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Optimal coil design for maximum power transfer efficiency in resonantly coupled systems.

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Optimal Coil Design for Maximum Power Transfer Efficiency in Resonantly Coupled Systems

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***Abstract*—Maintaining maximum power transfer efficiency (PTE) is one of the main challenges in resonant inductive power transfer (IPT) systems. Maximum PTE can be achieved if the coupling between transmitter and receiver coils is strong. One way of achieving this is to geometrically optimise a coil by employing small ohmic resistance combined with high self-inductance. In this paper a design method for an optimum coil geometry which offers maximum PTE has been introduced. The proposed technique, in addition to minimising the system's physical size, provides high level of PTE for both strongly- and loosely-coupled links. A design example for a typical IPT system is presented that shows, with a proper selection of strong coupling factor (e.g.: C = 220.), the designed coil geometry can provide maximum PTE of 95.4% for coupling coefficient K = 1. Also, for a loose inductive link with K = 0.215, maximum calculated and measured PTE values are 89% and 86%, respectively.**

I. Introduction

Resonant inductive coupling is a well established method of wireless power transfer (WPT). The operation principal of this signal (i.e.: power or data.) transfer technique is based on resonant coupling of a magnetic field between a primary and a secondary coil. This wireless coupling operation makes inductive power transfer (IPT) ideal for diverse short-range and mid-range WPT applications, such as: wireless battery charging, implantable biomedical devices, RFID sensors, etc.

A major WPT challenge is achieving maximum power transfer efficiency (PTE) [1]. PTE is affected by: coil size (e.g.: diameter, length, number of turns, etc.), transmission medium distance and attenuation level, plus terminating circuitry of both receiver and transmitter.

This paper discusses an optimum coil design procedure. The novel approach seeks to optimise coil geometry while maintaining maximum PTE. The technique lends itself well to IPT applications where the physical size and shape of transceiver is limited.

II. Theoretical Analysis

Operating a transceiver inductive link at magnetic resonance maximises the linking action. A simplified "lumped resonant IPT system" is shown in Fig. 1. In this model both primary and secondary coils are considered identical. The LC-tanks are tuned to an operating resonant frequency f_o ($\omega_o = 2\pi f_o = 1/\sqrt{LC}$) [1], where L is the inductance value of the primary and secondary coils, and capacitor C is their respective resonance pair. The primary side is considered as an ac voltage source, V_s, with an impedance, R_s. The secondary side R_L represents a load (dc or ac). R is the winding ohmic resistance of the primary and the secondary coils. M is the mutual inductance between coils.

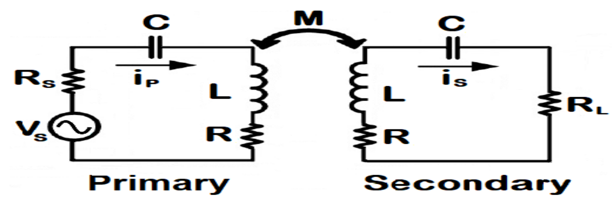

Fig. 1. Circuit model.

A. Maximum Power Transfer Efficiency Formulation

At resonance the power transfer efficiency, η, of the transceiver system in Fig. 1 can be expressed as:

$$\eta = \frac{K^2 Q_p Q_s}{1 + K^2 Q_p Q_s}\left(1 - \frac{Q_s}{Q_i}\right) \quad (1)$$

where K is the coupling coefficient between the coils, $K = M/L$. The primary and secondary loaded quality factors are included as: $Q_p = \omega_o L/(R + R_s)$ and $Q_s = \omega_o L/(R + R_L)$. Q_i is the intrinsic Q factor of the coil, $Q_i = \omega_o L/R$.

From (1) a condition which guarantees maximum PTE (i.e.: $\eta = 1$.) can be expressed as:

$$\begin{cases} Q_i - Q_s \equiv Q_i \\ K^2 Q_p Q_s \equiv 1 + K^2 Q_p Q_s. \end{cases}$$

These require both the following inequalities to be satisfied:

$$\begin{cases} Q_i \gg Q_s & (2) \\ K^2 Q_p Q_s \gg 1. & (3) \end{cases}$$

With proper coil wire selection inequality (2) is met, i.e.: a low ohmic resistance yields large Q_i. Inequality (3) is the electrical representative [1] of "Strong Coupling Conduction" [2]. A strong coupling link between transmitter and receiver coils requires –

1) An inductive link with high coupling coefficient "K": The transmission medium gap and coil mutual orientation sets K between 0 and 1 (i.e.: $0 \leq K \leq 1$.).

2) A transceiver system with low terminating impedances (R_s&R_L): The optimum low values for R_s and R_L will vary depending on system power level and application.

3) Transmitter and receiver coils with high self-inductance and small ohmic resistance: Equation (4) expresses coil (i.e.: coreless solenoid.) self-inductance as [3]:

$$L = \frac{\mu_o \pi N^2 r_c^2}{\sqrt{4r_c^2 + l_c^2}} \tag{4}$$

in which μ_o is free space permeability, N is number of turns, r_c is coil radius and l_c is coil length (where $l_c \gg r_c$). Also, the coil ohmic resistance can be calculated from [2]:

$$R = \frac{N r_c \sqrt{2\omega_o \mu \rho}}{d_w} \tag{5}$$

where μ and ρ are permeability and resistivity of winding wire material, and d_w is wire diameter. From (4), increasing N and r_c improves coil self-inductance. However, increasing these values has the detrimental effect of raising the coil ohmic resistance.

The challenge is to design a geometrically optimised coil with maximised self-inductance and low ohmic resistance.

B. Optimum Coil Design for Strong Coupling

Finding an optimal coil geometry to maximise PTE for a certain $R_s \& R_L$ requires exploiting the coil self-inductance to yield an ohmic resistance that provides strong coupling between primary and secondary coils.

In selecting a suitable coil ohmic resistance we introduce a constant parameter C ($C \gg 1$) to represent the "Strong Coupling Factor". Thus (3) can be restated as:

$$K^2 Q_p Q_s = C. \tag{6}$$

For a given $R_s \& R_L$ one can initially consider $K = 1$, leading (6) to:

$$Q_p Q_s = C \;\Rightarrow\; \omega_o^2 L^2 = C(R + R_s)(R + R_L). \tag{7}$$

As the coil requires its maximum self-inductance value we solve (7) for R to yield the ohmic resistance which maximises PTE for a given IPT system.

The coil self-inductance, (4), is a factor of three variables; N, r_c and l_c. Replacing N in (4) with its equivalent from (5) permits L to restated based on r_c as:

$$L = \frac{\mu_o \pi R^2 d_w^2 r_c}{\sqrt{(2\omega_o \mu \rho)(R^2 d_w^4 + 8 r_c^4 \omega_o \mu \rho)}}. \tag{8}$$

In a tightly wound coil $l_c = N d_w$. Differentiating (8) with respect to r_c provides an r_c which maximizes L:

$$r_c = \sqrt[4]{\frac{R^2 d_w^4}{8\omega_o \mu \rho}} = \frac{l_c}{2} \;\Rightarrow\; L_{max} = \sqrt[4]{\frac{\mu_o^4 \pi^4 d_w^4 R^6}{128 \omega_o^3 \mu^3 \rho^3}}. \tag{9}$$

Substituting L with L_{max} in (7) yields:

$$\left(\frac{\omega_o^2 \mu_o^2 \pi^2 d_w^2}{4(\sqrt{2\omega_o \mu \rho})^3}\right) R^3 - CR^2 - C(R_s + R_L)R - CR_sR_L = 0. \tag{10}$$

Solving (10) for R determines the ohmic resistance. Having R permits r_c and N to be calculated from (9) and (5) as the next section will show.

III. Design Example & Experimental Results

To investigate the maximum PTE between coils using the proposed design method an IPT system, based on Fig. 1, was physically constructed with $R_s = 3\ \Omega$, $R_L = 3.5\ \Omega$ at $f_o = 1.06$ MHz. For a nominal C value ($C = 220$) the coil ohmic resistance from (10) is $R = 152.3$ mΩ. The coil winding conductor used is 0.8 mm (dia) copper wire. For this R value $r_c = 9.5$ mm, $N = 23.8$ and $l_c = 19.0$ mm.

As shown in Fig. 2, for the physical circuit parameters and a strong inductive link (i.e.: $K = 1$.), the maximum calculated PTE (via MATLAB) is 95.4%. Maximum PTE is seen to drop with a reduction in coupling coefficient, K; however, the maximum achievable PTE still stays centered around the designed coil geometry (i.e.: $r_c = 9.5$ mm.). Fig. 3 compares calculated and measured PTE values for a loosely-coupled inductive link with $K = 0.215$. For $r_c = 9.5$ mm the calculated maximum PTE is 89% and the measured maximum PTE, from the physical circuit, is 86%. Tests on physical geometries either side of $r_c = 9.5$ mm show falling PTE values, thus validating the design method proposed.

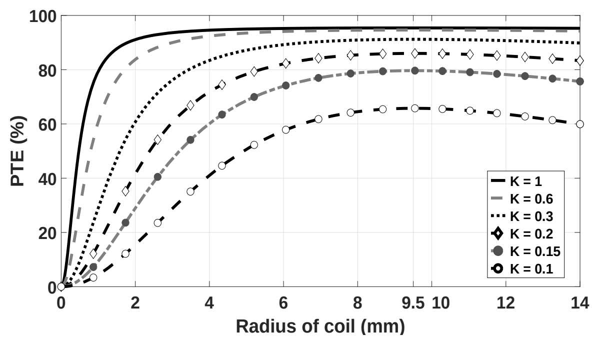

Fig. 2. Maximum PTE for different coupling coefficients (K).

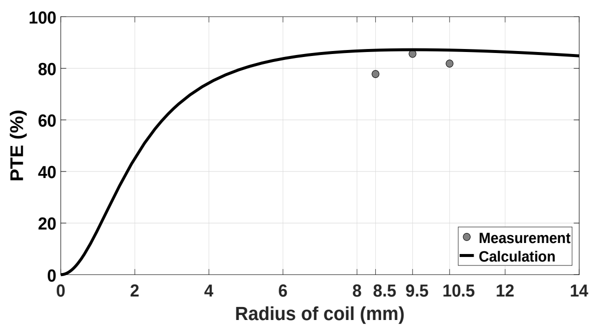

Fig. 3. Calculated & measured efficiency for $K = 0.215$.

IV. Conclusion

A novel design approach has been shown to optimise coil geometry in both strongly- and loosely-coupled IPT inductive links to yield maximum PTE. Simulated and practical measurements produced optimised results within 3% difference.

References

[1] M. Kiani and M. Ghovanloo, "The circuit theory behind coupled-mode magnetic resonance-based wireless power transmission", *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 59, pp. 2065–2074, 2012.

[2] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, "Wireless power transfer via strongly coupled magnetic resonances", *Sci. Exp.*, vol. 317, pp. 83–86, 2007.

[3] M. Sadiku, *Elements of electromagnetics*. Oxford university press, 2015.