Structural Time-dependent Reliability Assessment with A New Power Spectral Density Function

Cao Wang, S.M.ASCE[1], Hao Zhang, M.ASCE[2] and Michael Beer, Dr.Eng., M.ASCE[3]

ABSTRACT

An important ingredient of time-dependent reliability analysis of civil structures is to choose a proper model for the applied loads. The stochastic process theory has been widely used in existing studies to perform structural time-dependent reliability analysis. However, the use of many types of power spectral density function leads to an inefficient calculation of structural reliability. This paper proposes an analytical method for structural reliability assessment, where a new power spectral density function is developed to enable the reliability analysis to be conducted with a simple and efficient formula. A non-Gaussian load process, if present, is first converted into an "equivalent" Gaussian process to improve the assessment accuracy. Illustrative examples are presented to demonstrate the applicability of the proposed method. Results show that a greater autocorrelation in the load process leads to a smaller failure probability. The structural reliability may be significantly overestimated if one simply treats the non-Gaussian load process as a Gaussian one. Moreover, the impact of modeling the load process as a continuous process or a discrete one on structural reliability is also investigated.

[1]Ph.D. student, School of Civil Engineering, Univ. of Sydney, Sydney, NSW 2006, Australia. ORCID: http://orcid.org/0000-0002-2802-1394. Email: cao.wang@sydney.edu.au

[2]Associate Professor, School of Civil Engineering, Univ. of Sydney, Sydney, NSW 2006, Australia (corresponding author). Email: hao.zhang@sydney.edu.au

[3]Professor of Uncertainty in Engineering and Head, Institute for Risk and Reliability, Leibniz Univ. Hannover, 30167 Hannover, Germany; Institute for Risk and Uncertainty, Univ. of Liverpool, Liverpool L69 3BX, United Kingdom; International Joint Research Center for Engineering Reliability and Stochastic Mechanics (ERSM), Tongji Univ., Shanghai 200092, China. E-mail: beer@irz.uni-hannover.de

Keywords: Time dependent reliability; Stochastic process; Outcrossing rate; Load autocorrelation

INTRODUCTION

Civil structures and infrastructures are subjected to both environmental attacks (e.g., Chloride-induced corrosion to RC structures) and severe load effects (e.g., over-weighted traffic loads to bridges) during their service life. Such factors may essentially impair the structural service reliability. A probability-based approach should be used to evaluate the serviceability level and remaining life of an engineered structure (Mori and Ellingwood 1993; Enright and Frangopol 1998; Akiyama and Frangopol 2014; Wang et al. 2017). The basic concept of structural reliability assessment is to examine whether the load effect ($\mathcal{S}$) exceeds the structural resistance (load-bearing capacity, $\mathcal{R}$). Both $\mathcal{R}$ and $\mathcal{S}$ are practically uncertain due to the randomness arising from structural geometry, material strength, load volume, and others. Mathematically, the structural failure probability, $\mathbb{P}$, is estimated by $\mathbb{P} = \Pr(\mathcal{R} - \mathcal{S} < 0)$, where Pr denotes the probability of the event in the bracket. For the reliability assessment of a structure within a specific reference period (e.g., during its lifetime), however, both the resistance and the external loads may vary with time and thus cannot be simply represented by a single random variable. Under this context, let $\mathcal{R}(t)$ and $\mathcal{S}(t)$ denote the resistance and load effect at time t, respectively. The time-dependent reliability within a service period of $[0, T]$, $\mathbb{L}(T)$, is given by

$$\mathbb{L}(T) = \Pr\left\{\mathcal{R}(t) > \mathcal{S}(t), \forall t \in [0, T]\right\} = \int_0^T \int_{Z(t)>0} f_{Z(t)}(z(t))\mathrm{d}[z(t)]\mathrm{d}t \tag{1}$$

where $Z(t) = \mathcal{R}(t) - \mathcal{S}(t)$ is the limit state function at time t, and $f_{Z(t)}$ is the probability density function (PDF) of $Z(t)$, which also varies with t. By definition, the time-dependent failure probability, $\mathbb{P}(T)$, is the complementary of $\mathbb{L}(T)$, i.e., $\mathbb{P}(T) = 1 - \mathbb{L}(T)$. Note that Eq. (1) indeed involves a multi-fold integral, as well as the potential association between different folds, and thus is often difficult or even impossible to solve directly. Specifically, in terms of the external loads, both the non-stationarity and the temporal autocorrelation should be considered in a reasonable manner. As such, some simplifications have been introduced to achieve a practical yet sufficiently accurate solution to the reliability problem (Mori and Ellingwood 1993; Melchers 1999; Li et al. 2005; Li et al. 2015; Wang et al. 2016; Wang and Zhang 2018). One of the existing methods to model the external loads is to employ a discrete stochastic process (e.g., a Poisson process) to represent the occurrence of significant loads that may impair structural safety directly. A remarkable work was done by Mori and Ellingwood (1993), who considered a stationary Poisson process for the loads, and proposed a closed-form solution for structural time-dependent reliability,

$$\mathbb{L}(T) = \exp\left\{\lambda \int_0^T F_S[r_0 \cdot g(t)]\mathrm{d}t - \lambda T\right\} \tag{2}$$

where r_0 is the initial resistance, λ is the mean occurrence rate of the Poisson process (i.e., on average λ event(s) occurs within a unit time), F_S is the cumulative density function (CDF) of each load effect, and $g(t)$ is the deterioration function of resistance (i.e., the ratio of resistance at time t to the initial resistance). Li et al. (2015) further proposed a generalized form of Eq. (2), where the non-stationarity in the load stochastic process was also considered. Moreover, note that the autocorrelation in the load process also arises due to common physical-based causes (e.g., Ellingwood and Lee 2016). Conceptually, the correlation between two load effects at two different time points is expected to decrease as the time separation increases. A frequently-used model takes the form of (e.g., Li et al. 2016b)

$$\rho(\tau) = \exp(-k \cdot \Delta\tau) = \exp(-k|\tau_1 - \tau_2|) \tag{3}$$

where $\rho(\tau)$ is the linear correlation coefficient between two loads with a time separation (or a spatial distance) of $\Delta\tau$, k is the scale factor accounting for the correlation changing rate, τ_1 and τ_2 are the two occurring times of loads. Eq. (3) is, however, only valid for a continuous process as a discrete load process is unavoidably associated with intermittence. Wang and

Zhang (2018) proposed a model to describe the autocorrelation in a discrete process, and investigated the impact of load temporal correlation on structural time-dependent reliability. Ellingwood and Lee (2016) studied the autocorrelation in the hurricane wind process, where an auto-regressive model was used to measure the autocorrelation in the wind loads.

The aforementioned discrete load processes, however, may fail to describe the cases where the load effect is applied continuously to a structure (e.g., underground poles subjected to earth pressure). Fig. 1 shows a conceptual comparison between a continuous load process (Fig. 1(a)) and a discrete one (Fig. 1(b)). For use in structural reliability assessment, a continuous load process could be transformed to a discrete one, where only the significant load events (e.g., with a magnitude that exceeds a pre-defined threshold) are considered. While this approach has been used in the literature (e.g., Mori and Ellingwood 1993; Li et al. 2015), the error induced by such an approximation in structural reliability remains unaddressed.

For a continuous load process which is applied uninterruptedly, the main characteristics of the process can be captured by the statistics including the mean value, variance and autocorrelation. Further, the structural time-dependent reliability analysis can be transformed into a problem of a stochastic process crossing a predefined barrier level (e.g., the resistance) (Grigoriu 1984; Engelund et al. 1995; Li et al. 2016b). The solution is usually referred to as "first passage probability". This method has been widely used in the literature to estimate the reliability of civil structures and infrastructure subject to continuous loads (Hagen and Tvedt 1991; Ferrante et al. 2005; Li et al. 2005; Pillai and Veena 2006). For example, Li et al. (2005) developed a method for reliability analysis considering a non-stationary Gaussian vector process. Beck and Melchers (2005) investigated the error introduced in the calculation of the upcrossing rate in the presence of a random barrier. The load stochastic process has been, for the most part, modeled as Gaussian in existing studies, which may differ significantly from the realistic case since a Gaussian (normal) distribution may lead to a non-positive value of the load effect, inconsistent with the physical-based properties. Li et al. (2016a) developed a closed-form solution to the "first passage probability" considering a non-stationary lognormal distribution. The Nataf transformation method can be used to convert a nonnormal stochastic process into a normal one (e.g., Zheng and Ellingwood 1998), which is applicable for cases where the load process follows an arbitrary distribution (e.g., a Weibull or Extreme Type I distribution, as has also been widely used in existing studies (Melchers 1999; Tang and Ang 2007)). However, existing approaches for reliability assessment considering the temporal autocorrelation in the load process are complicated, with which the application of reliability assessment in practical use may be difficult. A model of load autocorrelation is essentially desirable to enable feasible compatibility to practical cases and also an efficient approach of structural reliability assessment.

This paper develops a method for structural time-dependent reliability analysis, where, in order to achieve a simple and efficient solution to the structural reliability, a new power spectral density function of the load process is proposed, containing two parameters that can be calibrated in an explicit form. Illustrative examples are presented to demonstrate the applicability of the proposed method and to investigate the role of stochastic load process in structural reliability. The difference between the reliabilities associated with a discrete load process and a continuous one is also discussed.

STOCHASTIC PROCESS-BASED RELIABILITY ASSESSMENT

Gaussian process of loads

The time-dependent reliability based on the stochastic process theory has been well documented in the literature (Grigoriu 1984; Engelund et al. 1995; Li et al. 2016b) and is introduced briefly in this section. Consider the case where the load process in Eq. (1) is Gaussian. Let

$$Z(t) = \mathcal{R}(t) - \mathcal{S}(t) = \Omega(t) - X(t) \tag{4}$$

where $\Omega(t) = \mathcal{R}(t) - \mathbb{E}[\mathcal{S}(t)]$ and $X(t) = \mathcal{S}(t) - \mathbb{E}[\mathcal{S}(t)]$, with $\mathbb{E}$ denoting the mean value of the random variable in the bracket. With this, $X(t)$ in Eq. (4) is a stationary Gaussian process with a mean value of 0 and a standard deviation of $\sigma_X = \sigma_S$, where σ_S is the standard deviation of $\mathcal{S}(t)$. Fig. 2 presents an illustration of the upcrossing rate-based reliability problem. The positive upcrossing rate of $X(t)$ relative to $\Omega(t)$ at time t, $\nu^+(t)$, is estimated by (e.g., Lutes and Sarkani 2004)

$$\begin{aligned}\lim_{\mathrm{d}t\to 0}\nu^+(t)\mathrm{d}t &= \Pr\left\{\Omega(t) > X(t)\bigcap\Omega(t+\mathrm{d}t) < X(t+\mathrm{d}t)\right\}\\ &= \Pr\left\{\Omega(t+\mathrm{d}t) - \dot{X}(t)\mathrm{d}t < X(t) < \Omega(t)\right\}\\ &= \int_{\dot{\Omega}(t)}^{\infty}\left[\dot{X}(t)-\dot{\Omega}(t)\right] f_{X\dot{X}}\left[\Omega(t),\dot{X}(t)\right]\mathrm{d}\dot{X}(t)\mathrm{d}t\end{aligned} \tag{5}$$

where $\dot{X}$ (or $\dot{\Omega}$) denotes the derivative of X (or Ω). Rearranging Eq. (5) gives

$$\nu^+(t) = \int_{\dot{\Omega}(t)}^{\infty}\left(\dot{X}-\dot{\Omega}\right) f_{X\dot{X}}\left(\Omega,\dot{X}\right)\mathrm{d}\dot{X} \tag{6}$$

Since $X(t)$ is a 0-mean stationary Gaussian process, $X(t)$ are $\dot{X}(t)$ are mutually independent, with which one has

$$f_{X\dot{X}}(x,\dot{x}) = \frac{1}{2\pi\sigma_X\sigma_{\dot{X}}}\exp\left\{-\frac{1}{2}\left(\frac{x^2}{\sigma_X^2}+\frac{\dot{x}^2}{\sigma_{\dot{X}}^2}\right)\right\} \tag{7}$$

where $\sigma_{\dot{X}}$ is the standard deviation of $\dot{X}(t)$. Substituting Eq. (7) into Eq. (6) gives

$$\nu^+(t) = \frac{1}{2\pi\sigma_X}\exp\left[-\frac{\Omega^2(t)}{2\sigma_X^2}\right]\cdot\left\{\sigma_{\dot{X}}\exp\left(-\frac{\dot{\Omega}^2(t)}{2\sigma_{\dot{X}}^2}\right)-\sqrt{2\pi}\dot{\Omega}(t)\left[1-\Phi\left(\frac{\dot{\Omega}(t)}{\sigma_{\dot{X}}}\right)\right]\right\} \tag{8}$$

where $\Phi(\,)$ is the CDF of standard normal distribution. Assuming that the upcrossings of $X(t)$ to $\Omega(t)$ are temporally independent and are rare (e.g., at most one upcrossing may occur during a short time interval), the Poisson point process can be used to model the occurrence of the upcrossings. Let N_T denote the number of upcrossings during time interval $[0,T]$, and it follows,

$$\Pr(N_T = i) = \frac{1}{i!}\left\{\int_0^T\nu^+(t)\mathrm{d}t\right\}^i\exp\left\{-\int_0^T\nu^+(t)\mathrm{d}t\right\} \tag{9}$$

for $i = 0, 1, 2, \ldots$. Further, the structural reliability during $[0, T]$ is the probability of $N_T = 0$, i.e.,

$$\mathbb{L}(T) = [1 - \mathbb{P}(0)] \exp\left\{-\int_0^T \nu^+(t)\mathrm{d}t\right\} \tag{10}$$

where $\mathbb{P}(0)$ is the failure probability at initial time. Specifically, as $\mathbb{P}(0)$ is typically small enough, one has (Engelund et al. 1995; Melchers 1999)

$$\mathbb{L}(T) = \exp\left\{-\int_0^T \nu^+(t)\mathrm{d}t\right\} \tag{11}$$

Eq. (11) presents the time-dependent reliability for a reference of T years. The derivation of $\nu^+(t)$ in Eq. (11) has been based on the assumption of a Gaussian process of loads. This may lead to a significantly biased estimate of structural reliability in many cases where the load effect follows a non-Gaussian distribution such as a lognormal, Weibull or Extreme Type I distribution. A more generalized case will be discussed subsequently, where the load process may follow an arbitrary distribution. Finally, it is noticed that the resistance deterioration process is assumed to be deterministic in this paper; for cases where the uncertainties associated with the deterioration are non-negligible and shall be taken into account, one may use the total probability theorem to obtain the "expectation" of the structural reliability (Rackwitz 2001).

Arbitrary stochastic process of loads

In this section, the time-dependent reliability in the presence of an arbitrary stochastic process of loads is discussed. First, reconsider the time-variant limit state function $Z(t)$ in Eq. (4). Note that

$$\Pr[Z(t) > 0] = \Pr[\mathcal{R}(t) - \mathcal{S}(t) > 0] = \Pr\left\{\Phi^{-1}\left[F_{S(t)}(\mathcal{R}(t))\right] - \mathcal{Q}(t) > 0\right\} \tag{12}$$

where $\mathcal{Q}(t) = \Phi^{-1}\left[F_{S(t)}(\mathcal{S}(t))\right]$. With this, the term $\mathcal{Q}(t)$ is assigned as a standard Gaussian process, and further an "equivalent resistance" is defined as $\mathcal{R}^*(t) = \Phi^{-1}\left[F_{S(t)}(\mathcal{R}(t))\right]$. In such a way, the time-dependent reliability analysis is transformed into solving a standard "first passage probability" problem. That is, Eqs. (8) and (11) apply in the presence of the "equivalent" resistance and load.

A key step herein is to find the correlation in $\mathcal{Q}(t)$ provided that the correlation in $\mathcal{S}(t)$ is known. Suppose that the correlation coefficient between $\mathcal{S}_i = \mathcal{S}(t_i)$ and $\mathcal{S}_j = \mathcal{S}(t_j)$ is ρ_{ij}, and the correlation coefficient between the corresponding $\mathcal{Q}_i = \mathcal{Q}(t_i)$ and $\mathcal{Q}_j = \mathcal{Q}(t_j)$ is ρ'_{ij}. The relationship between ρ_{ij} and ρ'_{ij} can be determined by (Liu and Der Kiureghian 1986; Melchers 1999)

$$\rho_{ij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Theta_1\Theta_2 \cdot \Psi(y_1, y_2; \rho'_{ij})\mathrm{d}y_2\mathrm{d}y_1 \tag{13}$$

in which Θ_1, Θ_2 and Ψ are given by

$$\Theta_1 = \frac{F_{S_i}^{-1}(\Phi(y_1)) - \mathbb{E}(\mathcal{S}_i)}{\sqrt{\mathbb{V}(\mathcal{S}_i)}}; \tag{14a}$$

$$\Theta_2 = \frac{F_{S_j}^{-1}(\Phi(y_2)) - \mathbb{E}(\mathcal{S}_j)}{\sqrt{\mathbb{V}(\mathcal{S}_j)}}; \tag{14b}$$

$$\Psi(y_1, y_2; \rho'_{ij}) = \frac{1}{2\pi\sqrt{1-\rho'^2_{ij}}} \exp\left\{\frac{y_1^2 - 2\rho'_{ij}y_1y_2 + y_2^2}{2(1-\rho'^2_{ij})}\right\} \tag{14c}$$

where $F_{S_i}^{-1}$ is the inverse of the CDF of $\mathcal{S}_i$, and $\mathbb{V}(\,)$ denote the variance of the random variable in the bracket. Equations. (13) and (14) are the key component of the Nataf transformation (i.e., the transformation from $\mathcal{S}(t)$ to $\mathcal{Q}(t)$ herein) addressing the autocorrelation structure of the Gaussian process $\mathcal{Q}(t)$. Eq. (13) indicates that ρ'_{ij} depends on the COV (coefficient of variation) of $\mathcal{S}_i$ and $\mathcal{S}_j$ only if ρ_{ij} is given.

It is noticed that the method of "equivalent" resistance and load is a generalized form of the "translation process" method developed by Grigoriu (1984), where a constant barrier level was considered. Moreover, Grigoriu (1984) also suggested that the use of a Nataf transform method results in a negligible error in the estimate of upcrossing rate for many common distribution types such as Weibull, Extreme Type I, lognormal and Gamma, implying the feasibility of the Nataf transformation-based method in dealing with practical reliability problems with a non-Gaussian load process. Kim and Shields (2015) presented a further development on Grigoriu's translation processes for strongly non-Gaussian processes, where the transformation was realized with an iteration-based simulation approach that considers the autocorrelation function of the stochastic process. However, a simulation-based method may limit the applicability of reliability assessment in practical use due to the relatively low efficiency compared with a closed-form solution.

RELIABILITY WITH A CONTINUOUS OR A DISCRETE LOAD PROCESS

Recall that the time-dependent reliability problem has been addressed in Eqs. (2) and (11), respectively. The former considers a discrete load process where only the significant load events that may impair the structural safety directly are incorporated, while the later is derived based on a continuous load process. The difference between the two types of load model is discussed in this section.

First, consider the CDF of $\max\{X(t)\}$ within a time duration of Δ, $F_{X_{\max|\Delta}}$, where $X(t) = \mathcal{S}(t) - \mathbb{E}[\mathcal{S}(t)]$ is the normalized load process (c.f. Eq. (4)). In the presence of a continuous Gaussian load process, with Eqs. (8) and (11), let $\Omega(t) = x$ and $\dot{\Omega}(t) = 0$, which corresponds to the case of a constant boundary, one has

$$F_{X_{\max|\Delta}}(x) = \exp\left\{-\frac{\sigma_{\dot{X}}}{2\pi\sigma_X}\exp\left(-\frac{x^2}{2\sigma_X^2}\right)\Delta\right\} \tag{15}$$

Further, as Δ is small enough (Newland 1993)

$$F_{X_{\max|\Delta}}(x) \approx 1 - \frac{\sigma_{\dot{X}}\Delta}{2\pi\sigma_X}\exp\left(-\frac{x^2}{2\sigma_X^2}\right) \tag{16}$$

which yields a Rayleigh distribution. Eq. (16) suggests that the maximum load effect within a time interval that is sufficiently short necessarily follows a Rayleigh distribution, if the continuous load process is Gaussian. For a discrete load process, e.g., a Poisson process, however, the distribution of $\max\{X(t)\}$ within a short time interval of Δ is given by

$$F_{X_{\max|\Delta}}(x) = 1 - \lambda\Delta \cdot (1 - F_S(x)) \tag{17}$$

where λ is the mean occurrence rate of the Poisson process, and F_S is the CDF of load magnitude conditional on the occurrence of one load event. Eq. (17) indicates that the CDF of maximum load is eventually dependent on F_S, and thus may vary for different distributions of each load event. Letting the two CDFs of maximum load in Eqs. (16) and (17) be equal yields

$$F_S(x) = 1 - \frac{\sigma_{\dot{X}}}{2\pi\lambda\sigma_X} \exp\left(-\frac{x^2}{2\sigma_X^2}\right) \tag{18}$$

Eq. (18) suggests that if a continuous Gaussian process is transformed to a discrete one, the CDF of the load effect conditional on the occurrence of one load event simply follows a Rayleigh distribution.

For the more generalized case of a non-Gaussian load process, $X(t)$ can be converted into a Gaussian process $\mathcal{Q}(t)$, as discussed before. With this, for a reference period of Δ, the CDF of $\max\{X(t)\}$ is given by

$$F_{X_{\max|\Delta}}(x) = \Pr\left\{\bigcap_{0\le t\le\Delta} \left(\Phi^{-1}[F_S(\mathcal{S}(t))] < \Phi^{-1}(F_S(x))\right)\right\} \tag{19}$$

Let $x^* = \Phi^{-1}(F_S(x))$, and Eq. (19) becomes

$$\begin{aligned} F_{X_{\max|\Delta}}(x) &= \exp\left\{-\frac{\sigma_{\dot{Q}}\Delta}{2\pi}\exp\left(-\frac{x^{*2}}{2}\right)\right\} \\ &\approx 1 - \frac{\sigma_{\dot{Q}}\Delta}{2\pi}\exp\left(-\frac{x^{*2}}{2}\right) \\ &= 1 - \frac{\sigma_{\dot{Q}}\Delta}{2\pi}\exp\left\{-\frac{[\Phi^{-1}(F_S(x))]^2}{2}\right\} \end{aligned} \tag{20}$$

It should be noted that Eq. (20) is only valid when x is large enough. Eq. (20) implies that when the load process is non-Gaussian, the maximum load effect within a time interval does not necessarily follow a Rayleigh distribution. The distribution type in Eq. (20) is the original development of the present paper and is referred to as "Pseudo-Rayleigh distribution" by the authors. Nonetheless, the distribution type of $\max\{X(t)\}$ is determined if $X(t)$ is continuous, which again differs from the case of a discrete load process.

Next, the difference between the reliabilities associated with a discrete load process and a continuous one is discussed. For simplicity, the load process is assumed to be Gaussian. With a discrete load process, the time-dependent reliability within $[0, T]$ is estimated by

$$\mathbb{L}_{\mathrm{d}}(T) = \Pr\left[\bigcap_{0<t\leq T}(\Omega(t) - X_{\max} > 0)\right] = \exp\left[-\frac{\sigma_{\dot{X}}}{2\pi\sigma_X}\int_0^T \exp\left(-\frac{\Omega^2(t)}{2\sigma_X^2}\right)\mathrm{d}t\right] \tag{21}$$

which takes a similar form of Eq. (11) with a different upcrossing rate $\nu^+(t)$ in Eq. (8). In fact, Eq. (8) can be rewritten as

$$\nu^+(t) = \frac{\sigma_{\dot{X}}}{2\pi\sigma_X}\exp\left[-\frac{\Omega^2(t)}{2\sigma_X^2}\right]\cdot h(z) \tag{22}$$

where

$$h(z) = \exp\left(-\frac{z^2}{2}\right) - \sqrt{2\pi}z\left[1 - \Phi\left(z\right)\right] \tag{23}$$

with $z = z(t) = \frac{\dot{\Omega}(t)}{\sigma_{\dot{X}}}$. Intuitively, for a constant barrier level, $z = 0$ since $\dot{\Omega}(t) = 0$, with which $h(z) = 1$, consistent with the results in Gomes and Vickery (1977).

By noting that z is typically negative as $\dot{\Omega}(t) < 0$ and that $h(z)$ is a monotonically decreasing function of z, $h(z) \geq h(0) = 1$ for $\forall z < 0$. For simplicity, Eq. (22) is rewritten as $\nu^+(t) = \nu_0^+(t)\cdot h(z)$. According to Eq. (11), the time-dependent reliability with a continuous load process is given by

$$\mathbb{L}(T) = \exp\left\{-\int_0^T \nu^+(t)\mathrm{d}t\right\} = \exp\left\{-\int_0^T \nu_0^+(t)h(z)\mathrm{d}t\right\} \tag{24}$$

With the mean value theorem for integrals (e.g., Comenetz 2002), there exists a real number

$z_0 \in [\min_{t=0}^{T} z(t), \max_{t=0}^{T} z(t)]$ such that

$$\mathbb{L}(T) = \exp\left\{-h(z_0) \cdot \int_0^T \nu_0^+(t)\mathrm{d}t\right\} = [\mathbb{L}_\mathrm{d}(T)]^{h(z_0)} \leq \mathbb{L}_\mathrm{d}(T) \tag{25}$$

Thus, it can be concluded that the choice of a discrete load model overestimates the structural safety or equivalently, underestimates the failure probability, if the realistic load process is continuous. In fact, with Eq. (25), since $\mathbb{P}_d(T) = 1 - \mathbb{L}_d(T)$ is typically small enough for well-designed structures, one has

$$\mathbb{P}(T) = 1 - [\mathbb{L}_\mathrm{d}(T)]^{h(z_0)} = 1 - [1 - \mathbb{P}_\mathrm{d}(T)]^{h(z_0)} \approx h(z_0) \cdot \mathbb{P}_\mathrm{d}(T) \tag{26}$$

which implies that the failure probability is underestimated by a factor of $\frac{1}{h(z_0)}$ if the continuous load process is modeled as a discrete one. It is noticed, however, that the difference between $\mathbb{P}(T)$ and $\mathbb{P}_d(T)$ may be fairly small for many practical cases where $h(z_0)$ is close to 1.0; this point will be further discussed in the following.

A NEW POWER SPECTRAL DENSITY FUNCTION

In stochastic process theory based time-dependent reliability analysis, one of the crucial ingredients is the modeling of the autocorrelation in the load process. For a stationary process, say, $X(t)$, the autocorrelation is only dependent on the time separation τ but not the absolute time. With this, the autocorrelation in $X(t)$ is defined as $R(\tau) = \mathbb{E}[X(t)X(t+\tau)] = R(-\tau)$ (Newland 1993). An illustrative example is presented in Fig. 3, which shows the dependence of autocorrelation in the hurricane load process on the time interval between two successful hurricane events (Ellingwood and Lee 2016). The autocorrelation decreases sharply at the early stage where τ is relatively small, and converges to zero latter with a fluctuation along the horizontal axis. Such an autocorrelation function also applies to many other types of external loads which are affected by common underlying causes (Wang and Zhang 2018).

The spectral density function of $S(\omega)$, which is a Fourier transform of $R(\tau)$, also provides a tool to describe the statistical characteristics of $X(t)$. Mathematically, one has

$$R_X(\tau) = 2\int_0^\infty S(\omega)\cos(\omega\tau)\mathrm{d}\omega \tag{27a}$$

$$\sigma_{\dot{X}}^2 = R_{\dot{X}}(0) = -\frac{\mathrm{d}^2 R_X(0)}{\mathrm{d}\tau^2} = 2\int_0^\infty \omega^2 S(\omega)\mathrm{d}\omega \tag{27b}$$

Eq. (27b) implies that a spectral density function, $S(\omega)$, consequently gives an estimate of the standard deviation of $\dot{X}(t)$. However, since an improper integral is involved in Eq. (27b), an arbitrary form of $S(\omega)$ does not necessarily lead to a converged form of $\sigma_{\dot{X}}$. For example, if $R(\tau)$ takes the form of $R(\tau) = \sigma_X^2 \exp(-k\tau)$ (c.f. Eq. (3)), where σ_X is the standard deviation of $X(t)$, it follows (e.g., Zheng and Ellingwood 1998)

$$S(\omega) = \frac{1}{\pi}\int_0^\infty R(\tau)\cos(\tau\omega)\mathrm{d}\tau = \frac{k\sigma_S^2}{\pi(k^2+\omega^2)} \tag{28}$$

with which Eq. (27b) does not converge. Furthermore, even for some spectral density functions that result in a converged $\sigma_{\dot{X}}$, the integral operation in Eq. (27b) may be inefficient when used in the structural reliability assessment in Eq. (11) (that is, a two-fold integral will be involved in Eq. (11) if substituting Eqs. (8) and (27b) into Eq. (11)), especially for use in practical engineering.

In an attempt to achieve a simple and convergent form of Eq. (27b), a new power spectral density function is developed in this section, which takes the form of

$$S(\omega) = \frac{a}{\omega^6 + b}, \quad -\infty < \omega < +\infty \tag{29}$$

where a and b are two constants. It can be seen that Eq. (29) satisfies the basic properties of a power spectral density function: it's an even function of ω (i.e., $S(-\omega) = S(\omega)$) and positive (this is satisfied by noting that both a and b are positive values, see Eq. (35) below).

With the proposed spectral density function in Eq. (29), according to Eq. (27), it follows

$$R(\tau) = R(\tau, b) = 2a \cdot \int_0^\infty \frac{1}{\omega^6 + b} \cos(\omega\tau) \mathrm{d}\omega \tag{30a}$$

$$\sigma_X^2 = R(0, b) = 2a \cdot \int_0^\infty \frac{1}{\omega^6 + b} \mathrm{d}\omega = \frac{2a\pi}{3b^{5/6}} \tag{30b}$$

The integral operation involved in Eq. (30a) can be solved in a closed form. To begin with, one has

$$R(1, b) = \frac{2a\pi}{12b^{5/6}} \exp\left(-\frac{b^{1/6}}{2}\right) \cdot \left[2\exp\left(-\frac{b^{1/6}}{2}\right) + 4\cos\left(\frac{\sqrt{3}}{2}b^{1/6} - \frac{\pi}{3}\right)\right] \tag{31}$$

Further, it is easy to find that

$$R(\tau, b) = \tau^5 \cdot R(1, b\tau^6) \tag{32}$$

As such, Eq. (30) provides a straightforward approach to find a and b in the density function $S(\omega)$, provided that the autocorrelation function in the load process is known. It is noticed that while the autocorrelation function in Eq. (32) has been derived directly based on Eq. (29) rather than from a physics-based case, Eq. (32) nevertheless is feasible to capture different dependence scenarios of load autocorrelation on the time separation that decreases sharply at the early stage and subsequently fluctuates along the time axis with a decreasing magnitude. This fact is guaranteed by noting that in Eq. (32), the magnitude of $R(\tau, b)$ is controlled by the term $\exp\left(-\frac{b^{1/6}\tau}{2}\right)$, which is a monotonically decreasing function of τ with a given b, while the fluctuation of $R(\tau, b)$ is posed by the term $2\exp\left(-\frac{b^{1/6}\tau}{2}\right) + 4\cos\left(\frac{\sqrt{3}}{2}b^{1/6}\tau - \frac{\pi}{3}\right)$.

For illustration purpose, Fig. 4 shows the dependence of $R(\tau)$ on the time separation τ for $b = 30, 60$ and 90, respectively, assuming $a = 1$ for all the three cases. The autocorrelation decreases sharply at the early stage where τ is relatively small, and converges to zero soon with a fluctuation along the horizontal axis. The overall trends in Fig. 4 coincide well with that in Fig. 3. Moreover, it is seen that the different values of b result in different shapes of the autocorrelation function, indicating that the proposed spectral density function enables freedom for different depending scenarios of $R(\tau)$ on the time separation τ.

With the autocorrelation in $X(t)$ addressed, one can further find the correlation coefficient in $X(t)$, $\rho(\tau)$, by $\rho(\tau) = R(\tau)/\sigma_X^2$. For instance, for a unit time separation of $\tau = 1$, one has

$$\rho(1, b) = \frac{1}{4} \exp\left(-\frac{b^{1/6}}{2}\right) \cdot \left[2 \exp\left(-\frac{b^{1/6}}{2}\right) + 4 \cos\left(\frac{\sqrt{3}}{2} b^{1/6} - \frac{\pi}{3}\right)\right] \tag{33}$$

Mathematically, it is easy to see that $\lim_{b\to 0} \rho(1, b) = 1$ and $\lim_{b\to\infty} \rho(1, b) = 0$. Eq. (33) can be simply extended to other values of τ by noting that

$$\rho(\tau) = \rho(\tau, b) = \frac{R(\tau, b)}{\sigma_X^2} = \frac{\tau^5 \cdot R(1, b\tau^6)}{\sigma_X^2} \tag{34}$$

Further, with $S(\omega)$ taking the form of Eq. (29), it follows

$$\sigma_{\dot{X}}^2 = 2a \cdot \int_0^\infty \frac{\omega^2}{\omega^6 + b} \mathrm{d}\omega = \frac{\pi a}{3\sqrt{b}} \tag{35}$$

It can be seen from Eq. (35) that both a and b are positive real numbers due to the fact that $\sigma_{\dot{X}}^2$ is a positive real number. Furthermore, with Eq. (35), it is easy to see that Eq. (8) has a simple form with only fundamental algebras involved, which is beneficial for the application of structural reliability assessment when substituting Eq. (8) into Eq. (11). The applicability of the proposed power density function will be demonstrated in the next section. It is emphasized, finally, that for the case where the load process is non-Gaussian, the proposed density function also applies, if both the resistance and load effect are converted to the "equivalent" ones respectively, as discussed above.

NUMERICAL EXAMPLE

In this section, an illustrative example is presented to demonstrate the applicability of the proposed power spectral density function in structural time-dependent reliability assessment, and to investigate the role of load autocorrelation in structural safety.

Consider a structure subjected to the joint effect of both a dead load $\mathcal{D}$ and a continuous lateral load $\mathcal{H}$ (due to, e.g., the lateral earth pressure (Clayton et al. 2014)). Table 1 presents the probability distribution of the resistance and loads, with a load combination as follows (ASCE standard 7, ASCE 2002),

$$0.75\mathcal{R}_n = 0.9\mathcal{D}_n + 1.6\mathcal{H}_n \tag{36}$$

where $\mathcal{R}_n$ is the nominal resistance, $\mathcal{D}_n$ is the nominal dead load, and $\mathcal{H}_n$ is the nominal lateral load. Assume that $\mathcal{D}_n = \mathcal{H}_n$.

The initial resistance and dead load are modeled as deterministic, due to the fact that the randomness associated with the live loads contributes to the majority of the overall uncertainties for most engineered structures (e.g., Ellingwood et al. 1982; Ellingwood and Hwang 1985). The initial resistance has a value of 1.1 times the nominal resistance reflecting the modeling bias. The dead load is approximated by the nominal value which coincides well with many *in-situ* surveys. The live load in Table 1 in fact represents the "arbitrary point-in-time" load having a value that would be measured if the load process were to be sampled at some specific time instants.

A reference period of 50 years (i.e., T is up to 50 years) is considered in the following analysis. Moreover, taking into account the operational environmental factors that are responsible for the deterioration of structural resistance (e.g., the corrosion of steel bars in RC structures due to the ingression of Chloride in marine/coastal areas (Pang and Li 2016)), it is assumed that the structural resistance degrades linearly by 20% over a reference period of 50 years. The autocorrelation coefficient in the lateral load process is assumed to be 0.3 for a time separation of 1 year (i.e., $R(1\,\text{year}) = 0.3\sigma_H^2$, where σ_H is the standard deviation of $\mathcal{H}$). It is emphasized that while a lognormal stochastic load process (that is, the load process evaluated at an arbitrary time follows a lognormal distribution) is considered herein, the method in this paper is also applicable for loads with other distribution types such as a

Weibull or Extreme Type I distribution (Melchers 1999; Tang and Ang 2007).

Note that the lateral load $\mathcal{H}$ follows a lognormal distribution, and thus is transformed into a standard normal distribution $\mathcal{H}^*$ by $F_H(\mathcal{H}) = \Phi(\mathcal{H}^*)$, where F_H is the CDF of $\mathcal{H}$. With this, according to Eq. (13), the autocorrelation coefficient in the process $\mathcal{H}^*(t)$ for a time separation of 1 year is found to be $\dfrac{\ln(1+0.3c_H^2)}{\ln(1+c_H^2)} = 0.3241$, where c_H is the COV of $\mathcal{H}$. As such, with Eq. (30), the two parameters a and b can be found numerically as 18.1 and 78.7 respectively for $\mathcal{H}^*$. Fig. 5 shows the autocorrelation coefficient in $\mathcal{H}^*$ as a function of time difference τ, where an exponential decay model is also presented for comparison. It can be seen that with both types of correlation coefficient function, the autocorrelation in the load process diminishes rapidly for τ being up to three years. to have a similar shape overall. Moreover, in Fig. 5, the autocorrelation coefficient in $\mathcal{H}(t)$ assuming a Gaussian process of $\mathcal{H}(t)$ is also plotted, as well as an exponential law of the autocorrelation decay in the "assumed" normal $\mathcal{H}(t)$. The difference between the time-variation scenarios of correlation coefficient functions associated with $\mathcal{H}^*$ and normal $\mathcal{H}$ is negligible.

The spectral density function takes the form of Eq. (29), with which the autocorrelation coefficient in $\mathcal{H}^*(t)$ is modeled by Eq. (34). With the two parameters a and b obtained, one can simulate a sample sequence of $\mathcal{H}^*(t)$ and correspondingly, $\mathcal{H}(t)$. Since $\mathcal{H}^*(t)$ is a standard Gaussian process, one has (Newland 1993)

$$\mathcal{H}^*(t) \sim \sqrt{\frac{2}{N}} \cdot \sum_{j=1}^{N} \cos(\omega_j t + \theta_j) \tag{37}$$

where N is a sufficiently large integer, ω_j is a real random variable with a PDF of $S(\omega)$ (Note that the standard deviation of $\mathcal{H}^*$ is 1.0, and thus $\int_{-\infty}^{\infty} S(\omega)\mathrm{d}\omega = 1$), and θ_j is a random variable that is uniformly distributed in $[0, 2\pi]$. The simulation method for ω_j is discussed in Appendix I. Fig. 6 demonstrates sample sequences for $\mathcal{H}^*(t)$ and $\mathcal{H}(t)$ (normalized by $\mathcal{H}_n$), respectively. Such realizations in Fig. 6 provide a straightforward impression on the time-variation of the stochastic process with certain statistical characteristics.

Fig. 7(a) shows the time-dependent failure probabilities for reference periods up to 50 years, assuming a mean lateral load of $0.4\mathcal{H}_n$, $0.5\mathcal{H}_n$ (as in Table 1) and $0.6\mathcal{H}_n$, respectively. A greater load magnitude leads to a higher probability of failure. For reference periods exceeding 10 years, the logarithmic failure probability increases approximately linearly with time, which is consistent with the observations in Li et al. (2015). For comparison purpose, Fig. 7(b) presents the time-dependent failure probabilities assuming a Gaussian process of loads. It can be seen from the comparison between Figs. 7(a) and (b) that the assumption of a Gaussian load process underestimates the failure probability compared with the lognormal load process. This observation can be explained by examining the upper tail behaviour of a normal distribution and a lognormal distribution, as shown in Fig. 8. With the same mean value and standard deviation, a lognormal distribution has a longer upper tail compared with a normal distribution, and thus results in a greater probability that the random variable exceeds a given threshold. Specifically, suppose that the structural failure probability is represented by $F(1.0\mathcal{H}_n)$, where F is the CDF of either a lognormal or a normal distribution in Fig. 8. For the case of $0.4\mathcal{H}_n$, the failure probability associated with a lognormal load is 0.015, which is approximately 10 times of that associated with a normal distribution. This fact indicates that treating a non-Gaussian load process as Gaussian may result in significant error in the estimate of structural reliability.

In order to investigate the impact of load autocorrelation on structural time-dependent reliability, Fig. 9 presents the time-dependent failure probabilities for different cases of correlation coefficients in load: case (1) $\rho(1\,\text{year}) = 0.1$, case (2) $\rho(1\,\text{year}) = 0.3$ (the same as before) and case (3) $\rho(1\,\text{year}) = 0.5$. Correspondingly, the autocorrelation coefficients in $\mathcal{H}^*$ are 0.1107, 0.3241 and 0.5278 for a time separation of 1 year. Further, with Eq. (33), the parameter b is found as 371.1, 78.7 and 16.4 respectively for the three cases. In Fig. 9, the failure probability increases exponentially with T for reference periods exceeding 10 years, which is consistent with the observation from Fig. 7(a). Moreover, Fig. 9 suggests that a stronger autocorrelation in loads leads to a smaller failure probability. This can be explained by considering an extreme case where the structural survival is represented by $S_1 < r \bigcap S_2 < r$, where r is the resistance (a deterministic value), S_1 and S_2 are two identically distributed loads with a CDF of F. For the case of fully correlated S_1 and S_2, the failure probability is simply $1-F(r)$, which is greater than that associated with independent S_1 and S_2 (i.e., $1 - F^2(r)$). Fig. 9 on one hand implies the importance of identifying the load autocorrelation in an accurate estimate of structural reliability, and on the other hand suggests that for cases where only insufficient load information is available, the assumption of a weak autocorrelation in loads leads to a relatively conservative estimate of structural reliability.

By noting that the load process follows a lognormal distribution, as summarized in Table 1, the CDF of maximum load effect within a reference period of Δ can be found through Eq. (20). Fig. 10 plots the CDFs of maximum load for cases of $\rho(1 \text{ year}) = 0.1, 0.3$ and 0.5, respectively. A stronger load autocorrelation leads to a shorter upper tail of the CDF, and subsequently results in a smaller exceeding probability given a predefined threshold. This observation is consistent with the one from Fig. 9 that a greater load autocorrelation leads to a smaller failure probability.

Finally, the difference between the failure probabilities associated with a discrete load process and a continuous one is discussed. The failure probabilities are calculated with Eqs. (21) and (26), respectively. For the three cases in Fig. 7(a), the difference between $\mathbb{P}(T)$ and $\mathbb{P}_d(T)$ is found to be negligible. For instance, for a reference period of 50 years, if the mean value of $\mathcal{H}(t)$ is $0.5\mathcal{H}_n$, then $\mathbb{P}(T)$ and $\mathbb{P}_d(T)$ are equal to 0.036 and 0.035, respectively (with a difference of less than 2%). This small difference can be explained as follows. Consider a Gaussian load process, with which the term z in Eq. (23) is rewritten as follows,

$$z = \frac{\dot{\Omega}(t)}{\sigma_{\dot{X}}} = \frac{\dot{\Omega}(t)}{\sqrt{\frac{\pi a}{3\sqrt{b}}}} = \frac{\sqrt{2}\dot{\Omega}(t)}{\sigma_X b^{1/6}} \tag{38}$$

With the structural configuration in Table 1, for the typical cases where $\rho(1 \text{ year}) \leq 0.8$

(correspondingly, $b \geq 0.73$ according to Eq. (33)),

$$0 > z \geq \frac{-\sqrt{2} \cdot 0.2/50 \cdot \left(1.1 \cdot \frac{0.9\mathcal{D}_n + 1.6\mathcal{H}_n}{0.75}\right)}{0.5 \cdot 0.5\mathcal{H}_n \cdot 0.73^{1/6}} = -0.0874 \tag{39}$$

with which $\frac{1}{h(z_0)} \in [0.8981, 1]$. This fact implies that the difference between $\mathbb{P}(T)$ and $\mathbb{P}_d(T)$ has a maximum of approximately 10%. In fact, even for an extreme case where the resistance degrades severely by 50% over a reference period of 50 years, the maximum difference between the two failure probabilities is about 20%. As a result, it can be concluded that a continuous load process can be reasonably modeled by a discrete process where only significant load events are considered.

CONCLUSIONS

This paper has proposed a method to estimate the structural time-dependent reliability in the presence of a new power spectral density function, which yields a simple and efficient solution to the structural reliability. Illustrative examples are presented to demonstrate the applicability of the proposed method. The following conclusions can be drawn from this paper.

1. The structural time-dependent reliability analysis in the presence of a non-Gaussian load process can be transformed into a standard "first passage probability" problem by introducing an "equivalent" load. Provided that the autocorrelation in the load process is known, the correlation coefficient function in the "equivalent" load process can be uniquely determined.
2. Some types of power spectral density function of a stochastic process may result in a non-convergent estimate of the standard deviation of the process's derivative, and thus cannot be used in reliability assessment directly (c.f. Eqs. (8) and (11)). The proposed spectral density function as in Eq. (29), however, enables an analytical estimate of the stochastic process's characteristics, and further yields a closed-form formula of structural time-dependent reliability.

3. If the load process is non-Gaussian, simply assuming a Gaussian process for loads may lead to a significantly biased estimate of structural reliability. This fact indicates the importance of properly addressing the distribution type of the load process.
4. A stronger load autocorrelation leads to a smaller failure probability. For cases where the load information is insufficient, the assumption of a weak autocorrelation in loads results in a relatively conservative estimate of structural reliability.
5. The impact of choosing a continuous or a discrete load model on structural reliability is compared. The former leads to a specific distribution type (not necessarily Rayleigh if the load process is non-Gaussian) of maximum load effect during a time interval of interest. The assumption of a discrete stochastic process for loads overestimates the structural safety compared with that associated with a continuous load model. The difference is, however, negligible for most engineering cases, and thus the two methods of modeling load process can be used exchangeably for the purpose of structural safety assessment.

APPENDIX I. ON THE SAMPLING OF A RANDOM VARIABLE WITH A KNOWN PDF

In this section, the sampling of a random variable with a known PDF is discussed. The rejection method can be used to sample a random variable with a known PDF but follows an irregular distribution (Ross 2014).

First, consider a random variable X with a standard deviation of σ_X and a PDF of $f_X(x) = \frac{a_0}{x^6+b} = \frac{S(x)}{\sigma_X^2}$, where $S(x)$ is as in Eq. (29), and $a_0 = \frac{a}{\sigma_X^2}$. Clearly, one can show that $\int_{-\infty}^{\infty} f(x)\mathrm{d}x = \int_{-\infty}^{\infty} \frac{S(x)}{\sigma_X^2}\mathrm{d}x = 1$. For further derivation, an auxiliary random variable Y is introduced, which has a PDF of $f_Y(y) = \frac{\sqrt{b}/\pi}{y^2+b}$. The CDF of Y is $F_Y(y) = \int_{-\infty}^{y} \frac{\sqrt{b}/\pi}{z^2+b}\mathrm{d}z = \frac{1}{\pi}\left(\arctan\left(\frac{y}{\sqrt{b}}\right) + \frac{\pi}{2}\right)$. Mathematically, it can be proven that

$$S(y) = \frac{a_0}{y^6+b} \leq \frac{a_0(b+1)\pi}{b^{1.5}} \cdot f_Y(y) \tag{40}$$

With this, the procedure of sampling a realization x for X is as follows,

- Simulate two random numbers u_1 and u_2 that are uniformly distributed in $[0,1]$.
- Set $y = \sqrt{b}\tan\left(u_1\pi - \frac{\pi}{2}\right)$.
- If $u_2 \leq \frac{S(y)}{\frac{a_0(b+1)\pi}{b^{1.5}} \cdot f_Y(y)}$, then set $x = y$; otherwise return to step 1 (i.e. re-sample u_1 and u_2).

This procedure has been used in the sampling of $\mathcal{H}^*(t)$ and $\mathcal{H}(t)$ in Fig. 6.

ACKNOWLEDGEMENTS

The research described in this paper was supported by the Faculty of Engineering and IT PhD Research Scholarship (SC1911) from the University of Sydney. This support is gratefully acknowledged. The authors would like to acknowledge the thoughtful suggestions of two anonymous reviewers, which substantially improved the present paper.

REFERENCES

Akiyama, M. and Frangopol, D. M. (2014). "Long-term seismic performance of rc structures in an aggressive environment: Emphasis on bridge piers." *Structure and Infrastructure Engineering*, 10(7), 865–879.

ASCE (2002). *Minimum design loads for buildings and other structures*. American Society of Civil Engineers.

Beck, A. T. and Melchers, R. E. (2005). "Barrier failure dominance in time variant reliability analysis." *Probabilistic engineering mechanics*, 20(1), 79–85.

Clayton, C. R., Woods, R. I., Bond, A. J., and Milititsky, J. (2014). *Earth pressure and earth-retaining structures*. CRC Press.

Comenetz, M. (2002). *Calculus: the elements*. World Scientific Publishing Co Inc.

Ellingwood, B., MacGregor, J. G., Galambos, T. V., and Cornell, C. A. (1982). "Probability based load criteria: load factors and load combinations." *Journal of the Structural Division*, 108(5), 978–997.

Ellingwood, B. R. and Hwang, H. (1985). "Probabilistic descriptions of resistance of safety-related structures in nuclear plants." *Nuclear Engineering and Design*, 88(2), 169–178.

Ellingwood, B. R. and Lee, J. Y. (2016). "Life cycle performance goals for civil infrastructure: intergenerational risk-informed decisions." *Structure and Infrastructure Engineering*, 12(7), 822–829.

Engelund, S., Rackwitz, R., and Lange, C. (1995). "Approximations of first-passage times for differentiable processes based on higher-order threshold crossings." *Probabilistic Engineering Mechanics*, 10(1), 53–60.

Enright, M. P. and Frangopol, D. M. (1998). "Service-life prediction of deteriorating concrete bridges." *Journal of Structural Engineering*, 124(3), 309–317.

Ferrante, F., Arwade, S., and Graham-Brady, L. (2005). "A translation model for non-stationary, non-gaussian random processes." *Probabilistic Engineering Mechanics*, 20(3), 215–228.

Gomes, L. and Vickery, B. (1977). "On the prediction of extreme wind speeds from the parent distribution." *Journal of Industrial Aerodynamics*, 2(1), 21–36.

Grigoriu, M. (1984). "Crossings of non-gaussian translation processes." *Journal of Engineering Mechanics*, 110(4), 610–620.

Hagen, Ø. and Tvedt, L. (1991). "Vector process out-crossing as parallel system sensitivity measure." *Journal of Engineering Mechanics*, 117(10), 2201–2220.

Kim, H. and Shields, M. D. (2015). "Modeling strongly non-gaussian non-stationary stochastic processes using the iterative translation approximation method and karhunen–loève expansion." *Computers & Structures*, 161, 31–42.

Li, C.-Q., Firouzi, A., and Yang, W. (2016a). "Closed-form solution to first passage probability for nonstationary lognormal processes." *Journal of Engineering Mechanics*, 142(12), 04016103.

Li, C.-Q., Lawanwisut, W., and Zheng, J. (2005). "Time-dependent reliability method to assess the serviceability of corrosion-affected concrete structures." *Journal of Structural Engineering*, 131(11), 1674–1680.

Li, Q., Wang, C., and Ellingwood, B. R. (2015). "Time-dependent reliability of aging structures in the presence of non-stationary loads and degradation." *Structural Safety*, 52, 132–141.

Li, Q., Wang, C., and Zhang, H. (2016b). "A probabilistic framework for hurricane damage assessment considering non-stationarity and correlation in hurricane actions." *Structural Safety*, 59, 108–117.

Liu, P. and Der Kiureghian, A. (1986). "Multivariate distribution models with prescribed marginals and covariances." *Probabilistic Engineering Mechanics*, 1(2), 105–112.

Lutes, L. D. and Sarkani, S. (2004). *Random vibrations: analysis of structural and mechanical systems*. Butterworth-Heinemann.

Melchers, R. (1999). *Structural reliability analysis and prediction*. Wiley, New York.

Mori, Y. and Ellingwood, B. R. (1993). "Reliability-based service-life assessment of aging concrete structures." *Journal of Structural Engineering*, 119(5), 1600–1621.

Newland, D. E. (1993). *An introduction to random vibrations, spectral & wavelet analysis (third edition)*. Pearson Education Limited, Edinburgh Gate, Harlow, England.

Pang, L. and Li, Q. (2016). "Service life prediction of rc structures in marine environment using long term chloride ingress data: Comparison between exposure trials and real structure surveys." *Construction and Building Materials*, 113, 979–987.

Pillai, T. M. and Veena, G. (2006). "Fatigue reliability analysis of fixed offshore structures: A first passage problem approach." *Journal of Zhejiang University, Science A*, 7(11), 1839–1845.

Rackwitz, R. (2001). "Reliability analysis – a review and some perspectives." *Structural Safety*, 23(4), 365–395.

Ross, S. M. (2014). *Introduction to probability models (tenth edition)*. Academic press.

Tang, W. H. and Ang, A. (2007). *Probability concepts in engineering: Emphasis on applications to civil & environmental engineering*. Wiley Hoboken, NJ.

Wang, C., Li, Q., and Ellingwood, B. R. (2016). "Time-dependent reliability of ageing structures: an approximate approach." *Structure and Infrastructure Engineering*, 12(12), 1566–1572.

Wang, C. and Zhang, H. (2018). "Roles of load temporal correlation and deterioration-load dependency in structural time-dependent reliability." *Computers & Structures*, 194, 48–59.

Wang, C., Zhang, H., and Li, Q. (2017). "Reliability assessment of aging structures subjected to gradual and shock deteriorations." *Reliability Engineering & System Safety*, 161, 78–86.

Zheng, R. and Ellingwood, B. R. (1998). "Stochastic fatigue crack growth in steel structures subject to random loading." *Structural Safety*, 20(4), 303–323.

List of Tables

1 Probabilistic models of resistance and loads 27

TABLE 1: Probabilistic models of resistance and loads

Item	Mean	COV	Distribution
Initial resistance	$1.10\mathcal{R}_n$	0	Deterministic
Dead load	$1.00\mathcal{D}_n$	0	Deterministic
Lateral load	$0.50\mathcal{H}_n$	0.5	Lognormal

List of Figures

1 A comparison between a continuous load process and a discrete one. 29

2 Illustration of the outcrossing rate of stochastic process $X(t)$ relative to $\Omega(t)$. 30

3 Autocorrelation in hurricane load effects (after Ellingwood and Lee 2016). . . 31

4 Dependence of $R(\tau)$ on τ for different values of b. 32

5 Autocorrelation functions in both $\mathcal{H}^*(t)$ (solid line) and Gaussian $\mathcal{H}(t)$ (dashed line). 33

6 Sample sequences of $\mathcal{H}(t)$ (normalized by $\mathcal{H}_n$) and $\mathcal{H}^*(t)$, respectively. 34

7 Time-dependent failure probability for periods up to 50 years. 35

8 Upper tail behaviour of the CDF of $\mathcal{H}$ (normalized by $\mathcal{H}_n$). 36

9 Dependence of failure probability on the autocorrelation in load process. . . 37

10 The CDF of $\max\{\mathcal{H}(t)\}$ (normalized by $\mathcal{H}_n$) during a unit time $\Delta = 1$. . . . 38

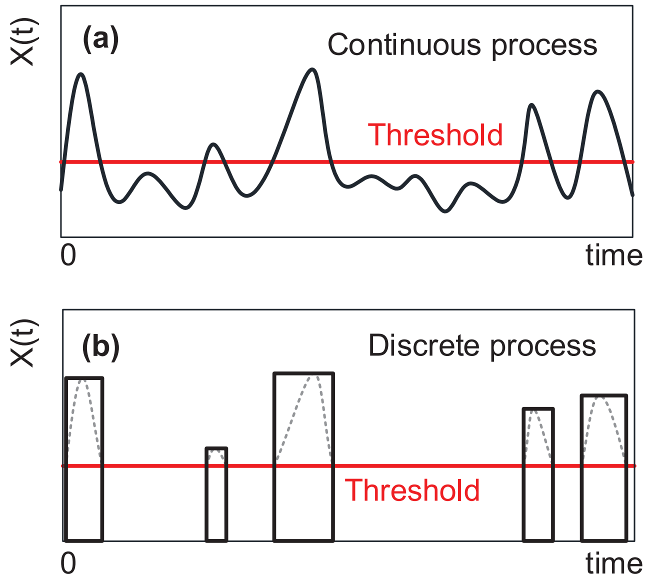


FIG. 1: A comparison between a continuous load process and a discrete one.

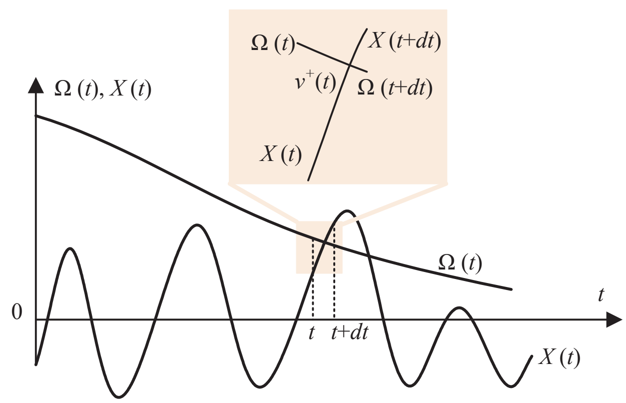


FIG. 2: Illustration of the outcrossing rate of stochastic process $X(t)$ relative to $\Omega(t)$.

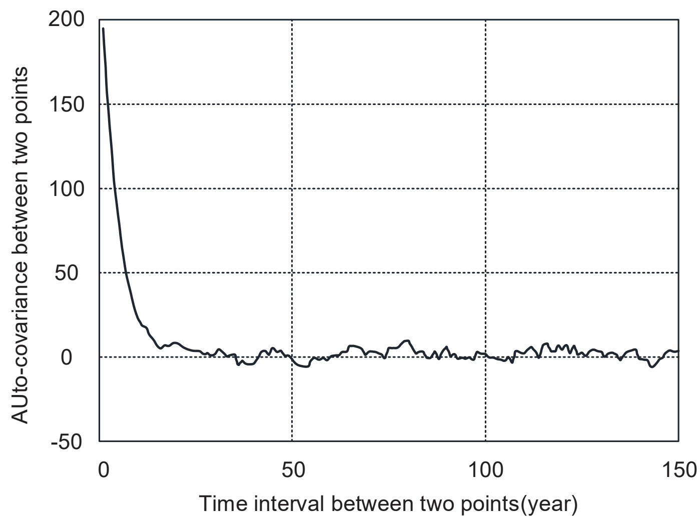


FIG. 3: Autocorrelation in hurricane load effects (after Ellingwood and Lee 2016).

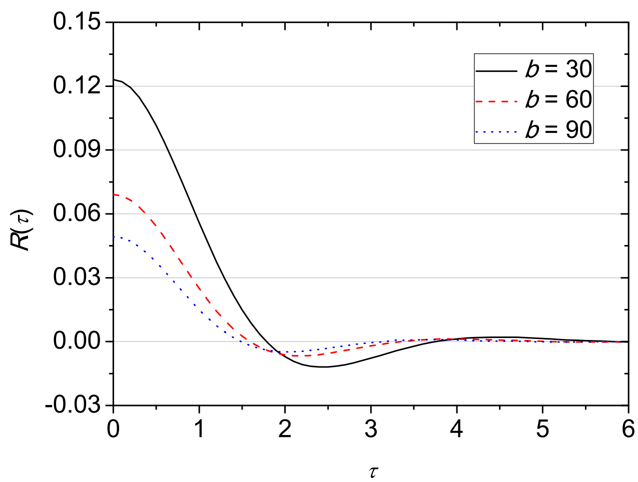


FIG. 4: Dependence of $R(\tau)$ on τ for different values of b.

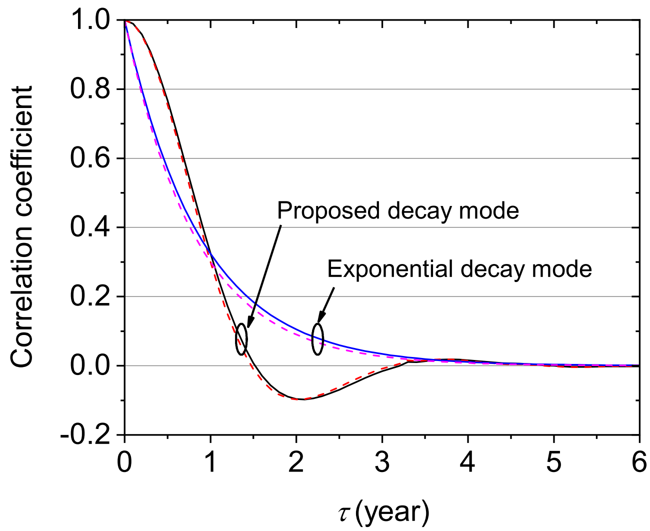


FIG. 5: Autocorrelation functions in both $\mathcal{H}^*(t)$ (solid line) and Gaussian $\mathcal{H}(t)$ (dashed line).

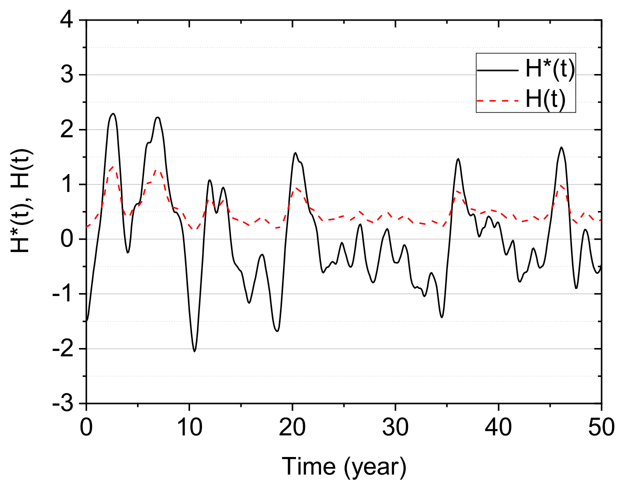
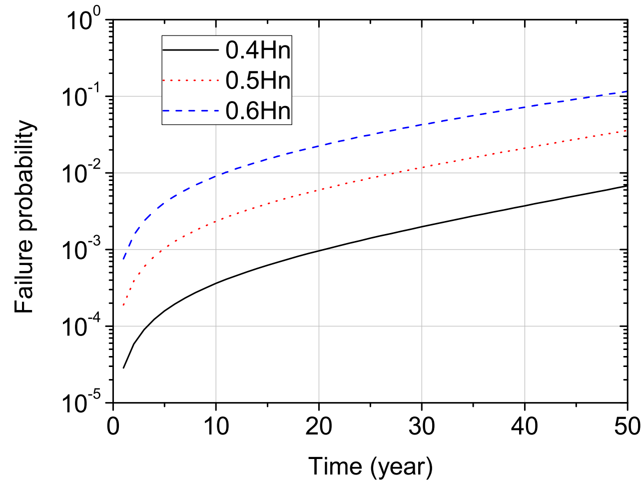
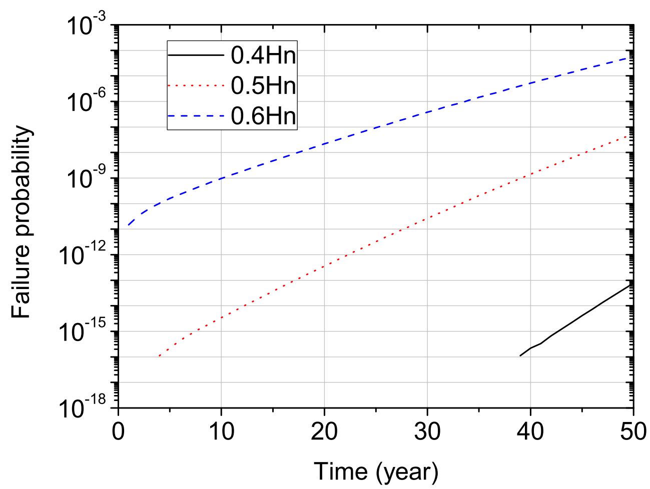


FIG. 6: Sample sequences of $\mathcal{H}(t)$ (normalized by $\mathcal{H}_n$) and $\mathcal{H}^*(t)$, respectively.



(a) $\mathcal{H}$ follows a lognormal distribution as summarized in Table 1



(b) Assuming a Gaussian process of $\mathcal{H}(t)$

FIG. 7: Time-dependent failure probability for periods up to 50 years.

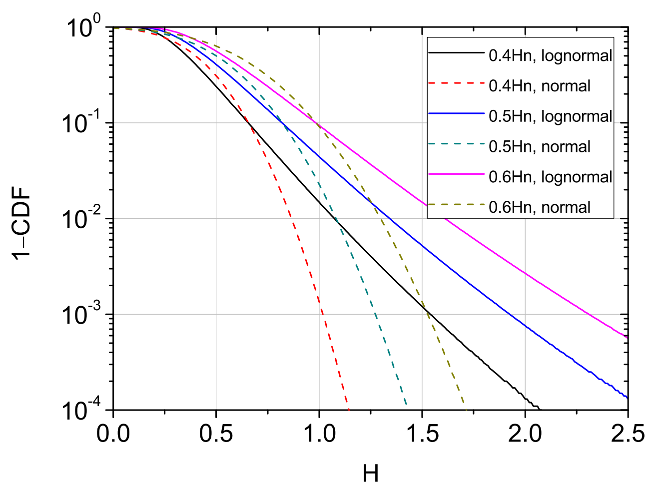


FIG. 8: Upper tail behaviour of the CDF of $\mathcal{H}$ (normalized by $\mathcal{H}_n$).

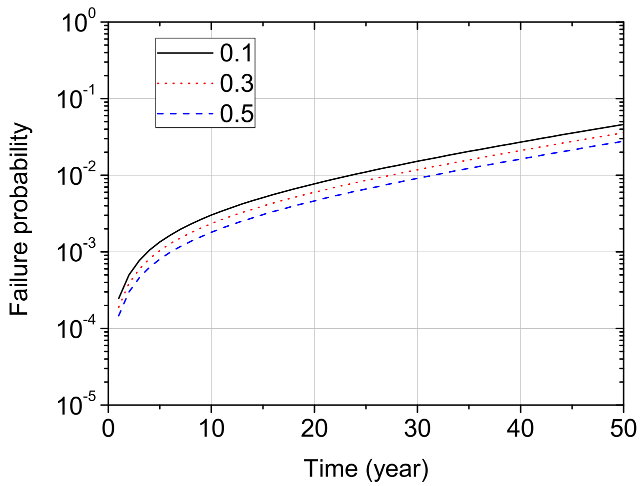


FIG. 9: Dependence of failure probability on the autocorrelation in load process.

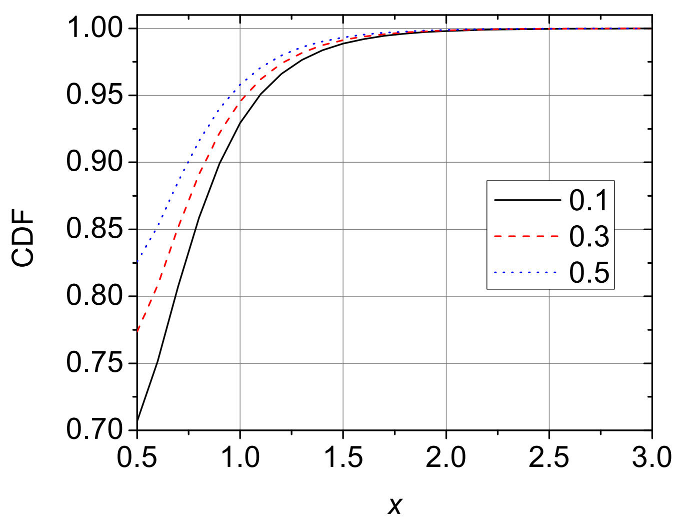


FIG. 10: The CDF of $\max\{\mathcal{H}(t)\}$ (normalized by $\mathcal{H}_n$) during a unit time $\Delta = 1$.