

Decentralized Control of a Multivariable System

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According to the equations (1) and (3), the closed loop system can be represented as follows:

$$\dot{\hat{x}}_i = (A_{i1} + K_{i1})\hat{x}_1 + (A_{i2} + K_{i2})\hat{x}_2 + \dots + (A_{in} + K_{in})\hat{x}_i + g_i, \quad (i=1,2,\dots,n). \quad (4)$$

It can be seen from equation (4) that the system "plant + controller" will be dynamically decoupled on each channel if the following conditions are achieved:

$$A_{ij} + K_{ij} = 0, \quad (i, j=1,2,\dots,n, \quad i \neq j). \quad (5)$$

In order to obtain the desirable motion of the system according to the reference model (2), the following relation should be provided:

$$A_{ii} + K_{ii} = A_{ii}^m. \quad (6)$$

III. SYNTHESIS OF ADAPTIVE CONTROL ALGORITHMS

The variable parameters of the plant (1) can be represented as follows:

$$A_{ij} = A_{ij}^0 + \Delta A_{ij},$$

where

$A_{ij}^0 = \text{const.}$, ($i, j=1,2,\dots,n$), A_{ij}^0 - are constant matrices whose coefficients correspond to the nominal operation mode of the subsystem,
 $\Delta A_{ij} = \Delta A_{ij}^0(t)$, ΔA_{ij}^0 - are deviations of plant parameters from the nominal operation mode.

The controller parameters can be represented analogously:

$$K_{ij} = K_{ij}^0 + \Delta K_{ij},$$

where

$K_{ij}^0 = \text{const.}$, ($i, j=1,2,\dots,n$), K_{ij}^0 - are matrices of constant coefficients of the controller,
 $\Delta K_{ij} = \Delta K_{ij}(t)$, ΔK_{ij} - are matrices of adjustable coefficients of the controller.

Thus, the motion of the subsystems' states on a non-nominal operation mode can be represented as follows:

$$\dot{x}_i = \sum_{j=1}^n (A_{ij}^0 + \Delta A_{ij}(t))x_j + u_i \quad (i=1,2,\dots,n). \quad (7)$$

The feedback control is:

$$u_i = g_i + \sum_{j=1}^n (K_{ij}^0 + \Delta K_{ij}(t))x_j. \quad (8)$$

Substituting equation (8) in (7) one can obtain the equation of motion of the system "plant + controller" on each channel:

$$\dot{x}_i = \sum_{j=1}^n (A_{ij}^0 + \Delta A_{ij}(t))x_j + \sum_{j=1}^n (K_{ij}^0 + \Delta K_{ij}(t))x_j + g_i. \quad (9)$$

The model motion equation on each channel can be chosen as follows:

$$\dot{x}_i^m = A_{ii}^m x_i^m + \sum_{\substack{j=1 \\ j \neq i}}^n (A_{ij}^0 + K_{ij}^0)x_j + g_i. \quad (10)$$

Comparing equations (9) and (10) and taking into account that $A_{ii}^0 + K_{ii}^0 = A_{ii}^m$, we can obtain error motion equation:

$$e_i = A_{ii}^m e_i + (\Delta A_{ii}(t) + \Delta K_{ii}(t))x_i + \left(\sum_{\substack{j=1 \\ j \neq i}}^n \Delta A_{ij}(t) + \sum_{\substack{j=1 \\ j \neq i}}^n \Delta K_{ij}(t) \right) x_j, \quad (11)$$

where

$$e_i = x_i - x_i^m. \quad (12)$$

Let's introduce the following notation:

$$Y_{ij} = Y_{ij}(t) = \Delta A_{ij}(t) + \Delta K_{ij}(t). \quad (13)$$

Therefore, the error equation (11) can be represented as:

$$\dot{e}_i = A_{ii}^m e_i + Y_{ii} x_i + \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} x_j. \quad (14)$$

The adaptive control algorithms can be specified in the following form:

$$\frac{d}{dt} \Delta K = \Psi. \quad (15)$$

(Here and further we omit indexes i, j for simplicity, unless it will be required).
Given that parameters' deviation $\Delta A = \Delta A(t)$ is differentiable in time:

$$\frac{d}{dt} \Delta A(t) = R(t). \quad (16)$$

Therefore, according to equations (12) – (16) the following can be obtained:

$$\begin{aligned} \dot{e} &= A^m e + Y e, \\ Y &= \Psi + R. \end{aligned} \quad (17)$$

where $R = R(t)$.

The equations (17) and (2) describe the dynamic motion of the adaptive control system with the reference model.

Given that the matrix Ψ is a function of error deviation e and time t , and $\Psi(e, t) = 0$ at $t = 0$.

In this case the state and parameters error motions

$$e \equiv 0, \quad Y \equiv 0, \quad (18)$$

with $R(t) \equiv 0$ (according to the hypothesis of quasi-stationary) is the solution of the system (17).

It is suggested to use the second method of Lyapunov in order to obtain the adaptive control algorithms from the condition of zero stability solution (18) of the system (17) [1], [2], [3] and [4].

The quadratic function can be chosen as follows:

$$V = \gamma e^T P e + \text{tr}(Y Y^T), \quad (19)$$

where

γ -const., P - is a symmetric matrix.

The time derivative for the function V is obtained as follows:

$$\begin{aligned} \dot{V} &= \gamma \dot{e}^T P e + \gamma e^T P \dot{e} + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \\ &\gamma (A^m e + Y x)^T P e + \gamma e^T P (A^m e + Y x) + \\ &\text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \gamma [(A^m e)^T + (Y x)^T] P e + \\ &\gamma e^T P (A^m e + Y x) + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \\ &\gamma [e^T (A^m)^T + x^T Y^T] P e + \gamma e^T P A^m e + \\ &\gamma e^T P Y x + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \\ &\gamma e^T (A^m)^T P e + \gamma x^T Y^T P e + \gamma e^T P A^m e + \\ &\gamma e^T P Y x + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T) = \end{aligned}$$

$$\begin{aligned} &\gamma e^T [(A^m)^T P + P A^m] e + \gamma x^T Y^T P e + \\ &\gamma e^T P Y x + \text{tr}(\dot{Y} Y^T + Y \dot{Y}^T). \end{aligned} \quad (20)$$

Following some intermediate matrix manipulations we obtain:

$$\Psi = -\gamma P e x^T. \quad (21)$$

The derivative of the Lyapunov function is represented as:

$$\dot{V} = \gamma e^T Q e, \quad (22)$$

where $Q = (A^m)^T P + P A^m$, Q - is the negative definite matrix.

Taking into account that A^T is a Hurwitch matrix, the following is obtained:

$$\dot{V} \leq 0. \quad (23)$$

Therefore, according to equations (17), (15) and using conditions of quasi-stationary of the system, one can obtain the adaptive control algorithms as:

$$\Delta \dot{K} = -\gamma P e x^T \quad (24)$$

It follows from the above that motion of the system (17) with the algorithms (24) is stable and conditions (5) and (6) are satisfied.

CONCLUSUONS

The adaptive algorithms described in this paper allow one to achieve independent control of separate subsystems with uncertain parameters in a complex multi-axis machinery. The desirable behavior of each sub-system can be obtained according to a reference model of the relevant sub-system. The stability of the overall system is guaranteed according to the stability criteria based on the Lyapunov function.

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