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A possible long-lived asteroid population at the equilateral Lagrangian points of Saturn

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ABSTRACT

The Lagrangian equilateral points of a planetary orbit are points of equilibrium that trail at 60°, ahead (L4) or behind (L5), the trajectory of a planet. Jupiter is the only major planet in our Solar system harbouring a known population of asteroids at those locations. Here we report the existence of orbits close to the Lagrangian points of Saturn, stable at time-scales comparable to the age of the Solar system. By scaling with respect to the Trojan population we have estimated the number of objects that would populate the regions, which gives a significant figure. Moreover, mutual physical collisions over the age of the Solar system would be very rare, so the evaporation rate of this swarm arising from mutual interactions would be very low. A population of asteroids not self-collisionally evolved after their formation stage would be the first to be observed in our planetary system. Our present estimations are based on the assumption that the capture efficiency at Saturn's equilateral points is comparable with the one corresponding to Jupiter, thus our figures may be taken as upper limits. In any case, observational constraints on their number would provide fundamental clues to our understanding of the history of the outer Solar system. If they existed, the surface properties and size distribution of those objects would represent unusually valuable fossil records of our early planetary system.

Key words: celestial mechanics – minor planets, asteroids – planets and satellites: formation – Solar system: formation.

1 INTRODUCTION

A number of investigations have been carried out to understand why asteroids have not being found at the Lagrangian points of the other major planets. Did the orbital evolution in those kind of orbits drive objects away from those regions or there existed some particular condition at the origin of our Solar system which inhibited the capture of asteroids there? (Yoder 1979; Marzari & Scholl 1998, 2000; Shoemaker, Shoemaker & Wolfe 1989)

The orbits at the triangular points of Saturn are known to be unstable as a result of different dynamical reasons (Marzari & Scholl 1998, 2000; de la Barre, Kaula & Varadi 1996; Innanen & Mikkola 1989; Everhart 1973), however some stable ones over a time-scale ≈400 Myr are known to exist (de la Barre et al. 1996). Marzari & Scholl (1998) put forward the idea that the rapid accretion of the gaseous component of the major planets favoured the capture of surrounding primordial objects in Trojan-like orbits. According to their models the capture efficiency of Saturn and Jupiter are of the same order, unless the conditions for the objects to be captured are met just when those planets are close to their present relative location, i.e. near the 2:5 mean-motion resonance. In this case the perturbations by Jupiter reduce the capture efficiency of Saturn in some 50 per cent, mostly owing to the effect of secular resonances appearing mostly at low libration amplitudes.

On the other hand, selection effects and/or the low number of specifically designed observational surveys may have conspired against the discovery of objects in such a region.

In Section 2 we present the method used to locate the stable niches around the equilateral points of Saturn. This method, together with a scaling argument with respect to the Trojan ones, provides the size of the putative population, which is presented in Section 3. Also a selection effect caused by the overlapping of the target regions with the Galactic plane is discussed. Next, in Section 4, the self-collisional dynamics is investigated and finally the relevance of our results with respect to the study of the origin of the Solar system is discussed in Section 5.

2 DYNAMICAL STABILITY

To locate the stable niches around the equilateral points of Saturn (L4S and L5S) we built both proper frequency and proper elements diffusion maps (Nesvorny & Ferraz-Mello 1997; Morbidelli 1997). For irregular orbits some quantities behave like constants of motion over limited time-spans. The random diffusion of these

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quasi-constants of motion or its associated frequencies over time, gives an indication of how dependent is the orbital evolution on the initial conditions (Nesvorny & Ferraz-Mello 1997; Laskar 1993). The diagram is constructed by plotting a diffusion coefficient, which is an estimation of the dynamical time-scale, as a function of some initial conditions.

The orbits of virtual particles in a grid have been integrated using a symplectic mapping (Wisdom & Holman 1991) including the perturbations of the four major planets. The rest of the initial conditions are chosen as in Holman & Wisdom (1993), i.e. initial inclination and eccentricity equal to those of Saturn. The integration is stopped if the particle enters within the Hill's sphere (Danby 1988) of a planet. The time-step used was 0.1 yr for all objects. The signal analysed was that of the elements

$$p = e\sin(\varpi)$$

$$q = e\cos(\varpi), \qquad (1)$$

where e is the eccentricity and ϖ the longitude of the perihelion. The proper frequency associated with longitude of perihelion of the orbit is computed using the frequency modified Fourier transform method (FMFT) (Šidlicovský & Nesvorny 1997; Nesvorny & Ferraz-Mello 1997) over a running window of time length $T_1 = 2\,\mathrm{Myr}$ every $T_2 = 1$ Myr. These values have been chosen such that the most important secular frequencies in the region can be accurately computed. As a test, the diffusion coefficient of Jupiter and Saturn has been calculated, giving a value which would imply a dynamical time-scale of the order of the age of the Solar system. A dispersion of the frequencies, ϵ, is computed from the values obtained from 10 successive windows. In Fig. 1 we plot a diffusion coefficient, $D = \epsilon - log_{10}(T_2)$, which is taken as an indicator of the decimal logarithm of the inverse of the dynamical time-scale of the orbit (Melita & Brunini 2000). The same diffusion coefficient can be computed by analysing the dispersion of the amplitude of the signal, which corresponds to the proper eccentricity. Also the diffusion of the proper semi-major axis, a_P, has been calculated, but in the case of a_P, it has been computed as $a_\mathrm{P} = sup(a)$ (Laskar 1995; Froeschle & Lega 1996) over the same running windows. Thus, the diffusion of proper elements over various planes have been explored, all giving very similar figures.

A further test for our methods have been performed by building the correspondent diffusion diagrams around the Lagrangian points of Jupiter, L4J and L5J, where the Trojan asteroids are located. In both, the Jupiter and Saturn cases, our results agree nicely with previous studies (de la Barre 1996; Holman & Wisdom 1993).

Once the stable niches in the Saturn case were identified, we integrated the orbits of particles in the most stable of them for 1 Gyr. We have also 'cloned' some of those initial conditions, by varying the initial semi-major axis and relative longitude with respect to Saturn. By this method we have concluded that a good estimation of the mean size of the correspondent stable niches corresponding to each of the stable orbits is the half distance between the particles in those variables.

Fig. 1 displays the diffusion diagram for L4S as well as the

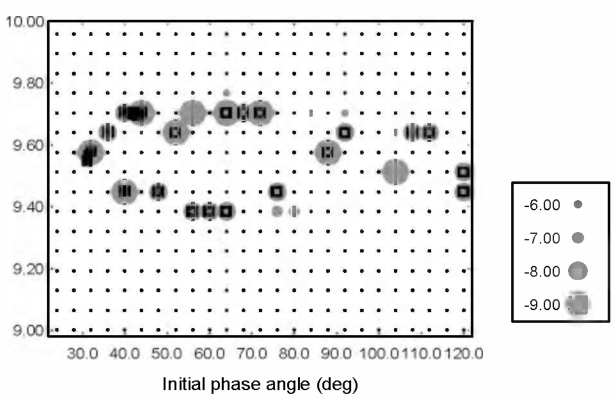

Figure 1. Proper frequency diffusion map of L5S as a function of the initial semi-major axis and the initial phase angle $\sigma = \lambda - \lambda_\mathrm{S}$, where λ is the mean longitude of the particle and λ_S is the mean longitude of Saturn. The radii of the markers are proportional to the upper limit of D (see text for details) as indicated. Only particles that survive after 1 Gyr are plotted. Particles with values of $D < -9.65$ would be stable over the age of the Solar system. Dots indicate the rest of the initial conditions explored. Square markers indicate the initial conditions integrated further. Solid-markers indicate particles that survive for 1 Gyr. Hollow markers are particles that do not survive after this time-scale.

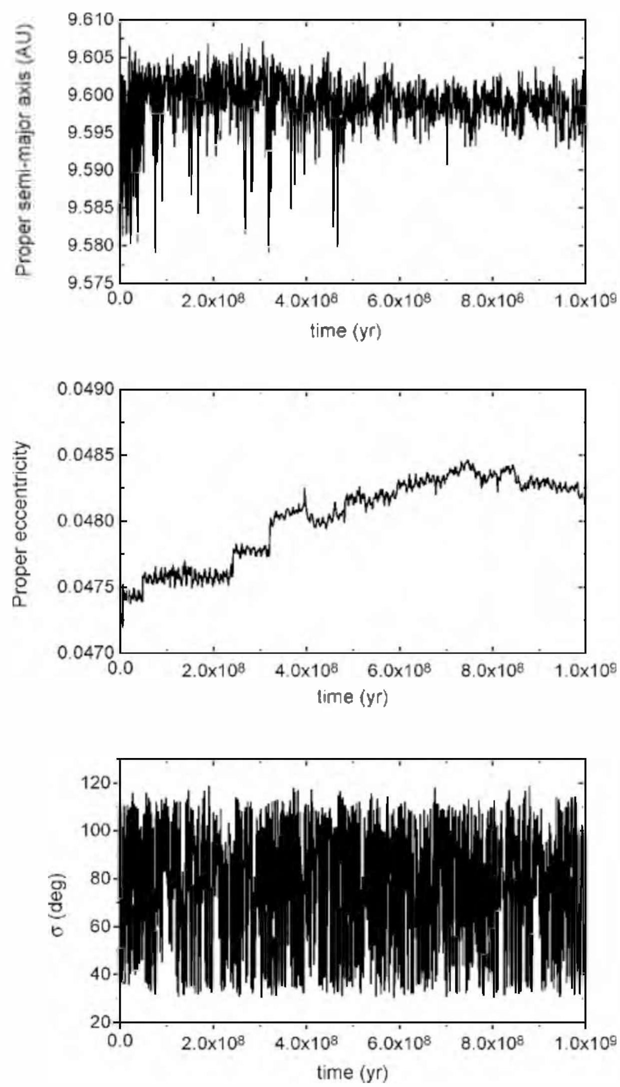

Figure 2. Proper semi-major axis, proper eccentricity and phase angle, σ, of one of the long-term stable particles. The proper semi-major axis, a_P, is numerically obtained as the $sup(a)$ over a running window of 10 Myr, where a is the semi-major axis. The proper eccentricity is computed using the FMFT method as the amplitude of the complex signal composed by $h = e\sin(\varpi)$, $k = e\cos(\varpi)$ over the aforementioned running window, where ϖ is the longitude of perihelion of the orbit.

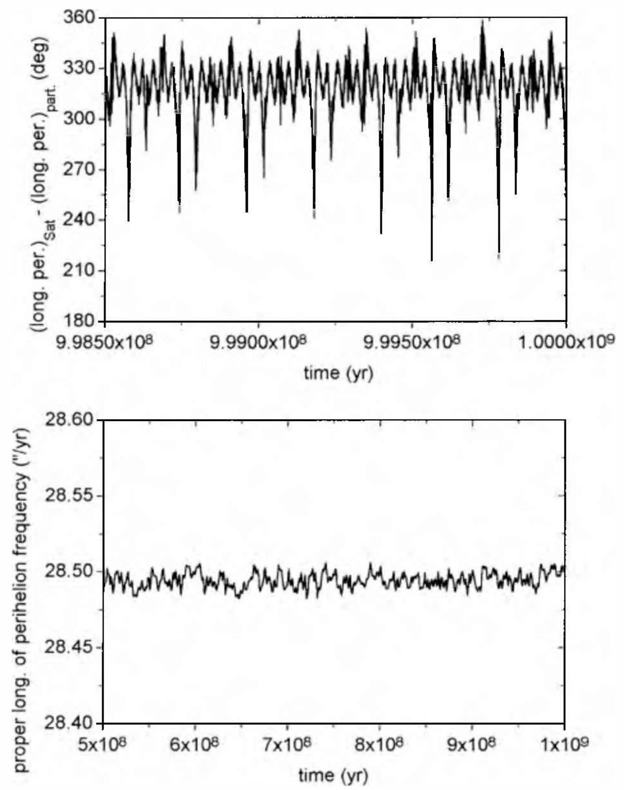

Figure 3. Detail of the evolution of the relative perihelion longitude and the frequency of the longitude of perihelion.

initial location of particles that survived for more than 1 Gyr of simulated time.

In Fig. 2 the orbital evolution of one of the long-lived particles is shown. Although the integration time-span is still smaller than the age of the Solar system, the dynamical behaviour of their orbits would indicate that all the particles would survive for another 3.5 Gyr (see Fig. 2), as indicated by the value of their dynamical diffusion coefficient. Moreover, the perihelion longitude steadily librates about the one of Saturn (see Fig. 3) with its frequency remaining nearly constant throughout all the integration time-span, which would prevent these objects (all showing very similar dynamical features) to approach other secular resonances, which have been described as a common cause of the instability of *Trojans* of Saturn at low inclinations and large libration amplitudes (Marzari & Scholl 2000). Besides, the dynamical evolution of these long term stable orbits is very different than the unstable ones reported in Marzari & Scholl 2000. In any case, to understand better the main characteristics of these orbits, a more detailed study will be presented in a forthcoming publication.

3 AN ESTIMATE OF THE NUMBER OF OBJECTS

The current number of objects that could be found in L4S can be estimated by scaling with the correspondent stable regions of the Trojans (Holman 1997), if a similar origin for both populations is assumed. This number depends on the size distribution of each population and since the one at L4S is unknown, a range shall be given. The number of objects greater than a given physical radii R, orbiting around L4J at a given time, $N^R_{\mathrm{L4J}}(t)$, can be expressed as

$$N^R_{\mathrm{L4J}}(t) = \sigma^R_{\mathrm{L4J}} \quad A_{\mathrm{L4J}} \quad f_{\mathrm{L4J}}(t), \tag{2}$$

where σ^R_{L4J} is the initial mass density of objects with radii greater than R at L4J, A_{L4J} is the initial geometrical area, and $f_{\mathrm{L4J}}(t)$ is the fraction of objects that would survive in the region up to that instant. Disregarding the loss of objects by non-dynamical reasons, as well as the change in the size distribution, we may estimate the number of asteroids orbiting around L4S at a given time by

$$N^R_{\mathrm{L4S}}(t) = N^R_{\mathrm{L4J}}[(1-q_J)/(1-q_S)](s^R_{\mathrm{L4S}}/s^R_{\mathrm{L4J}}) \times (A_{\mathrm{L4S}}/A_{\mathrm{L4J}}) \times [f_{\mathrm{L4S}}(t)/f_{\mathrm{L4J}}(t)], \tag{3}$$

where we have assumed that the size distribution at L4J and L4S are of the form r^{-q_J} and r^{-q_S} respectively. The ratio $f_{\mathrm{L4S}}(t)/f_{\mathrm{L4J}}(t)$ is estimated using the ratio between the number of particles survived after 1Gyr in each correspondent stable niche. In the case of L4J we computed the location of the stable region with an integration of ~11 Myr and the number of surviving particles after this time-scale is scaled to the number of surviving ones after 1 Gyr using a probability of diffusion from the Trojan region given by Levison, Shoemaker & Shoemaker (1997). Assuming that the primordial surface mass density in the Solar system had a radial dependence as r^{-2} (Weidenschilling 1977) and if possible values of q_S range between $3 < q_S < 5$, the number of asteroids at L4S with diameters greater than 5 km would be between 2240 ± 490 and 10200 ± 2190, where the error bars are computed using similar scaling arguments.

Some remarks should be made regarding this estimation. The scaling of the stable niches has been carried out by comparing two sets of initial conditions at low eccentricity and inclination. From other studies we know that stable niches exist at greater eccentricities (de la Barre et al. 1996) but our own investigations would indicate that the region at L4S becomes more unstable at greater inclinations. Regarding the other equilateral Lagrangian point of Saturn, L5S, a similar method would give a number of objects some 60 per cent smaller than the number at L4S, but better statistics would be needed to determine this figure better. Nevertheless, differences between the dynamical characteristics of the orbits at the L4 and L5 points of the major planets are already known to exist (Holman & Wisdom 1993).

A short remark should be made regarding the physical processes lying behind our scaling argument. As it was mentioned earlier, Marzari & Sholl (1998) notice that, because of secular resonant effects, a considerable number of objects are drawn out of the *Saturn–Trojan* orbits as Jupiter and Saturn approach the commensurability of periods giving rise to the great inequality. A number of processes (as the gravitational interaction with the surrounding nebular gas and collisions with smaller bodies) would have produced significant radial migrations among the outer planets (see for example Brunini & Fernandez 1999, Ward 1997), it is unlikely that may have acquired their gaseous envelope around their present relative location. Anyhow, when approaching it, a clearing of the *Trojan* population of Saturn would have occurred and only objects at the long-term orbits would have survived for the rest of the age of the Solar system.

Since in our argument, we compare the sizes of the niches of the long-term orbits, we assume that the physical collisions and gravitational encounters that would occur when the objects are dynamically evaporating, would not produce significatively different effects on the long-term stable niches of the equilateral points of both planets; this assumption should be tested since there is a size difference between them. Besides, a mild approach to the present relative location would not produce the same results as a very violent (non-adiabatic) one.

Thus, observational constraints on the characteristics of the Saturn's *Trojan* populations will contribute with important clues of the early conditions of the outer Solar system. If no object is present today at L4S or L5S, this would mean that those early conditions were such as to wipe out the whole population there. With this constraint in mind, it would be feasible to try to reproduce those conditions to have a better understanding of the formation scenario of the outer planets. If a population is found, its size distribution as well as the physical properties of the objects would represent invaluable fossil records of the early outer Solar system.

The apparent magnitude of an object is related to its absolute magnitude through the relation

$$m_R = H_R + 5\log_{10}(r\,\Delta), \tag{4}$$

where r is the heliocentric distance and Δ is the geocentric distance. If we assume for the objects at L4S and L5S the same albedo as for the Trojans, and neglecting phase-angle effects, the correspondent apparent magnitude of objects at L4S and L5S with a diameter of 8 km is $m_R = 24$ (Shoemaker et al. 1989). Then, if we also assume a similar distribution of inclinations as for the Trojans, between 0.1 and 0.5 (depending on q_S) objects per squared degree with $m_R < 24$ would be expected.

The target region where these objects should be looked for, lies close to the ecliptic. Assuming large libration amplitudes as the ones found in our simulations, 1/3 of the ecliptic area would be covered by the orbits of these objects. Since nearly 100 squared degrees of the ecliptic has been searched for Kuiper Belt objects (KBOs), finding some 300 KBOs (see for example Jewitt, Luu & Trujillo 1998, Jedicke & Herron 1997, Kowal 1989) to a limiting magnitude of $m_R < 24$ and 1/3 of this region ($\approx 30\,\mathrm{deg}^2$) would overlap with the L4S and L5S target region, thus between 3 and 15 objects should have been found there. However, it is also interesting to notice that the target region has being intercepted by the Galactic plane ever since 1992 (see Fig. 4), when the first KBO was discovered (Jewitt & Luu 1992). Let us recall that this selection effect prevented the detection of the inner satellites of Uranus up to 1998 (Gladman et al. 1998) and could also be the cause of the difference in the number of Trojans discovered at the L4J and L5J swarms (Shoemaker et al. 1989). Thus, either the presence of the Milky Way prevented these objects to be discovered or by the scaling arguments we have overestimated the size of the population. Up to now, a survey specially designed to discover *Trojans* of Saturn was unsuccessful to make a detection (Chen et al. 1997), however the region in the sky covered by this search (20°) is much smaller than the one covered by the stable orbits.

4 COLLISIONAL DYNAMICS

Since the stable niches at L4S are very small, the orbits contained within are very similar. Hence, relative velocities between the objects orbiting in them will be very small and collisions very rare. The parameter giving the number of physical encounters per year per physical cross-section per number of pairs of objects in the population is p, the intrinsic probability of collision. For the stable niches at L4S, p as well as the most probable relative velocity, v_r, have been computed using the method by Marzari et al. (1997), which is numerically based and it was specifically designed to take into account the particular characteristics of the Trojan orbits. We have obtained a value of $p = 3\times10^{-19}$ $\mathrm{km^{-2}\,yr^{-1}}$ and $v_r = 0.56\,\mathrm{km\,s^{-1}}$. The average number of encounters that an object of physical radius R would suffer, $\mathrm{d}N_c(t)$, is given by:

$$\frac{\mathrm{d}N_c}{\mathrm{d}t}(t) = pSn(t), \tag{5}$$

where S is the joint cross-section and $n(t)$ is the number of colliding objects at a given time. After an initial clearing stage where the objects at very unstable locations rapidly evaporate, the population at L4S would have evolved according to a slow diffusion. We have modelled this diffusion process by making an exponential fit with the number of surviving particles in our numerical integration (see Fig. 5). To obtain the total number of collisions over the age of the Solar system we integrate expression (5) over time. The values obtained are very low. For example, objects of $R = 7.5$ km, would in average suffer between 0.13 collisions for $q_S = 3$ to 0.2 for $q_S = 5$ with objects of diameter of 1 km or more, over the age of the Solar system. Hence, if there are any objects left at those orbits at the end of the clearing stage, which would have occurred right after the major planets acquired much of its present mass, then there is a very good chance that they could be found today because self-interactions and even

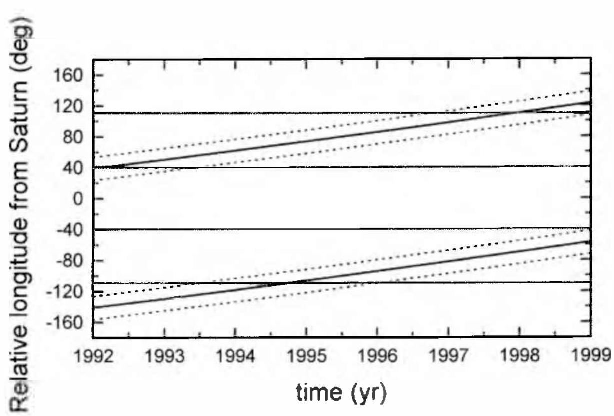

Figure 4. Relative longitude between Saturn and the intersection points between the ecliptic and the Galaxy. The libration regions of the long-term stable orbits are indicated. A mean galactic width of 30° is indicated.

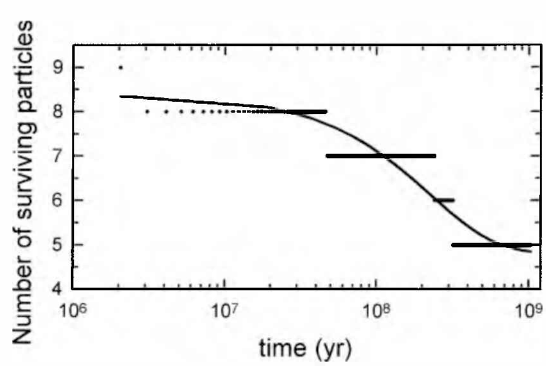

Figure 5. Number of surviving particles as a function of time in the slow diffusion regime. The curve of the exponential fit, $n(t) = N0 + N1\exp(-t/\tau)$, is shown, where $\tau = 2.2810^8$ yr, $N0 = 4.8$ and $N1 = 3.58$.

perturbations by passing objects (LP comets or escapees from the Kuiper belt) are negligible.

5 DISCUSSION

Here, we have considered the possibility that some primordial objects would have survived in the vicinity of the Lagrangian equilateral points of Saturn. Our results indicate that around L4S there are small stable regions that may potentially harbour a residual population. Specific surveys should be directed to them, as well as to other long-term stable regions of similar characteristics (Holman 1997). Naturally, the hypothetical discovery of an asteroid swarm at any of the triangular points of the orbit of a major planet other than Jupiter, will provide fundamental clues to the study of our home planetary system. The case analysed turned out to be particularly interesting because it would be an asteroid population not to have undergone a self-collisional process, therefore not only those bodies would be very lowly altered by fragmentation events but also their mass distribution would be the same as the one of the primordial objects in the region! The chances that these swarms would survive the conditions in the early Solar system have to be explored, as it has been done with other long lived orbital groups (Brunini & Melita 1998). Even if no objects are to be found, this fact would represent a constraint to be met by any hypothetical scenario of the major-planetary formation process. However it should be stressed that the long-term stable orbits would not be the only source of objects at the equilateral points of Saturn, since temporal captures of objects could also be possible, as there is a steady flux of comets from the Kuiper belt. Hence, a putative discovery would also help to understand better the Kuiper belt and the Centaur populations.

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