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Effect of the plasticity model on the yield surface evolution after abrupt strain-path changes

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Abstract. Abrupt strain path changes without elastic unloading have been used in the literature to investigate the yield surface of sheet metals, both experimentally and theoretically. Such pioneering studies emphasized an apparent non-normality of the plastic strain rate tensor with respect to the trace of the yield surface in stress space, following such a strain-path change. They inspired numerous subsequent developments of plasticity models including non-associated flow rules. In this paper, this type of abrupt strain-path changes is investigated using state-of-the-art plasticity models. The aim is to emphasize the respective contributions of elasticity, isotropic-kinematic hardening, and rate sensitivity, to the apparent violation of the normality condition. The results show that these classical ingredients of plasticity models significantly contribute to the apparent vertex and loss of normality. These effects are quantified for typical sheet metals subject to biaxial-to-shear orthogonal strain path change.

1. Introduction

An abrupt strain-path change without elastic unloading has been used to determine the shape of the subsequent yield surface after an initial loading. Plasticity theoretical calculations as well as experimental validations have shown an apparent non-normality of plastic flow, with a vertex formed on the yield surface [1, 2]. Based on these pioneering studies, numerous phenomenological and physical plasticity models including non-associated flow rules were developed in last two decades, e.g., in [3-6].

According to these research results, some reasons have been pointed out to explain the presence of a vertex and of the loss of normality, like the role of elasticity and rate sensitivity. However, the respective contributions to non-normality of these ingredients were not investigated in detail, and need to be further clarified. In particular, a majority of the former studies consider only isotropic hardening, thus the influence of kinematic hardening, if any, was not studied yet.

This paper analyzes and compares the effects on the apparent non-normality on subsequent yield surface from two factors of constitutive modeling: isotropic-kinematic hardening and rate sensitivity after abrupt strain-path change. These effects are quantified for typical sheet steel subject to biaxial-to-shear strain path change. The aim of the investigation is to clarify which fraction of the observed results needs to be described by specifically-added non-normality effects, which otherwise may be overestimated. The apparent non-normality is simply quantified by the angle between the actual strain rate tensor and the trace of the subsequent loading path, which is supposed to stay close to the yield surface, as illustrated by angle β in Figure 1. The material model used for the investigation is

described in Section 2, while Section 3 describes the numerical simulations and the results, leading to the final conclusions.
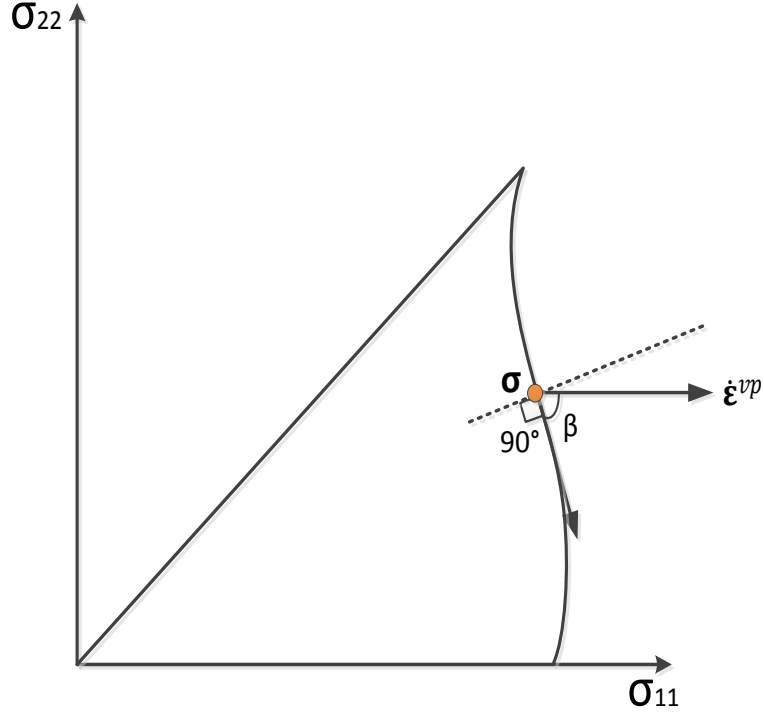

Figure 1. Schematic illustration of apparent non-normality situation after abrupt strain-path change, following the experimental results reported in [2].

2. Elaso-visco-plastic model

In sheet metal forming processes, the material usually undergoes large deformation and its mechanical behavior is described by rate constitutive equations. In this paper, the total strain rate tensor $\dot{\boldsymbol{\varepsilon}}$ is additively decomposed in elastic strain rate $\dot{\boldsymbol{\varepsilon}}^e$ and visco-plastic strain rate $\dot{\boldsymbol{\varepsilon}}^{vp}$. A hypo-elastic law linearly relating the Cauchy stress rate is expressed as

$$\dot{\boldsymbol{\sigma}} = \mathbf{C} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{vp}) \tag{1}$$

where $\mathbf{C}$ is the fourth-order elasticity tensor.

The plastic flow rule defines the direction of the visco-plastic strain rate as a function of the stress tensor components. The equivalent stress σ_{eq} employed here is von Mises for simplicity

$$\sigma_{eq} = \sqrt{\frac{3}{2}(\boldsymbol{\sigma}' - \mathbf{X}) : (\boldsymbol{\sigma}' - \mathbf{X})} \tag{2}$$

where $\boldsymbol{\sigma}'$ denotes the deviatoric part of the Cauchy stress tensor and $\mathbf{X}$ is a second-order tensor which describes kinematic hardening. The plastic flow rule can be written as

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \dot{\varepsilon}_{eq}^{vp}\,\mathbf{V}\,,\mathbf{V} = \frac{\partial \sigma_{eq}}{\partial \boldsymbol{\sigma}} = \frac{3}{2}\frac{(\boldsymbol{\sigma}' - \mathbf{X})}{\sigma_{eq}} \tag{3}$$

where $\dot{\varepsilon}_{eq}^{vp}$ is the equivalent visco-plastic strain rate which is defined by the following relationship

$$\dot{\varepsilon}_{eq}^{vp} = \dot{\varepsilon}^* sinh(\frac{\sigma^*}{k^*}) \tag{4}$$

where k^* and $\dot{\varepsilon}^*$ are material parameters. The scalar "overstress" σ^* denotes the increase of stress intensity due to visco-plastic strain rate so that

$$\sigma_{eq} - \sigma_0 - R - \sigma^* \leq 0 \tag{5}$$

In this equation, σ_0 is a material parameter designating the initial yield stress, and R is the isotropic hardening of the material.

In the framework of the developed constitutive model, isotropic hardening law is implemented in the generic form

$$\dot{R} = h_R \dot{\varepsilon}_{eq}^{vp} \tag{6}$$

which can describe various hardening models like Voce, Swift, or their combinations.

In this paper, the kinematic hardening described by the backstress tensor $\mathbf{X}$ is governed by

$$\dot{\mathbf{X}} = C_x(X_{sat}\mathbf{N} - \mathbf{X})\dot{\varepsilon}_{eq}^{vp} \tag{7}$$

where C_x and X_{sat} are material parameters and $\mathbf{N}$ is the unit tensor parallel to the visco-plastic flow direction.

3. Numerical model and simulation results

According to the experimental study with metal sheets in [2], abrupt strain path change simulations were performed by a specifically developed program. Among possible two-step loading processes, the first step of loading considered in the simulations was a biaxial loading prescribed by $\dot{\varepsilon}_{11}=\dot{\varepsilon}_{22}>0$. When the nominal strains reach the values $\varepsilon_{11}=\varepsilon_{22}=0.01$, the applied total strain rate is abruptly converted to $\dot{\varepsilon}_{22} = -\dot{\varepsilon}_{11}$ with $\dot{\varepsilon}_{11}>0$.

In order to investigate the respective non-normality contributions of isotropic-kinematic hardening and rate sensitivity, three simulation groups which include its corresponding material parameters, C_x, X_{sat} and k^*, were simulated and analyzed. It is noteworthy that the isotropic hardening model was readjusted for each simulation in order to make sure that the stress-strain response during the first loading path is identical for all the simulations in the investigation.

3.1. Apparent loss of normality caused by rate sensitivity

The stress path for abrupt strain path change following equibiaxial pre-strain loading and the plastic flow directions are shown in Figure 2 for different values of the strain rate sensitivity parameter k^*. Only isotropic hardening is considered in these simulations. It is obvious that rate sensitivity plays a significant role in the shape of the subsequent yield surface with a severe blunt "corner" effect. This effect also implies an apparent non-normality, with angle β significantly larger than 90° during the transient phase after strain-path change. Both effects increase when the rate sensitivity parameter increases (or, alternatively, when the strain rate increases). After the transient zone following the strain-path change, the direction of plastic strain rate tends to become perpendicular to the stress path.
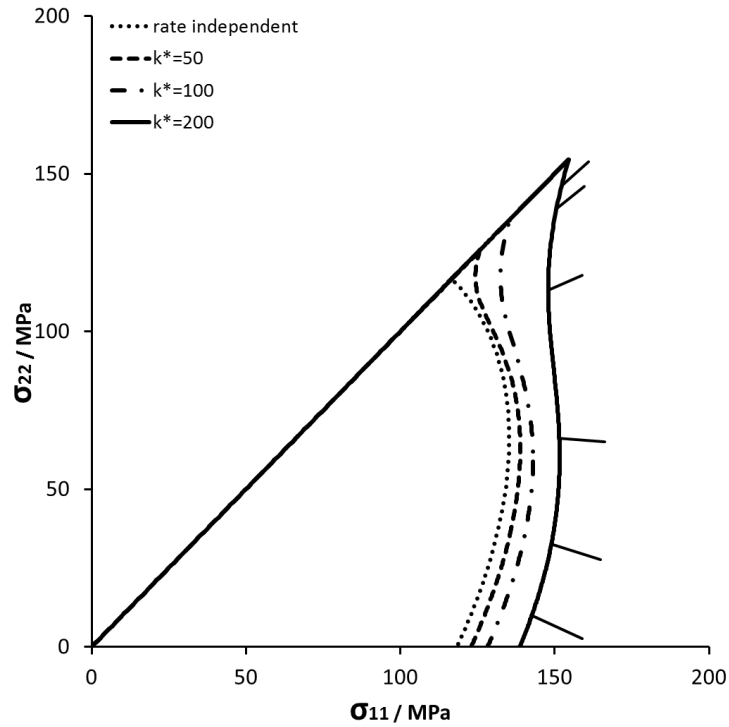
Figure 2. Influence of the rate sensitivity parameter k^*. Stress path for abrupt strain path change following equibiaxial pre-strain loading, and directions of plastic strain rate (for $k^*=200$).

3.2. Apparent loss of normality due to kinematic hardening

Figure 3 shows the stress path for abrupt strain path change following equibiaxial pre-strain loading and corresponding plastic flow directions, when kinematic hardening is added along with isotropic hardening. In these simulations, X_{sat} =50MPa. It appears that kinematic hardening has an influence on the apparent non-normality of plastic strain-rate direction with respect to the stress path. This effect appears with a delay after the strain-path change; this delay decreases when C_x increases. This effect does not seem to have been investigated deeply in the literature. Parameter X_{sat} has a similar influence, as displayed in Figure 4, with larger effects corresponding to larger values of X_{sat}.
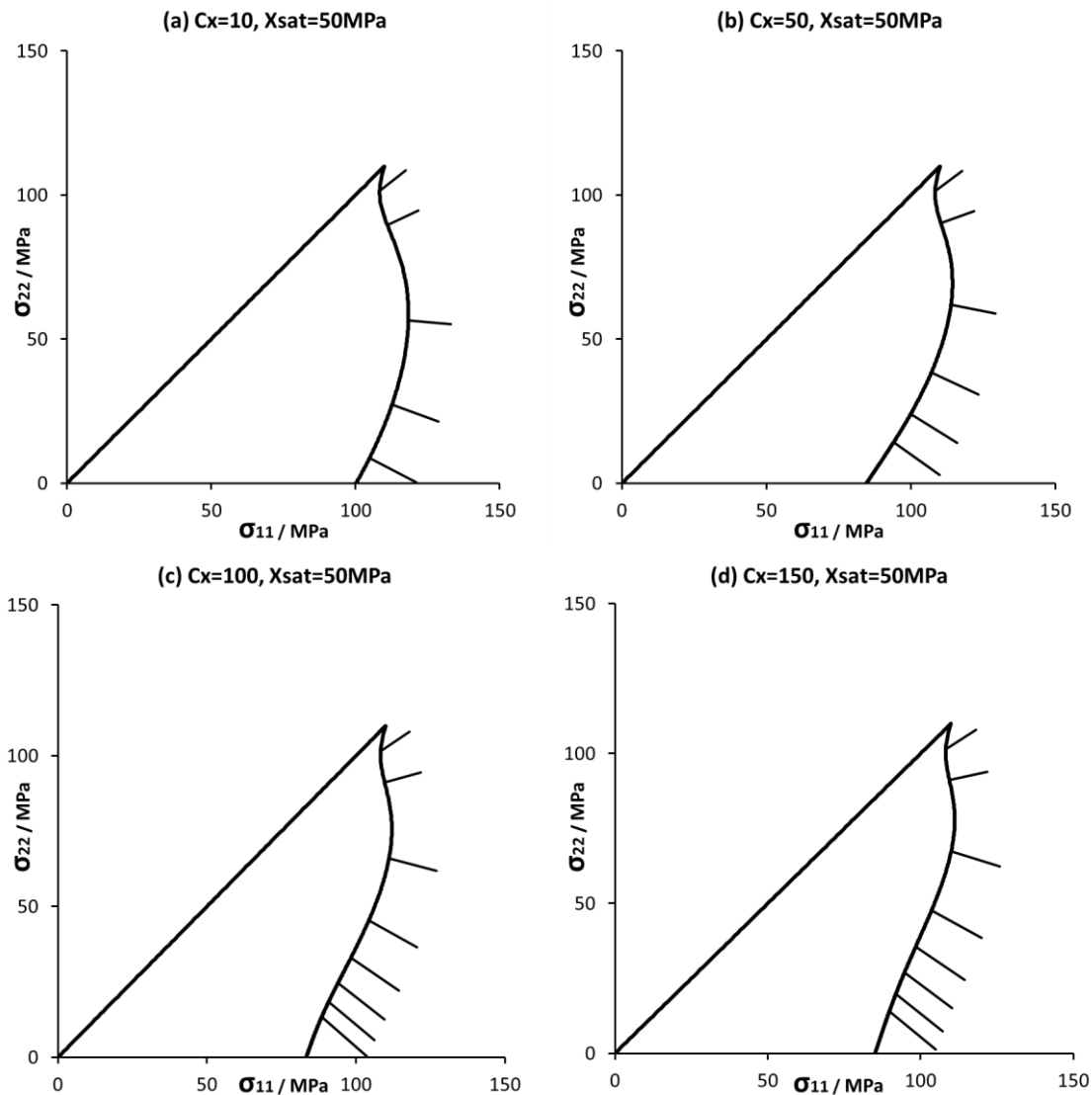
Figure 3. Influence of the kinematic hardening parameter C_x. Stress path for abrupt strain path change following equibiaxial pre-strain loading. The direction of plastic flow is also indicated, at specified values of subsequent plastic strain.

4. Conclusion

An elasto-visco-plastic model including isotropic-kinematic hardening and rate sensitivity was used to simulate a typical abrupt strain-path change as described in [1, 2]. The apparent non-normality observed in this configuration was shown to depend, at least partly, on the constitutive response of the material according to classical plasticity modeling. The material's strain rate sensitivity had a strong influence during the early transition zone after strain-path change, while kinematic hardening contributed to this apparent non-normality at a later stage. These findings will be further explored in order to assess the part of this apparent non-normality phenomenon that would correspond to a specific non-normality behavior of the material.
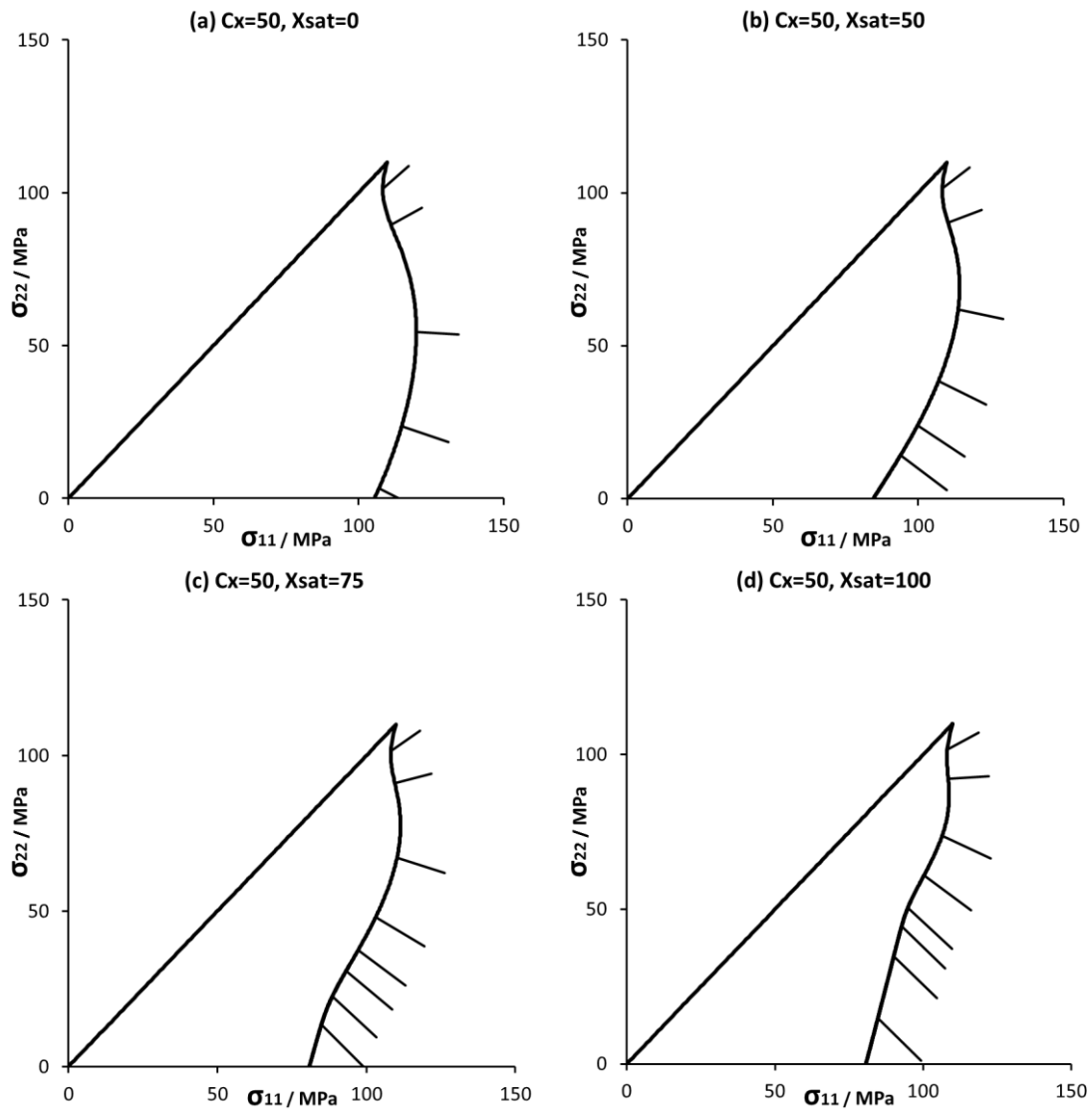
Figure 4. Influence of kinematic hardening parameter X_{sat}. Stress path for abrupt strain path change following equibiaxial pre-strain loading, and plastic strain rate directions.

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