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Water Distribution Networks Optimization Using Disjunctive Generalized Programming

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Water Distribution Networks (WDN) are systems of water distribution used in industrial processes and urban centers. The optimal WDN design can be very effective in saving energy, specifically in pumping service, to carry water to nodes of demand, at appropriated velocities and pressures. Indirectly, it can contribute in reducing liquid pollution and accidents caused by pressure overestimation in nodes. The design of WDN can be treated as an optimization problem with a Mixed Integer Nonlinear Programming (MINLP) formulation. The objective function, to be minimized is the WDN cost, given by the product of the pipe diameters and their lengths. The problem constraints are the mass balances in each node, the energy balances in the WDN loops and pressure and velocities limits. A set of commercial diameters is available, with proper costs and rugosity coefficients. The majority of paper published in this research field use external hydraulic simulators and meta-heuristic methods to solve the optimization problem. In the current paper a mathematical model using a deterministic Mathematical Programming approach is proposed and all variables are simultaneously optimized, avoiding the use of external software for pressure and velocities calculations. Two case studies were used to test the model applicability and coded in GAMS, using the global optimization solver BARON. Results showed that for both cases global optima was achieved, proving that it is possible to solve the problem, independently of external hydraulic simulator.

1. Introduction

Water is the most important fluid used in industrial processes and in urban centers or irrigation systems. Water supply systems are fundamental in the industrial society and supply services must provide water in appropriated quality, pressure and velocities, in piping, pumps, valves, reservoirs, meters and other accessories. Pumping stations are responsible by the water network pressurization and elevated reservoirs are normally used. The topology heterogeneity in the zones to which water must be supplied must also be considered and systems working only by gravity are frequently used (Zhang et al., 2018). The network is formed by pressure nodes (reservoirs or points of demand) and pipes linking these nodes. Loops can exist in the WDN and looped problems present difficulties in solving, when flow directions are not defined.

The design of WDN can be formulated as a constrained optimization problem, whose objective function to be minimized is the network cost, composed by the product of pipes and their diameter costs. The constraints are the mass balances in the nodes, the energy balance in the loops and pressure and velocities limits. In general, a set of commercial diameters is available and the network pipe diameters must belong to this set. Nonlinear equations are available to calculate the nodes pressures and water velocities. The problem is nonlinear and nonconvex and, because of this complexity, global optimization techniques are not frequently used in its solution. Besides, hydraulic simulators are frequently used to circumvent the nonlinearity problem.

Stochastic approaches have been proposed, to solve the problem and the majority of the published papers using this type of solution uses also, hydraulic simulators. De Corte and Sorensen (2016) presented an overview of the metaheuristic techniques developed for the WDN design optimization problem, citing some of the most important works using Ant Colony Optimization (ACO), Genetic Algorithms (GA), Harmony Search (HS), Particle Swarm Optimization (PSO), Simulated Annealing (SA), Tabu Search (TS) among other techniques.

Important deterministic approaches were also proposed. Alperovits and Shamir (1977) proposed a Linear Programming Gradient (LPG) to the WDN optimization. Hansen *et al.* (1991) presented a successive linear programming (SLP) approach with a local search algorithm to solve the problem. Sarbu and Borza (1997) proposed an improved Mixed Integer Linear Programming (MILP) model, which was the extension of the model proposed by Costa et al. (2001) and used Branch and Bound for the solution of the problem. Good upper and lower bound estimates are necessary in order to find good solutions. Shamir and Howard (1968) and Watanatada (1973) proposed NLP models, using the Newton-Raphson method to solve the system of equations. Authors related difficulties in finding optimal values and problems with no solution. D'Ambrosio et al. (2014) presented an MINLP model and Spatial Branch and Bound and piecewise linear relaxations were used. In the present paper an MINLP model is proposed to the optimal design of looped WDN. A deterministic Mathematical Programming approach is proposed to solve the model using a global optimization solver in GAMS. The novelty of the paper is that it is not necessary to use hydraulic simulators to calculate pressures and velocities and the hydraulic equations are solved simultaneously in the optimization problem. Two examples from the literature were used to test the model.

2. Optimization model

The proposed MINLP model is based on the papers of Surco et al. (2017) and Surco et al. (2018). The objective function to be minimized is the WDN cost, composed by the pipes' diameters cost. The constraints are the mass balances in the nodes and the energy balances in the loops, as well as pressures and velocities limits. The following sets, parameters and variables are defined:

Sets:

K	Set of nodes
J	Set of pipes
D	Set of diameters
Γ	Set of loops
A	Set of pumps
PPD_γ	Set of positive pressure drops in a loop γ
NPD_γ	Set of negative pressure drops in a loop γ
FI_k	Set of fluid streams that enter in the node k
FO_k	Set of fluid streams that leave the node k

Parameters:

L	Pipe length
D	Commercial diameter
$dmd(k)$	Node demand
pr_{min}	Minimum allowed pressure
v_{min} and v_{max}	Minimum and maximum allowed velocities
ω, α, β	coefficients of the Hazzen-Williams equation
C	Hazzen-Williams rugosity coefficient
elv	Node elevation
a	Pump
Ep	Pump energy

Variables:

C_T	Total cost
h_f	Pressure drop
pr	Pressure in the node
q	Volumetric flow rate
v	Velocity
x	Diameter
y	Binary variable
Y	Boolean variable
σ	Rugosity
λ	WDN Cost

To each diameter is associated a cost per length, $Cost(D_i)$ and a rugosity coefficient, $C_i, \forall i \in D$

The objective function must consider the sum of all tube diameters and their costs:

$$C_T = \sum_{j \in J} L_j\, Cost(x_j), \qquad \forall j \in J \tag{1}$$

The node demand is given by the difference between the inlet flow rate and the outlet flow rate.

$$\sum_{j \in FI_k} q_j - \sum_{j \in FO_k} q_j = dmd(k), \forall\, k \in k \tag{2}$$

The sum of the pressure drops in the links for the nodes belonging to a loop must be equal to the energy liberated by a pump, if it exists:

$$\sum_{j \in PPD_\gamma} h_f(j) - \sum_{\epsilon NPD_\gamma} h_f(j) = \sum_{a \in A} E_P^a(\gamma), \qquad \forall\, \gamma \in \Gamma \tag{3}$$

The pressure in any point in the WDN must be less or equal to a minimum limit:

$$pr_{min}(k) \leq pr(k), \qquad \forall\, k \in K \tag{4}$$

The flow velocities must also attend minimum and maximum limits:

$$v_{min} \leq v_j \leq v_{max}\,, \qquad \forall\, j \in J \tag{5}$$

The most used equation to the pressure drop calculations is the Hazen-Williams:

$$h_f(j) = \frac{\omega L_j q_j^\alpha}{C_j^\alpha x_j^\beta}, \qquad \forall\, j \in J \tag{6}$$

The parameters ω, α and β depend on the unities system used. Savic and Walters (1977) present different equations and coefficients, with the most used unity systems used in this type of problem.

To the pressure calculations it must be considered the existing pressure in each node. Considering τ_k a flowrate path, initiating in the reservoir and finishing in a node $k, \forall\, k \in K$, if the node $k = k_r$ corresponds to the reservoir, then its pressure is given by:

$$pr(kr)\; =\; elv(kr)\; =\; elv(re) \tag{7}$$

For the other nodes, the pressure is defined as follows:

$$pr(k) = -\sum_{j \in \tau_k} h_f(j) + [elv(re) - elv(k)]\,, \qquad \forall\, k \in K,\; k \neq k_r \tag{8}$$

The flow velocities can be calculated by:

$$v_j = \frac{4q_j}{\pi x_j^2}, \qquad \forall\, j \in J \tag{9}$$

The optimization model can be described as:

$$\min Eq(1) \tag{10}$$

s. t. Eq(2) to Eq(9)

3.1 MINLP disjunctive formulation

Given the pipes sequence and $y_{i,j}$ the binary variables associated to the pipe j and diameter D_i, λ_j and σ_j the cost and rugosity associated to the same diameter, the following disjunctions are valid:

$$\bigvee_{i \in D, j \in J} \begin{bmatrix} Y_{ij} \\ x_j = D_i \\ \lambda_j = L_j Cost(D_i) \\ \sigma_j = R_i \end{bmatrix} \tag{11}$$

These disjunctions must satisfy the following equations:

$$x_j = \sum_{i \in D} D_i y_{i,j} \tag{12}$$

$$\lambda_j = L_j \sum_{i \in D} Cost(D_i) y_{i,j} \tag{13}$$

$$\sigma_j = \sum_{i \in D} R_i y_{i,j} \tag{14}$$

Using a Big M formulation, the MINLP model is:

$$\min C_T = \sum_{j \in J} \lambda_j \tag{15}$$

$$s.t. \sum_{j \in VE_k} q_j - \sum_{j \in VS_k} q_j = dmd(k) \tag{16}$$

$$\sum_{j \in PPD_\gamma} h_f(j) - \sum_{j \in NPD_\gamma} h_f(j) = \sum_{a \in A} E_P^a(\gamma) \tag{17}$$

$$pr_{min}(k) \leq pr(k) \tag{18}$$

$$v_{min} \leq v_j \leq v_{max} \tag{19}$$

$$x_j = \sum_{i \in D} D_i y_{i,j} \tag{20}$$

$$\lambda_j = L_i \sum_{i \in D} Cost(D_i) y_{i,j} \tag{21}$$

$$\sigma_j = \sum_{i \in D} R_i y_{i,j} \tag{22}$$

$$x_j - D_i \leq M\left(1 - y_i^j\right) \tag{23}$$

$$\lambda_j - L_j Cost(D_i) \leq M\left(1 - y_i^j\right) \tag{24}$$

$$\sigma_j - R_i \leq M\left(1 - y_i^j\right) \tag{25}$$

3. Case studies

3.1 Case Study 1

Case study 1 is a well-known benchmark problem, named *Too Loop* WDN, proposed by Alperovits and Shamir (1977). Figure 1-a) presents the WDN topology with flow directions and nodes demands and elevations. The network presents 1 reservoir, 8 links (1,000 m length each one) and 7 nodes. The minimum pressure required in each node is 30 water column meters and the velocity limits are 0.3 m/s and 3 m/s. The coefficients of Hazen-Williams equation are $\omega = 10.674$, $\alpha = 4.871$ and $\beta = 1.852$ and the dimensionless Hazen-Williams roughness coefficient C is 130 for all pipes. A set composed by 14 commercial diameters (mm) is available, with their respective costs ($), between parenthesis: D = {25.4 (2), 50.8 (5), 76.2 (8), 101.6 (11), 152.4 (16), 203.2 (23), 254.0 (32), 304.8 (50), 355.6 (60), 406.4 (90), 457.2 (130), 508.0 (170), 558.8 (300), 609.6 (550)}.
The problem was solved using the solver BARON in GAMS, and the solution (cost of $ 419,000) was the same obtained in other works using single pipe, like Savic and Walters (1977), Liong and Antiquzzaman (2004), Mohan and Babu (2010) and Surco et al. (2017). Velocities (m/s) and pressure drops (m) calculated for each one of the 8 pipes are given in Table 1.

3.2 Case Study 2

This problem was proposed by Fujiwara and Khang (1990) and the WDN has 32 nodes (1 reservoir), 34 pipes and 3 loops and is presented in Figure 1 – b), jointly with node demands and elevations. The reservoir is at 100 m and all other nodes are at the soil level. The Hazzen-Williams non-dimensional rugosity coefficient for all pipes is 130 and Hazzen-Williams equation parameters are $\omega = 10.9031$, $\alpha = 4.871$ and $\beta = 1.852$. The set of available diameters (m) is: D = {.3048; .4064; .5080; .6096; .7620; 1.016}. The problem was solved using BARON in GAMS and the results found are better than the literature for the same parameters. Table 2 presents

the optimal pipes diameters (m) and a costs comparison with the literature in its last line. As it can be noted, there are some differences in the diameters, among the three solutions. The cost achieved in the present paper ($ 6.183.10^6$) is better than the values of $ 6.220.10^6$, obtained by Liong and Atiquzzaman (2004) and of $ 6.220.10^6$, obtained by Savic and Walters (1997).

Table 1: Two loop WDN velocities and pressure drops

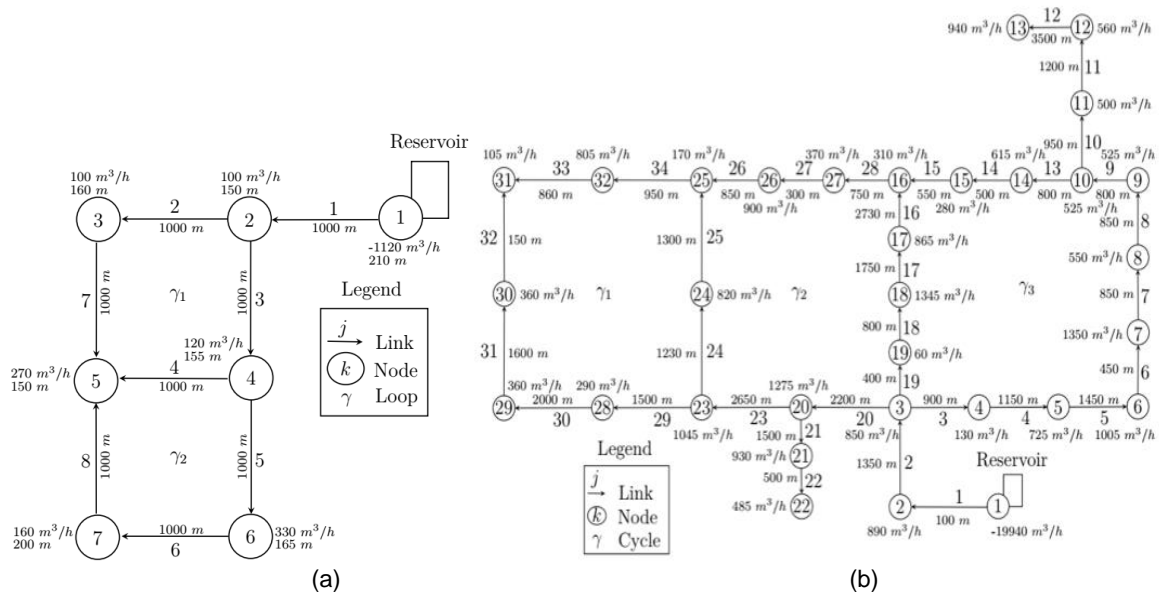| Pipe	1	2	3	4	5	6	7	8
v (m/s)	1.90	1.85	1.46	1.12	1.14	1.10	1.30	0.31
h_f (m)	6.76	12.79	4.80	14.65	3.00	4.90	6.66	6.75

Figure 1: a) Two Loop WDN. b) Hanoi WDN.

Table 2: Diameters (m) comparison for the Hanoi WDN

Pipe	Savic and Walters (1997)	Liong and Atiquzzaman (2004)	Present paper	Pipe	Savic and Walters (1997)	Liong and Atiquzzaman (2004)	Present paper
1	1.016	1.016	1.016	18	0.6096	0.7620	0.6096
2	1.016	1.016	1.016	19	0.6096	0.7620	0.6096
3	1.016	1.016	1.016	20	1.016	1.016	1.016
4	1.016	1.016	1.016	21	0.5080	0.5080	0.5080
5	1.016	1.016	1.016	22	0.3048	0.3048	0.3048
6	1.016	1.016	1.016	23	1.016	0.7620	1.016
7	1.016	1.016	1.016	24	0.7620	0.7620	0.7620
8	1.016	0.7620	1.016	25	0.7620	0.6096	0.7620
9	0.7620	0.7620	1.016	26	0.5080	0.3048	0.6096
10	0.7620	0.7620	0.7620	27	0.3048	0.5080	0.3048
11	0.7620	0.7620	0.6096	28	0.3048	0.6096	0.3048
12	0.6096	0.6096	0.6096	29	0.4064	0.4064	0.4064
13	0.4064	0.4064	0.4064	30	0.4064	0.4064	0.4064
14	0.4064	0.3048	0.3048	31	0.3048	0.3048	0.3048
15	0.3048	0.3048	0.3048	32	0.3048	0.4064	0.4064
16	0.4064	0.6096	0.3048	33	0.4064	0.5080	0.4064
17	0.5080	0.7620	0.5080	34	0.5080	0.6096	0.6096
Cost ($.10^6$)					6.195	6.220	6.183

4. Conclusions

Although water distribution networks design problem is not new, when it is formulated as an optimization problem, hydraulic calculus is complex due to the nonlinearities and nonconvexities involved in the Hazzen-Williams equation. Most of the published papers in the literature use meta-heuristic techniques, which are not able to ensure global optimization and use auxiliary software in calculating pressures and velocities. In the present paper the synthesis of WDN was formulated as an optimization problem. An MINLP model was proposed and reformulated using Generalized Disjunctive Programming. Two case studies from the literature were used to test the model applicability. Both were solved using the global optimization solver BARON in GAMS. The solutions of both problems are the best found in the literature. One can conclude that this kind of formulation is innovative and properly adequate to solve this type of problem, despite the difficulties present due to its nonlinearities and nonconvexities. Besides, by reformulating the original optimization problem using GDP, it is possible to achieve global optimum solutions as the problems tested in the paper. The great novelty and the main contribution of this paper is that the model developed is able to calculate pressure and velocities, being not necessary the use of additional software to the hydraulic calculus.

Acknowledgments

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