle time for the vendors, as the total production capacity of the vendors exceeds the buyer's demand more and more. Figure 4 illustrates the development of the total system costs of the models for different values of p. Obviously, model ID with equal-sized shipments leads to the highest costs for all examined values of p. It can also be noted that the resulting total system costs of model DD with equal-sized shipments are always lower than for model ID with equal-sized shipments. Thus, in the scenario studied here, for the equal-sized shipment policy, model DD should be preferred. For $170 \leq p \leq 210$, models DD and ID with geometrically increasing batch shipments led to the lowest total system costs, and for $220 \leq p \leq 240$, models DD with equal-sized and geometrically increasing batch shipments led to the lowest total system costs. Another interesting result again is that for $170 \leq p \leq 210$, models DD and ID with geometrically increasing batch shipments led to the same results. Again, the ratio of the buyer's inventory carrying cost to the inventory carrying cost of the vendors is responsible for this result. Appendix B gives an explanation for these results and shows that in many cases, the two policies are identical, i.e. the number of batch shipments m and the value for λ are the same. These findings confirm the results obtained by Viswanathan (1998), who showed that none of the batch shipment policies dominates the respective other in the single vendor-single buyer case. As a result, it is necessary to identify the best shipment policy for each application individually. The non-monotonic behavior of the total system cost can be explained as follows: In our example, we consider p-values with $p \in \{170; 180; 190; \ldots; 240\}$. Especially the boundaries of the set represent "extreme" values for the structure of the supplier base, where for $p = 170$, three vendors are just enough to meet the demand of the buyer, whereas for $p = 240$, two vendors are almost able to meet the demand of the buyer. Regarding the β_i-values in Table B.1 in Appendix B, one can see that for small values of p, the order quantity is equally divided among the vendors, whereas for high values of p, two vendors produce almost the entire order quantity, and only a small quantity is assigned to the remaining vendor. Using these two extreme situations and switching to the equal-sized batch shipment policy for high p-values, the system is able to avoid the increase in inventory in the total system although the order quantities are relatively high. In the middle part of the p set, the buyer's order quantity is still assigned in a relatively equal way to the vendors, with the variance in the vendors' production quantities becoming larger as p increases. Since the sum of the vendors' production rates is already significantly higher than the buyer's demand rate in the middle part of the p set, inventory is built up very quickly in the system. Therefore, the system reacts with a reduction of the order quantity, which leads to higher ordering cost. Nevertheless, the system appears to be more flexible in the extreme situations, i.e., with p-values close to the boundaries of the set mentioned above, which helps to lower total cost.

Figure 5 illustrates the number of shipments per lot for different values of p. In all cases, the number of shipments decreased roughly for $170 \leq p \leq 210$ and increased for $220 \leq p \leq 240$. This behavior can be explained as follows: First, an increase in p leads to a shorter production time for a given batch size at vendor i, which entails lower inventory in the system. In this case, the system is able to reduce transportation cost by lowering m and by consequently trading off transportation against inventory carrying costs. However, once the gap between the total production rate of the vendors and the demand rate of the buyer becomes too large, the vendors push too much inventory into the system, which gives the system an incentive to increase the number of batch shipments again. Smaller batch shipments, which result in the latter case, lead to an earlier initiation of the consumption phase and a smaller increase in the system inventory during the production cycle. Finally, we observed that for large production rates, the system switches to an equal-sized batch shipment policy to avoid that all vendors initiate their production cycles (too) early, which would be the consequence of a (very) small first batch shipment. The result is an initially slow increase in the system inventory (cf. Figures 1 and 2).

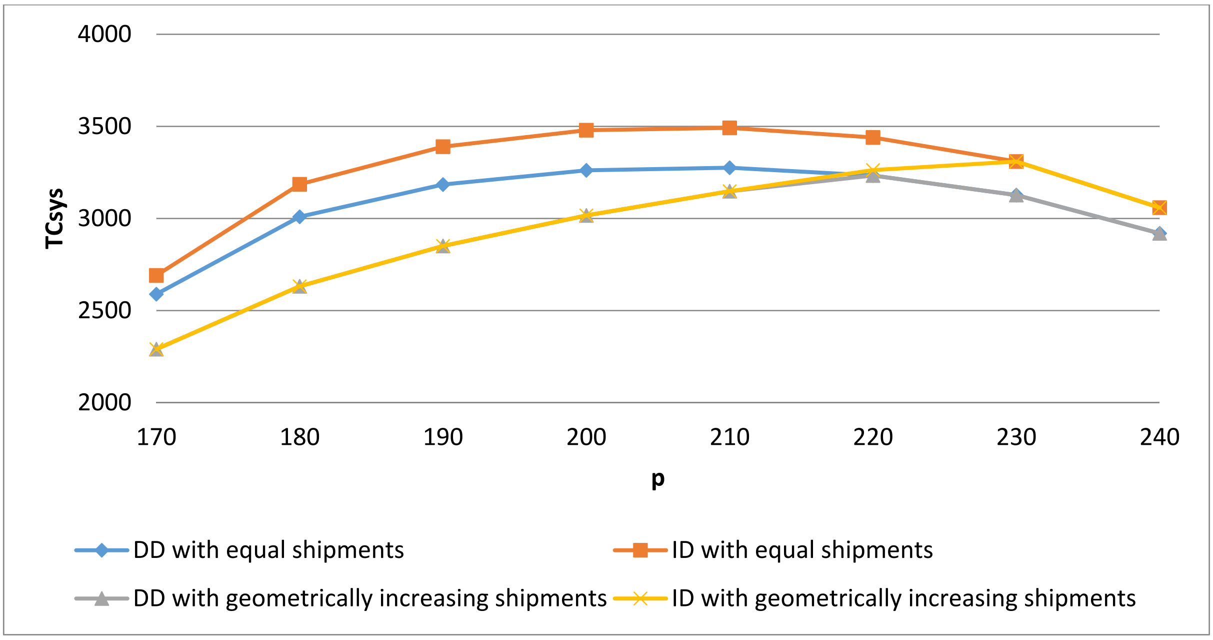

Figure 4: *Influence of different values of p on the total system costs*

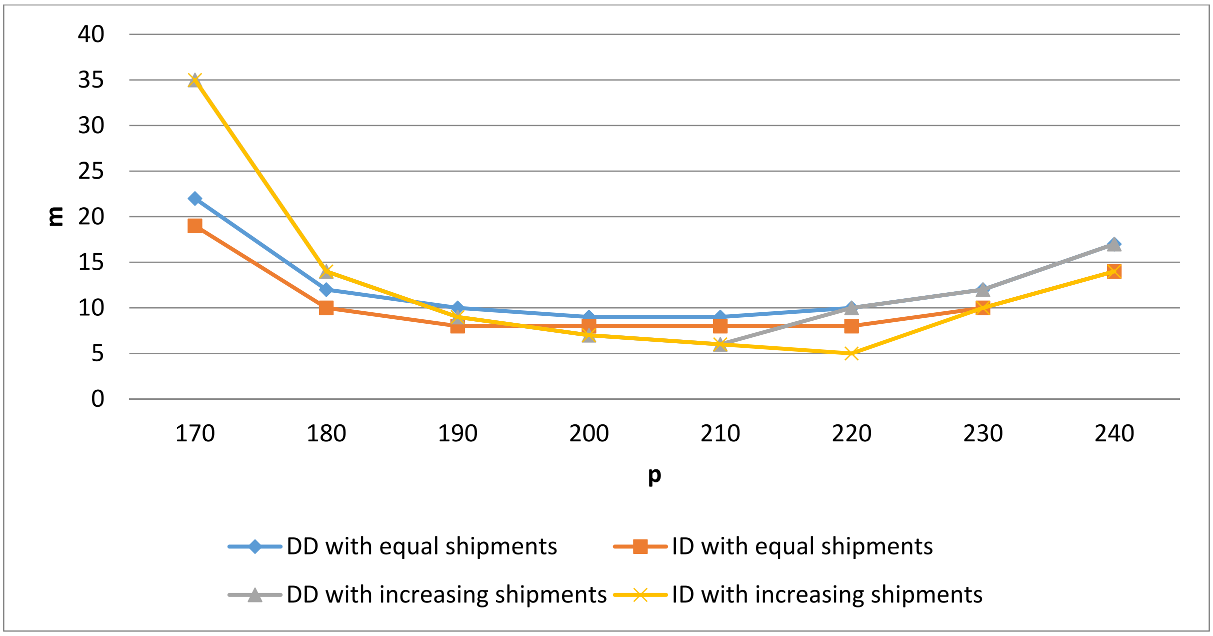

Figure 5: *The influence of different values of p on the number of batch shipments*

Since the focus of our paper is on the shipment strategies, we also investigate the influence of different values for the aggregated handling and transportation cost per delivery c_T ($c_T \in \{5; 10; 15; \ldots; 50\}$) on the total system costs of the different models and on the number of batch shipments (The exact results for the total system costs and the number of batch shipments can be found in Appendix B. The values for the β_i and λ are also presented in Appendix B.). In addition, we change the value of $p = 200$ to $p = 220$ because our earlier results indicate that this value might lead to interesting results (change in the cost behavior in Figure 4). Figure 6 illustrates the development of the total system costs of the models for different values of c_T. It shows that model ID with equal-sized shipments is dominated by all other policies for the given data set. For $20 \leq c_T \leq 50$, models DD and ID with geometrically increasing batch shipments led to the same results and dominate model DD with equal-sized shipments.

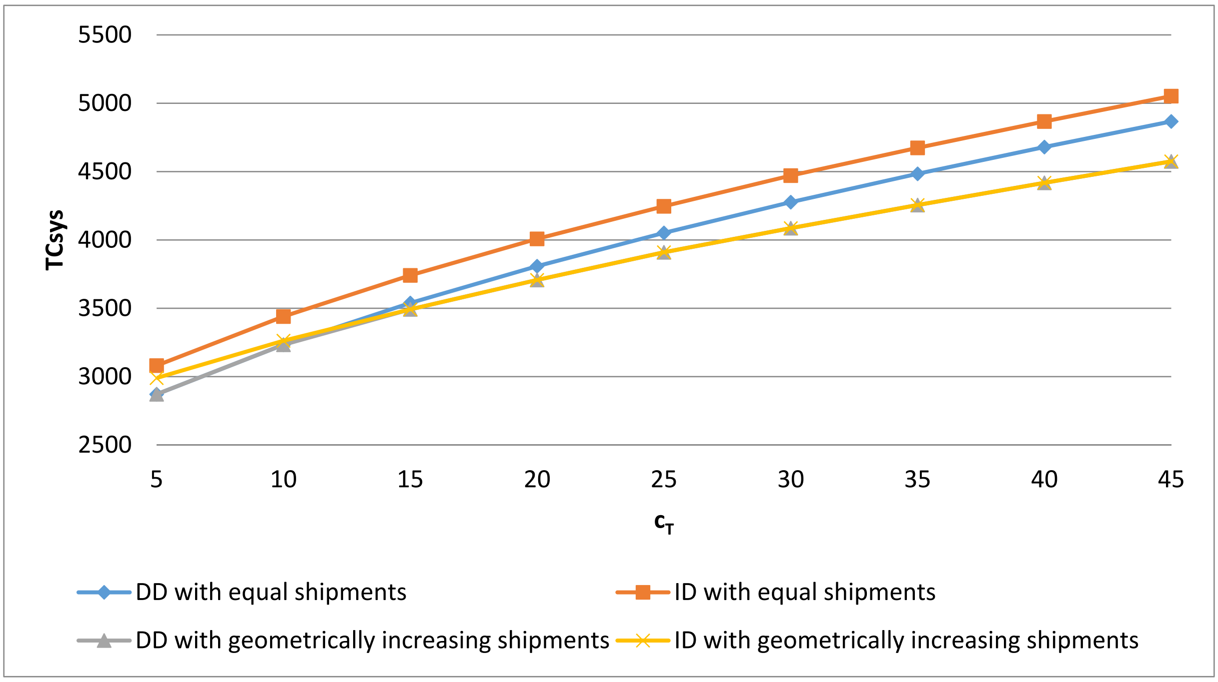

Figure 6: *The influence of different values of c_T on the total system costs*

Figure 7 illustrates the development of the number of batch shipments for different values of c_T. Since only the handling and transportation cost per delivery are changed in the data set, the results are not surprising and can be summarized as follows: The higher the handling and transportation cost, the fewer batches are shipped from the vendors to the buyer. In addition, the policies using geometrically increasing batch shipments made fewer shipments on average than the models using an equal-sized shipment policy.

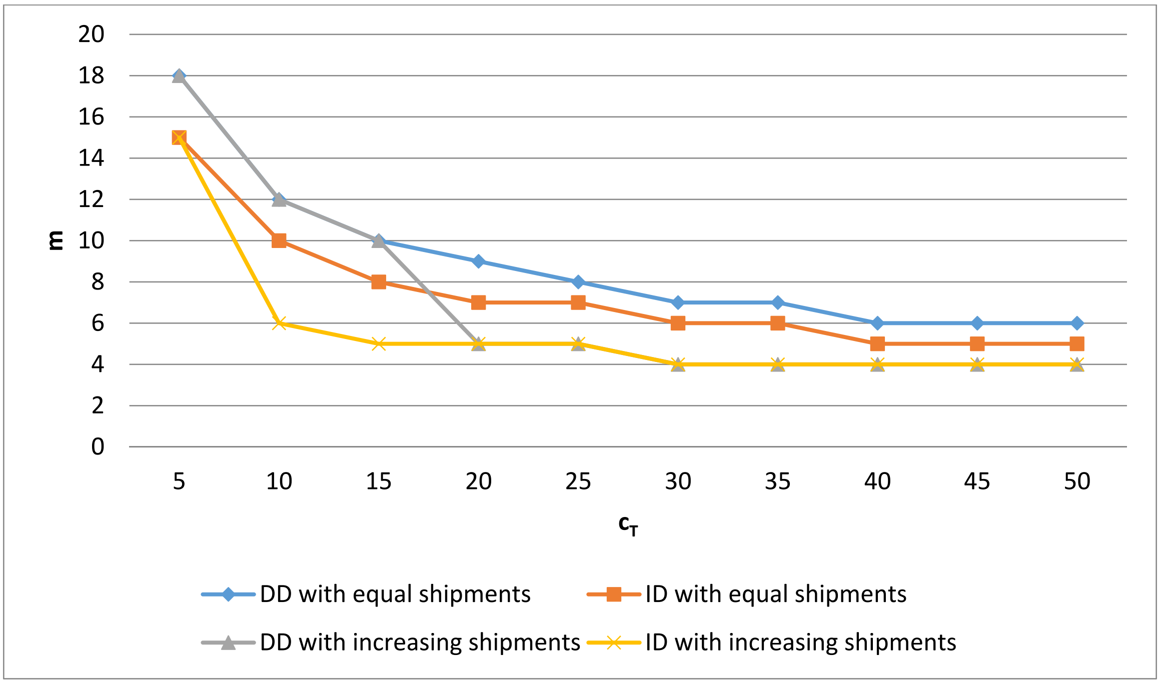

***Figure 7:** The influence of different values of c_T on the number of batch shipments*

Finally, we changed the unit inventory carrying charges per unit of time at the buyer and the vendors to $h^b = 2.5$ and $h^v = 4.0$ and investigated their influence in combination with different values of p (see Figure 8) and c_T (see Figure 9) on the total system costs. Since the holding costs at the vendors are higher than at the buyer now, Figure 8 shows that the behavior of the system changed as compared to the results shown in Figure 4. Now, model DD with equal-sized shipments leads to the highest total costs for all examined values of p. For $170 \leq p \leq 210$, models DD and ID with geometrically increasing shipments led to the lowest total system costs. For $220 \leq p \leq 240$, model ID with equal-sized shipments and model ID with geometrically increasing shipments led to the lowest total system costs. As in the former example, the fact that models with equal-sized and geometrically increasing batch shipments led to the same results for $p \geq 230$ can be explained by the fact that the lowest total cost occurred for $\lambda = 1$ in this case.

Figure 9 illustrates the influence of different values of c_T on the total system costs. As in Figure 8, model DD with equal-sized shipments led to the highest total costs in all examined cases, whereas model ID with geometrically increasing shipments led to the lowest total costs for all values of c_T. In addition, it can be seen that for $5 \leq c_T \leq 25$, model ID with equal-sized shipments led to lower total costs than model DD with geometrically increasing shipments. These results change for c_T-values equal to or larger than 30.

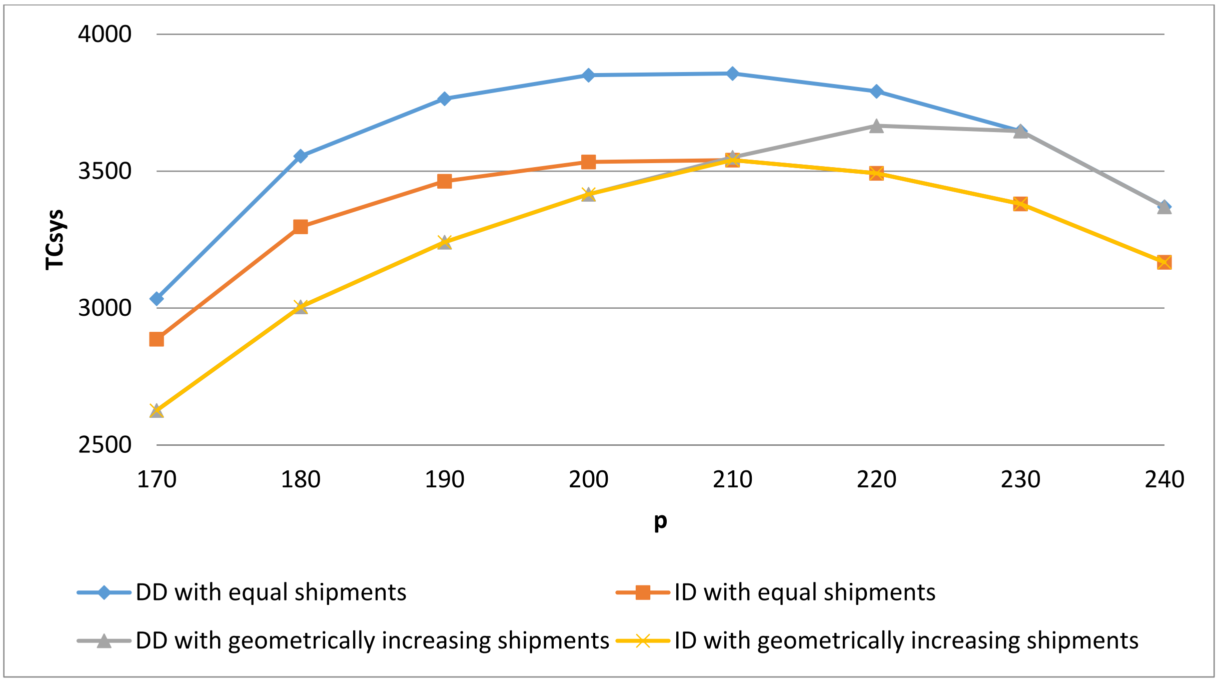

Figure 8: *The influence of varying values of p on the total system costs for* $h^v > h^b$

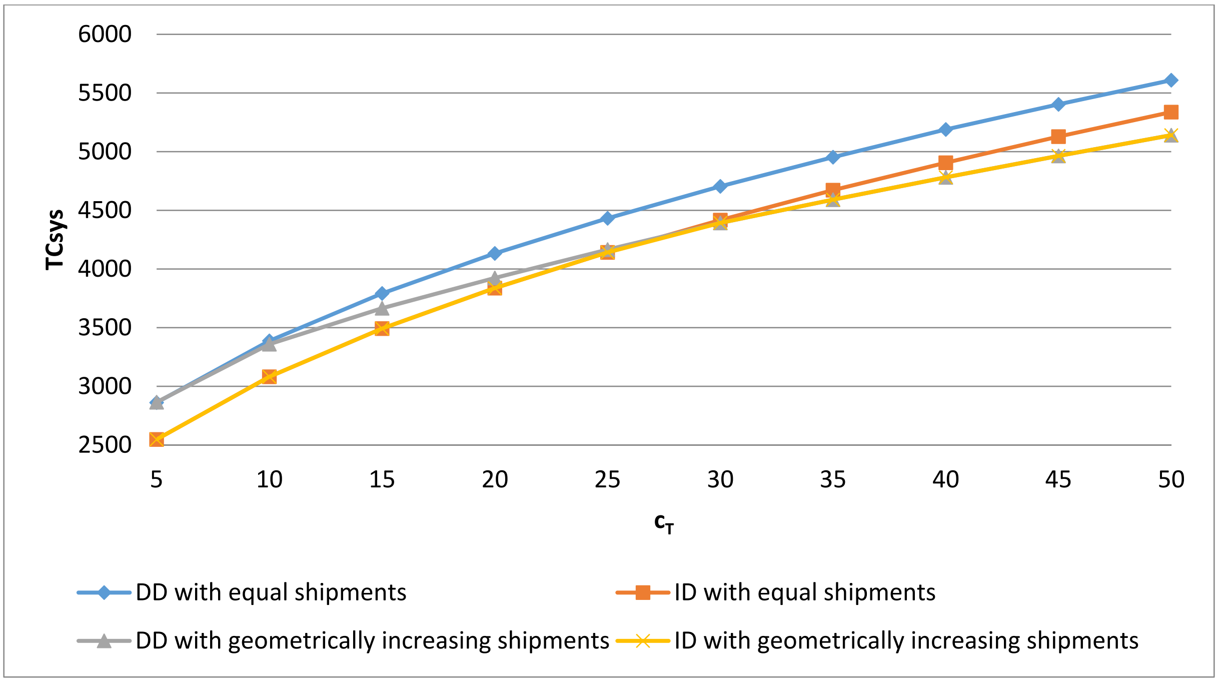

Figure 9: *Influence of different values of* c_T *on the total system costs*

7 Conclusion

This paper studied a supply chain consisting of multiple vendors supplying a single product to a single buyer. The paper generalized the models of Glock (2012a) by assuming that consecutive batch shipments from the vendors to the buyer increase by a fixed factor. For the generalized models, solution procedures were suggested. In comparison to the case of equal-sized batch shipments discussed by Glock (2012a), using the batch shipment policies proposed in this paper may help to reduce total system costs.

The results of numerical experiments indicate that none of the models dominates the respective other models in all possible scenarios. Obviously, the relative advantage of a model depends on the given cost parameters. For instance, the relative magnitude of the unit inventory carrying charges on the vendors' and the buyer's side were shown to affect the relative performance of the models significantly. An interesting result is that in some cases, the geometrically increasing batch shipment policy had no cost advantage over the equal-sized batch shipment policy of the same delivery structure, such that the parameter that influences the increase of the unequal-sized batch shipments was set to 1. These findings confirm earlier results obtained by Viswanathan (1998) for the single vendor-single buyer case.

As compared to the models developed by Glock (2012a), permitting the delivery of geometrically increasing batch shipments gives additional flexibility to the system to reduce the cycle surplus at the buyer. Especially in situations where keeping inventory at the buyer is expensive or where the buyer intends to move to just-in-time deliveries with low inventory levels, reducing the cycle surplus at the buyer and moving the majority of inventory to the vendors may be desirable. The models proposed in this paper support the management of production and inventory replenishment decisions in such a scenario. Another result worth mentioning is that always one of the models ID or DD with equal-sized batch shipments was dominated by the three other models in all cases we investigated, depending on the given cost parameters. Even though this result was derived from numerical examples and may not be valid in general, it may still indicate that the use of the affected coordination policies should be very carefully evaluated in practice, as other policies may lead to better results.

To increase the scope of our analysis, the models proposed in this paper could be extended in various ways. For example, it would be interesting to consider heterogeneous vendors or to assume that the buyer faces a pool of pre-selected vendors, out of which a set of vendors has to be selected. For the case of heterogeneous vendors, it would also be interesting to relax the assumption of an equal number of batches for all vendors. Clearly, this relaxation would significantly increase the complexity of the model and may make other solution procedures for finding an optimal value for m necessary. In addition, studying a multiple vendor-multiple buyer planning situation could be of interest, especially in case the buyers and/or the vendors compete with each other. For such a supply chain, it would be interesting to investigate whether one of the batch shipment policies dominates the others, or whether completely different policies should be implemented. Permitting a combination of geometrically increasing and equal-sized batch shipments could also lead to good results. In such a situation, the first x shipments would increase in size according to a fixed factor, and the last $(m - x)$ shipments would be of equal sizes (see, e.g., Goyal, 2000). Since the focus of our paper was on the total system costs, it would also be interesting to study the division of the additional profit generated by the system as a result of cooperation between the vendors and the buyer. To ensure that all supply chain members benefit from cooperation, and to give all of them an incentive to participate in the cooperation, it may be necessary to implement suitable incentive systems. An overview of contracts that might serve as an incentive mechanism in the scenario investigated in this paper is provided by Tsay et al. (1998). We leave these and other extensions for future research.

References

Agrawal, K., Raju, A., 1996. Improved joint economic lot size model for purchaser and a vendor. In: Khan, M.K., Wright, C.S., Whalley, R. (Eds.), Advanced Manufacturing Processes, Systems, and Technologies, ASME Books, London, 579-587.

Banerjee, A., 1986. A joint economic-lot-size model for purchaser and vendor. Decision Sciences, 17(3), 292-311.

Banerjee, A., Burton, J.S., 1994. A coordinated order-up-to inventory control policy for a single supplier and multiple buyers using electronic data interchange. International Journal of Production Economics, 35(1-3), 85-91.

Bazan, E., Jaber, M.Y., Zanoni, S., 2015. Supply chain models with greenhouse gases emissions, energy usage and different coordination decisions. Applied Mathematical Modelling, 39(17), 5131-5151.

Ben-Daya, M., Al-Nassar, A., 2008. An integrated inventory production system in a three-layer supply chain. Production Planning and Control, 19(2), 97-104.

Bogaschewsky, R.W., Buscher, U.D., Lindner, G., 2001. Optimizing multi-stage production with constant lot size and varying number of unequal sized batches. Omega, 29(2), 183-191.

Chatterjee, A.K., Ravi, R., 1991. Joint economic lot-size model with delivery in sub-batches. Operations Research, 28(2), 118-124.

Costantino, N., Pellegrino, R., 2010. Choosing between single and multiple sourcing based on supplier default risk: a real options approach. Journal of Purchasing and Supply Management, 16(1), 27-40.

Gill, P.E., Murray, W., Wright, M.H., 1981. Practical optimization. Academic Press, London and New York.

Glock, C.H., 2009. A comment: "Integrated single vendor-single buyer model with stochastic demand and variable lead time". International Journal of Production Economics, 122(2), 790-792.

Glock, C.H., 2011. A multiple-vendor single-buyer integrated inventory model with a variable number of vendors. Computers & Industrial Engineering, 60(1), 173-182.

Glock, C.H., 2012a. Coordination of a production network with a single buyer and multiple vendors. International Journal of Production Economics, 135(2), 771-780.

Glock, C.H., 2012b. The joint economic lot size problem: a review. International Journal of Production Economics, 135(2), 671-686.

Glock, C.H., Kim, T., 2014. Shipment consolidation in a multiple-vendor-single-buyer integrated inventory model. Computers & Industrial Engineering, 70, 31-42.

Glock, C.H., Kim, T., 2015. Coordinating a supply chain with a heterogeneous vehicle fleet under greenhouse gas emissions. The International Journal of Logistics Management, 26(3), 494-516.

Goyal, S.K., 1977. An integrated inventory model for a single supplier-single customer problem. International Journal of Production Research, 15(1), 107-111.

Goyal, S.K., 1995. A one-vendor multi-buyer integrated inventory model: a comment. Decision Sciences, 19, 236-241.

Goyal, S.K., Nebebe, F., 2000. Determination of economic production-shipment policy for a single-vendor-single-buyer system. European Journal of Operational Research, 121(1), 175-178.

Goyal, S.K., 2000. On improving the single-vendor single-buyer integrated production inventory model with a generalized policy. European Journal of Operational Research, 125(2), 429-430.

Hill, R.M., 1997. The single-vendor single-buyer integrated production-inventory model with a generalized policy. European Journal of Operational Research, 97(3), 493-499.

Hill, R.M., 1999. The optimal production and shipment policy for the single-vendor single-buyer integrated production-inventory problem. International Journal of Production Research, 37(11), 2463-2475.

Jaber, M.Y., Glock, C.H., El Saadany, A.M.A., 2013. Supply chain coordination with emissions reduction incentives. International Journal of Production Research, 51(1), 69-82.

Jaber, M.Y., Goyal, S.K., 2008. Coordinating a three-level supply chain with multiple suppliers, a vendor and multiple buyers. International Journal of Production Economics, 116(1), 95-103.

Joglekar, P.N., 1988. Comments on a quantity discount pricing model to increase vendor profits. Management Science, 34(11), 1391-1398.

Joglekar, P.N., Tharthare, S., 1990. The individually responsible and rational decision approach to economic lot sizes for one vendor and many purchasers. Decision Sciences, 21(3), 492-506.

Kim, T., Goyal, S.K., 2009. A consolidated delivery policy of multiple suppliers for a single buyer. International Journal of Procurement Management, 2(3), 267-287.

Leung, K.N.F., 2010. A generalized algebraic model for optimizing inventory decisions in centralized or decentralized multi-stage multi-firm supply chain. Transportation Research Part E: Logistics and Transportation Review, 46(6), 896-912.

Ouyang, L.-Y., Wu, K.-S., Ho, C.-H., 2004. Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time. International Journal of Production Economics, 92(3), 255-266.

Pan, J.C.-H., Yang, J.-S., 2002. A study of an integrated inventory with controllable lead time. International Journal of Production Research, 40(5), 1263-1273.

Seliaman, M.E., Ahmad, A.R., 2009. A generalized algebraic model for optimizing inventory decisions in a multi-stage complex supply chain. Transportation Research Part E: Logistics and Transportation Review, 45(3), 409-418.

Tsay, A.A., Nahmias, S., Agrawal, N., 1998. Modeling supply chain contracts: a review. In: Tayur S., Ganeshan R., Magazine M. (Eds), Quantitative Models for Supply Chain Management. Kluwer, Boston (Chapter 10).

Viswanathan, S., 1998. Optimal strategy for the integrated vendor-buyer inventory model. European Journal of Operational Research, 105(1), 38-42.

Woo, Y.Y., Hsu, S.-L., Wu, S., 2000. Order processing cost reduction in a joint vendor-buyer inventory system via the application of information technology. The Engineering Economist, 45(4), 350-365.

Appendix

Appendix A

Figure A.1 shows the objective function Eq. (27) as a function of m for given parameters.

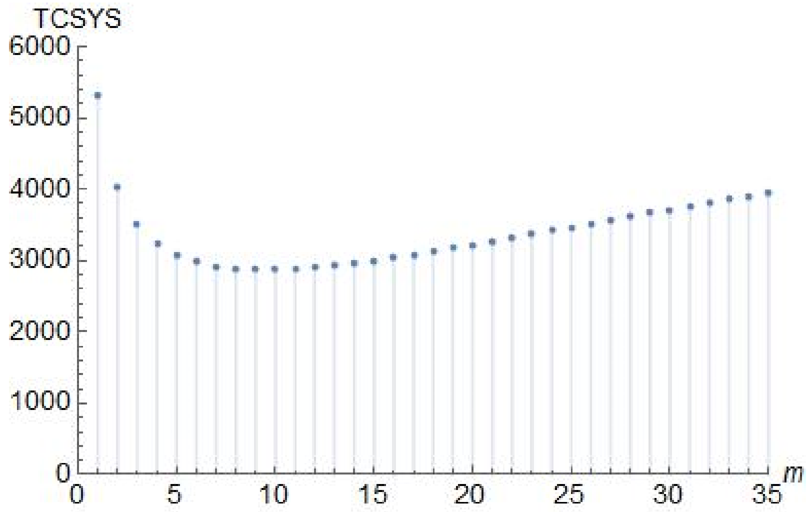

Figure A.1: *Graphical illustration of the objective function for different numbers of batch shipments*

Figure A.2 illustrates the objective function Eq. (27) for varying values of λ and given parameters.

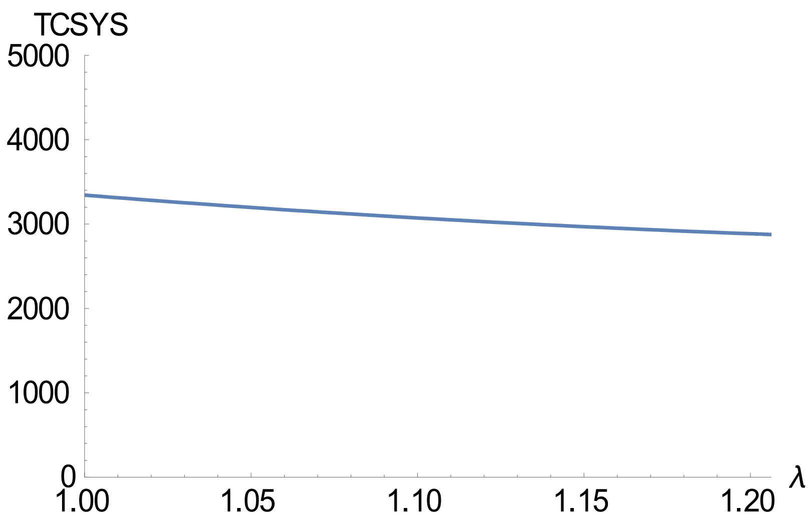

Figure A.2: *Graphical illustration of the objective function for different values of* λ

Appendix B

Table B.1: *Results of the numerical study for different values of* p

p		DD with equal shipments	ID with equal shipments	DD with increasing shipments	ID with increasing shipments
170	TC^{sys}	2589.15	2690.96	2290.64	2290.64
	TC^{v}	2210.61	1973.39	1890.57	1890.57
	TC^{b}	378.54	717.57	400.06	400.06
	m	22	19	35	35
	λ	1.00	1.00	1.03	1.03
	β_i	(0.34, 0.32, 0.34)	(0.34, 0.32, 0.34)	(0.33, 0.33, 0.34)	(0.33, 0.33, 0.34)
	Q	2471.86	2127.49	4070.92	4070.92
180	TC^{sys}	3009.30	3185.78	2632.03	2632.03
	TC^{v}	2579.19	2165.98	2119.50	2119.50
	TC^{b}	430.12	1019.80	512.53	512.53
	m	12	10	14	14
	λ	1.00	1.00	1.12	1.12
	β_i	(0.36, 0.28, 0.36)	(0.36, 0.28, 0.36)	(0.32, 0.33, 0.35)	(0.32, 0.33, 0.35)
	Q	1379.06	1161.41	1747.70	1747.70
190	TC^{sys}	3184.26	3390.00	2850.70	2850.70
	TC^{v}	2724.49	2247.38	2253.19	2253.19
	TC^{b}	459.77	1142.63	597.51	597.51
	m	10	8	9	9
	λ	1.00	1.00	1.22	1.22
	β_i	(0.38, 0.24, 0.38)	(0.38, 0.24, 0.38)	(0.31, 0.33, 0.36)	(0.31, 0.33, 0.36)
	Q	1161.97	958.70	1219.00	1219.00
200	TC^{sys}	3261.50	3479.40	3016.66	3016.66
	TC^{v}	2771.27	2262.65	2350.55	2350.55
	TC^{b}	490.23	1216.76	666.11	666.11
	m	9	8	7	7
	λ	1.00	1.00	1.32	1.32
	β_i	(0.4, 0.2, 0.4)	(0.4, 0.2, 0.4)	(0.30, 0.33, 0.36)	(0.30, 0.33, 0.36)
	Q	1065.46	934.07	1002.76	1002.76
210	TC^{sys}	3275.59	3492.06	3148.20	3148.20
	TC^{v}	2767.48	2266.80	2424.80	2424.80

	TC^b	508.11	1225.26	723.40	723.40
	m	9	8	6	6
	λ	1.00	1.00	1.42	1.42
	β_i	(0.42, 0.16, 0.42)	(0.42, 0.16, 0.42)	(0.29, 0.33, 0.37)	(0.29, 0.33, 0.37)
	Q	1060.88	930.68	889.40	889.40
220	TC^{sys}	3233.33	3439.95	3233.33	3262.80
	TC^v	2721.98	2261.36	2721.98	2482.54
	TC^b	511.35	1178.59	511.35	780.26
	m	10	8	10	5
	λ	1.00	1.00	1.00	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.12, 0.44, 0.44)	(0.29, 0.33, 0.38)
	Q	1144.33	944.78	1144.33	789.20
230	TC^{sys}	3127.26	3309.41	3127.26	3309.41
	TC^v	2617.37	2195.49	2617.37	2195.49
	TC^b	509.89	1113.93	509.89	1113.93
	m	12	10	12	10
	λ	1.00	1.00	1.00	1.00
	β_i	(0.46, 0.08, 0.46)	(0.46, 0.08, 0.46)	(0.08, 0.46, 0.46)	(0.08, 0.46, 0.46)
	Q	1327.04	1118.02	1327.04	1118.02
240	TC^{sys}	2919.39	3058.79	2919.38	3058.78
	TC^v	2420.17	2093.03	2420.16	2093.02
	TC^b	499.22	965.76	499.22	965.76
	m	17	14	17	14
	λ	1.00	1.00	1.00	1.00
	β_i	(0.48, 0.04, 0.48)	(0.48, 0.04, 0.48)	(0.04, 0.48, 0.48)	(0.04, 0.48, 0.48)
	Q	1806.88	1503.86	1806.89	1503.87

Table B.2: *Results of the numerical study for different values of* c_T

c_T		DD with equal shipments	ID with equal shipments	DD with increasing shipments	ID with increasing shipments
5	TC^{sys}	2398.51	2614.49	2398.51	2614.49
	m	18	15	18	15
	λ	1.00	1.00	1.00	1.00
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.12, 0.44, 0.44)	(0.12, 0.44, 0.44)
10	TC^{sys}	2871.13	3081.21	2871.13	2990.11
	m	12	10	12	6
	λ	1.00	1.00	1.00	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.12, 0.44, 0.44)	(0.29, 0.33, 0.38)
15	TC^{sys}	3233.33	3439.95	3233.33	3262.80
	m	10	8	10	5
	λ	1.00	1.00	1.00	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.29, 0.33, 0.38)
20	TC^{sys}	3539.18	3741.32	3492.31	3492.31
	m	9	7	5	5
	λ	1.00	1.00	1.53	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.29, 0.33, 0.38)	(0.29, 0.33, 0.38)
25	TC^{sys}	3808.33	4008.44	3707.64	3707.64
	m	8	7	5	5
	λ	1.00	1.00	1.53	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.29, 0.33, 0.38)	(0.29, 0.33, 0.38)
30	TC^{sys}	4051.80	4246.66	3909.89	3909.89
	m	7	6	4	4
	λ	1.00	1.00	1.53	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.29, 0.33, 0.38)	(0.29, 0.33, 0.38)
35	TC^{sys}	4276.77	4470.98	4086.37	4086.37
	m	7	6	4	4
	λ	1.00	1.00	1.53	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.29, 0.33, 0.38)	(0.29, 0.33, 0.38)
40	TC^{sys}	4484.28	4673.98	4255.53	4255.53
	m	6	5	4	4
	λ	1.00	1.00	1.53	1.53

	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.29, 0.33, 0.38)	(0.29, 0.33, 0.38)
45	TC^{sys}	4679.81	4866.94	4418.22	4418.22
	m	6	5	4	4
	λ	1.00	1.00	1.53	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.29, 0.33, 0.38)	(0.29, 0.33, 0.38)
50	TC^{sys}	4867.50	5052.53	4575.13	4575.13
	m	6	5	4	4
	λ	1.00	1.00	1.53	1.53
	β_i	(0.44, 0.12, 0.44)	(0.44, 0.12, 0.44)	(0.29, 0.33, 0.38)	(0.29, 0.33, 0.38)