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On the dimensionality of inference in credal nets with interval probabilities

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Statement

The number of decision variables (or dimensionality) required to compute the **inference in two-state credal networks** with interval probabilities grows at most **linearly with the number of nodes** directly connected to the queried variable.

Strategy and proof

We prove this statement by means of the *interval gradient* on a *vacuous credal* network. A vacuous credal network is a network whose probabilities are in the open interval (0, 1). The interval gradient is obtained from the derivatives of the independent inputs over the open interval. x_k is the k^{th} independent input.

$$\{x\}^{\downarrow} = \left\{ x_k : \left.\frac{\partial P_{infer}(x_k)}{\partial x_k}\right|_{(0,1)} < 0, k = 1, \dots, D \right\} \quad (1)$$

$$\{x\}^{\uparrow} = \left\{ x_k : \left.\frac{\partial P_{infer}(x_k)}{\partial x_k}\right|_{(0,1)} > 0, k = 1, \dots, D \right\} \quad (2)$$

$$\{x\}^{M} = \{x\}^{\downarrow} \cup \{x\}^{\uparrow}, \qquad \{k\}^M = \{k : x_k \in \{x\}^M\} \quad (3)$$

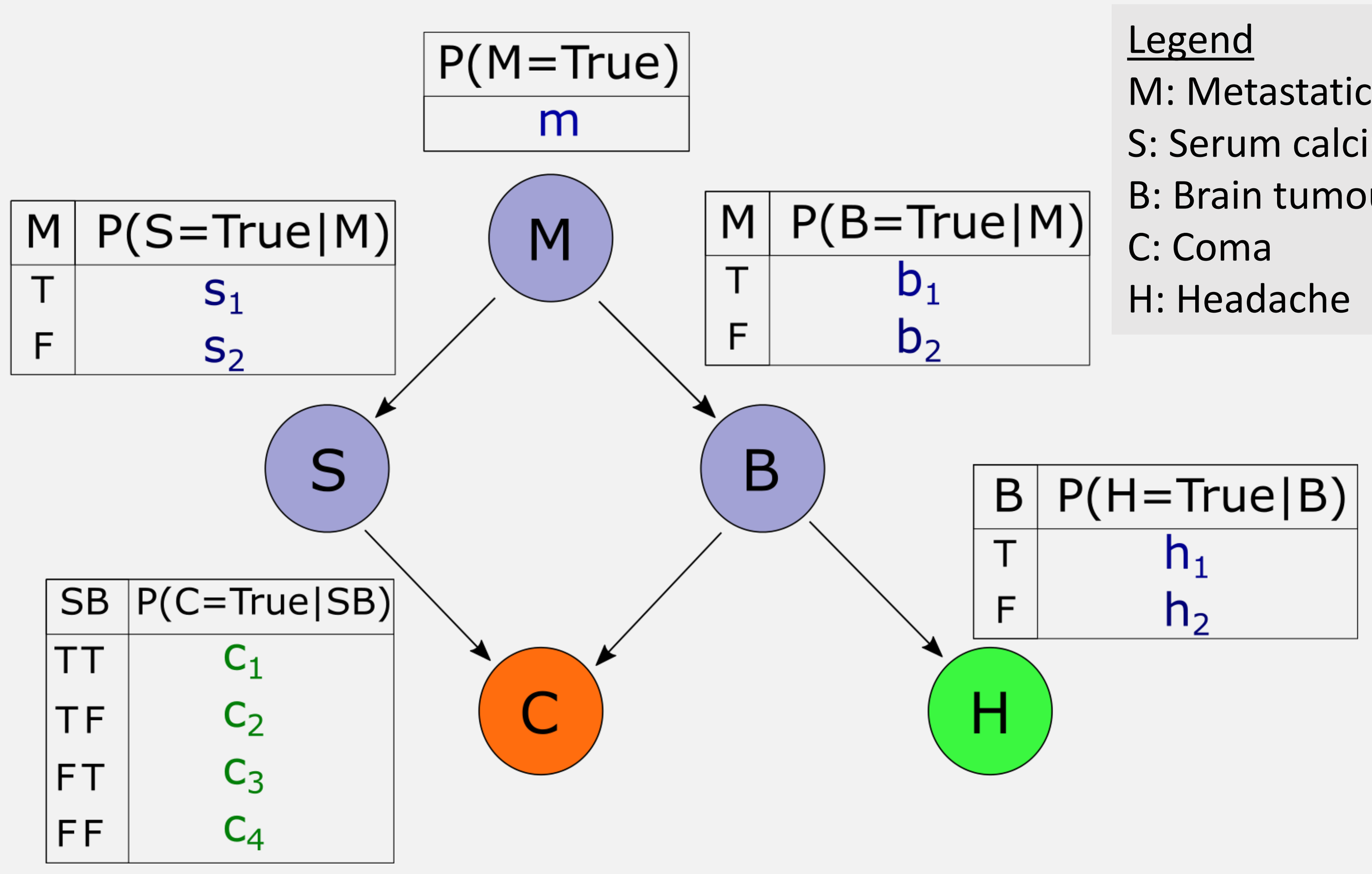
$$R = D - \#\{k\}^M \quad (4)$$

The proof needs specialisation on the network under study, however coefficients can be stored upfront on recurring architectures. In (4) the integer R is the reduced dimension of the credal network.

Example: *Multiply-connected network*

Queries:

$\overline{P}(C\|H)$	$x_{\{k\}^M} = \{\mathbf{c}_{1:4}\}$	$x_{\{k\}^{\neg M}} = \{\mathbf{m}, \mathbf{s}_{1:2}, \mathbf{b}_{1:2}, \mathbf{h}_{1:2}\}$
$\overline{P}(S\|C)$	$x_{\{k\}^M} = \{\mathbf{s}_{1:2}, \mathbf{c}_{1:4}\}$	$x_{\{k\}^{\neg M}} = \{\mathbf{m}, \mathbf{b}_{1:2}\}$
$\overline{P}(H)$	$x_{\{k\}^M} = \{\mathbf{h}_1, \mathbf{h}_2\}$	$x_{\{k\}^{\neg M}} = \{\mathbf{m}, \mathbf{b}_{1:2}\}$



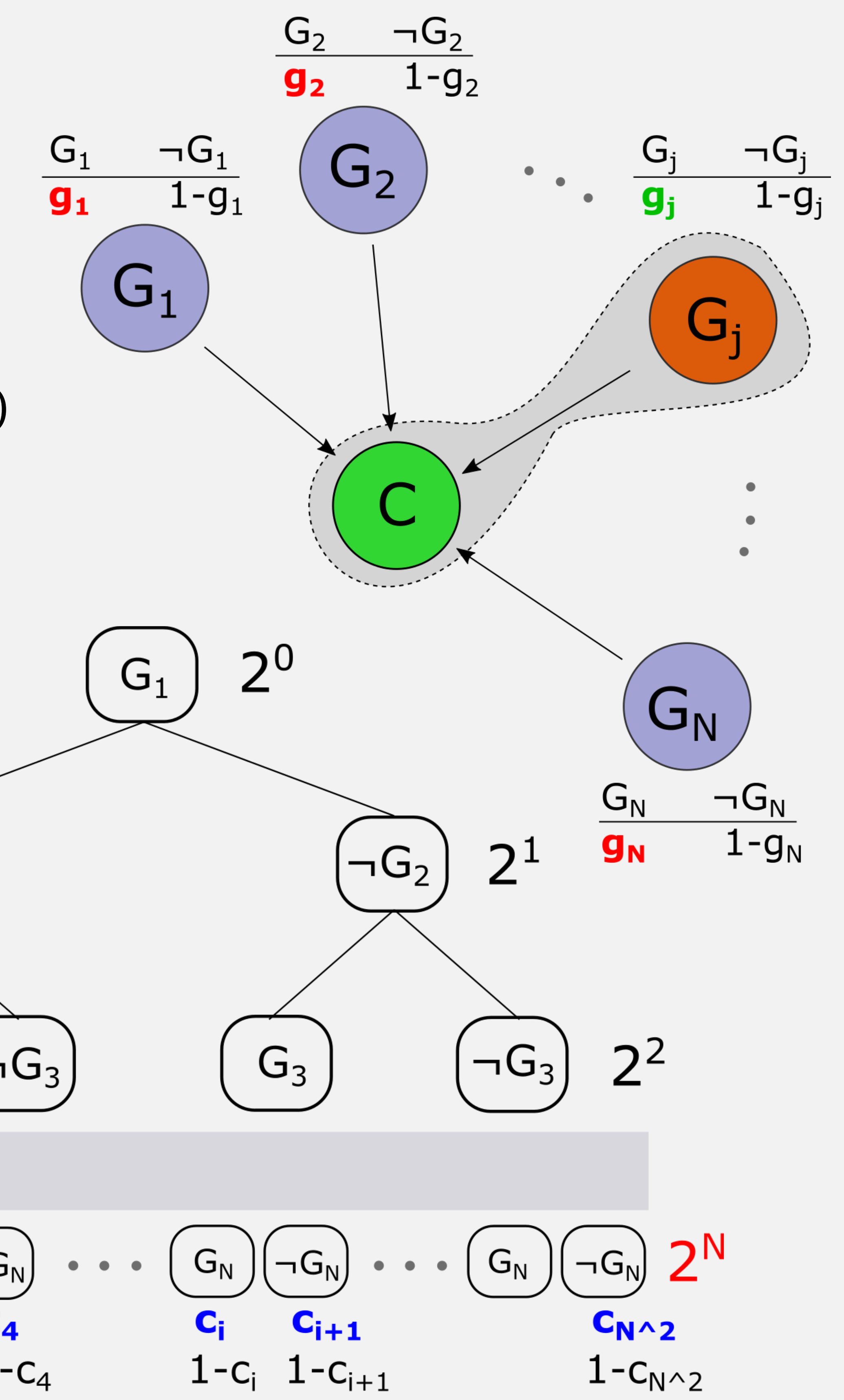
Legend
M: Metastatic cancer
S: Serum calcium increase
B: Brain tumour
C: Coma
H: Headache

General example

$$\overline{P}(G_j|C) =$$
$$= \overline{P}_j(x_k) = ?$$

Ex.: $x_{k=1} = g_1$

$g_j = P(G_j = True)$



Algorithm

$$P(G_j|C) = \frac{\sum_{\{g_1,\dots,g_N\}\setminus g_j} P(G_1,\dots,G_N,C)}{P(C)} = \frac{P(G_j,C)}{P(C)} =$$

$$= P(X_k) = \frac{P(G_j,C)}{P(C)}$$

1. $x_k = \{g_1, g_2, \dots, g_j, \dots, g_N, c_1, c_2, \dots, c_{2^N}\}$

2. $k^{\{M\}} = \left\{ k : \left.\frac{\partial P_j(x_k)}{\partial x_k}\right|_{(0,1)} \setminus \{0\} \right\}$

3. $x_k = \{g_1, g_2, \dots, \mathbf{g_j}, \dots, g_N, \mathbf{c_1}, \mathbf{c_2}, \dots, \mathbf{c_{2^N}}\}$

4. $x_{\{k\}^M} = \{\mathbf{g_j}, \mathbf{c_1}, \mathbf{c_2}, \dots, \mathbf{c_{2^N}}\}$ $\quad x_{\{k\}^{\neg M}} = \{g_1, g_2, \dots, g_N\}$

5. $\underline{P}(x_k) = \min_{k \in \{k\}^{\neg M}} P(x_k) \qquad \overline{P}(X_k) = \max_{k \in \{k\}^{\neg M}} P(x_k)$

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