# The Thin Red Line, Molinism, and the Flow of Time

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**Abstract**

In addressing the problem of the (in)compatibility of divine foreknowledge and human freedom, philosophers of religion encounter problems regarding the metaphysics and structure of time. Some models of temporal logic developed for completely independent reasons have proved especially appropriate for representing the temporal structure of the world as Molinism conceives it. In particular, some models of the Thin Red Line (TRL) seem to imply that conditionals of freedom are true or false, as Molinists maintain. Noting the resemblance between Molinism and TRL models, Restall (2011) has advanced some criticisms of Molinism that have also been leveled against TRL models. In particular, Restall believes that the implication $p \to \mathbf{HF}p$ is not true in TRL models. Because Molinists must also accept that this implication is not true, this is a problem for them. We will show that Restall's criticism is wide of the mark. Firstly, it will be demonstrated that in many open future models (not just TRL) the implication $p \to \mathbf{HF}p$ is invalid. Secondly, while it is possible to account for this implication, some modifications are required in respect of the branching time semantics. In proposing one such modification, we show that this new

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semantics can be adopted by advocates of the TRL and, as a consequence, by Molinists as well. We conclude that the principle stated by Restall is either a problem for many open future models (not just for Molinists) or can be accounted for by these models and so is not a problem for Molinists either.

In addressing the centuries-old problem of the (in)compatibility of divine foreknowledge and human freedom, philosophers of religion encounter problems regarding the metaphysics and structure of time. As divine foreknowledge concerns past knowledge of what will happen in the future, this connection is entirely natural. As we will see, some models of temporal logic developed for completely independent reasons have proved especially appropriate for representing the temporal structure of the world as Molinism conceives it. In particular, some models of the Thin Red Line (TRL), according to which there is a true future among the possible futures, seem to imply that some conditionals of freedom, such as "if the agent a were in conditions $C$, she would freely choose to do $\varphi$", are true or false, as Molinists maintain. Noting the resemblance between Molinism and TRL models, Restall (2011) has advanced some criticisms of Molinism that have also been leveled against TRL models. In particular, Restall believes that the implication between $p$ and "it has always been true in the past that in the future $p$" is not true in TRL models. Because Molinists must also accept that this implication is not true, Restall maintains that this is a problem for Molinists, given the plausibility of this implication. The present aim is to show that Restall's criticism is wide of the mark. It will be demonstrated that *many* open future models (not just TRL) run into the problematic implication between $p$ and "it has always been true in the past that in the future $p$". While it is possible to account for this implication, some modifications are required in respect of the branching time semantics. In proposing one such modification, we do not contend that this is the only possibility, but we show that this new semantics can be adopted by advocates of the TRL and, as a consequence, by Molinists as well. We conclude that the principle stated by Restall is either a problem for *many* open future models (not just for Molinists) or can be accounted for by these models and so is not a problem for Molinists either.

The structure of the paper is as follows. Sections 1 and 2 compare the indeterminist Aristotelian model and the TRL model. In section 3, the TRL model is shown to be especially apposite for Molinism. Restall's argument is presented in section 4, and in section 5, the problem advanced by Restall is

shown to be common to other open future models, including the Aristotelian model. In section 6, a semantics is proposed that can solve Restall's problem, which can be implemented by both Aristotelian indeterminists and TRL theorists and, therefore, by Molinists as well. Conclusions are presented in section 7.

# 1 Models of the Future

There is broad consensus that propositions regarding the past can be true or false. However, propositions concerning future contingents continue to provoke discussion. One possibility is the *determinist*[1] conception, according to which the future is determined by the past. In this case, propositions concerning the future are true or false, as are those concerning the past, because what will happen tomorrow is already determined today. Suppose, however, that the world is indeterministic, and that the future, unlike the past, is open. This means that many future histories are possible, and that the past does not determine what will happen. Which truth value should we then assign to propositions concerning the future in an indeterminist framework? There are at least two possible positions; either propositions concerning the future are untrue (the Aristotelian solution)[2] or they can be true (the Thin Red Line solution).

The Aristotelian position[3] is grounded on the intuition that no future his-

[1]Sometimes, the term "fatalism" is used in this context, invoking a distinction between *logical* and *theological* fatalism. We favor the term "determinism" because there is at least a sense in which fatalism is not equivalent to the idea that there is a unique future history. On this view, a certain state of the world is destined to occur, regardless of any choices made by agents. This implies a *certain form* of determinism (in that a certain fate is decided), but this does not impede the existence of alternatives – that is, of different possibilities that lead to the same final outcome. To formally capture this idea of fatalism, it is necessary to assume that the past is branching – in other words, that many histories can converge on a single instant.

[2]Under the label "Aristotelism" we collect the positions according to which propositions regarding future contingents cannot be true. By "untrue" we mean that these propositions can be considered either false (Peircean semantics, cf. Prior 1967: 128-9 and, more recently, Todd 2016)) or neither true nor false (supervalutionism, cf. Thomason 1970, 1984). For present purposes, these two alternatives can be treated on par because both suppose that there is no true future history that is privileged over the others.

[3]We will use the term "Aristotelian" without adopting any stance about the historical question of Aristotle's actual theory (on this issue, see Crivelli 2004: 198—226)

tory is privileged over any other because the past does not establish which will ensue. If a proposition is true in some possible futures and false in others, it seems we cannot say what will happen in the future *tout court*. Thus, an indeterminist and open conception of the future seems to imply that propositions concerning the future cannot be true. However, many scholars have claimed that the truth of propositions about the future is not incompatible with indeterminism and libertarian freedom.[4] A common motto among these scholars is that the future is "determinate but not determined." The basic idea is as follows. Suppose that Ann is a libertarian[5] agent, and that the actual situation in the world does not determine whether or not Ann will drink a beer tomorrow. Tomorrow, however, Ann will have to choose whether to drink a beer, and her choice will eliminate the other alternative. Suppose that the truth of the proposition "Ann will drink a beer tomorrow", when evaluated at the present time, *depends* on the choice Ann makes tomorrow. If Ann chooses to drink a beer tomorrow, the proposition is true at the present time; if Ann chooses otherwise, the proposition is false. Obviously, Ann's choice is epistemologically inaccessible today, but this does not preclude that Ann will choose in a certain way tomorrow. If the truth of the proposition at the present time depends on Ann's future choice then the proposition "Ann will drink a beer tomorrow" has a truth value, even though we do not know which one. We can preserve bivalence even in a libertarian and indeterminist framework, in which the present does not determine which among the possible future histories will become true.

If a future history is to become true, it is privileged, on the basis that this history will become true while the others will be pruned. In other words, the future history that occurs is the actual future history; the others are just possible futures that will not become real. The existence of a true future history is not at odds with indeterminism because it is precisely the fact that the agent will freely choose in a certain way that privileges that history over the others. This privileged history is usually called the Thin Red Line (TRL)[6] to distinguish it metaphorically from other histories.

From here, we will ignore the determinist model, according to which there

[4]This thesis has been proposed by many different scholars in different contexts; see for instance Barnes & Cameron (2009), Merricks (2009), Øhrstrøm (2009), Malpass & Wawer (2012) Rosenkranz (2012), Borghini & Torrengo (2013), Wawer (2014).

[5]A libertarian agent is an agent that can perform (at least) a free action in the following sense: the agent determines the action and the agent could do otherwise.

[6]The expression "Thin Red Line" was introduced by Belnap & Green (1994).

is only one possible future history; we will instead compare the two indeterminist models outlined above: the Aristotelian model, according to which there are several possible future histories, none of which is privileged over the others; and the TRL model, according to which there is a true future history that is privileged over the others. The next section elaborates a formal treatment of these two models.

## 2 The two indeterminist models compared

To begin, some basic notions of temporal logic will be introduced to characterize the two positions in question.[7] A branching time structure ($\mathsf{BT}$) is a couple $\mathfrak{B} = \langle T, < \rangle$, where $T$ is a non-empty set of instants and $<$ is a relation defined on $T$. Intuitively, the instants are possible instantaneous states of the world and $<$ is the relation of temporal precedence. This relation is therefore asymmetric and transitive and satisfies (at least) the conditions of *Backward Linearity* ($BL$) and *Historical Connectedness* ($HC$):

**(BL)** $\forall t, t_1, t_2 (t_1 < t \wedge t_2 < t) \Rightarrow (t_1 = t_2 \vee t_1 < t_2 \vee t_2 < t_1)$

In words, two instants of the past of $t$ are either identical or ordered by $<$; this implies that, for every instant $t$, there is one and only one past history.

**(HC)** $\forall t_1 \forall t_2 \exists t (t \leq t_1 \wedge t \leq t_2)$

HC asserts that all the instants are connected in the past; the maximal subsets of instants linearly ordered in $T$ are referred to as *histories* – the possible courses of events in the world. Ours is a propositional language that includes a possible infinite set of propositional variables ($Var$) and two temporal operators **P** and **F**. We can define an evaluation function $V : Var \longmapsto \wp(T)$ that maps every propositional letter $p$ onto a set of instants at which $p$ is true. A model $BT$ is, then, a couple $\langle \mathfrak{B}, V \rangle$. According to the indeterminist intuition, there is only one past history but many possible future histories. We can represent this situation as follows.

Let us suppose to evaluate the propositions in the schema with respect to the instant $t_0$. While the two future histories $h_1$ and $h_2$ differ only from $t_0$

[7] See, for instance, Burgess (1979); Thomason (1984); Øhrstrøm (1981, 2009); Belnap *et al.* (2001), Hasle and Øhrstrøm (2005).

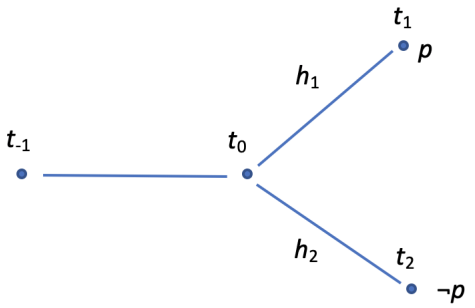



Figure 1:

onward, they share all the states of affairs at instants prior to $t_0$. In $h_1$ at the moment $t_1$, the proposition $p$ is true; in $h_2$ at the moment $t_2$, the proposition $\neg p$ is true. The problem is how to evaluate the proposition $\mathbf{F}p$ – that is, in the future, $p$ – with respect to $t_0$. Indeed, while there is a state of affairs in $h_1$ that makes $p$ true, there is no such state of affairs in $h_2$.

One possibility would be to evaluate the proposition not only with respect to a time but also with respect to a history (Ockhamist semantics):

$$
\begin{array}{lcl}
M, t/h \vDash p & \Leftrightarrow & t \in V(p) \\
M, t/h \vDash \neg\varphi & \Leftrightarrow & M, t/h \nvDash \varphi \\
M, t/h \vDash \varphi \wedge \psi & \Leftrightarrow & M, t/h \vDash \varphi \textit{ and } M, t/h \vDash \psi \\
M, t/h \vDash \mathbf{P}\varphi & \Leftrightarrow & \exists t'(t' < t \wedge M, t'/h \vDash \varphi \\
M, t/h \vDash \mathbf{F}\varphi & \Leftrightarrow & \exists t'(t' > t \wedge t' \in h \wedge M, t'/h \vDash \varphi)
\end{array}
$$

In such a case, $\mathbf{F}p$ will be true at $t_0/h_1$ because there is a subsequent instant in $h_1$ at which $p$ is true. However, it is false at $t_0/h_2$ because there is no instant in $h_2$ subsequent to $t_0$ at which $p$ is true:

$M, t_0/h_1 \vDash \mathbf{F}p$ and $M, t_0/h_2 \vDash \mathbf{F}\neg p$

However, relativizing the evaluation in terms of histories does not address the problem of the truth of propositions about the future *tout court*. Consider a contingent proposition $p$, such as "Ann drinks a beer." It is intuitively possible that Ann will or will not drink a beer. When it is asked whether

or not Ann will drink a beer (i.e., whether $\mathbf{F}p$), an answer that relativizes the truth of $\mathbf{F}p$ to histories is not entirely satisfactory. Indeed, a reply such as "There is a history in which Ann drinks a beer, and there is a history in which she does not drink a beer" does not seem helpful; we would like to know what Ann will decide *tout court*, not with respect to a history.

So, when $p$ is true in some future histories and false in other histories, which is the truth value of the proposition $\mathbf{F}p$ with respect to the moment $t_0$? As noted above, there are two possible alternatives. The first of these is that this proposition is untrue;[8] in this framework, the clause of the future is

$$M, t \vDash \mathbf{F}p \quad \Leftrightarrow \quad \forall h \exists t'(t' > t \wedge t' \in h \wedge M, t'/h \vDash p)$$

If $p$ is not true in all histories radiating from $t_0$, then at least two options are possible. On the first option, $\mathbf{F}p$ is false (Peircean model). In this case, all propositions concerning future contingents are false. On the second option, $\mathbf{F}p$ is false if in all histories radiating from $t_0$ $p$ is false and neither true nor false if there is at least one history in which $p$ is true and at least one history in which $\neg p$ is true (supervalutationism).

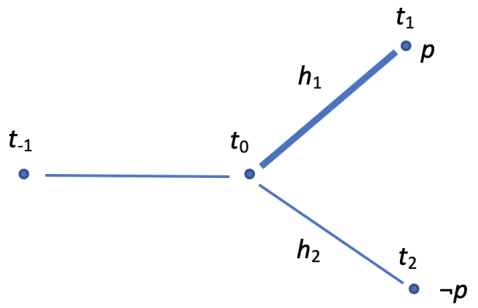
The $\mathsf{TRL}$ model is different; it assumes that the future is branching, as in the Aristotelian model, but presupposes that one of these histories has a privileged status with respect to the others. In other words, it is the history of the world that will come true. A $\mathsf{TRL}$ structure is a couple $\mathfrak{T} =< \mathfrak{B}, \mathsf{TRL} >$, where $\mathfrak{B}$ is a branching structure and $\mathsf{TRL}$ is a privileged history:

$$\begin{array}{lcl}
M, t \vDash^{\mathsf{TRL}} p & \Leftrightarrow & t \in V(p) \\
M, t \vDash^{\mathsf{TRL}} \neg\varphi & \Leftrightarrow & M, t \nvDash^{\mathsf{TRL}} \varphi \\
M, t \vDash^{\mathsf{TRL}} \varphi \wedge \psi & \Leftrightarrow & M, t \vDash^{\mathsf{TRL}} \varphi \textit{ and } M, t \vDash^{\mathsf{TRL}} \psi \\
M, t \vDash^{\mathsf{TRL}} \mathbf{P}\varphi & \Leftrightarrow & \exists t'(t' < t \wedge M, t' \vDash^{\mathsf{TRL}} \varphi) \\
M, t \vDash^{\mathsf{TRL}} \mathbf{F}\varphi & \Leftrightarrow & \exists t'(t' > t \wedge t' \in \mathsf{TRL} \wedge M, t' \vDash^{\mathsf{TRL}} \varphi)
\end{array}$$

The relevant clause is the one concerning the future: $\mathbf{F}\varphi$ is true if there is a time in the privileged future history at which $\varphi$ is true. Notice that, in this

[8]Recall that by "untrue" we mean either false or neither true nor false. As Todd (2016) points out, there are some analogies between the positions according to which the propositions concerning the future lack a truth value or are false and the respective positions of Strawson and Russell regarding which truth value, if any, to assign to a proposition such as "The actual king of France is bald", expressed at a time when France is a republic. However, Schoubye and Rabern (2017) show that the standard arguments for Russell's treatment of definite descriptions fail to apply to the treatment of the future operator.

version of the TRL model, the Thin Red Line is a constituent of the structure and not of the valuation; it is the world that has a unique privileged history.



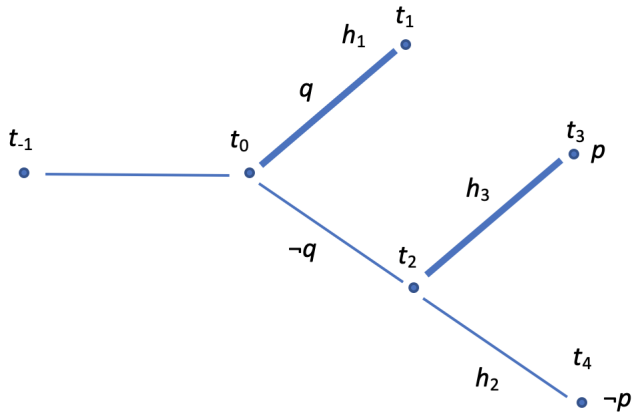

In this schema, the history $h_1$ is marked in red because it is the history that the agent will make actual by choosing $p$. Of course, the agent could have chosen otherwise, but she will not.

We will not discuss which of these two models best accounts for the idea of an open future. One of the objections leveled against the TRL model is that it does not take the openness of the future sufficiently seriously and that it is a disguised form of determinism (cf. Belnap & Green 1994, McFarlane 2003). If what an agent will do is already determinate, she cannot really do otherwise; possible futures that differ from the TRL would not really be open to the agent. Advocates of the TRL reply that choosing one alternative does not mean that the agent could not have chosen another. If it is only contingently true that the agent will choose an alternative over the others, there is no necessity, and the openness of the future is preserved (cf. Øhrstrøm 2009, Rosenkranz 2012). We will not take a side in this debate because our interest lies elsewhere: to show that some TRL models are compatible with Molinism and to evaluate some objections that have been advanced against these models.[9]

[9]The TRL model presented here suffices for *theological* Ockhamism (see for instance Plantinga 1986). Theological Ockhamists are committed to the claim that God believed yesterday that a certain agent $a$ will do $\varphi$ tomorrow. Since God's beliefs are infallible, it follows that it is true today that agent $a$ will do $\varphi$ tomorrow, even though $a$, being

# 3 The TRL+ Model

Suppose that Mary has invited Ann to a party. Ann must decide whether to accept the invitation or to stay home to read a novel. Suppose that Ann decides to dedicate herself to reading in the TRL. What would Ann have done if she had gone to the party? She would have had to decide whether to drink a beer or not. What would she have decided to do? The situation is as follows.




The moment $t_0$ is the moment at which Ann decides to stay at home to read a novel ($q$) or to go to the party ($\neg q$). As Ann will choose to read a

free, could have done otherwise. Therefore, although many different alternatives are open to $a$, one of these is privileged because it is the alternative that $a$ will choose, and this is guaranteed by divine foreknowledge. One might believe that every account of divine foreknowledge must accept a TRL model; indeed, if God is prescient and infallible, He already knows what agents will do in the future. Therefore, it must already be true that they will decide in certain ways rather than others. This seems to commit us to a TRL model. However, this is not a necessary consequence if a timeless solution to the problem of divine foreknowledge and human freedom is accepted. In the timeless model, God is out of time, and it is therefore unnecessary to claim that God knew yesterday what agents would do tomorrow. This timeless solution is demonstrably compatible with the Aristotelian model of contingent futures (cf. Authors, 20XX). In the next section, we will show that Molinists require a more demanding framework than the TRL model presented here. This more demanding framework will be referred to as TRL+.

novel, the history $h_1$ is marked. However, if Ann had gone to the party, she would have had to decide whether to drink a beer ($p$) or not ($\neg p$)—that is, to make $h_3$ or $h_2$ true. What would she have done in such a circumstance?

According to Belnap and Green (1994), the TRL theory should not only account for sentences such as

1. Ann will read a novel and she will not go to the party

but also for sentences such as

2. Ann will read a novel and she will not go to the party but, if she had gone to the party, she would have drunk a beer.[10]

In other words, for every bifurcation of the tree, whether in the actual history of the world or not, it is necessary to assign a truth value to the proposition that describes what an agent will do in that circumstance. Among advocates of TRL, one of the main aims is to preserve bivalence. However, bivalence must be preserved not only for sentences such as (1) but also for sentences such as (2).

Øhrstrøm (2009) has advanced a model that solves this problem.[11] In a nutshell, Øhrstrøm's idea is not to consider the TRL semantically as a history of the world but as a function (indicated here in lower case, trl) that takes times and yields histories. In other words, for every bifurcation, trl defines the TRL relative to that bifurcation, as in the schema above.

Formally, trl must satisfy two principles:

**i.** The instant at which the function is defined must be a member of the local TRL: $t \in \mathsf{trl}(t)$

**ii.** $t_1 < t_2 \wedge t_2 \in \mathsf{trl}(t_1) \Rightarrow \mathsf{trl}(t_1) = \mathsf{trl}(t_2)$; that is, if a particular history $h$ is picked as the TRL of an instant, then all later instants in $h$ also pick it as their TRL.

[10] Actually, Belnap and Green consider sentences such as: "Ann will read a novel and she will not go to the party. It is, however, possible that she will go the party, and then *later* she will drink a beer". In the text, we use counterfactuals in order to show the relevance of Belnap and Green's objection for Molinism.

[11] Before Øhrstrøm, a similar solution was proposed by McKim and Davis (1976).

Local TRLs enable us to solve cases such as Ann's and to respond to Belnap and Green's criticism of the concept of TRL. It is worth noting that, in this model, there is no unique marked history (*the* TRL) but a function defined on elements of the structure. We call this model TRL+.

According to TRL+, the clause for the future is

$$M, t \vDash^{\mathsf{TRL}+} \mathbf{F}\varphi \quad \Leftrightarrow \quad \exists t'(t' > t \wedge t' \in \mathsf{trl}(t) \wedge M, t' \vDash \varphi)$$

As noted above, the instant at which Ann chooses in a certain way determines, by definition, a local TRL, which allows to evaluate also with respect to instants that do not belong to the true future. Local TRLs have raised some concerns (cf. Belnap and Green 1994 and Belnap, Perloff, Xu 2001). These criticisms of TRL+ are not of interest here; in demonstrating that Restall's objection to Molinism is orthogonal to the problems raised by TRL+, we will show that the issues he raises are common to many other branching frameworks and are not specific to TRL+. In sketching a solution to Restall's objections, we will show that this solution can be adopted by advocates of both the Aristotelian and the TRL model, confirming that Restall's question is not pertinent to Molinism or to the idea of a true future *per se*.

# 4 Criticism of the Molinist model

According to Restall (2011), the Molinist needs precisely what the TRL+ model offers. The Molinist maintains that the so-called counterfactuals of freedom have a truth value; these counterfactuals take the following form:

3. In circumstance $C$, agent $a$ would have done $\varphi$.

The truth of counterfactuals of freedom must hold in general.[12] For any circumstance in which an agent makes a free choice, a true future must exist

[12]The Molinist maintains that counterfactuals of freedom (CF) are eternally known by God. They are the objects of middle knowledge, in as much as it is intermediate between the knowledge of eternal and immutable truths and the knowledge of contingent truths. By knowing the eternal truths and the CFs concerning every possible agent before the creation of the world, God knows the best world to be created because, for every possible world, He has foreknowledge of how free agents will behave in that world and of the outcomes of their actions. Being perfectly good, God can therefore choose the best of all possible worlds. From the Molinist perspective, it is obviously crucial that the truth of CFs is compatible with human freedom. The thesis that there is no opposition between the truth of CFs and human freedom is perfectly characterized by TRL+, as for any choice

– that is, it must be true that the agent will choose in a certain way, even if she could have done otherwise. Within the model, this means that, for any fork (i.e., for any choice), there exists a TRL: a history that the agent, in that circumstance, will make true. The TRL+ model perfectly accounts for the truth of counterfactuals such as (3); indeed, the second sentence of (2) is a counterfactual of freedom.[13]

Relying on Belnap and Green (1994), Restall (2011) argues against TRL+, and thus against Molinism, because it does not satisfy the following very plausible principle[14] :

4. $p \rightarrow \mathbf{HF}p$

That is, if at a certain moment it is true that $p$, it also has always been true in the past that, in the future, it would be true that $p$. For instance, if today it is true that Ann is at home reading a novel, then yesterday it was true that today Ann would be at home. According to Restall:

> The failure of this principle is why the thin red line is to be rejected (...) Why accept this principle? (...) Here, I will just give one argument to the effect that this principle is valid. Suppose at point $c$ [a past moment], as I stand looking at my tie collection, my son Zachary and my spouse Christine are there, and Zachary says 'Dad will wear a green tie' and Christine says, 'Greg will wear a brown tie'. Then, retrospectively, from the point of view of $g$ [the actual moment], what Zachary said at $c$ was correct, and what Christine said was incorrect. (p. 234)

Consider now why principle (4) is not satisfied in TRL+. Looking at the last picture and, in particular, the instant $t_3$ in history $h_3$. At this pair instant-history, it is true that $p$ (i.e., that Ann is drinking her beer at the party). To check whether, in the past, it was always true that Anna would

made by an actual or possible agent, it is already true (before the choice) that the agent will freely choose in a certain way. For a complete introduction to Molinism, see Flint (1998). See also Craig (1991), chap. XIII.

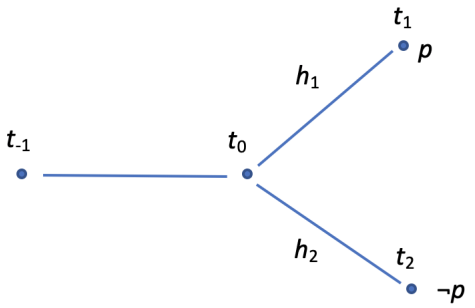
[13]Actually, the Molinist needs a more complex model constituted by many trees. There are as many trees as there are possible initial states of the world, but we can overlook this complication here because, for our purposes, the resulting model would not be conceptually richer than what we are discussing.

[14]The operator $\mathbf{H}$ is the dual of the operator $\mathbf{P}$: $\mathbf{H}\varphi \equiv \neg\mathbf{P}\neg\varphi$

drink a beer at the party, we have to go back to the instant $t_0$. At this instant, is it true that, in some future instant, Ann is going to drink a beer at the party? The answer is negative. At the instant $t_0$, there exists a true future ($h_1$), where Ann is at home reading the novel. So, in the past of $t_3$, it is not true that, in the future, it is true that $p$. Therefore, principle (4) is not valid and, given its plausibility, Restall claims that this is a good reason to refuse both TRL+ and Molinism.

# 5 A problem for the Aristotelian model too

For the moment, we will not respond to Restall's criticism. Instead, we will demonstrate that Restall's problem affects the Aristotelian indeterminist model too. Let us consider the following situation.




At $t_0$, the agent was free to choose to go to the party ($p$) or to remain at home ($\neg p$). Now, let us evaluate Restall's principle, at $t_2$,

5. $M, t_2 \vDash \neg p \rightarrow \mathbf{HF}\neg p$

Clearly, $\neg p$ holds at $t_2$. Now, if check if $\mathbf{HF}\neg p$ holds. Accordingly, $\mathbf{F}\neg p$ must always be true in the past of $t_2$. So, the point is if $M, t_0 \vDash \mathbf{F}\neg p$ holds. However, as we have seen in the Aristotelian model, the existence of a history in which $p$ is true entails that $\mathbf{F}\neg p$ is untrue at $t_0$. This means that the Aristotelian model shares the same problem as the Molinist model in

falsifying the plausible principle that if something is true today, it was true yesterday that it would be true today. The objection does not apply to the "supervaluational" Aristotelian models which we discuss in the next section.

As a way out, the principle $p \rightarrow \mathbf{HF}p$ can be made true by adopting the solution previously outlined: the evaluations must be relativized with respect to two parameters, where the situation is as follows:

6. $M, t_2/h_2 \vDash \neg p \rightarrow \mathbf{HF}\neg p$

It is easy to check that things work here; we have that $M, t_0/h_2 \vDash \mathbf{F}\neg p$. However, $M, t_0/h_2 \vDash \mathbf{F}\neg\varphi \Leftrightarrow \exists t'(t' > t_0 \wedge t' \in h_2 \wedge M, t'/h_2 \vDash \neg\varphi)$. Clearly, in history $h_2$ there is an instant at which the agent is at home, and this suffices to secure the principle's validity. However, the point is the same as before; this framework does not capture the intuitive meaning of the future. On this construal, Restall's principle should turn out as follows: if $p$ is true in a history, then it is true in the past of that history that $p$ will be true in the future of that history. This is surely correct but not very informative.

In conclusion, validating Restall's principle is a problem for both the "Molinist" TRL+ framework and for the indeterminist Aristotelian framework (and, as can readily be shown, for the "pure" TRL system). In a nutshell, the problem concerns the construal of a possibility that was open in the past but is now closed. For this reason, to (at least partially) account for Restall's intuition, we believe it is necessary to refer to a further element.

# 6 A possible way out

One can react in a number of ways to the failure of $p \rightarrow \mathbf{HF}p$ in the branching time semantics under consideration here. For example, one might argue that this principle, however intuitive, is not inescapable, and that if abandoning it is the cost of a branching semantics, then that is a cost worth paying. Alternatively, one might try to modify the semantics to account for this principle. In fact, there are some proposals in literature that aim to reform both the Aristotelian and the TRL model and that validate the principle $p \rightarrow \mathbf{HF}p$. As for the Aristotelian model, Thomason (1970, 1984) puts forward the idea of restricting the valuation at a moment $t$ just to the histories that pass through $t$. In other terms, $t$ is the *perspective* from which propositions are evaluated (Thomason 1984: 145) Furthermore, the evaluation is in fact a supervaluation: a proposition $\varphi$ is true a $t$ if it is true in *every* history

that pass trough $t$, false if it is false in *every* history that pass through $t$, indeterminate otherwise. On this view, $p \rightarrow \mathbf{HF}p$ is valid because at any time $t$ at which $p$ is true, also $\mathbf{HF}p$ is, given that the evaluation is restricted to the histories that pass through $t$.

As for the TRL model, the proposals of Malpass and Wawer (2012) and Wawer (2014) make the principle under examination true. The main strategy of these papers is to reject the relativization of the TRL to times and to keep the original idea of a unique TRL. Clearly, if a formula is evaluated at a moment belonging to the TRL, then the principle is valid. For the evaluation to moments external to the TRL, Malpass and Wawer (2012) use Thomason's method of supervaluations, whereas Wawer (2014) try to rephrasing the formulas evaluated at moments not belonging to the TRL as modalized formulas evaluated at moments belonging to the TRL.

So, within both the Aristotelian and the TRL models, it is possible to make adjustments to make the principle valid.[15] Here, we will propose a new method of accounting for the principle. We believe that this is an interesting alternative to the models of Thomason, Malpass and Wawer. First, it is a general method, which is not internal to the Aristotelian or TRL framework, but can be applied to both. Second, it is able to account for Molinist conditionals of freedom. This is not the case on the alternative models. For instance, according to Malpass and Wawer (2012), the sentences containing a future operator, if evaluated at a moment external to TRL, are true iff some fact obtains in the future of every history radiating from that moment. Therefore, the sentence "if Ann had gone to the party, she would have drunk a beer" is true iff Ann drinks a beer in the future of every history passing through the moment at which Ann is at the party – which is a counterfactual moment, external to the TRL. However, to guarantee Ann's freedom of choosing otherwise and not to drink a beer, there should be at least a history in which Ann does not drink a beer. But, if Ann drinks a beer in some histories that radiate from the moment at which Ann is at the party, and Ann does not drink a beer in some other histories radiating from that moment, the proposition $\mathbf{F}$(Ann drinks a beer), evaluated at that point, is deprived of truth value, according to the model of Malpass and Wawer (2012). However, if we wish to use the TRL model to account for Molinism and conditionals of freedom, it is necessary to say that the conditional is true, even thought Ann does not drink beer in every history. This means that it is necessary

[15] Thanks to an anonymous referee for emphasizing this point.

to keep the TRL+ model, in which the TRL is relativized to instants of time to make conditionals of freedom true or false. Our proposal has the advantage of justifying the principle $p \rightarrow \mathbf{HF}p$ within TRL+, i.e. of justifying the principle within a model in which the conditionals of freedom have a truth value.

Our framework has no new element.[16] However, we believe that the *combination* of elements it uses is new. It is based on the idea that formulas are evaluated in a model, at a time $t$, from a certain perspective or context. The idea of a perspective or context of use circulate in the branching time semantics in very different forms. We have seen that it is essential in Thomason (1970, 1984) to account for the principle $p \rightarrow \mathbf{HF}p$. The model of Belnap, Perloff, Xu (2001) uses the parameter of the context and that of Wawer (2012) the context of use. Here we will adopt a notion of perspective close to that of MacFarlane (2003, 2014). According to MacFarlane, every proposition must be evaluated at two different times, which he calls context of assessment and context of use. Also in our proposal the evaluation occurs at two different times. However, our model differs from that of MacFarlane for different reasons. One of the most important points is that it allows the evaluation of formulas at points different from those connected to the perspective in order to account for conditionals of freedom. Another substantial difference is that MacFarlane considers his framework as *alternative* to that of the TRL, whereas ours is fully compatible with the TRL model.

Let us take into account the closed formula $\varphi$. $\varphi$ has a certain truth value at $t$. As said above, our semantical framework introduces another ingredient: we evaluate $\varphi$ at $t$ from the temporal perspective $t'$ (which, of course, might coincide with $t$). Roughly, the idea is to consider the perspective $t'$ as the point at which the world is arrived, that is, the present moment.[17] We will use the perspective to prune respectively the histories and the TRLs on which formulas are evaluated. Let us start to see how this works within the Aristotelian model.

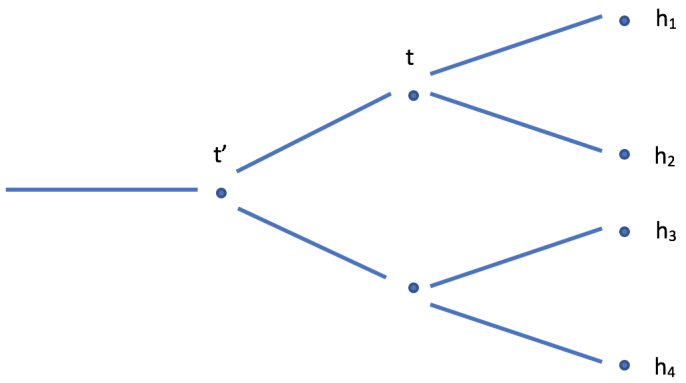
[16] We want to thank an anonymous referee for the criticisms and suggestions about our formal framework.

[17] Friends of a dynamic and realist metaphysics of time could construe the idea of perspective we are presenting in strong sense. However, our semantic allows also an indexical reading according to which the perspective indicates that instant we consider our 'now', without any metaphysical privilege.

## 6.1 Perspective and Aristotelian model

Let $H_t = \{h | t \in h\}$ be the bundle of histories at $t$, that is, the set of histories which pass through $t$. Our model is, then, constituted by the structure $\mathsf{BT}$, the evaluation function $V$, and two temporal indices: the instant of evaluation and the perspective from which one evaluates. We sharply distinguish two kinds of propositions: *factual* propositions, which concern things that happen at the present, the past and the future of a given perspective, and *counterfactual* propositions, which do not concern what it is happening, happened or will happen from the given perspective, but what could or could have happen from another perspective. To give an interpretation to counterfactual propositions is particularly important here because conditionals of freedom are counterfactual propositions. In this section, we address factual proposition within the Aristotelian model, in section 6.2, we will deal with factual propositions within the $\mathsf{TRL}$ model, section 6.3 concerns counterfacual propositions in both models.

The evaluations of factual propositions in the Aristotelian model are always relativized to the *intersection* of the (bundle of) histories which pass through the moment of evaluation $t$ and the histories which pass through the perspective $t'$. Since we are evaluating factual propositions, we suppose that this intersection is never empty, that is that the moment of evaluation is connected with the temporal perspective (i.e. $(t > t') \vee (t' > t) \vee (t' = t)$). This seems a reasonable condition because factual propositions concern what happens at a certain time or in the past or in the possible futures of that time (the perspective). Because the histories on which we evaluate must pass through the perspective, certain branches are *pruned*. Let us see an example:

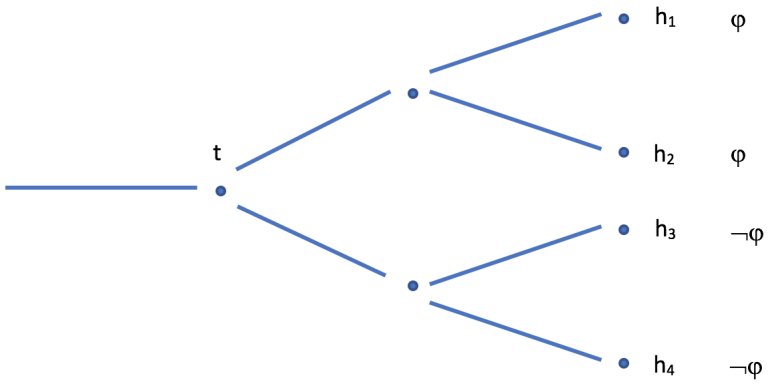



In this schema we have four histories; let us suppose we evaluate the formula $\varphi$ at $t$, from the perspective $t'$: $M, t, t' \vDash \varphi$. So we have the following truth conditions:

$$
\begin{array}{lcl}
M, t, t' \vDash p & \Leftrightarrow & \forall h \in H_t \cap H_{t'}, t \in V(p) \\
M, t, t' \vDash \neg\varphi & \Leftrightarrow & \forall h \in H_t \cap H_{t'}, M, t, t' \nvDash \varphi \\
M, t, t' \vDash \varphi \wedge \psi & \Leftrightarrow & \forall h \in H_t \cap H_{t'}, M, t, t' \vDash \varphi \text{ and } M, t, t' \vDash \psi \\
M, t, t' \vDash \mathbf{P}\varphi & \Leftrightarrow & \forall h \in H_t \cap H_{t'}, \exists t'' < t, M, t'', t \vDash \varphi
\end{array}
$$

Notice that our evaluation at times and perspectives is analogous to a standard evaluation as far as evaluations not regarding the future are concerned.[18] Things change when we consider the evaluation of future tense. The idea is, in a nutshell, the following:

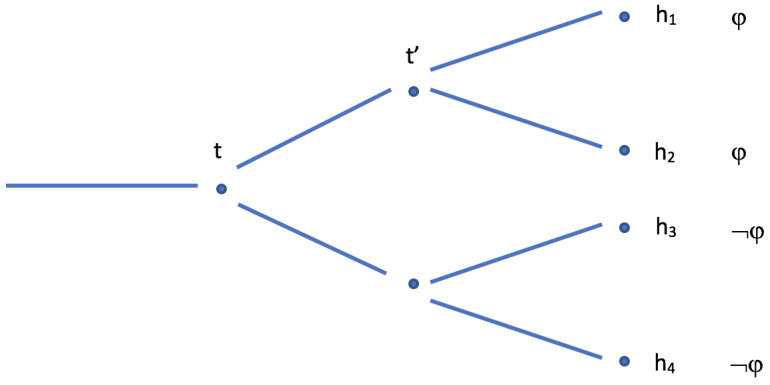
[18]Since in these cases $(H_t \cap H_{t'}) \subseteq H_t$, the quantification over the intersection is superfluous and it would be sufficient to quantify over the histories in $H_t$. However, we stick to this formalization for symmetry with the future case.




Here, we have a branching structure in which there are four histories: in the first two, it is true that $\varphi$ at some times; in the second, it is true that $\neg\varphi$ at other times. Now, let us hypothesize we want evaluate $\mathbf{F}\varphi$ at $t$ from the perspective $t$. We have, then:

$$M, t, t \vDash \mathbf{F}\varphi \quad \Leftrightarrow \forall h \in H_t \cap H_t \exists t' > t, M, t', t \vDash \varphi$$

Obviously, this does not hold since there exist two histories in the intersection on which $\neg\varphi$ is true. But now, let us suppose that the perspective changes (i.e. time flows) and the schema becomes:




This schema is perfectly similar to the previous except for the perspective of the evaluation, which is now $t'$.

$$M, t, t' \vDash \mathbf{F}\varphi \quad \Leftrightarrow \forall h \in H_{t'} \cap H_t \exists t'' > t, M, t'', t' \vDash \varphi$$

Time flowed – so to speak – and the amount of histories in the intersection decreased. From the perspective of $t'$, therefore, $\varphi$ will happen at $t$. So, for example, while from the perspective of yesterday, it was not true that Ann would be at home today, from the perspective of today, when Ann has already decided to stay at home, it was true yesterday that Ann would stay at home. So, our framework incorporates MacFarlane's intuition that the evaluation at a certain time of a formula containing a future operator changes depending on the perspective that is assumed.[19]

Let us see, now, what happens to the crucial principle $p \rightarrow \mathbf{HF}p$. Let us assume to evaluate at $t'$ from the perspective of $t'$; we have, then, that

$$M, t', t' \vDash p \rightarrow \mathbf{HF}p$$

So, let us assume $M, t', t' \vDash p$; then, it must be always true in the past that $\mathbf{F}p$. Let us consider the instant $t$ previous to $t'$:

$$M, t, t' \vDash \mathbf{F}p \quad \Rightarrow \forall h \in H_{t'} \cap H_t \exists t'' > t, M, t'', t' \vDash p$$

Of course, this holds because $p$ holds in every history that passes through $t'$ at a moment subsequent to $t$. In particular, it holds at $t'$ itself. Since the evaluation is always restricted to the histories in the intersection $H_{t'} \cap H_t$, if $p$ holds at $t$, then it is true at every moment in the past of $t$ that $p$ will hold in the future.

The principle $p \rightarrow \mathbf{HF}p$ is valid also in Thomason's supervalutationist framework. However, in the framework we present here it is possible to say that the same utterance is true or false or indeterminate depending on the perspective from which it is considered. Suppose that yesterday Paul said: "Ann will stay at home and she will not go to the party". If Ann is free to stay at home or not and if a supervalutationist semantics of the future is assumed, it is intuitively true that Paul's assertion has no truth value until Ann's decision to stay at home. But after that decision, *this same assertion* acquires a truth value. In our framework, as in MacFarlane's relativist framework, it is possible to account for this intuition, whereas in Thomason's framework formulas cannot change their truth value at a time in dependence of a certain perspective.

[19] When $\varphi$ does not hold in the future of every history of the intersection, we have, as before, two options: either we can state that $\mathbf{F}\varphi$ is false (Peirceanism) or we can use supervalutations.

## 6.2 Perspective and TRL

In this section, we will extend our framework of evaluation at times and perspectives to TRL-semantics. Our general idea for TRL semantic mirrors that of the Aristotelian semantics. In the latter case, histories that do not pass through the moment of evaluation and the perspective are pruned. In TRL+ case, instead, TRL-histories that do not pass through these two points are pruned. Recall that in TRL+, for every point of the world there is a local TRL relative to that point. Then, we suppose that the TRL on which formulas are to be evaluated must necessarily pass through the moment of evaluation and the perspective.[20] To do this, we introduce an auxiliary notion, that of a TRL relative, not to an instant of time, but to a segment of the tree, in particular to the segment that connects the perspective and the time of evaluation. What is the TRL relative to a certain interval of time? It seems intuitive to say that it is the TRL relative to the last moment of that interval. Since the last moment of the interval that connects the perspective and the moment of evaluation can be either the perspective or the moment of evaluation, we can define the TRL relative to this interval – which we call $\mathsf{TRL}^*$ – as follows:

$$TRL^*(t,t') \Leftrightarrow \begin{cases} TRL(t) & \text{if } t \geq t' \\ TRL(t') & \text{if } t' > t \end{cases}$$

Now, we have all the elements for defining the semantics of TRL+ in our framework:

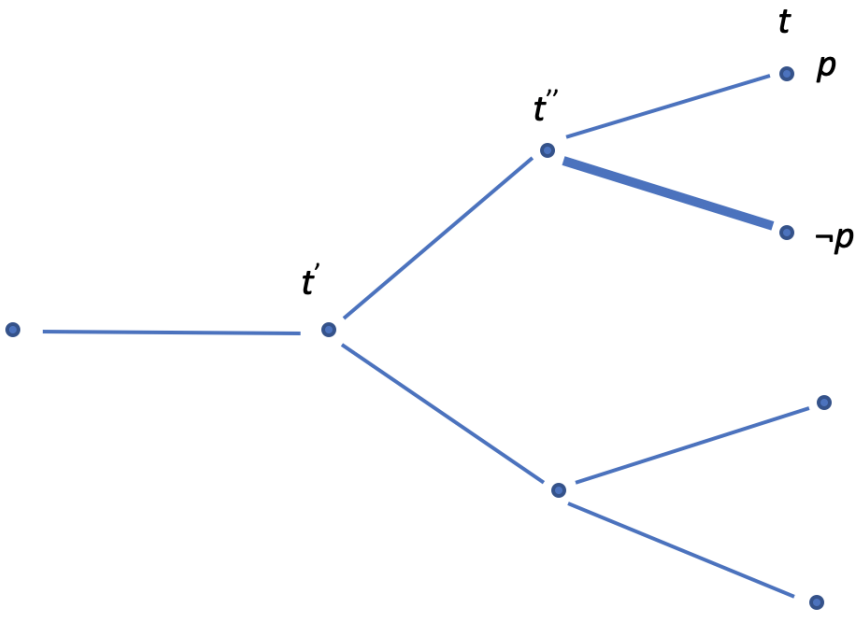
[20] Recall that the moment of evaluation and the perspective must be connected. It might be reasonable to introduce further conditions on their relationship. For example, it seems to be reasonable that the perspective $t'$ must belong to the TRL of the point of evaluation $t$ (i.e. $t' \in \mathsf{TRL}(t)$). It would be unnatural if the "present" were not on the TRL of a past point of evaluation. This is especially true if the perspective is interpreted in a realist and dynamical sense, as a point that moves on the tree. In this case, if the principle is not accepted, some counterintuitive consequences follow. For example, it might be the case that it is true today that Ann will drink a beer tomorrow. Nevertheless, when time flows and tomorrow becomes the present time, it is false that Ann drinks a beer. However, we will put aside this matter here. Our aim is to show that the TRL+ model can validate the principle of retrogradation of truth and the relationships between the perspective and the moment of evaluation are orthogonal to this problem.

$$
\begin{array}{lcl}
M,t,t' \vDash p & \Leftrightarrow & t \in V(p) \\
M,t,t' \vDash \neg\varphi & \Leftrightarrow & M,t,t' \nvDash \varphi \\
M,t,t' \vDash \varphi \wedge \psi & \Leftrightarrow & M,t,t' \vDash \varphi \text{ and } M,t,t' \vDash \psi \\
M,t,t' \vDash \mathbf{P}\varphi & \Leftrightarrow & \exists t'' < t, M,t'',t' \vDash \varphi \\
M,t,t' \vDash \mathbf{F}\varphi & \Leftrightarrow & \exists t'' > t, t'' \in TRL^{*}(t,t') \text{ and } M,t'',t \vDash \varphi
\end{array}
$$

Obviously, the crucial clause is that regarding the future. $\mathbf{F}\varphi$ is true at an instant and a perspective iff there exists an instant subsequent to the instant of evaluation that belongs to the $\mathsf{TRL}$ relative to the segment $t - t'$ at which $\varphi$ is true. If the perspective follows the instant of evaluation, then $\mathsf{TRL}$ that must be considered is that relative to the perspective. This implies that all the $\mathsf{TRLs}$ relative to moments previous to the perspective will not be considered in the evaluation.

If the perspective $t'$ follows or is equal to the moment of evaluation $t$ or coincides with it, then the principle $p \rightarrow \mathbf{HF}p$ is valid. Suppose $p$ is true at $t$. Then, the principle is true if at every instant previous to $t$, $\mathbf{F}\varphi$ is true. At each of these instant $t''$, the formula $\mathbf{F}\varphi$ is true iff there is an instant subsequent to $t''$ and belonging to $\mathsf{TRL}^{*}(t'',t')$ at which $p$ is true. Now, the $\mathsf{TRL}$ relative to the segment $t'' - t'$ necessarily pass through $t$ (because $t'' < t < t'$) and, thus, through an instant at which $p$ is true. This validates the principle.

Notice, however, that the principle fails to be valid if the moment of evaluation follows the perspective. It is easy to see why:




Suppose, again, that $t$ is the moment of evaluation and $t'$ the perspective and that $t > t'$. Furthermore, suppose that there exists a moment $t''$ such that $t > t'' > t'$. If $p$ is true at $t$, to validate the principle, $\mathbf{F}p$ must be true at every moment that precedes $t$. In particular, it must be true at $t''$. Now, $\mathbf{F}p$ is true at $t''$, given a perspective $t'$, if $p$ is true at at least one of the instants belonging to $\mathsf{TRL}^*(t'', t')$ that follows $t''$. However, since $t'' > t'$, the $\mathsf{TRL}$ of the segment $t'' - t'$ is equal to the $\mathsf{TRL}$ of $t''$. But it is possible that the $\mathsf{TRL}$ relative to $t''$ is different from that relative to $t$ and that in such $\mathsf{TRL}$ $p$ is not true. In such case, $\mathbf{F}p$ would be false, invalidating the principle.

Is the failure of the principle under the condition $t > t'$ a problem? We do not believe that to be the case. Restall himself seems to suggest, at least implicitly, something like this:

> [R]etrospectively, from the point of view of $g$ [the actual moment], what Zachary said at $c$ was correct, and what Christine said was incorrect. (p. 234)

The key word is the adverb "retrospectively"; the retrogradation of truth is an intuitive principle if, say, things *happened* in a certain way. It is because, finally, Greg wore the green tie that it is true that it was true that he would wear the green tie. Our semantics is perfectly capable of characterizing this state of affairs.

Finally, we introduce the operator $\Box$. $\Box p$ at a certain moment $t$ means that $p$ will necessarily happen in the future of $t$, so that the future of $t$ is settled with respect to $p$.[21] We suppose that $\Box$ is sensitive only to the moment of evaluation and insensitive to the perspective. Formally:

$$M, t, t' \vDash \Box\varphi \quad \Leftrightarrow \forall h \in H_t, \exists t'' > t \wedge t'' \in h, M, t'', t \vDash \varphi$$

As expected, $\mathbf{F}p \rightarrow \Box p$ is valid in the Aristotelian framework, but it is not valid in the TRL+ framework. More interestingly, however, while the principle $p \rightarrow \mathbf{HF}p$ is valid in both frameworks (if the point of evaluation is in the past of the perspective), the principle $p \rightarrow \mathbf{H}\Box p$ fails to be valid in either framework. Every point in the past at which $\neg p$ is true at a moment following that point is sufficient to render the principle invalid. And this seems reasonable. If Ann has decided to stay at home and not to go to the party, it has always been true that she would stay at home, but it has not always been settled that she would stay there: she had a chance to go after all.

In conclusion, the semantical framework sketched here makes the principle $p \rightarrow \mathbf{HF}p$ true in certain cases; if Restall's thesis is that the principle must be logically valid, then our model clearly fails to satisfy his requirement. However, it is plausible to construe the principle as concerning what is now happening and, then, what was *retrospectively* true that it would happen. If this interpretation is sound, our framework both preserves the branching structure of time and makes the principle true in those cases that align with Restall's intended meaning.

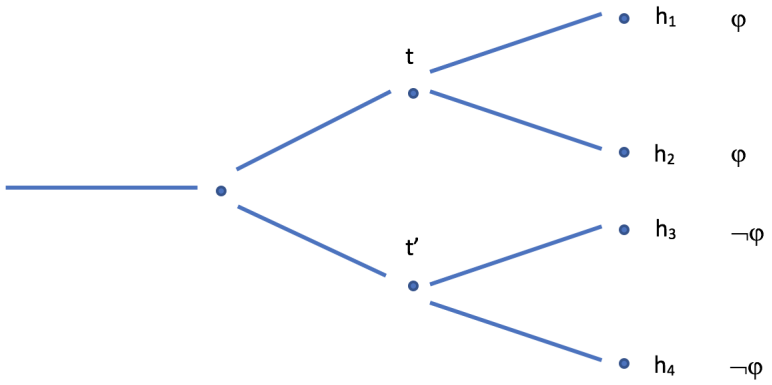
It follows that our semantics can address Restall's problem independently of one's position in relation to the Thin Red Line; we can admit the existence of a local true future or we can maintain an Aristotelian position; finally, it

[21] For similar operators, cf. Belnap, Perlof and Xu (2001), p. 161 and Wawer (2014), p. 371. Notice that we could also introduce the operator *Unsettled*, but it would not be the dual of $\Box$. The future of $t$ is unsettled or contingent with respect to $t$ iff there are some histories radiating from $t$ in which it is true at a moment subsequent to $t$ and some histories radiating from $t$ in which it is false at a moment subsequent to $t$. Consequently, $p$ is unsettled iff also $\neg p$ is.

can be incorporated within Thomason's supervalutationist framework. In all cases, our semantics allows the principle Restall wishes to preserve to be made true without abdicating to branching time.

## 6.3 Counterfactual cases

Suppose that we wish to evaluate propositions that do not concern the present, the past or the future of a certain perspective, but what would happen if things were gone in a different way from they went (from a certain perspective). Intuitively, things happened in this way, but how $\varphi$ would have been if, at that time, the events were different? These are, clearly, counterfactual situations.




Let us hypothesize that the perspective of the evaluation is $t'$ – in other terms: that the world "arrived" to $t'$ – but we want to evaluate a formula at $t$. For instance, at $t'$ Ann is reading a book at home because she decided not to go the party. But suppose that she had gone to the party, instead. At $t$ Ann is at the party and she has to decide whether to drink a beer or not. What would have Ann decided if she had gone to the party?

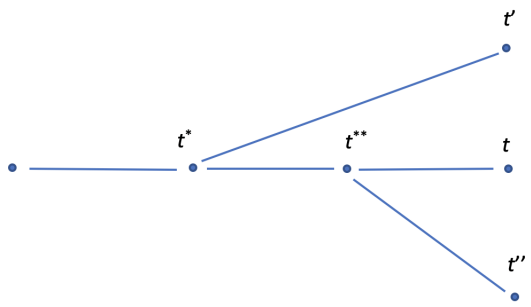
We will provide here a semantics for *some* kinds of counterfactuals, the conditionals of freedom. In particular, our analysis is restricted to *historical counterfactuals* (cf. Placek & Müller 2007), i.e. counterfactuals that concern historical or real possibilities. We further restrict our analysis, within this particular class, to the counterfactuals that concern the decisions that free agents would take in some counterfactual situations.

Roughly, our idea is to shift the original perspective to another, close, perspective, which represent the time the world would have arrived to, if things were different. In particular, we would like to know how things go if the perspective were the counterfactual moment at which Ann is at party. We need, then, a function that shift the perspective from the actual one to a counterfactual one in which Ann is at the party, i.e. in which the antecedent of the counterfactual is true. We can, then, evaluate the consequent of the counterfactual from that perspective. Formally, we can characterize this intuition by introducing a *Counterfactual Perspective Function*, $f_{CP}$, which takes an instant and yields an "analogous" instant that belongs to an alternative branch. In other words, the function takes the "present" moment and maps it on another possible present moment. More particularly[22], $f_{CP}$ is a function that takes two arguments, the present perspective and the antecedent of the conditional, and gives another perspective as value: this perspective is a moment at which the antecedent of the conditional is true and which is, among the moments at which the conditional is true, the moment "closest" to the present moment.

There are, of course, many ways to account for the closeness or distance between two moments belonging to different histories. Here is an idea.[23] A counterfactual moment $t'$ is closer to the actual moment $t$ than another moment $t''$, if the history of the world that arrives at $t'$ is more similar to the history of the world that arrives at the present moment $t$ than the history of the world that arrives at the moment $t''$. More specifically, suppose that the histories that contain $t$ and $t'$ bifurcate at $t^*$ and the histories that contain $t$ and $t''$ bifurcate at $t^{**}$. Then, $t'$ is a closer counterfactual moment than $t''$ if the segment $t^* - t'$ is more similar to the segment $t^* - t$ than the segment $t^{**} - t''$ to the segment $t^{**} - t$. We consider the notion of similarity between two segments of a history as primitive and unanalysed in our framework. Discussions about the similarity of segments are analogous to the discussions about similarity of worlds.

[22]The following interpretation of counterfactuals is inspired by Thomason and Gupta (1980).

[23]See Thomason and Gupta (1980), Placek and Müller (2007) and Wawer and Wroński (2015).




With this machinery, we are now ready to give the semantics of conditionals and of conditionals of freedom in particular. As for conditional in general, we propose the following formalization:

$$M, t, t' \vDash \varphi > \psi \quad \Leftrightarrow M, f_{CP}(t', \varphi), f_{CP}(t', \varphi) \vDash \psi$$

The function $f_{CP}$ shifts *both* the moment of evolution and the perspective to a moment at which the antecedent of the counterfactual, $\varphi$, is true and which is the moment closest to $t'$ among those at which $\varphi$ is true. The conditional $\varphi > \psi$ is true iff also the consequent, $\psi$, is also true at this moment.

Counterfactuals of freedom are a particular case of counterfactuals. Their general formulation is "If the agent $a$ were in the circumstances $C$, she would freely perform the action $Z$", that is, these conditionals say what an agent would freely choose to do, if the antecedent were true.[24] For instance, the counterfactual "If Ann were at the party, she would drink a beer" is true if 1) Ann had decided to go to the party and 2) if, once at the party, she freely chose to drink a beer rather than, for instance, a Coke. So we have to suppose that more than one possibility is open to Ann in the future of the counterfactual moment at which she is at the party, for instance the possibility of drinking a beer and that of drinking a Coke. More than one history passes through the counterfactual instant and Ann does different things in these histories at moments subsequent to the counterfactual moment. So a counterfactual of freedom has the following structure:

[24]For an introduction to counterfactuals of freedom, cf. Perszyk (2011).

$M, t, t' \vDash \varphi > \mathbf{F}\psi$

How to evaluate the formula $\mathbf{F}\psi$ depends on the interpretation of the future that we assume. According to the Aristotelian view, every history that passes through the counterfactual moment must contain a moment following the counterfactual moment at which $\psi$ is true. So, if $\varphi$ is true only in some but not all futures of the counterfactual moment, as required by the libertarian freedom of the agent, the conditionals of freedom are untrue according to this view. According to the TRL+ view, instead, the counterfactual of freedom is true if the local TRL radiating from the counterfactual moment contains a moment subsequent to the counterfactual moment at which $\psi$ is true. The two clauses are the following:

I) $M, t, t' \vDash \varphi > \mathbf{F}\psi \Leftrightarrow$
$\forall h \in H_{f_{CP}(t',\varphi)} \cap H_{f_{CP}(t',\varphi)} \exists t'' > f_{CP}(t', \varphi), M, t'', f_{CP}(t', \varphi) \vDash \psi$

II) $M, t, t' \vDash \varphi > \mathbf{F}\psi \Leftrightarrow$
$\exists t'' > f_{CP}(t', \varphi), t'' \in \mathsf{TRL}^*(f_{CP}(t', \varphi), f_{CP}(t', \varphi)), M, t'', f_{CP}(t', \varphi) \vDash \psi$

Notice that, since the perspective and the moment of evaluation coincide at counterfactual moments, the $\mathsf{TRL}$ at which the future clause has to be evaluated is that relative to the counterfactual perspective.

Notice also that the principle $p \rightarrow \mathbf{HF}p$ is validated also at counterfactual moments. In other words, if Ann had gone to the party, it would always have been true that she would go to the party.

Finally, if the antecedent of the conditional if true at the *actual* perspective, we believe that the conditional is true if its consequent is also true at the actual perspective. In this case, the point closest to the perspective at which the antecedent is true is the perspective itself.[25] So, we have to adjust our semantics of counterfactuals according to the following lines:

1. if $M, t, t' \vDash \varphi$, then $M, t, t' \vDash \varphi > \psi \Leftrightarrow M, t, t' \vDash \psi$
2. if $M, t, t' \nvDash \varphi$, then $M, t, t' \vDash \varphi > \psi \Leftrightarrow M, f_{CP}(t', \varphi), f_{CP}(t', \varphi) \vDash \psi$

Instead, if the antecedent is not true at any point – it is historically impossible –, we suppose that the conditional has no truth value: the function $f_{CP}$ gives no output and the conditional cannot be evaluated.

[25] So, we accept Lewis (1973)'s view that counterfactuals with true antecendents have a truth value, even though they are pragmatically infelicitous.

TRL+ is, then, perfectly apt to make Molinist conditionals of freedom true or false. If TRL+ modeled the actual structure of the world, God could know what every free agent would freely choose in every circumstance. Furthermore, TRL+ validates the intuitive principle $p \rightarrow \mathbf{HF}p$, when the perspective follows or coincides with the point of evaluation.

# 7 Conclusion

In this article, we have analyzed two open future models. In the Aristotelian model, future-tense propositions lack truth value (or, alternatively, they are all false). In the TRL model, future-tense propositions are true or false; however, according to advocates of TRL, this feature does not affect the openness of the future, as the real future is just one of many alternatives at agents' disposal. Restall argues that TRL+ cannot work, as it is unable to validate the retrogradation of truth: $p \rightarrow \mathbf{HF}p$. However, as we have shown that the Aristotelian model encounters the same problem, this criticism is not specific to Molinism but also applies to some open future models. We then sketched a semantical framework within which the truth of propositions is relativized to the point at which the world has arrived, where the specific features of TRL+ are preserved but $p \rightarrow \mathbf{HF}p$ is, at least in the relevant cases, validated. While Molinism can clearly be rejected for many reasons, we believe that Restall's criticism is not among them.

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