# Testing and comparing conditional risk-return relationship with a new approach in the cross-sectional framework

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## Abstract

This paper presents an innovative approach in examining the conditional relationship between beta and returns for stocks traded on S&P 500 for the period from July 2001 to June 2011. We challenge other competitive models with portfolios formed based on the Book Value per share and betas using monthly data. A novel approach for capturing time variation in betas whose pattern is treated as a function of market returns is developed and presented. The estimated coefficients of a nonlinear regression constitute the basis of creating a two factor model. Our results indicate that the proposed specification surpasses alternative models in explaining the cross-section of returns. The implications of this study show that the proposed new risk factors which found to be significant both in time-series and cross-section analyses provide valuable information in better understanding the characteristics of returns, targeting the reinforcement of stock market efficiency and the capital allocation procedure.

Keywords: Cross-sectional regression; CAPM; S&P 500;

## 1. Introduction

Tests of the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) are based on the assumption that the market risk, measured by $\beta$, remains constant over time. However, empirical investigations, such as Blume (1971), Levy (1974), Fabozzi and Francis (1977), Harvey (1989), Ferson and Harvey (1991, 1993), Huang (2001), Woodward and Anderson (2009), document that the estimated beta coefficients exhibit significant time variation. Therefore, a reliable test of CAPM should take into account that $\beta$ is nonstationary (Huang, 2001) whereas the question

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of what risks explain asset pricing anomalies remains at the core of financial research (Lin and Zhang, 2013).

In this paper, we provide an innovative approach which in view of betas instability, it tries to capture their time variation, considering their pattern as a function of market returns. The procedure is free from subjective bias problems related to the selection of a critical threshold of market returns while at the same time it takes into consideration nonlinearities observed in betas (Lin et al., 1992). In addition, the procedure adequately recognizes beta's behavior at the tails of the distribution by giving equal weight to market returns found far from the mean. Hence, it helps a loss-averse investor who may be interested in more about the size of the left tail of the return distribution (Ait-Sahalia and Brandt, 2001) and consequently the beta's behavior at this distribution's part. We then construct a Two Factor Model (TFM) whose variables aim at measuring asymmetric and constant systematic risk. The variables are formed by grouping stocks with specific characteristics relative to the estimated beta coefficients. Building such a model we facilitate individual investors and financial institutions to capital allocation. For example, scrutinizing individual assets among thousands of them that exist worldwide is a difficult task. However, this complexity is reduced by labeling assets (Boyer, 2011) as we do with our stocks.

We motivate our analysis not only by the model's promising results in the time series context (Messis and Zapranis, 2014)[1] but also by the fact that stocks with different betas in 'up' and 'down' markets might accommodate different fundamental characteristics and hence different risk levels. A number of studies have tried to capture betas variability but to our knowledge no model accommodates both asymmetric and systematic risk with a clear and sufficient way. In this way, we extend the analysis of Ang, Chen and Xing (2006) (hereafter ACX) to allow for a combination of two different risks into a single model.

We challenge the CAPM, the Fama and French three factor model (FF3FM) (Fama and French, 1996), the Premium Labor- model (PLM) (Jagannathan and Wang, 1996) (hereafter JW) and the Arbitrage Pricing Theory (APT) (Ross, 1976) both

[1] The TFM in time series regressions gives lower mispricing errors, lower values of the Gibbons, Ross, Shanken (1989) test and $R^2$'s similar to those of FF3FM. The examined portfolio was among others that of momentum whose effects challenged the three factor model.

conditionally[2] and unconditionally with book value per share (BVps) and beta based portfolios. The results indicate that the proposed explanatory variables are priced and that this specification surpasses competitive models, in explaining the cross-section of return. In particular, we find that different risk factors are priced by the market. Our model outperforms the CAPM and the PLM both in terms of $R^2$ and F-test values while this model and the FF3FM present *p*-values of the F-test (pv-F) lower than 5% significant level when running unconditional cross-sectional regressions. The TFM's asymmetric risk is priced and the market risk premium appears the expected positive sign for all models apart from those of CAPM and FF3FM when the BVps portfolios are employed. In relation to the APT, we show that this model better explains the portfolio returns when the NSI is used for capturing market risk premium instead of the S&P 500. In the conditional cross sectional regressions for the same portfolios once again we observe the significance of the model's factors. For the beta based portfolios the market risk premium of the TFM and the CAPM is almost identical both conditionally and unconditionally. We therefore interpret our findings as evidence that the two variables which by construction accommodate different beta characteristics can better explain the cross-section of expected stock returns. We also study the economic interpretation of our factors in terms of alphas giving special attention to the Neutral Stock Index (NSI) which measures constant systematic risk. Annualized alpha of the index with respect to the benchmark index is substantial. It produces an alpha of 10.8%, which is much higher than the one reported by Moreira and Muir (2017) using volatility-managed portfolios (4.9%) and certainly well above 400 US equity mutual fund whose average annualized alpha is negative (-4.65%) (Frijns et. al, 2013). For our second index named as 'SMISI' (i.e. Superior minus Inferior Stock Index) and aims at capturing asymmetric risk, we demonstrate that this index acts as an 'insurance' portfolio. Daniel et. al (2002) report that an 'insurance' portfolio must provide high returns in bad states of the world and low returns in good states of the world. Indeed, we find that this portfolio gives positive returns for negative S&P market returns and almost zero returns for positive benchmark's returns. In addition, the portfolio's factor loadings are negative and statistically significant which constitute another characteristic of 'insurance' portfolios.

[2] Lewellen and Nagel (2006) argue that conditional asset pricing models perform well due to the cross-sectional design adopted for testing these models and the failure to impose theoretical constraints in the estimation process. Hence, they propose that time-series tests are more suitable to these models.

ACX through a simplistic mechanism provide the theoretical framework of how downside versus upside risk may be priced differently. The authors report that assets characterized by greater downside than upside risk are unattractive to hold because their payoffs are very low at the moment when the investors' wealth is low. Hence, in equilibrium and when disappointment aversion preferences (Gul, 1991) among investors are present, they should be compensated by higher expected returns, for holding stocks with high downside risk. ACX directly estimate downside risk which is priced in the cross-section of stock returns and it is different from all well-known effects (i.e. size, book to market, momentum etc.).

Even though, the authors find that stocks with high downside betas have high unconditional average returns, they also find that stocks sorted on realized $\beta^{+}$ (i.e. stocks that exhibit higher betas in up markets rather than in down markets) do not gain lower returns as expected. Along the same line are the results of the Investment Insights (2011) published by the Perkins Investment Management which depict that high quality stocks have offered considerable return advantages over longer time periods since they offer stronger downside protection and solid upside capture which are also accompanied by lower risk. These empirical evidences could be attributed to idiosyncratic risk. For example, ACX demonstrate that the asset's idiosyncratic risk (i.e. CAPM alpha) decreases as the difference between stocks sorted according to $\beta^{+} - \beta^{-}$ increases. In traditional portfolio theory, idiosyncratic risk, which cannot be hedged, is not important to diversified investors. However, Mendenhall (2004) argues that idiosyncratic risk may be relevant to unbiased investors such as arbitrageurs who are considered to be highly specialized and hold a few, relatively large positions of stocks at any one time. The author also points out that the models of Shleifer and Vishny (1997) and Wurgler and Zhuravskaya (2002) imply that stocks with higher idiosyncratic risk should be less attractive to arbitrageurs. From this point of view, when equivalent earnings surprises happen, arbitrageurs may take smaller positions in stocks with higher idiosyncratic risk. If other investors underreact to earnings announcements, then high idiosyncratic-risk stocks will be more mispriced and therefore exhibit greater price movement in the direction of an earnings surprise. The earnings surprise in turn will affect betas in the expected direction (Ball et al.,1993).

This study adds to the literature with different ways. In particular our primary goal is to compare directly the benefits of combining systematic and asymmetric risk into a single model. This kind of combination gives valuable information of better understanding the characteristics of returns, targeting the reinforcement of stock market efficiency. From this point of view, we also extend the literature of time varying betas and capital allocation through the proposed novel method of capturing betas variability and through the process of labeling assets respectively. Moreover, the model's new information which might be easily accessible to the average investor, mutual funds and policy makers provides bubbles less opportunities to be formed and to be sustained.

The paper is organized as follows. Next section develops the methodology for the model's empirical examination. In section 3 our empirical results are demonstrated. More precisely, in section 3.1 the data are described while in section 3.2 the empirical results of the conditional and unconditional cross-section regressions are presented. Finally, section 4 concludes the paper.

## 2. Methodological framework

In this section, we provide the methodology for building our two factor model. The methodology is constituted by a two-step procedure. At the first step, we estimate beta coefficients through a nonlinear model and at the second step we group our stocks into portfolios in order to form the TFM.

The CAPM is a set of predictions concerning equilibrium of expected return on risky assets (Bodie et al., 2002). In the cross sectional context, the model states that differences in average returns depend linearly and solely on asset betas (Cuthbertson and Nitzsche, 2004). Betas are estimated running the following time series regression for each security or portfolio *i*:

$$R_{i,t} - R_{,ft} = a_i + \beta_i (R_{m,t} - R_{f,t}) + e_{i,t} \tag{1}$$

where $R_{i,t} - R_{f,t}$ is the excess return of asset *i*, $R_{m,t} - R_{f,t}$ is the market excess return, $\beta_i$ is the systematic risk and $a_i$ and $e_{i,t}$ are assumed to be zero according to the model.

### *2.1 Capturing betas' variability with a novel approach*

In the proposed novel method a two-step procedure is applied to capture any variations in beta coefficients. In the first step, the beta coefficients from equation (1) are estimated. Using the standard OLS method and daily returns for a time interval of three years[3], we estimate the first beta coefficient of period *t*. Next, the rolling regression method, which is a common procedure for assessing time-varying betas (Ang and Chen, 2007) is employed. More precisely in order to obtain the second value of beta, the first observation is dropped and a new one is added to the end of the sample. The procedure is followed for a five-year period estimating the respective betas of each day. The previous procedure results to 1,250 betas which we rank them in ascending order relative to the market return on day *t*=1…1,250. More precisely, for the 5 year period we construct an interval between the minimum and the maximum market return which we split into discrete subintervals. For each market return discrete subinterval we find the corresponding betas. Then, the averaged values of the estimated betas for each market return discrete subinterval are calculated. This ensures that equal weights will be given at each observation catching up any differences in each and every market condition. At the same time, any subjective bias at the selected market interval is avoided. The discrete subintervals of the market return are generally different for each period. Their number is determined by the extent to which a given period is more or less volatile since the range of market returns is different in each case.

The reason for which we chose to model the $\beta$ coefficient by giving equal weights at each market return discrete interval stems from the fact that the highest number of market returns lie around its mean in a normal distribution. To be more specific, a way of estimating realized betas in up and down markets is the use of different cutoff points. For example, Bawa and Lindenberg (1977) measure downside beta $(\beta^{-})$ as follows:

$$\beta^{-} = \frac{\operatorname{cov}\left(R_i, \mathrm{R}_m \mid \mathrm{R}_m < \mu_m\right)}{\operatorname{var}\left(R_m \mid \mathrm{R}_m < \mu_m\right)}. \tag{2}$$

with $\mu_m$ being the average market excess return as cutoff point. ACX define in a similar manner the upside beta as:

$$\beta^{+} = \frac{\operatorname{cov}\left(R_i, \mathrm{R}_m \mid \mathrm{R}_m > \mu_m\right)}{\operatorname{var}\left(R_m \mid \mathrm{R}_m > \mu_m\right)}. \tag{3}$$

The fact that the CAPM assumes that beta remains constant for any given market return, $R_m$, means that for any market return in a supposed interval [-10%,…+10%]

[3] Daves et al., (2000) show that daily returns data of three years time interval give the best daily beta predictions.

the beta is equal (i.e. $\beta_{i|R_m=-10\%} = \beta_{i|R_m=-9\%} = ... = \beta_{i|R_m=0\%} ... \beta_{i|R_m=+9\%} = \beta_{i|R_m=+10\%}$). Thus, the beta coefficient is a straight line across the horizontal axis of market returns. However, as empirical findings indicate, this is not the case as $\beta_{i|R_m=-10\%} \neq \beta_{i|R_m=-9\%} \neq ... \neq \beta_{i|R_m=0\%} ... \beta_{i|R_m=+9\%} \neq \beta_{i|R_m=+10\%}$. Furthermore, for a given market return $R_m$ in a particular period (for example the average market excess return, $\mu_m$) there is one probability such that $\Pr(R_m = \mu_m) = \frac{n(\mu_m)}{\text{N}}$ with $n(\mu_m)$ being the number of observations that approximate the average market excess return and $N$ being the total market return observations for that particular period. However, due to the fact that most of returns lie around zero, modeling $\beta$ without assigning specific weights at each market return discrete interval, it is rather difficult to efficiently recognize beta's behavior for market returns far from the mean, especially when 'large' shocks take place. Hence, other approximations cannot fully absorb them adequately. In addition, it is well known that computing other risk measures such as VaR, historical quantiles are used under the assumption that any return in a particular period is equally likely (Engle and Manganelli, 2004). Following this assumption we model the $\beta$ coefficient by giving equal weights at each market return discrete interval.

After constructing the proposed variables, a question arises regarding the form of beta coefficient as a function of the sorted market return, $R_{ms}$, (i.e. $\overline{\beta} = f(R_{ms})$, (Faff and Brooks, 1998)). Lin et al., (1992) suggest that beta mean fluctuates around an upward or downward parabolic trend pattern. Hence, the functional form of $f(\cdot)$ is approached by:

$$\overline{\beta}_{i,[R^-_{ms,j}, R^+_{ms,j}]} = \alpha e^{(b^* R_{ms,j} + c R^2_{ms,j} + u_i)} \tag{4}$$

where $\alpha$, $b^*$, $c$ are the coefficients to be estimated for each stock $i$, $R_{ms,j}$ is the sorted market return in the subinterval $j$ common to all stocks $i$, $\overline{\beta}_i$ are the average betas of stock $i$ corresponding to each market return subinterval $\left[R^-_{ms,j}, R^+_{ms,j}\right]$ and $u_i$ are the residuals of stock $i$. We do not make any assumption about the residuals distribution since we are interested only in the magnitude of the estimated coefficients. Furthermore, we expect the $c$ coefficient to play an important role in capturing nonlinearities[4] of betas. However, in order to build our portfolios, we are interested only in the magnitude of $b^*$ and not *c*.

Through linearization and assuming that beta coefficients are non-negative, as usually happens in financial contexts (Andersen et al., 2006), equation (4) can be written as:

$$\ln\left(\bar{\beta}_{i,\left[R^-_{ms,j},R^+_{ms,j}\right]}\right) = \ln(\alpha) + b^* R_{ms,j} + cR^2_{ms,j} + u_i. \quad (5)$$

From the above equation, $b^*$ is used for constructing our basic indices as we explain later. The exponential constant term could be thought of being a constant beta coefficient when the remaining terms are zero. Messis and Zapranis (2016) use these specific coefficients for predicting stock betas and compare their accuracy prediction with other well-known models. The results indicate that the new approach overwhelms the other models in longer samples. If $f(\mathrm{R}_{ms})$ is continuous in the interval $\left[R^-_{ms}, R^+_{ms}\right]$ and differentiable then $1/\bar{\beta}\cdot(\partial\bar{\beta}/\partial R_{ms}) = (b^* + 2cR_{ms})$ or $\partial\bar{\beta}/\partial R_{ms} = \bar{\beta}(b^* + 2cR_{ms})$. For $\bar{\beta} > 0$ and $c > 0$ ($c < 0$) the function is convex (concave) as the second derivative is positive (negative). The concavity (convexity) of the function could indicate the investors' willingness to take less (more) risk at the extremes. From this point of view the *c* coefficient might be another risk characteristic that we left for future research.

### *2. 2. Constructing the Two-Factor Model*

Consider for example that we are able to split the constituents of S&P 500 or any other benchmark, into three groups according to the estimated $b^*$ coefficient of equation (5). The first group contains stocks that present similar characteristics in that they have on average nearly zero $b^*$ coefficients[5]. The other two groups are constituted from stocks with varying betas. In this case, $b^*$ coefficients are positive

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[4] The model of equation (5) is a log-linear model without *c* coefficient and all nonlinear effects could be absorbed through the remaining coefficients giving misleading results.

[5] Our results indicate that the $b^*$ coefficients are nearly zero with the average value for the whole examined period being 0.000012 or 0.00%.

and negative[6]. Then, it is possible to define the market return as $R_{m,t} = \Sigma_j w_{j,t} R_{j,t}$ for $j = 1,2,3$ with $w_{j,t}$ and $R_{j,t}$ to denote the weight and returns of group *j* respectively.

Based on the first group of stocks, we construct the first index, which we call it 'Neutral Stock Index' (NSI). This index targets in measuring the constant systematic risk. Hence, it is free from stocks with different betas in 'up' and 'down' markets since its $b^*$ coefficients are nearly zero on average. In the specific case where the assumption of constant betas coming from the CAPM holds then the NSI resembles to the general index of S&P 500, as we explain later in this section.

The second and third group contains stocks with positive and negative $b^*$ coefficients. We use these groups for the construction of the second index named as 'SMISI' (i.e. Superior minus Inferior Stock Index). This index aims at capturing the risk associated with 'Superior' and 'Inferior' stocks (i.e. asymmetric risk). It represents the difference in returns between the 30% of stocks with the highest $b^*$ (i.e. second group) and the 30% of stocks with the lowest $b^*$ (i.e. third group). The remaining 40% of the stocks goes to the construction of the NSI. We expect that stocks with the highest $b^*$ coefficients would have higher risk-adjusted returns compared to those ones with the lowest $b^*$ coefficients[7]. The intuition behind this stems from the fact that at each state of market return the expected return of security *i* is higher. Consider for example two stocks with the same regular beta (i.e. the exponential constant term of equation (5)) which for our example we assume it 1.5, but with different $b^*$ coefficients. The first stock's $b^*$ coefficient is 0.10 and the second stock's $b^*$ coefficient is -0.10. This means that if the expected next period market return is 5% then the expected payoff of the first stock is 10% (i.e. $2 \times 5\%$) and of the second stock is 5% ($1 \times 5\%$). In a similar manner, if the expected next period market return is -5% then accordingly the first stock's payoff is -5% and the second stock's payoff is -10%. So, we could say that 'Superior' stocks are described by increasing beta coefficient as market return increases. The reverse holds for

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[6] The average values of the $b^*$ coefficients for the 'Superior' and 'Inferior' stocks are 1.03% and -0.96% respectively. The null hypothesis of the zero mean is rejected at all levels of significance.

[7] Our results indicate that portfolios sorted on the basis of the highest $b^*$ coefficients produce higher statistically significant constants (α's) and higher Sharpe and Treynor ratios.

‘Inferior[8]’ stocks. A ‘Superior’ stock as mentioned earlier should include the characteristics that lead to higher returns than its competitors. For example, it could be a stock with relatively low leverage. Hence, in bad states of the world its beta coefficient would not increase as much as a stock with high leverage values (Jagannathan and Wang, 1996). ‘Superior’ stocks should be more attractive to arbitrageurs. If no surprise happens to fundamentals those stocks might continue to behave in a similar manner through the process of intentional or spurious herding the distinction of which has been emphasized by Bikhchandani and Sharma (2000).

After describing what our two indices represent, we now complete the theoretical background by decomposing the CAPM. It is well known that we can set the model’s intercepts to zero allowing us to write:

$$R_{j,t} = \beta_{j,m} R_{m,t} + \tilde{e}_{j,t} \ , \tag{6}$$

where $\beta_{j,m}$ is the beta for group $j$ and $\tilde{e}_{j,t}$ is the residual term for the same group. Following Campbell et al. (2001), we rewrite equation (6) as follows:

$$R_{j,t} = R_{m,t} + e_{j,t} \ . \tag{7}$$

In Equation (7), which is also referred as ‘market –adjusted-return model (Campbell et al., 1997), $e_{j,t}$ denotes the difference between the return of group $j$ (i.e. $R_{j,t}$) and the market return $R_{m,t}$. Combining equation (6) with (7) we have:

$$\begin{aligned} &R_{j,t} = R_{m,t} + e_{j,t} \Rightarrow e_{j,t} = R_{j,t} - R_{m,t} \Rightarrow \\ &e_{j,t} = (\beta_{j,m} R_{m,t} + \tilde{e}_{j,t}) - R_{m,t} \Rightarrow e_{j,t} = \tilde{e}_{j,t} + R_{m,t}(\beta_{j,m} - 1). \end{aligned} \tag{8}$$

From the above equation, $R_{j,t}$ which denotes NSI in this case, is equal to the benchmark index only when $\beta_{j,m} = 1$ or $R_{m,t} = 0$. In the long run, we expect betas to not deviate from equilibrium values and remain constant (Adrian and Franzoni, 2009), indicating that $\beta_{j,m} = 1$. On the contrary, when betas fluctuate around an upward or downward parabolic trend pattern then $e_{j,t} \neq \tilde{e}_{j,t}$. If the CAPM is misspecified and $\tilde{e}_{j,t}$ contain information that it is not captured by the model then $\tilde{e}_{j,t}$ should be priced by any other factor that contains such information. Hence, we expect $\tilde{e}_{j,t}$ to depend on

[8] ‘Inferior’ and ‘Superior’ stocks could be also defined interchangeably as those with ‘Higher Downside Risk’ (HDR) and ‘Higher Upside Risk’ (HUR) respectively.

the SMISI index since it may contain the residuals' information. Chen (1983) uses the CAPM's residuals to compare APT with CAPM in a cross sectional context.

From the above analysis, we are able now to empirically implement the TFM. The model's tested equations in the time series (equation 9) and the cross-sectional context (equation 10) respectively are given as follows:

$$R_{i,t} - R_{f,t} = a_i + c_i SMISI_t + n_i NSI_t + e_{i,t}\,. \qquad (9)$$

$$r_i = \lambda_o + \lambda_{smisi}\hat{c}_i + \lambda_{nsi}\hat{n}_i + z_i \qquad (10)$$

## 3. Empirical Results

### *3.1. Data description*

The constituents of the S&P 500 are used in our dataset. The S&P 500 index is employed as a benchmark index since it is a good representation of the overall U.S. stock market. The rate of return of each security, $R_i$, at time $t$ is calculated as $R_{it} = P_{it} / P_{it-1} - 1$ while it is adjusted for splits and changes in capital structures. From the estimation of returns dividends were omitted as their inclusion would add little to the overall variability (Lo and MacKinley, 1988). Our dataset runs from 1991 for consistency purposes with the other competitive models, whereas the testing period spans from July 2001 to June 2011. This period covers two significant events. The first one is 9/11 which adversely affected the global economy and the financial markets around the world (Choudhry, 2005) while the second one refers to the global financial crisis of 2008. The risk free rate is the 3-month US Treasury bill[9]. In order to construct the variables used in the TFM we first employ daily observations for estimating the $b^*$ coefficients, as mentioned in the previous sections. To include a stock in the 'Superior' or 'Inferior' portfolios for a given year, its $\beta$ should be statistically significant, at least at 10% level, for the whole 5-year period[10]. This way, we ensure that each $\beta$ coefficient coming from the CAPM (equation 1) has explanatory power and that it can be used for estimation purposes.

The monthly return observations of the FF3FM are retrieved from the authors' internet homepage[11]. For the PLM we use the same variables as in JW. The bond yields of BAA and AAA used as the premium in the PLM. Similarly, the per capita

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[9] Using the 30-day T-Bill to construct the excess returns do not change our results

[10] The number of stocks that have been removed according to this filtering approach never surpassed the 5% of the sample. Those stocks do not belong to any of the three portfolios since they don't provide any information about the behaviour (i.e. increasing, decreasing or constant) of systematic risk. Hence, there might be other factor(s) that explain the returns of those stocks.

[11] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

monthly income series was obtained from the Federal Reserve Bulletin published by the Board of Governors of the Federal Reserve System and was used as the labor variable. Following JW, the growth rate in labor income is computed as: $R_t^{labor} = [L_{t-1} + L_{t-2}]/[L_{t-2} + L_{t-3}]$, where $L_{t-1}$ is the per capita labor income at month *t*-*1*, which becomes known at the end of month *t*.

In order to apply the APT, the selected macroeconomic variables that were used as independent variables in the time series regressions are presented in Table 1. The choice of macroeconomic variables has been made arbitrarily, in the sense that they influence the securities in the same degree, implying that all securities operate in the same economic environment and that the particular variables are important to the whole economy. However, some of them are similar to those employed by Clare and Thomas (hereafter CT) (1994). The macroeconomic time-series were obtained from the Federal Reserve Bank of St. Louis. Time-series such as output or inflation are used with one time lag in order to make these variables contemporaneous with series of portfolio returns (Chen et al., 1986; Clare and Thomas, 1994). For example, the announcement of January's inflation is done in February and hence investors revise stock prices accordingly in February.

The models are tested on two different portfolios sorted on the historical beta coefficients and the Book Value per share (BVps). The beta based portfolios are formed following the standard methodology of Fama and MacBeth (1973) (hereafter FMcB). This methodology has been criticized for different reasons (see for example Roll (1977) and Isakov (1999)). However, it is still widely used in most empirical studies (Fraser et al., 2004) for testing models in the cross-sectional framework. According to this method, the first five years of monthly observations (i.e. *t*-120,…, *t*-61) are used to estimate the ordinary CAPM betas for each security. After estimating the stocks' $\beta_i$ coefficients from equation (1), the stocks were ranked on the basis of estimated betas and were assigned to one of the ten portfolios. The first portfolio consists of stocks with the lowest betas, while the last portfolio consists of stocks with the highest betas. This process was repeated for each subsequent year in our data set. Hence, a time-series of monthly returns from July 1996 to June 2011 for each of the ten portfolios was obtained. Next, the beta of each portfolio is estimated over the second period of 5 years (i.e. *t*-60,…, *t*-1) by regressing the realized portfolio returns on the market index in order to reduce the 'errors in variables' problem (Clare and

Thomas, 1994). The BVps sorted portfolios are formed every calendar year, starting in 2001. The BVps data were obtained from Compustat.

**Table 1**: Macroeconomic variables

| Variable | Symbol | Form | Series ID |
|---|---|---|---|
| Default risk | (BAA-LTGB) | FD | |
| Term structure | (LTGB-TB3M) | FD | |
| 3 month Treasury bill rate | (TB3M) | FD | TB3MS |
| Gold price | (GP) | FD | GOLDPMGBD228NLBM |
| Real retail sales | (RRS) | FDL | RRSFS |
| Industrial production | (IP) | FDL | INDPRO |
| Oil price | (OIL) | FD | MCOILBRENTEU |
| Unemployment | (UNEM) | FDL | UNRATENSA |
| M3 | (M3) | L | MABMM301USM657S |
| Exchange rate | (EXR) | FDL | EXUSUK |
| Consumer price index | (CPI) | FDL | CPIAUCNS |
| Exports/Imports | (EXPIMP) | FD [L(Exp/Imp)] | BOPGEXP (BOPGIMP) |
| Yield on Long-term GB | (LTGB) | FD | 10YCMR |
| Excess market return | (MR) | L | |

Notes: The sample period is from July 1996 to June 2011. L, FD and FDL are for level, first differences and first differences of the log respectively for the selected time series. The series ID concern the identification code given by the Federal Reserve Bank of St. Louis.

Following Fraser et al. (2004), we repeat this procedure by updating the beta estimates on a monthly basis. Thus, time-series of risk premiums of the models are generated. The test of significance of the risk premia is performed as in FMcB and CT as follows:

$$t_{\lambda} = \frac{\hat{\lambda}}{s\left(\hat{\lambda}\right)/\sqrt{n}} \tag{11}$$

In the above equation, $\hat{\lambda}$ is the mean value of the estimated risk premium, $s\left(\hat{\lambda}\right)$ is the standard deviation and $n$ is the number of observations. The variables are priced over the estimation period at the 10% level as in CT.

The relatively low number of available stocks at the very early stage of the sample could cause survivorship bias problems. To examine possible effects related to survivorship bias, we also form small and big sample portfolios. The small sample portfolios contain those stocks that were used in the construction of the TFM at the very beginning of the sample. Note that eight years of daily observations are used for obtaining betas.  Hence, the relatively low number of used stocks in the small sample up to 2006 is mainly due to this limitation. The dataset increases significantly from early 2000. Figure 1 depicts the number of shares contained in the two samples. The big sample portfolios (medium grey) contain stocks with at least 60 monthly return observations that were used to estimate $\beta$'s and build beta based portfolios. Those stocks were also employed to form portfolios sorted on BVps.

**Figure 1**: Number of shares in analysis


Tables 2 and 3 provide summary statistics of average portfolio returns for the two samples. For each portfolio, the tables show the mean monthly returns in excess of the 3-month Treasury bill, the standard deviation of the monthly excess returns and the *t*-statistics associated with the hypothesis of zero portfolio returns. Both tables exhibit the positive differences in returns between the lowest and highest BVps portfolios and highest and lowest beta sorted portfolios. The highest (lowest) beta sorted portfolio has an average excess return of 1.30% (0.56%) per month, with a corresponding difference in betas of 0.56. The evidence demonstrates that there is a reward for holding higher-beta stocks but as ACX who find similar results, report that this evidence per se does not validate CAPM, because the model predicts that only the $\beta$ coefficient should explain a firm's expected return. The pattern of portfolio returns between the big and the small samples is similar. A deviation is observed between the 9$^{th}$ and 10$^{th}$ decile of small sample beta sorted portfolios.

**Table 2:** Summary statistics for Simple Monthly Excess Returns (in Percent) for the portfolios formed using Book value per share and beta coefficients: 07/01-06/11, 120 Months, Big sample (429 shares on average per year).

| | Deciles | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **BV per share** | **Low** | | | | | | | | | **High** |
| Mean | 1.91 | 1.42 | 0.99 | 0.95 | 1.06 | 0.97 | 0.92 | 0.63 | 0.79 | 0.34 |
| Std. Dev. | 6.20 | 5.37 | 5.03 | 5.44 | 5.86 | 4.87 | 5.41 | 5.56 | 5.31 | 6.25 |
| *t*-statistics | 3.38 | 2.89 | 2.15 | 1.92 | 1.98 | 2.18 | 1.87 | 1.24 | 1.64 | 0.60 |
| **Beta** | **Low** | | | | | | | | | **High** |
| Mean | 0.56 | 0.81 | 0.60 | 0.98 | 0.85 | 0.97 | 0.97 | 1.08 | 1.25 | 1.30 |
| Std. Dev. | 3.30 | 3.83 | 4.04 | 4.32 | 5.00 | 5.20 | 5.99 | 6.50 | 7.11 | 9.85 |
| *t*-statistics | 1.85 | 2.31 | 1.63 | 2.48 | 1.87 | 2.05 | 1.78 | 1.81 | 1.92 | 1.44 |

**Table 3:** Summary statistics for Simple Monthly Excess Returns (in Percent) for the portfolios formed using Book value per share and beta coefficients: 07/01-06/11, 120 Months, Small sample (257 shares on average per year).

| | Deciles | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **BV per share** | **Low** | | | | | | | | | **High** |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Mean | 1.26 | 0.95 | 0.77 | 0.72 | 0.68 | 0.91 | 0.59 | 0.58 | 0.83 | 0.16 |
| Std. Dev. | 5.79 | 5.16 | 4.91 | 5.60 | 5.64 | 4.79 | 6.27 | 5.34 | 4.99 | 6.53 |
| *t*-statistics | 2.39 | 2.02 | 1.71 | 1.40 | 1.32 | 2.08 | 1.03 | 1.19 | 1.83 | 0.27 |
| **Beta** | **Low** | | | | | | | | | **High** |
| Mean | 0.42 | 0.61 | 0.49 | 1.01 | 0.66 | 0.83 | 0.79 | 0.77 | 1.13 | 0.73 |
| Std. Dev. | 3.58 | 3.72 | 4.04 | 4.30 | 4.78 | 5.46 | 6.40 | 6.84 | 7.41 | 9.76 |
| *t*-statistics | 1.27 | 1.80 | 1.34 | 2.57 | 1.50 | 1.66 | 1.36 | 1.24 | 1.67 | 0.82 |

The estimated average betas produced by CAPM are depicted in Table 4. Our results indicate that there are not significant differences in estimated betas within portfolios formed on BVps. For example, even though the difference in returns between the 1st and 10th decile is 1.57% per month, the corresponding difference in betas is only 0.23. On the contrary, the estimated $\beta$ coefficients of portfolios sorted on betas seem to vary significantly from a low of 0.47 to a high of 1.65 indicating that higher return is associated with higher risk. Furthermore, we have to note that the slopes at both samples follow identical pattern.

**Table 4:** The estimated average slopes for the portfolios formed using Book value per share and beta coefficients. Both samples are included.

| | Deciles | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| BVps Big | 1.21 | 1.00 | 0.96 | 0.99 | 1.04 | 0.90 | 0.95 | 0.94 | 0.85 | 0.98 |
| Beta Big | 0.47 | 0.64 | 0.73 | 0.79 | 0.89 | 0.91 | 1.02 | 1.15 | 1.26 | 1.65 |
| BVps Small | 1.17 | 0.94 | 0.87 | 1.05 | 0.97 | 0.91 | 1.18 | 0.91 | 0.79 | 1.07 |
| Beta small | 0.48 | 0.57 | 0.68 | 0.69 | 0.79 | 0.95 | 1.17 | 1.30 | 1.40 | 1.82 |

Next, we examine whether a survivorship bias exist. In table 5 we present the results. Following Banz and Breen (1986) we examine whether the returns for each portfolio are different over the 120 months period. For brevity reasons, we report only the results of the Gibbons, Ross, Shanken (1989) (GRS) test of the zero $\alpha$'s hypothesis[12]. The table depicts that jointly $\alpha$'s are different from zero and statistically significant differences in returns between the big and the small sample exist. However, a more closely examination of portfolios indicates that only in three cases $\alpha$'s are different from zero for the BVps portfolios. Similarly for the beta portfolios, $\alpha$'s are different from zero in only one case. At this point, we should also mention that statistical results in tests of asset pricing are sensitive to the weight choice. Plyaka et. al. (2014) show the existence of substantial difference in the performance of equal and value-weighted portfolios. However, a large number of papers on empirical asset

[12] The remaining regression results are available from the authors upon request.

pricing uses equal-weighted mean returns. For robustness purposes, we report the results of equal and value-weighted portfolio returns in the Appendix.

**Table 5:** GRS test for testing the restriction that all ten alphas are jointly zero (constants in percent, std. errors in parentheses)

| | Deciles | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | GRS test |
| BV per share | 0.65 | 0.47 | 0.22 | 0.23 | 0.38 | 0.06 | 0.33 | 0.05 | -0.04 | 0.18 | 3.37 |
| | (0.23) | (0.15) | (0.15) | (0.16) | (0.16) | (0.14) | (0.19) | (0.11) | (0.14) | (0.11) | |
| Beta | 0.14 | 0.20 | 0.11 | -0.03 | 0.20 | 0.14 | 0.18 | 0.30 | 0.12 | 0.57 | 2.71 |
| | (0.13) | (0.16) | (0.16) | (0.13) | (0.15) | (0.12) | (0.17) | (0.19) | (0.18) | (0.14) | |

*3.2. Time series regressions of the two Indices on S&P 500*

In section 2.2, we argued that if the NSI resembles to the benchmark index then we expect betas to not deviate from equilibrium values and remain constant. This assumption implies that $\beta_{j,m} = 1$ in equation (6). To this effect we run time series regressions of NSI and SMISI on the S&P 500 for different subsamples. Both series are found to be stationary at all significant levels. In Table 6 we present the results. The findings demonstrate that the $\beta$ coefficient when the NSI is used as dependent variable is unity as expected when the total sample is employed while it remains close to it for the two subsamples. An interesting finding is the positive and statistically significant $\alpha$'s for all examined periods. This fact indicates that we are able to construct an index with specific characteristics that tracks the benchmark but has positive idiosyncratic risk. In other words, the NSI gives a positive Jensen's alpha reaching as high as 0.009 on a monthly basis. Multiplying this quantity by 12 we take an annualized alpha equal to 10.8%. This magnitude is more than double relative to the volatility-managed portfolios of Moreira and Muir (2017) even though our historical sample is not of equal length. Their portfolios produce an alpha of 4.9 % annually without taking into account transaction costs whereas they outperformed other competitive models[13]. Furthermore, our alpha is well above the average alpha of 400 US equity mutual funds whose value is negative (-4.65% p.a.). Frijns et. al. (2013) report that the majority of those mutual funds are tracking the market[14] with an

[13] Volatility-managed portfolios found to work better relative to the model of downside risk of Ang et al. (2006), the model of disaster risk of Lettau et al. (2014) and other microfinance models such as the habits model of Campbell and Cochrane (1999), the long-run risk model of Bansal et. al (2012), the time varying rare disasters model of Wachter (2013) and the intermediary-based model of He and Krishnamurthy (2013).

[14] Funds use different tracking error minimization techniques to weighting stocks and build portfolios that replicate an index targeting at the same time to higher expected returns. In addition, the reaction of funds may vary through time, depending on whether the market is bearish or bullish (Jawadi and Knanniche, 2012; Frijns et. al., 2013).

average $\beta$ of 0.969 which is close enough to that of NSI (1.007) when the whole period is considered. As far as the regression results of the SMISI index are concerned, we observe that this index works as an 'insurance' portfolio. Hence, the negative and statistically significant slopes found in two out of three cases protect investors in 'bad' states of the world when the marginal utility of wealth is high (Daniel et. al, 2002). Moreover, the portfolio's average monthly return in 'bad' ('good) states of the world is 1.10% (0.01%) when the corresponding S&P 500 average monthly return is -4.01% (2.99%) during the examined period.

**Table 6**: Regression results of indexes on S&P 500 for different periods

| | NSI | | | SMISI | | |
|---|---|---|---|---|---|---|
| Period | a | β | Rsq | a | β | Rsq |
| 7/2001-6/2006 | 0.008 (3.09)*** | 0.941 (14.3)*** | 0.78 | 0.010 (2.14)** | -0.101 (-0.83) | 0.01 |
| 7/2006-6/2011 | 0.009 (5.03)*** | 1.045 (28.8)*** | 0.93 | -0.000 (-0.16) | -0.341 (-3.73)*** | 0.19 |
| 7/2001-6/2011 | 0.008 (5.48)*** | 1.007 (29.1)*** | 0.87 | 0.004 (1.36) | -0.255 (-3.45)*** | 0.09 |

**,*** indicate statistically significant coefficient at the 5% and the 1% level respectively.

To test whether the SMISI index contains the information of residuals $\tilde{e}_{j,t}$, coming from the time series regression of the NSI on the S&P 500, we regress the residuals of the CAPM and market-adjusted model on the SMISI index. If the model is misspecified and $\beta_j$ do not capture all the information, then the respective residuals $\tilde{e}_{j,t}$ will no longer behave like white noise. Table 7 shows the results. We observe that the information is captured by SMISI when the two subsamples are employed. This fact indicates that when betas deviate from equilibrium values, the SMISI factor is able to capture any disequilibrium. However, when employing the total sample no longer the coefficient is different from zero. In addition, the same magnitude of the $\alpha$ coefficient is observed both in this regression and the previous one of table (6) when the NSI is used as dependent variable.

**Table 7**: Regression of residuals of the CAPM and market-adjusted-model on SMISI index

| Period | a | β | Rsq | a | β | Rsq |
|---|---|---|---|---|---|---|
| 7/2001-6/2006 | -0.003 (-1.42) | 0.255 (4.08)*** | 0.22 | 0.005 (2.26)** | 0.254 (4.08)*** | 0.22 |
| 7/2006-6/2011 | 0.000 (0.37) | -0.113 (-2.54)*** | 0.10 | 0.009 (5.21)*** | -0.116 (-2.59)** | 0.10 |
| 7/2001-6/2011 | -0.000 (-0.14) | 0.048 (1.18) | 0.02 | 0.008 (5.32)*** | 0.046 (1.14) | 0.01 |

**,*** indicate statistically significant coefficient at the 5% and the 1% level respectively.

### *3.3. Unconditional and Conditional cross-section regressions*

In this section we try to identify risk premiums associated with factors other than the market risk. Panel A of Table 8 depicts the evidence of the unconditional cross-sectional regressions from July 2001 to June 2011. The methodology used for estimating cross-sectional regressions is that of FMcB since unconditional models can be consistently estimated by this method (Lettau and Ludvigson, 2001). As we can see, the coefficients $\lambda_0$ are not statistically different from zero for the BVps sorted portfolios. This is consistent with the Sharpe-Lintner hypothesis (SLH).

In the case of the CAPM the $R^2$ of the regression is only 0.7%. On the other hand, $R^2$ increases to 70% and 91% for the TFM and FF3FM respectively. Furthermore, these two models present *p*-values of the F-test (*pv*-F) lower than 5% significant level. The SMISI factor of the TFM is priced and the market risk premium has the expected positive and statistically significant sign, if we move to higher significant levels. In the case of the FF3FM only the SMB factor is priced while the market risk premium is not statistically different from zero. For the PLM we observe that the $R^2$ is relatively low while neither the labor nor the premium factors influence the returns. The results indicate that the proposed model outperforms the CAPM and the PLM both in terms of $R^2$ and F-test values.

In the next step we examine the results of the four models on the portfolios formed on $\beta$ coefficients. A closer inspection of Panel A of Table 8 reveals that in this case substantial differences can be observed in the results. The $R^2$ for every model has increased. It reaches as high as 84.9% for the PLM with the remaining models to follow closely. The *p*-values of the F-test are zero for all models. The intercepts of CAPM and TFM appear to be significant violating the SLH. Finally even though the FF3FM has high $R^2$ none of its factors are priced.

Panel B of Table 8 depicts the results from July 2006 to August 2011 period. The $R^2$ of the TFM remains high while the FF3FM loses power relatively to its previously observed $R^2$ value when the BVps portfolios are employed. In the case of CAPM, it continues to appear the lowest $R^2$ value while it also leaves unexplained returns, consistent with the results of JW. The PLM performs better only in terms of $R^2$ since the *p*-value is higher than all levels of significance. In addition, the model's intercept is statistically different from zero as the CAPM does The same tests[15] have been also carried out using the small sample. The findings differ significantly with regard to $R^2$

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[15] The results are available from the authors upon request.

values which appear to be lower. In relation to beta based portfolios, all models increase the $R^2$ values. However, the FF3FM leaves unexplained returns.

**Table 8**: Unconditional Cross-sectional regressions of CAPM, FF3FM, TFM and PLM.

| Panel A: 2001-2011 (BS) | $\lambda_0$ | $\lambda_{mar}$ | $\lambda_{SMISI}$ | $\lambda_{NSI}$ | $\lambda_{SMB}$ | $\lambda_{HML}$ | $\lambda_{labor}$ | $\lambda_{prem}$ | $R^2$ /(*p*v-F) |
|---|---|---|---|---|---|---|---|---|---|
| BV per share | 0.014 | -0.004 | | | | | | | 0.007 |
| | (0.75) | (-0.24) | | | | | | | (0.81) |
| | -0.009 | | 0.042 | 0.018 | | | | | 0.701 |
| | (-0.75) | | (3.74)* | (1.54) | | | | | (0.01) |
| | 0.009 | -0.008 | | | 0.023 | -0.002 | | | 0.914 |
| | (0.78) | (-0.57) | | | (2.53)* | (-0.30) | | | (0.00) |
| | 0.009 | 0.004 | | | | | -0.006 | 1.305 | 0.244 |
| | (0.03) | (0.11) | | | | | (-0.99) | (0.93) | (0.61) |
| Beta portfolios | 0.003 | 0.005 | | | | | | | 0.825 |
| | (2.98)* | (6.15)* | | | | | | | (0.00) |
| | 0.004 | | -0.002 | 0.005 | | | | | 0.828 |
| | (2.10)** | | (-0.34) | (3.11)* | | | | | (0.00) |
| | 0.002 | 0.008 | | | -0.002 | 0.001 | | | 0.833 |
| | (0.95) | (1.80) | | | (-0.36) | (0.48) | | | (0.00) |
| | 0.006 | 0.006 | | | | | 0.001 | 0.201 | 0.849 |
| | (1.36) | (2.37)** | | | | | (0.91) | (0.85) | (0.00) |
| Panel B: 2006-2011 (BS) | $\lambda_o$ | $\lambda_{mar}$ | $\lambda_{SMISI}$ | $\lambda_{NSI}$ | $\lambda_{SMB}$ | $\lambda_{HML}$ | $\lambda_{labor}$ | $\lambda_{prem}$ | $R^2$ /(*p*v-F) |
| BV per share | 0.030 | -0.019 | | | | | | | 0.456 |
| | (3.52)* | (-2.59)* | | | | | | | (0.03) |
| | 0.014 | | 0.022 | -0.006 | | | | | 0.762 |
| | (1.87) | | (4.45)* | (-0.78) | | | | | (0.00) |
| | 0.003 | 0.003 | | | 0.003 | -0.012 | | | 0.879 |
| | (0.47) | (0.45) | | | (0.73) | (-2.83)* | | | (0.00) |
| | 0.028 | -0.013 | | | | | 0.000 | 0.355 | 0.496 |
| | (2.32)** | (-0.96) | | | | | (0.64) | (0.21) | (0.21) |
| Beta portfolios | -0.002 | 0.009 | | | | | | | 0.836 |
| | (-1.58) | (6.39)* | | | | | | | (0.00) |
| | -0.002 | | -0.002 | 0.009 | | | | | 0.840 |
| | (-0.86) | | (-0.51) | (3.92)* | | | | | (0.00) |
| | 0.009 | -0.005 | | | 0.012 | -0.009 | | | 0.954 |
| | (2.44)* | (-1.05) | | | (3.64)* | (-2.72)* | | | (0.00) |
| | 0.005 | 0.008 | | | | | 0.001 | 0.389 | 0.904 |
| | (1.19) | (2.78)* | | | | | (0.95) | (2.05)* | (0.00) |

*,** depict significance at the 5% and the 10% level respectively.

Next, in Table 9 we present the unconditional cross-sectional regressions for the APT. Earlier in this paper, we argued that the NSI targets in measuring the constant systematic risk. We claimed that this index resembles to the general index of S&P 500 if the assumption of constant betas coming from the CAPM holds. Hence, for testing the APT we used apart from the S&P 500, the NSI in order to capture the market risk premium. Following Groenewold and Fraser (GF) (1997), in the first stage we estimate the factor sensitivities for each of the 20 portfolios (i.e. 10 portfolios sorted on $\beta$'s betas and another 10 sorted on BVps) and for each of the 13 factors using OLS. In this stage, we retain only those factors that are priced at the 5% level ensuring that the number of independent variables is lower than the number of dependent variables. By following this procedure different variables of Table 1 are excluded from the final step each time. However, gold price, unemployment, term structure and real retail sales were excluded from the final step most of the times. It is worth mentioning here that the excess market return is also included in the model, since the initial results without market return have shown very low performance of the APT. More precisely, the $R^2$ was always less than 30% when only macroeconomic variables were included.

Panel A1 and A2 of Table 9 demonstrate the results of the unconditional cross-sectional regressions from July 2001 to August 2011 when the S&P 500 and the NSI are used as benchmark respectively. In the tested portfolios insignificant risk factors (low *t*-ratios) are eliminated one at a time (Groenewold and Fraser, 1997). The $R^2$ values of the model are quite high for both portfolios and for both indexes while the *p*-values of the F-test are zero. However, in the case of the beta based portfolios, the APT model leaves unexplained returns when the S&P is used as benchmark with the constant being significant at all usually levels. Similar results are observed in our second tested period. This time unexplained returns are observed in the case of BV per share portfolios when the S&P 500 is employed. From this point of view, we could say that the APT better explains the portfolio returns when the NSI is used for capturing market risk premium. The results of Table 9 indicate that market risk appears the expected positive sign and it is priced most of the times followed by the M3 (i.e. money supply) factor. The exchange rate (EXR) appears to be significant in three cases followed by industrial production (IP) and default risk (DR) with two cases. Finally, the variables of 3 month treasury bill rate (TB3M), exports/imports (EXPIMP), yield on Long-term GB (LTGB) and consumer price index (CPI) are statistically significant one time.

**Table 9**: Unconditional Cross-sectional regressions of APT

| Panel A1: 2001-2011 (BS)-SP500 | $\lambda_0$ | $\lambda_{MR}$ | $\lambda_{M3}$ | $\lambda_{TB3M}$ | | | | $R^2$ /(*pv*-F) |
|---|---|---|---|---|---|---|---|---|
| BV per share | -0.010 | 0.015 | 0.445 | | | | | 0.835 |
| | (-1.13) | (1.93)** | (5.95)* | | | | | (0.00) |
| Beta portfolios | 0.006 | 0.005 | -0.077 | 0.190 | | | | 0.961 |
| | (5.31)* | (4.99)* | (-2.47)* | (3.89)* | | | | (0.00) |
| **Panel A2: 2001-2011 (BS)-NSI** | $\lambda_0$ | $\lambda_{NSI}$ | $\lambda_{M3}$ | $\lambda_{TB3M}$ | $\lambda_{DR}$ | $\lambda_{EXPIMP}$ | | $R^2$ /(*pv*-F) |
| BV per share | -0.007 | 0.017 | 0.416 | | | | | 0.851 |
| | (-0.79) | (1.99)** | (6.12)* | | | | | (0.00) |
| Beta portfolios | 0.002 | 0.004 | -0.160 | | -0.149 | 0.023 | | 0.941 |
| | (1.56) | (2.08)** | (-2.23)** | | (-2.41)** | (2.59)* | | (0.00) |
| **Panel B1: 2006-2011 (BS)-SP500** | $\lambda_0$ | $\lambda_{MR}$ | $\lambda_{M3}$ | $\lambda_{DR}$ | $\lambda_{IP}$ | $\lambda_{LTGB}$ | $\lambda_{EXR}$ | $R^2$ /(*pv*-F) |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| BV per share | 0.006 | | 0.260 | 0.123 | -0.004 | -0.161 | | 0.968 |
| | (9.39)* | | (10.7)* | (3.22)* | (-2.79)* | (-3.52)* | | (0.00) |
| Beta portfolios | 0.000 | 0.006 | | | | | -0.007 | 0.891 |
| | (0.19) | (2.78)* | | | | | (-1.90)** | (0.00) |
| Panel B2: 2006-2011 (BS)-NSI | $\lambda_0$ | $\lambda_{NSI}$ | $\lambda_{CPI}$ | $\lambda_{IP}$ | $\lambda_{EXR}$ | | | $R^2$ /(*pv*-F) |
| BV per share | -0.011 | 0.019 | 0.010 | 0.012 | 0.024 | | | 0.917 |
| | (-1.29) | (2.31)** | (5.17)* | (3.39)* | (3.42)* | | | (0.00) |
| Beta portfolios | 0.001 | 0.007 | | | -0.007 | | | 0.897 |
| | (0.63) | (4.21)* | | | (-2.01)** | | | (0.00) |

*,** depict significance at the 5% and the 10% level respectively.

The results of the conditional cross-sectional regressions are presented in Table 10. The average values of risk premia can be found in the first column while the second one shows their associated *t*-test (equation 11). In the third column, a test of risk premia normality is presented while in the final column the average GRS test coming from the time series first step regression is shown. First, we observe that in the case of BVps portfolios the variables of the TFM are priced though a proportion of portfolio returns left unexplained. The results are similar in the case of the CAPM. Moving to the FF3FM the market risk and the HML factor appear to be significant while the constant and the SMB are not statistically different from zero.

As for the PLM our results indicate that no factor is priced. We have to mention here that the *t*-statistics should be cared with caution. That is due to the fact that there are cases in which the distribution of the estimated risk premia are clearly not normal, a result consistent with that of CT when macro-economic variables were used.

In relation to the APT, we proceeded to model's estimation several times dropping those variables with insignificant risk premia in an attempt to identify a simplified version of the model. The evidence indicates that the market is still priced while two of the factors, the log of exports to imports (EXPIMP) and industrial production (IP) variables that previously priced in the unconditional setting found to be significantly different from zero at the 10% level (i.e. $|t|>1.30$ since this is a one-sided test). The positive signs of the coefficients for the log of exports to imports and industrial production seem to be correct, except that of the market index. Regarding the beta based portfolios almost no risk premia are statistically different from zero apart from the case of the PLM. In the APT if the market return is added as an additional risk factor, then the term structure factor becomes significant. However, we chose to not include the market return in the table since it is not significant at any level even though the constant term diminishes in magnitude.

The GRS test in the last column of Table 10 depict that TFM clearly outperforms CAPM and FF3FM models in the first step time series regressions. Similar results are also found in Messis and Zapranis (2014) when momentum portfolios are also taken into consideration in the context of time series regressions. We should note here that the GRS test is not available for the PLM and the APT. This happens because in the case of the PLM the regressions have been conducted separately for each one of the variables while in the case of the APT model different number of factors have been found to be significant for each examined portfolio. However, it is worth mentioning that when averaging the estimated constant terms coming from the time series regressions across both kinds of portfolios we find significant differences between the APT and the TFM. In the case of beta sorted portfolios the average unexplained returns of the APT reach as high as 0.76% per month, significantly higher than the 0.16% per month of the TFM. Accordingly, in the case of BVps portfolios the average unexplained returns are 0.65% and 0.31% for the APT and the TFM respectively.

**Table 10**: Estimated risk premia in conditional cross-section regression.

| Panel A: 2001-2011 (BS) | | | $\lambda_k$ | t | N[1] | GRS test |
|---|---|---|---|---|---|---|
| BVps | CAPM | $\lambda_0$ | -0.011 | -2.06* | 8.09 (4)* | |
| | | $\lambda_{mar}$ | 0.024 | 4.18* | 2.71 (4)* | 16.4 |
| | TFM | $\lambda_0$ | -0.018 | -2.77* | 6.12 (4)* | |
| | | $\lambda_{SMISI}$ | 0.019 | 2.56* | 1.56 (3)* | |
| | | $\lambda_{NSI}$ | 0.032 | 3.97* | 5.65 (3)* | 2.97* |
| | FF3FM | $\lambda_0$ | -0.004 | -0.53 | 11.9 (2) | |
| | | $\lambda_{mar}$ | 0.017 | 1.61* | 23.9 (2) | |
| | | $\lambda_{SMB}$ | 0.007 | 1.13 | 7.01 (3)* | |
| | | $\lambda_{HML}$ | -0.012 | -2.07* | 5.11 (4)* | 10.36 |
| | PLM | $\lambda_0$ | 0.004 | 1.03 | 11.7 (2) | |
| | | $\lambda_{mar}$ | -0.001 | -0.11 | 12.4 (3) | |
| | | $\lambda_{prem}$ | 0.000 | -0.11 | 21.0 (2) | |
| | | $\lambda_{labor}$ | 0.012 | 0.08 | 5.83 (2)* | N/A |
| | APT | $\lambda_0$ | 0.017 | 3.30* | 4.71 (4)* | |
| | | $\lambda_{mar}$ | -0.007 | -2.76* | 3.86 (3)* | |
| | | $\lambda_{EXPIMP}$ | 0.011 | 2.39* | 12.7 (4) | |
| | | $\lambda_{IP}$ | 0.002 | 1.36* | 3.25 (2)* | N/A |
| Panel B: 2001-2011 (BS) | | | $\lambda_k$ | t | N[1] | GRS test |
| Beta port. | CAPM | $\lambda_0$ | 0.003 | 0.88 | 7.56 (3)* | |

| | | | | | |
|---|---|---|---|---|---|
| | $\lambda_{mar}$ | 0.006 | 1.03 | 9.61 (4) | 11.9 |
| TFM | $\lambda_0$ | 0.004 | 0.80 | 18.2 (4) | |
| | $\lambda_{SMISI}$ | 0.001 | 0.08 | 2.67 (4)* | |
| | $\lambda_{NSI}$ | 0.005 | 0.62 | 20.1 (3) | 1.39* |
| FF3FM | $\lambda_0$ | 0.006 | 1.64* | 3.87 (2)* | |
| | $\lambda_{mar}$ | 0.002 | 0.47 | 8.03 (3) | |
| | $\lambda_{SMB}$ | 0.001 | 0.12 | 19.2 (2) | |
| | $\lambda_{HML}$ | -0.003 | -0.60 | 9.42 (3) | 5.52 |
| PLM | $\lambda_0$ | -0.008 | -1.46* | 13.9 (4) | |
| | $\lambda_{mar}$ | 0.025 | 3.49* | 3.55 (3)* | |
| | $\lambda_{prem}$ | 0.002 | 1.35* | 33.4 (2) | |
| | $\lambda_{labor}$ | 0.041 | 0.27 | 3.39 (3)* | N/A |
| APT | $\lambda_0$ | 0.010 | 1.92* | 5.18 (2)* | |
| | $\lambda_{TS}$ | 0.062 | 0.64 | 13.8 (3) | N/A |

* depicts significance at 10% level.
1. The $\chi^2$ test for normality with 2, 3 and 4 degrees of freedom (in parentheses the d.f. used) at the 5% level is 5.991, 7.815 and 9.488 respectively.

### *3.4. Portfolio and models' performance in extreme market conditions*

The results from our previous section indicate that portfolios formed with different criteria gain higher returns. Next, we examine if they are fundamentally riskier. According to Lakonishok et al. (1994) a portfolio would be fundamentally riskier if, first, underperforms the competitive one in some states of the world and second the underperformance would coincide with 'bad' states, in which the marginal utility of wealth is high, making the portfolio unattractive to risk-averse investors. In addition, Chan and Lakonishok (1993) state that downside risk is a major concern of money managers. Due to the fact that beta represents a stock's return sensitivity to market ups and downs, it is expected to be a good measure of downside risk. For this point of view, low beta portfolios should face lower downside risk than high beta portfolios. The opposite should happen when market rises.

Tables 11 and 12 present the results of the ten largest down and up-market months of both portfolios. We are able to distinguish between the two examined portfolios some very interesting characteristics. Firstly, in down markets, the lowest decile BVps portfolio appears to have lower returns with respect to the highest one. However, this fact could be explained in the case of beta based portfolios due to the lower beta coefficient, as presented previously in table 4. Instead, BVps portfolios do not exhibit such differences in the estimated betas that could explain those return divergences. Thus, there might be some other reason associated with this better performance. In up markets, the lowest BVps portfolio does not differentiate from the highest one though this is the case between the two extreme beta based portfolios.

**Table 11:** Ten largest down market months: Simple Monthly Excess market Return (in Percent) and returns on portfolios formed using Book value per share and beta coefficients.

| | | | Deciles | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Month | Market | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **BV per share** | | | **Low** | | | | | | | | | **High** |
| 1 | 10/08 | -17.0 | -20.8 | -18.4 | -16.0 | -18.0 | -21.3 | -15.6 | -20.6 | -23.1 | -22.3 | -25.7 |
| 2 | 9/02 | -11.1 | -5.5 | -6.1 | -6.9 | -7.7 | -8.8 | -10.6 | -8.7 | -11.7 | -10.1 | -11.0 |
| 3 | 2/09 | -11.0 | -6.0 | -5.6 | -7.3 | -13.0 | -12.7 | -10.6 | -8.1 | -12.4 | -14.1 | -20.2 |
| 4 | 9/08 | -9.2 | -8.8 | -11.5 | -10.5 | -10.6 | -10.1 | -9.6 | -11.4 | -10.2 | -11.0 | -5.6 |
| 5 | 6/08 | -8.8 | -9.2 | -6.9 | -8.4 | -6.1 | -9.9 | -9.6 | -10.1 | -8.8 | -6.5 | -8.7 |
| 6 | 1/09 | -8.6 | -5.1 | -3.9 | -2.9 | -5.9 | -8.0 | -9.2 | -8.8 | -6.5 | -8.7 | -15.0 |
| 7 | 9/01 | -8.4 | -14.6 | -10.3 | -13.4 | -12.9 | -14.9 | -11.8 | -13.9 | -12.5 | -8 | -9.2 |
| 8 | 5/10 | -8.2 | -8.0 | -5.3 | -6.3 | -6.6 | -7.8 | -7.3 | -7.8 | -8.1 | -7.8 | -8.3 |
| 9 | 7/02 | -8.0 | -7.6 | -6.0 | -7.7 | -9.2 | -11.4 | -8.3 | -13.6 | -10.4 | -12.7 | -9.9 |
| 10 | 11/08 | -7.5 | -11.5 | -7.8 | -11.7 | -11.8 | -9.7 | -4.1 | -7.0 | -10.0 | -8.7 | -12.0 |
| Average | | -9.8 | -9.7 | -8.2 | -9.1 | -10.2 | -11.5 | -9.7 | -11.0 | -11.4 | -11.2 | -13.1 |
| **Beta based** | | | **Low** | | | | | | | | | **High** |
| 1 | 10/08 | -17.0 | -14.1 | -13.4 | -17.7 | -16.6 | -18.5 | -19.1 | -22.3 | -25.0 | -27.0 | -25.8 |
| 2 | 9/02 | -11.1 | -5.8 | -4.0 | -6.3 | -8.5 | -8.6 | -7.3 | -8.4 | -10.8 | -10.3 | -14.7 |
| 3 | 2/09 | -11.0 | -10.9 | -11.8 | -10.3 | -9.3 | -12.2 | -13.7 | -14.1 | -8.9 | -9.6 | -10.8 |
| 4 | 9/08 | -9.2 | -6.7 | -6.2 | -6.0 | -6.6 | -9.2 | -6.9 | -8.1 | -13.1 | -15.6 | -18.0 |
| 5 | 6/08 | -8.8 | -7.8 | -7.8 | -9.1 | -6.9 | -11.3 | -7.7 | -10.3 | -8.5 | -11.4 | -10.3 |
| 6 | 1/09 | -8.6 | -2.8 | -4.6 | -4.1 | -9.6 | -9.4 | -9.0 | -13.5 | -10.0 | -6.0 | -4.1 |
| 7 | 9/01 | -8.4 | -5.0 | -8.6 | -7.7 | -9.7 | -8.2 | -10.5 | -8.1 | -14.4 | -17.8 | -25.8 |
| 8 | 5/10 | -8.2 | -5.0 | -5.4 | -6.8 | -6.9 | -6.9 | -7.4 | -9.0 | -9.4 | -6.9 | -9.9 |
| 9 | 7/02 | -8.0 | -6.8 | -11.2 | -6.3 | -5.6 | -10.2 | -9.5 | -11.1 | -14.3 | -8.7 | -12.8 |
| 10 | 11/08 | -7.5 | -4.0 | -5.1 | -6.6 | -4.8 | -7.8 | -12.9 | -11.2 | -12.2 | -10.6 | -17.0 |
| Average | | -9.8 | -6.9 | -7.8 | -8.1 | -8.4 | -10.2 | -10.4 | -11.6 | -12.7 | -12.4 | -14.9 |

**Table 12:** Ten largest up market months: Simple Monthly Excess market Return (in Percent) and returns on portfolios formed using Book value per share and beta coefficients.

| | | | Deciles | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Month | Market | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **BV per share** | | | **Low** | | | | | | | | | **High** |
| 1 | 4/09 | 9.4 | 19.4 | 14.5 | 15.4 | 18.4 | 18.9 | 15.5 | 17.1 | 20.3 | 18.1 | 23.3 |
| 2 | 9/10 | 8.7 | 11.7 | 13.3 | 10.2 | 10.7 | 11.9 | 9.4 | 9.3 | 9.6 | 9.6 | 7.9 |
| 3 | 3/09 | 8.5 | 11.0 | 11.3 | 9.3 | 9.2 | 11.7 | 7.0 | 9.6 | 9.4 | 9.9 | 14.7 |
| 4 | 10/02 | 8.5 | 11.6 | 7.8 | 3.6 | 9.0 | 6.4 | 5.7 | 3.3 | 4.7 | 2.3 | 4.3 |
| 5 | 4/03 | 8.0 | 8.8 | 5.8 | 9.5 | 8.5 | 8.4 | 9.1 | 9.7 | 7.8 | 7.3 | 10.9 |
| 6 | 7/09 | 7.4 | 8.7 | 10.6 | 9.1 | 10.2 | 11.2 | 9.2 | 9.2 | 10.0 | 9.7 | 8.4 |
| 7 | 11/01 | 7.4 | 11.3 | 11.0 | 8.3 | 9.3 | 9.2 | 6.7 | 8.6 | 7.0 | 4.6 | 8.0 |
| 8 | 7/10 | 6.9 | 7.5 | 6.4 | 8.1 | 6.6 | 8.6 | 6.3 | 8.0 | 5.9 | 8.5 | 7.7 |
| 9 | 12/10 | 6.5 | 6.0 | 7.0 | 7.3 | 6.1 | 6.9 | 6.1 | 6.6 | 9.3 | 7.5 | 8.6 |
| 10 | 3/10 | 5.9 | 7.7 | 7.3 | 6.7 | 7.3 | 7.0 | 7.7 | 6.7 | 6.5 | 6.6 | 7.2 |
| Average | | 7.7 | 10.4 | 9.5 | 8.7 | 9.5 | 10.0 | 8.3 | 8.8 | 9.1 | 8.4 | 10.1 |
| **Beta based** | | | **Low** | | | | | | | | | **High** |
| 1 | 4/09 | 9.4 | 3.2 | 8.6 | 10.4 | 14.2 | 17.1 | 21.4 | 26.9 | 23.0 | 18.9 | 31.9 |
| 2 | 9/10 | 8.7 | 6.0 | 6.6 | 8.1 | 8.8 | 10.9 | 10.9 | 12.2 | 12.9 | 13.3 | 13.3 |
| 3 | 3/09 | 8.5 | 2.7 | 6.4 | 7.2 | 10.2 | 10.0 | 9.0 | 11.0 | 10.7 | 17.0 | 17.9 |
| 4 | 10/02 | 8.5 | -0.8 | -0.2 | 1.1 | 4.1 | 7.5 | 5.9 | 4.1 | 6.5 | 9.8 | 20.1 |
| 5 | 4/03 | 8.0 | 4.4 | 5.3 | 7.0 | 7.0 | 5.8 | 5.1 | 8.1 | 12.6 | 10.5 | 16.4 |
| 6 | 7/09 | 7.4 | 5.5 | 6.1 | 7.8 | 7.6 | 7.9 | 8.1 | 9.1 | 8.7 | 17.6 | 19.2 |
| 7 | 11/01 | 7.4 | 0.9 | 5.0 | 3.3 | 7.8 | 8.6 | 7.8 | 6.8 | 8.8 | 12.1 | 19.6 |
| 8 | 7/10 | 6.9 | 2.9 | 4.7 | 5.5 | 5.3 | 6.0 | 7.2 | 9.7 | 12.1 | 8.9 | 10.7 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 12/10 | 6.5 | 4.6 | 4.6 | 5.9 | 7.0 | 6.2 | 7.8 | 7.4 | 7.5 | 7.4 | 15.0 |
| 10 | 3/10 | 5.9 | 3.6 | 3.3 | 4.7 | 6.4 | 6.6 | 6.9 | 8.0 | 7.2 | 10.7 | 13.0 |
| Average | | 7.7 | 3.3 | 5.0 | 6.1 | 7.8 | 8.7 | 9.0 | 10.3 | 11.0 | 12.6 | 17.7 |

Following Chan and Lakonishok (1993), we also perform cross-sectional regression to explore the models' performance during extreme market conditions. A large down (up) market is defined as a month where the market excess return is larger in magnitude than the median of those observations that are negative (positive). The median of negative markets was found to be -2.43% from 26 observations while the median of positive markets was 2.22% including 34 observations. The panel data method is employed in this case primarily due to the low number of observations that could be possibly distort the results. Our findings are presented in Tables 13 and 14.

The results indicate that all models leave unexplained returns in down markets while the $R^2$ values are similar. However, the models perform better in up markets. In the case of the APT, we chose to include the market factor in the case of beta sorted portfolios both in up and down markets since the findings without this particular factor were poor. Furthermore, the CAPM seems to work reasonable well at both up and down markets compared to the findings reported earlier in terms of $R^2$ values. As for the SMISI factor of the TFM, we found the expected negative (in the case of beta sorted portfolios) and positive sign in down and up markets respectively.

**Table 13**: Cross-sectional regression results classified by down market months

| Panel A:CAPM,TFM, FF,PL | $\lambda_0$ | $\lambda_{mar}$ | $\lambda_{SMISI}$ | $\lambda_{NSI}$ | $\lambda_{SMB}$ | $\lambda_{HML}$ | $\lambda_{labor}$ | $\lambda_{prem}$ | R² |
|---|---|---|---|---|---|---|---|---|---|
| BV per share | -0.067 | 0.004 | | | | | | | 0.829 |
| | (-6.75)* | (0.46) | | | | | | | |
| | -0.070 | | 0.013 | 0.008 | | | | | 0.831 |
| | (-6.26)* | | (1.41) | (0.69) | | | | | |
| | -0.033 | -0.029 | | | 0.011 | -0.019 | | | 0.840 |
| | (-3.16)* | (-2.50)* | | | (1.94** | (1.95)** | | | |
| | -0.067 | 0.005 | | | | | -0.062 | 0.001 | 0.829 |
| | (-6.92)* | (0.54) | | | | | (-0.40) | (1.08) | |
| Beta portfolios | -0.010 | -0.051 | | | | | | | 0.794 |
| | (-7.09)* | (-57.4)* | | | | | | | |
| | -0.013 | | -0.036 | -0.051 | | | | | 0.796 |
| | (-5.19)* | | (-2.00)* | (-24.1)* | | | | | |
| | -0.019 | -0.041 | | | -0.027 | 0.002 | | | 0.792 |
| | (-4.27)* | (-9.12)* | | | (-2.84)* | (0.31) | | | |
| | -0.005 | -0.055 | | | | | 0.182 | -0.001 | 0.799 |
| | (-2.21)* | (-20.1)* | | | | | (2.65)* | (-1.09) | |
| **Panel B: APT** | $\lambda_0$ | $\lambda_{mar}$ | $\lambda_{EXPIMP}$ | $\lambda_{IP}$ | $\lambda_{TS}$ | | | | R² |
| BV per share | -0.055 | -0.008 | -0.036 | -0.004 | | | | | |
| | (-4.69)* | (-0.74) | (-2.22)* | (-0.94) | | | | | 0.835 |
| Beta portfolios | -0.004 | -0.056 | | | 0.141 | | | | |
| | (-1.92)** | (-28.2)* | | | (1.03) | | | | 0.821 |

*,** depict significance at 5% and 10% respectively.

**Table 14**: Cross-sectional regression results classified by up market months

| Panel A: Best months | $\lambda_o$ | $\lambda_{mar}$ | $\lambda_{SMISI}$ | $\lambda_{NSI}$ | $\lambda_{SMB}$ | $\lambda_{HML}$ | $\lambda_{labor}$ | $\lambda_{prem}$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| BV per share | 0.013 | 0.049 | | | | | | | 0.820 |
| | (1.10) | (4.08)* | | | | | | | |
| | 0.006 | | 0.001 | 0.058 | | | | | 0.817 |
| | (0.69) | | (0.11) | (5.46)* | | | | | |
| | -0.000 | 0.061 | | | 0.015 | 0.007 | | | 0.821 |
| | (-0.01) | (4.42)* | | | (2.32)* | (1.12) | | | |
| | 0.013 | 0.049 | | | | | 0.075 | -0.000 | 0.821 |
| | (1.28) | (5.34)* | | | | | (0.69) | (-0.16) | |
| Beta portfolios | 0.008 | 0.052 | | | | | | | 0.646 |
| | (6.63)* | (53.6)* | | | | | | | |
| | 0.005 | | 0.053 | 0.058 | | | | | 0.666 |
| | (-2.01)* | | (1.89)** | (20.3)* | | | | | |
| | 0.031 | 0.034 | | | 0.035 | -0.041 | | | 0.667 |
| | (2.95)* | (3.89)* | | | (4.54)* | (-2.11)* | | | |
| | -0.005 | 0.061 | | | | | -0.618 | 0.001 | 0.675 |
| | (-1.03) | (11.9)* | | | | | (-8.25)* | (1.06) | |
| Panel B: APT | $\lambda_0$ | $\lambda_{mar}$ | $\lambda_{EXPIMP}$ | $\lambda_{IP}$ | $\lambda_{TS}$ | | | | $R^2$ |
| BV per share | 0.011 | 0.049 | -0.010 | -0.001 | | | | | |
| | (1.12) | (4.92)* | (-1.49) | (-0.17) | | | | | 0.821 |
| Beta portfolios | 0.001 | 0.058 | | | -0.478 | | | | |
| | (0.01) | (15.7)* | | | (-3.41)* | | | | 0.661 |

*,** depict significance at 5% and 10% respectively.

## 4. Conclusions

This paper examines the efficacy of different models to explain the relationship between expected returns and risk in the cross-sectional context. We introduce a novel approach which is primarily based on the time varying nature of betas. The new TFM incorporates two variables. The first one is the ‘SMISI’ and captures the risk associated with the difference between ‘Superior’ and ‘Inferior’ stocks whose betas are increasing and decreasing in market return respectively. The second variable, the ‘NSI’, is constituted from invariant betas and operates as the market factor. This index is also economic meaningful in that it produces substantial alphas reaching as high as 10.08% per annum. We tested the hypothesis that combining into a single model both asymmetric and systematic risk can better explain asset pricing anomalies. After providing the theoretical background and the motivation of the proposed approach our model was compared against four models previously presented in the literature, the CAPM, the FF3FM, the PL model and the APT.

Our results indicate that the proposed specification surpasses alternative models in explaining the cross-section of returns. The implications of this study show that the proposed new risk factors which found to be significant both in time-series and cross-section analyses provide valuable information in better understanding the characteristics of returns, targeting the reinforcement of stock market efficiency.

The study shows that in the cross-sectional analysis both conditionally and unconditionally, the stock market prices different risk factors. In the case of the unconditional cross-sectional regressions our results indicate that the proposed model outperforms, in the sense of statistically significant factors, the CAPM and the PLM while it possesses similar p-values of the F-test with the FF3FM. In addition the $R^2$ value is high in every case. In the case of the BVps portfolios, the SMISI factor is priced and the market risk premium has the expected positive sign apart from the case of the FF3FM. However, the market risk premium is not different from zero at any statistically significant level. In relation to the PLM, it has relatively low $R^2$ value while neither labor nor premium factors influence the returns. The results of the portfolios formed on beta coefficients depict that PLM increases its $R^2$ with the rest models to follow closely. In the case of the APT, different risk factors are priced for the two sets of portfolios. An interesting finding is the model's performance when the NSI is used as benchmark.

The conditional cross-sectional regressions in the case of BVps portfolios identify the power of the TFM variables in explaining asset returns even though a proportion of them was left unexplained. Unexplained returns are also observed in the case of the CAPM. The market and the HML factors in the FF3FM appear to be significant with the constant not being statistically different from zero. Regarding the PLM no factor is priced. In the APT model, two factors appear to be significant (i.e. Exports/Imports and Industrial Production) other than the market risk that also mentioned in the unconditional setting. For the beta sorted portfolios, almost no risk premia were found to be statistically different from zero in the case of the PLM. Finally, the GRS test calculated in the first step time-series regressions depict the outperformance of TFM in relation to CAPM and FF3FM models.

The results from our previous section indicate that portfolios formed with different criteria gain higher returns. Next, we examined if they are fundamentally riskier. In extreme market conditions, the selected portfolios appear to have a different reaction. A downward movement of the market has a lower impact on the lower portfolios than in the higher one. However, in an upward movement the lowest BVps portfolio does not differentiate from the highest one though this is the case between the two extreme beta sorted portfolios. The models' performance in extreme conditions show that all models in down months leave unexplained returns but they perform better in up months.

The implications of this study show that there are additional factors other than the market risk that affect stock returns. The new risk factors which found to be significant both in time series and cross section analyses, give valuable information of better understanding the characteristics of returns, targeting the reinforcement of stock market efficiency and at the same time facilitating investors and financial institutions to capital allocation.

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## References

Adrian, T., Franzoni, T. (2009). Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM. *Journal of Empirical Finance*, 16: 537-556.

Ait-Sahalia, Y., Brandt, M. (2001). Variable selection for portfolio choice. *Journal of Finance*, 56: 1297-1351.

Andersen, T., Bollerslev, T., Diebold, F., Wu, J. (2006). Realized beta: Persistence and predictability. *Advances in Econometrics*, 20: 1-39.

Ang, A., Chen, J. (2007). CAPM over the long run: 1926-2001. *Journal of Empirical Finance*, 14: 1-40.

Ang, A., Chen, J., Xing, Y. (2006). Downside risk. *Review of Financial Studies*, 19: 1191-1239.

Ball, R., Kothari, S., Watts, R. (1993). Economic determinants of the relation between earnings changes and stock returns. *Accounting Review*, 68: 622-638.

Bansal, R., Kiku, D., Yaron, A. (2012). An empirical evaluation of the long-run risks model for asset prices. *Critical Finance Review*, 1: 183-221.

Banz, R., Breen, W. (1986). Sample-dependent results using accounting and market data: Some evidence. *Journal of Finance,* 41: 779-793.

Bawa, V., Lindenberg, E. (1977). Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics*, 5: 189-200.

Bikhchandani, S., Sharma, S. (2000). Herd behaviour in financial markets: A review. IMF working paper, WP/00/48.

Blume, M. (1971). On the assessment of risk. *Journal of Finance,* 26: 1-10.

Bodie,Z., Kane, A., Marcus, A. (2002). Investments (5th ed). Mc Graw-Hill Companies.

Boyer, B. (2011). Style-related comovement: Fundamentals or labels? *Journal of Finance*, 66: 307-332.

Campbell, J., Cochrane, J. (1999). By force a habit: A consumption-based explanation of aggregate stock market behavior. Journal of Political Economy, 107: 205-251.

Campbell, J., Lettau, M., Malkiel, B., Xu, Y. (2001). Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *Journal of Finance*, 56: 1-43.

Campbell, J., Lo, A., MacKinlay, C. (1997). The econometrics of financial markets. Princeton University Press, Princeton, New Jersey.

Chan, L., Lakonishok, J. (1993). Are the reports of beta's death premature? *Journal of Portfolio Management*, 19: 51-62.

Chen, N., Roll, R., Ross, S. (1986). Economic forces and the stock market. *Journal of Business*, 59, 383-403.

Chen, N.-F. (1983). Some empirical tests of the theory of arbitrage pricing. *Journal of Finance*, 38: 1393-1414.

Choudhry, T. (2005). September 11 and time-varying beta of United States companies. *Applied Financial Economics*, 15:1227-1242.
Clare, A., Thomas, S. (1994). Macroeconomic factors, the APT and the UK stock market. *Journal of Business Finance and Accounting*, 21: 309-330.
Cuthbertson, K., Nitzsche, D. (2004). Quantitative financial economics (2nd ed.). John Wiley & Sons.
Daniel, K., Hirshleifer, D., Teoh, S. (2002). Investor psychology in capital markets: evidence and policy implications. *Journal of Monetary Economics*, 49: 139-209.
Daves, P., Ehrhardt, M., Kunkel, R. (2000). Estimating systematic risk: The choice of return interval and estimation period. *Journal of Financial and Strategic Decisions*, 13:7-13.
Engle, R., Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business and Economic Statistics*, 22: 367-381.
Fabozzi, F., Francis, J. (1978). Beta as a random coefficient. *Journal of Financial and Quantitative Analysis*, 13: 101-116.
Faff, R., Brooks, R. (1998). Time-varying beta risk for Australian industry portfolios: an exploratory analysis. *Journal of Business Finance and Accounting*, 25: 721-745.
Fama, E., French, K. (1996). Multifactor explanations of asset pricing anomalies. *Journal of Finance*, 51: 55-84.
Fama, R., MacBeth, J. (1973). Risk, return and equilibrium: Empirical tests. *Journal of Political Economy*, 81: 607-636.
Ferson, W., Harvey, C. (1991). The variation of economic risk premiums. *Journal of Political Economy*, 99: 385-415.
Ferson, W., Harvey, C. (1993). The risk and predictability of international equity returns. *Review of Financial Studies*, 6: 527-566.
Fraser, P., Hamelink, F., Hoesli, M., MacGregor, B. (2004). Time-varying betas and the cross-sectional return-risk relation: evidence from the UK. *European Journal of Finance*, 10: 255-276.
Frijns, B., Gilbert, A., Zwinkels, R. (2013). Market timing ability and mutual funds: a heterogeneous agent approach. *Quantitative Finance*, 13: 1613-1620.
Gibbons, R., Ross, S., Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica*, 57: 1121-1152.
Groenewold, N., Fraser, P. (1997). Share prices and macroeconomic factors. *Journal of Business Finance and Accounting*, 24: 1367-1383.
Gul, F. (1991). A theory of disappointment aversion. Econometrica, 59: 667-686.
Harvey, C. (1989). Time-varying conditional covariances in tests of asset pricing models. *Journal of Financial Economics*, 24: 289-317.
He, Z., Krishnamurthy, A. (2013). Intermediary asset pricing. *American Economic Review*, 103: 732-770.
Huang, H-C. (2001). Tests of CAPM with nonstationary beta. *International Journal of Finance and Economics*, 6: 255-268.
Investment Insights. (2012). CAPM and quality stocks: The problem with beta. PERKINS Investment Management, March 2012.
Isakov, D. (1999). Is beta still alive? Conclusive evidence from the Swiss stock market. *European Journal of Finance*, 5: 202-212.
Jagannathan, R., Wang, Z. (1996). The conditional CAPM and the cross-section of expected returns. *Journal of Finance*, 51: 3-53.
Jawadi, F., Khanniche, S. (2012). Modeling hedge fund exposure to risk factors. *Economic Modelling*, 29: 2003-1018. Lakonishok, J., Shleifer, A., Vishny, R. (1994). Contrarian investment, extrapolation, and risk. *Journal of Finance*, 49: 1541-1578.
Lettau, M., Ludvigson, S. (2001). Resurrecting the ( C )CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy*, 81: 1238-1287.
Lettau, M., Maggiori, M., Weber, M. (2014). Conditional risk premia in currency markets and other asset classes. *Journal of Financial Economics*, 114: 197-225.
Levy, R. (1971). On the short-term stationarity of beta coefficients. *Financial Analysts Journal*, 27: 55-62.
Lewellen, J., Nagel, S. (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics*, 82: 289-314.
Lin, W., Chen, Y., Boot, J. (1992). The dynamic and stochastic instability of betas: Implications for forecasting stock returns. *Journal of Forecasting*, 11: 517-541.
Lin, X., Zhang, L. (2013). The investment manifesto. *Journal of Monetary Economics*, 60: 351-366.
Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budget. *Review of Economics and Statistics*, 47: 13-37.

Lo, A., MacKinlay, C. (1988). Stock prices do not follow random walks: Evidence from a simple specification test. *Review of Financial Studies*, 1: 41-46.

Mendenhall, R. (2004). Arbitrage risk and post-earnings announcement drift. *Journal of Business*, 77: 875-894.

Messis, P., Zapranis, A. (2016). Forecasting time-varying daily betas: A new nonlinear approach. *Managerial Finance*, 42: 54-73.

Moreira, A., Muir, T. (2017).Volatility-managed portfolios. *Journal of Finance*, 72: 1611-1644.

Plyakha, Y., Uppal, R., Vilkov, G. (2014). Equal or value weighting? Implications for asset-pricing tests. Available at SSRN: http://dx.doi.org/10.2139/ssrn.1787045.Roll, R. (1977). A critique of the asset pricing theory's tests: Part I: on past and potential testability of the theory. *Journal of Financial Economics*, 4: 129-176.

Ross, S. (1976). The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*, 13: 341-360.

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19: 425-442.

Shleifer, A., Vishny, R. (1997). The limits of arbitrage. *Journal of Finance*, 52: 35-55.

Wachter, J. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *Journal of Finance*, 68: 987-1035.

Woodward, G., Anderson, H. (2009). Does beta react to market conditions? Estimates of 'bull' and 'bear' betas using a nonlinear market model with an endogenous threshold parameter. *Quantitative Finance*, 9: 913-924.

Wurgler, J., Zhuravskaya, K. (2002). Does arbitrage flatten demand curves for stocks? *Journal of Business*, 75, 583-608.

## Appendix

On tables A1 and A2 we report the results of the unconditional regressions of the tested models on the portfolios formed on the basis of BV per share and $\beta$ coefficients. For robustness purposes, these portfolios are constructed using both equal (EW) and value (VW)-weighting schemes. We employ stocks that have available market value observations on Datastream during the period from 2001 to 2010. Each year, the portfolio weights are rebalanced. The sample starts with 255 stocks traded on the S&P 500 and ends with 337. All metrics are calculated using monthly returns. Using a different sample, we observe that the results of EW are differentiated from those reported on the main text. However, the main conclusions do not change dramatically. One interesting point is that EW and VW portfolios give different results as expected. The VW scheme appears to have lower $R^2$ ratios than EW portfolios. In the case of APT (Table A2), the NSI and the S&P 500 present similar results for both weighting schemes and portfolios. However, the NSI gives higher $R^2$ values in three out of four cases compared to the S&P 500.

**Table A1**: Unconditional Cross-sectional regressions of CAPM, FF3FM, TFM and PLM.

| Panel A: 2001-2011 (BS) | $\lambda_0$ | $\lambda_{mar}$ | $\lambda_{SMISI}$ | $\lambda_{NSI}$ | $\lambda_{SMB}$ | $\lambda_{HML}$ | $\lambda_{labor}$ | $\lambda_{prem}$ | $R^2$ /(*pv*-F) |
|---|---|---|---|---|---|---|---|---|---|
| BV per share (EW) | -0.025 | 0.033 | | | | | | | 0.478 |
| | (-1.96)** | (2.71)* | | | | | | | (0.03) |
| | -0.022 | | -0.002 | 0.033 | | | | | 0.572 |
| | (-2.02)** | | (-0.18) | (2.92)** | | | | | (0.05) |
| | 0.011 | -0.013 | | | 0.032 | 0.006 | | | 0.934 |
| | (1.35) | (-1.46) | | | (7.64)* | (1.64) | | | (0.00) |
| | -0.006 | 0.019 | | | | | -0.004 | 0.645 | 0.816 |
| | (-0.59) | (1.93)** | | | | | (-1.88) | (2.62)* | (0.01) |
| BV per share (VW) | 0.013 | -0.007 | | | | | | | 0.092 |
| | (1.72) | (-0.90) | | | | | | | (0.39) |
| | 0.011 | | 0.016 | -0.005 | | | | | 0.212 |
| | (1.68) | | (1.30) | (-0.64) | | | | | (0.44) |
| | 0.021 | -0.016 | | | 0.014 | -0.000 | | | 0.846 |
| | (5.33)* | (-3.74)* | | | (4.81)* | (-0.09) | | | (0.00) |
| | 0.010 | 0.002 | | | | | 0.001 | 0.410 | 0.288 |
| | (1.25) | (0.16) | | | | | (0.43) | (1.13) | (0.53) |
| Beta portfolios (EW) | 0.002 | 0.007 | | | | | | | 0.881 |
| | (2.39)* | (7.72)* | | | | | | | (0.00) |
| | 0.003 | | -0.005 | 0.006 | | | | | 0.891 |
| | (2.83)* | | (-1.04) | (3.95)* | | | | | (0.00) |
| | 0.000 | 0.009 | | | -0.003 | 0.004 | | | 0.900 |
| | (0.15) | (3.07)* | | | (-0.66) | (1.25) | | | (0.00) |
| | -0.000 | 0.010 | | | | | 0.001 | -0.081 | 0.912 |
| | (-0.30) | (3.93)* | | | | | (0.74) | (-0.62) | (0.00) |
| Beta portfolios (VW) | 0.004 | 0.003 | | | | | | | 0.243 |
| | (2.55)* | (1.60) | | | | | | | (0.14) |
| | 0.004 | | -0.000 | 0.003 | | | | | 0.244 |
| | (2.62)* | | (-0.05) | (1.26) | | | | | (0.37) |
| | 0.004 | 0.002 | | | -0.000 | -0.000 | | | 0.288 |
| | (1.54) | (1.11) | | | (-0.27) | (-0.11) | | | (0.53) |
| | 0.000 | 0.006 | | | | | 0.001 | -0.097 | 0.371 |
| | (0.15) | (1.61) | | | | | (0.64) | (-0.62) | (0.39) |
| **Panel B: 2006-2011 (BS)** | $\lambda_0$ | $\lambda_{mar}$ | $\lambda_{SMISI}$ | $\lambda_{NSI}$ | $\lambda_{SMB}$ | $\lambda_{HML}$ | $\lambda_{labor}$ | $\lambda_{prem}$ | $R^2$ /(*pv*-F) |
| BV per share (EW) | -0.007 | 0.013 | | | | | | | 0.111 |
| | (-0.47) | (1.00) | | | | | | | (0.34) |
| | -0.009 | | -0.005 | 0.017 | | | | | 0.233 |
| | (-0.82) | | (-0.47) | (1.46) | | | | | (0.39) |
| | 0.010 | -0.006 | | | 0.001 | 0.014 | | | 0.773 |
| | (0.89) | (-0.47) | | | (0.23) | (4.36)* | | | (0.02) |
| | 0.000 | 0.015 | | | | | -0.000 | 0.706 | 0.731 |
| | (0.01) | (1.50) | | | | | (-0.17) | (3.65)* | (0.04) |
| BV per share (VW) | 0.009 | -0.002 | | | | | | | 0.016 |
| | (1.48) | (-0.36) | | | | | | | (0.72) |
| | 0.006 | | 0.017 | 0.001 | | | | | 0.357 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1.18) | | (1.95)** | (0.21) | | | | | (0.21) |
| | 0.013 | -0.006 | | | -0.004 | 0.009 | | | 0.564 |
| | (1.63) | (-0.70) | | | (-0.84) | (2.24)** | | | (0.14) |
| | 0.008 | 0.009 | | | | | -0.001 | 0.835 | 0.446 |
| | (1.47) | (0.98) | | | | | (-0.35) | (2.14)** | (0.28) |
| Beta portfolios (EW) | -0.003 | 0.010 | | | | | | | 0.897 |
| | (-2.41)* | (8.38)* | | | | | | | (0.00) |
| | -0.003 | | 0.000 | 0.011 | | | | | 0.906 |
| | (-1.63) | | (0.08) | (5.07)* | | | | | (0.00) |
| | -0.003 | 0.009 | | | 0.004 | 0.002 | | | 0.898 |
| | (-0.36) | (1.02) | | | (0.70) | (0.39) | | | (0.00) |
| | -0.005 | 0.008 | | | | | -0.001 | -0.127 | 0.923 |
| | (-1.86) | (3.42)* | | | | | (-1.28) | (-1.23) | (0.00) |
| Beta portfolios (VW) | 0.001 | 0.006 | | | | | | | 0.330 |
| | (0.47) | (1.98)** | | | | | | | (0.08) |
| | -0.000 | | 0.007 | 0.009 | | | | | 0.574 |
| | (-0.13) | | (1.57) | (3.05)* | | | | | (0.05) |
| | -0.007 | 0.015 | | | -0.006 | -0.007 | | | 0.713 |
| | (-0.82) | (1.77) | | | (-1.85) | (-1.00) | | | (0.05) |
| | 0.003 | 0.009 | | | | | 0.003 | 0.195 | 0.532 |
| | (0.78) | (2.22)** | | | | | (1.53) | (0.98) | (0.18) |

*,** depict significance at 5% and 10% respectively.

**Table A2** : Unconditional Cross-sectional regressions of APT

| Panel A: 2001-2011 (BS)-SP | $\lambda_0$ | $\lambda_{MR}$ | $\lambda_{M3}$ | $\lambda_{DR}$ | $\lambda_{RRS}$ | | $R^2$/(*pv*-F) |
|---|---|---|---|---|---|---|---|
| BV per share (EW) | -0.013 | 0.021 | 0.255 | | | | 0.669 |
| | (-1.26) | (1.99)** | (2.09)** | | | | (0.02) |
| BV per share (VW) | 0.021 | -0.014 | | -0.154 | | | 0.798 |
| | (4.94)* | (-3.12)* | | (-4.65)* | | | (0.00) |
| Beta portfolios (EW) | 0.002 | 0.006 | | | | | 0.892 |
| | (2.87)* | (8.14)* | | | | | (0.00) |
| Beta portfolios (VW) | 0.006 | | 0.132 | | -0.005 | | 0.505 |
| | (12.6)* | | (2.49)* | | (-2.21)** | | (0.08) |
| **Panel B: 2001-2011 (BS)-NSI** | $\lambda_0$ | $\lambda_{NSI}$ | $\lambda_{M3}$ | $\lambda_{DR}$ | $\lambda_{OIL}$ | $\lambda_{EXPIMP}$ | $R^2$/(*pv*-F) |
| BV per share (EW) | -0.011 | 0.022 | 0.241 | | | | 0.706 |
| | (-1.30) | (2.48)* | (1.92)** | | | | (0.01) |
| BV per share (VW) | 0.024 | -0.022 | | -0.142 | | 0.021 | 0.769 |
| | (3.95)* | (-2.58)* | | (-3.65)* | | (1.94)** | (0.02) |
| Beta portfolios (EW) | 0.003 | 0.007 | | | | | 0.896 |
| | (3.53)* | (8.32)* | | | | | (0.00) |
| Beta portfolios (VW) | 0.007 | | 0.158 | | 1.838 | | 0.566 |
| | (13.6)* | | (2.66)* | | (2.30)** | | (0.08) |

*,** depict significance at the 5% and the 10% level respectively.