MECHANICAL CHAINS: IDENTIFICATION OF DYNAMICAL RESPONSES

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Abstract: *Approaches for dynamical responses identification in example of system with three freedom degrees with tie between two of outside elements are offered. First approach bases on using of mathematical models-analogues of automation control theory, second approach bases on rules of transformation of design schemes of chains theory. Conversion from design schemes to equivalent in dynamical attitude of structural schemes of automation control is applied at using of structural method. Feedback tie in automation control system dedicated to the protection object is dynamical responses on external disturbance and at zero value of the disturbance disposed of the coerced elasticity of the system. The using of direct conversion of design schemes for the rules of chains theory shows identical results for both approaches.*

Keyboards: *dynamical responses, structural approaches, chains theory, feedback ties, coerced elasticity;*

INTRODUCTION

Vibroprotection systems (VPS) of technical objects are various enough and can contain in the structure various elements which often unite in mechanical chains. At presence in chains of inertial elements and periodic external influences dynamic interactions form the responses of communications defining a condition of object of protection, contacts to basic surfaces and level of power interactions between elements [3÷5].

Dynamic esponses of communication are simply enough defined in flat mechanisms which represent the closed mechanical chains with one or several motionless links. Kinetostatical calculations and methodical base of calculations are well enough developed and have found reflexion in works of domestic scientists [1, 7, 9].

At the same time, definition of responses of communications in mechanical oscillatory systems is developed to a lesser degree. Though to questions of dynamics of oscillative motions in mechanisms [7,8] and vibroprotection systems the considerable attention [9,10] is paid.

Specificity vibroprotection systems consists in prevalence of periodic forms of movements of object of protection and links ÂÇÑ that in practice often leads to disclosing of contacts, sample of backlashes and occurrence vibro-impact processes that is undesirable effects in respect of maintenance of reliability and

safety of operation of cars, the equipment and apparatus [11,12].

In article the methodological basis of definition of dynamic reactions in mechanical oscillatory systems with use of the structural mathematical models-analogues received by transformation by Laplace initial linear or linearized of the differential equations of movement is offered.

I. Formulation of the tasks. Consider as initial design scheme of vibroprotection system in view of mechanical oscillation system with three freedom degree (fig. 1). The system makes small movements under the influence of periodical and kinematical disturbances which can be applied to all mass-inertial elements - $Q_i \left(i = \overline{1,3} \right)$.

Besides accounting of action of kinematical disturbances $z_k \left(k = \overline{1,2} \right)$, which connected with bearing surfaces *I* and *II* (fig. 1), is possible. The fixed coordinate system y_1, y_2, y_3 is selected as the main; friction forces are considered negligible and elements movements with masses m_1, m_2, m_3 occur on vertical; stiffness coefficients of linear elastical elements are designated through k_1, k_2, k_3, k_4, k_5. Circuit *I* on fig.1 determines placement of vibroprotection device (VPD); protection object has a mass m3. Bearing surface *II* is introduced for creature more full scheme: at necessity k_4 and k_5, as well as Q_i and z_k, may take zero values. Write equations for kinetical and potential energies:

$$T = \frac{1}{2}m_1\dot{y}_1^2 + \frac{1}{2}m_2\dot{y}_2^2 + \frac{1}{2}m_3\dot{y}_3^2, \tag{1}$$

$$\Pi = \frac{1}{2}k_1 y_1^2 + \frac{1}{2}k_2(y_2 - y_1)^2 + \frac{1}{2}k_3(y_3 - y_2)^2 + \frac{1}{2}k_4 y_3^2 + \frac{1}{2}k_5(y_3 - y_1)^2. \tag{2}$$
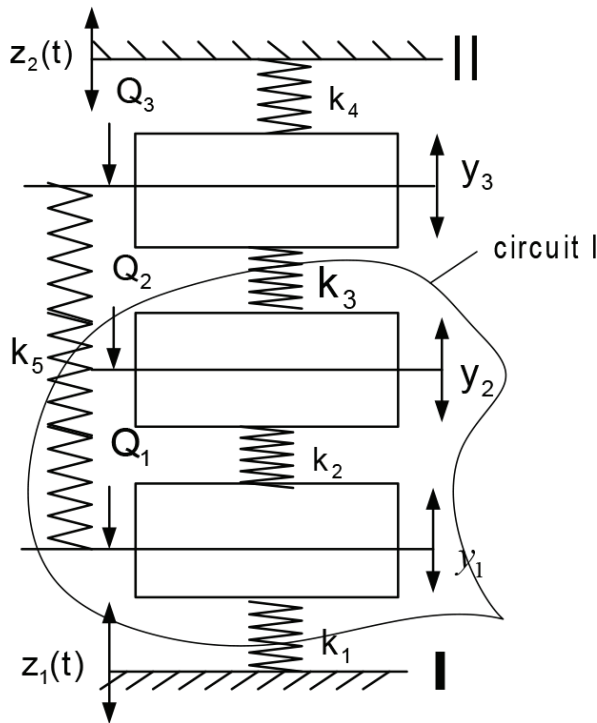

Fig. 1. Design scheme of vibroprotection systems with three freedom degrees

the standard technique [5] on the basis of a

For construction of mathematical model

formalism of LaGrange is used:

$$m_1\ddot{y}_1 + y_1(k_1 + k_2 + k_3) - k_2y_2 - k_5y_3 = k_1z_1; \tag{3}$$

$$m_2\ddot{y}_2 + y_2(k_2 + k_3) - k_2y_1 - k_5y_3 = 0; \tag{4}$$

$$m_3\ddot{y}_3 + y_3(k_3 + k_4 + k_5) - k_3y_2 - k_5y_1 = Q_3. \tag{5}$$

If to accept that $z_1=0$, $z_2=0$, and $Q_1\neq0$, $Q_2\neq0$, $Q_3 \neq 0$, communications between variables y_1, y_2, y_3 and Q_1, Q_2, Q_3 in the operational form can be presented in the form of following equations:

$$\tilde{y}_1 = \frac{\tilde{Q}_1(a_{22}a_{33} - a_{23}a_{32}) + \tilde{Q}_2(a_{13}a_{32} - a_{33}a_{12}) + \tilde{Q}_3(a_{12}a_{23} - a_{13}a_{22})}{A_0}, \tag{6}$$

$$\tilde{y}_2 = \frac{\tilde{Q}_1(a_{23}a_{31} - a_{31}a_{33}) + \tilde{Q}_2(a_{11}a_{33} - a_{13}a_{31}) + \tilde{Q}_3(a_{13}a_{21} - a_{11}a_{23})}{A_0}, \tag{7}$$

$$\tilde{y}_3 = \frac{\tilde{Q}_1(a_{21}a_{32} - a_{22}a_{31}) + \tilde{Q}_2(a_{12}a_{31} - a_{11}a_{32}) + \tilde{Q}_3(a_{11}a_{22} - a_{12}a_{21})}{A_0}, \tag{8}$$

where

$$A_0 = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{33}a_{12}a_{21} - \\ - a_{22}a_{13}a_{31} + a_{13}a_{21}a_{32} + a_{12}a_{23}a_{31}. \tag{9}$$

Is the characteristic equation of system (the system is symmetric: $a_{12}= a_{21}$, $a_{13}=a_{31}$, $a_{23}=a_{32}$); the badge ($\sim$) means the image on Laplace as in relation to (3) ÷ (5) transformation of Laplace is applied to the further calculations.

Using (6) ÷ (8), it is possible to construct enough great number of transfer functions.

For the further consideration it is accepted that on object of protection external periodic (harmonious) force $Q_3 \neq 0$ operates, other power factors are equal 0. The structural scheme of system corresponding to the equations (3) ÷ (5) is resulted on fig. 2.
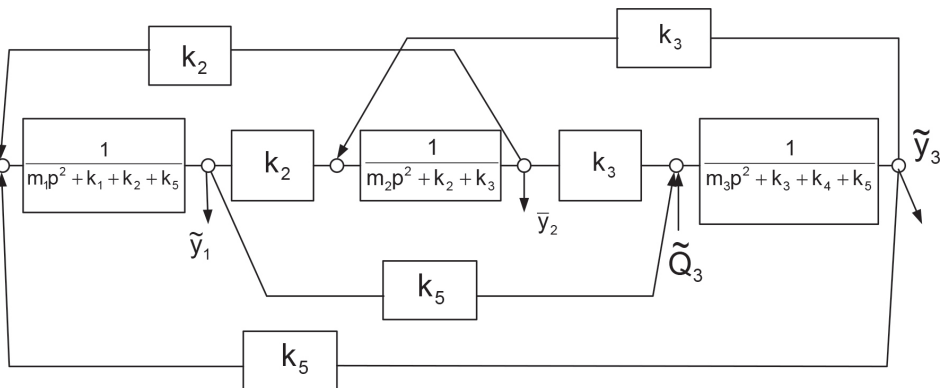

Fig. 2. Structural scheme for vibroprotection system on fig. 1 at $\tilde{Q}_3 \neq 0$

At action of external indignations dynamic responses arise between object of protection m_3 and a support *II*, and also between an inertial intermediate element m_1 VPD and a basic surface *I*. Dynamic responses will arise as well between separate elements of system.

II. Estimation of dynamical properties of system. Using equations (6) ÷ (8), we will write down for structural system on fig. 2 transfer functions:

$$W_1(p) = \frac{\tilde{y}_1}{\tilde{Q}_3} = \frac{a_{12}a_{23} - a_{13}a_{22}}{A_0}; \qquad (10)$$

$$W_2(p) = \frac{\tilde{y}_2}{\tilde{Q}_3} = \frac{a_{13}a_{21} - a_{11}a_{23}}{A_0}; \qquad (11)$$

$$W_3(p) = \frac{\tilde{y}_3}{\tilde{Q}_3} = \frac{a_{11}a_{22} - a_{12}a_{21}}{A_0}, \qquad (12)$$

where A_0 it is defined by equation (9). As the structural scheme on fig. 2 has communication between co-ordinates y_1 and y_3 ($k_5 \ne 0$) a particular interest represents a case loading $Q_2 \ne 0$. We will write down transfer functions of system for this case:

$$W_1'(p) = \frac{\tilde{y}_1}{\tilde{Q}_2} = \frac{a_{13}a_{32} - a_{12}a_{33}}{A_0}; \qquad (13)$$

$$W_2'(p) = \frac{\tilde{y}_2}{\tilde{Q}_2} = \frac{a_{11}a_{33} - a_{13}a_{31}}{A_0}; \qquad (14)$$

$$W_3'(p) = \frac{\tilde{y}_3}{\tilde{Q}_2} = \frac{a_{12}a_{31} - a_{11}a_{32}}{A_0}. \qquad (15)$$

Equations (10) ÷ (15) for transfer functions can be used for structural scheme transformation on fig. 2.

1. Let's consider a case $\tilde{Q}_3 \ne 0$ then (12) it is possible to transform to a kind

$$W_3(p) = \frac{\tilde{y}_3}{\tilde{Q}_3} = \frac{1}{a_{33} - \frac{a_{11}a_{23}^2}{A_1} - \cdots}$$

$$\cdots \frac{}{- \frac{a_{22}a_{13}^2}{A_1} + \frac{2a_{12}a_{23}a_{31}}{A_1}}, \qquad (16)$$

where $A_1 = a_{11}a_{22} - a_{12}^2$. $\qquad (17)$.

The detailed view (16) takes the form:

$$W_3^*(p) = \frac{\tilde{y}_3}{\tilde{Q}_3} = \frac{1}{m_3 p^2 + k_3 + k_4 + k_5 - \cdots}$$

$$\frac{}{- \frac{k_5^2(m_2 p^2 + k_2 + k_3)}{A_2} - \cdots} \qquad (18)$$

$$\frac{}{\cdots - \frac{k_3^2(m_1 p^2 + k_1 + k_2 + k_5)}{A_2} - \frac{2k_5 k_2 k_3}{A_2}}.$$

Here

$$A_2 = (m_1 p^2 + k_1 + k_2 + k_5)(m_2 p^2 + k_2 + k_3) - k_2^2, \quad (18')$$

and $p = j\omega$ – complex variable ($j = \sqrt{-1}$).

In turn, on equation (18) the structural scheme, as is shown in fig. 3 can be constructed. On this scheme the structural element corresponding to object of protection m3 is allocated. All other elements are grouped in a chain of feedback which, on the physical essence, represents the generalized spring with the stiffness defined by equation:

$$\tilde{k}_c' = (k_3 + k_4 + k_5)A_2 - \frac{k_5^2(m_2 p^2 + k_2 + k_3) + \cdots}{A_2}$$

$$\frac{\cdots + k_3^2(m_1 p^2 + k_1 + k_2 + k_5) - 2k_2 k_3 k_5}{} \qquad (19)$$
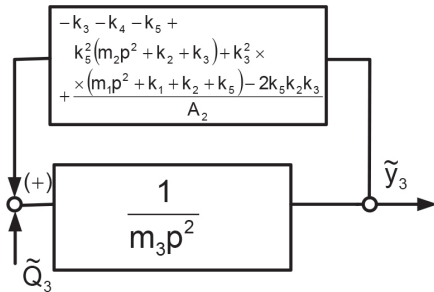

Fig. 3. Coerced structural scheme for identification dynamical response on protection object

Let's notice that feedback in the structural scheme (fig. 3) will have a positive sign. Dimension of an entrance signal in a return chain corresponds to displacement, and the target signal has dimension of force. That is feedback, in physical sense, reflects interaction with a basic surface II that corresponds to dynamic response

$$\tilde{R}_B = k_4 \tilde{y}_3 = k_4 W_3(p)\tilde{Q}_3. \qquad (19')$$

The second part of dynamic response is defined by interaction of elastic elements k_4 and k_5, and also an additional member who in equation (19) has a negative sign:

$$\tilde{R}_{m_3} = \left[\frac{(k_4 + k_3 + k_5)A_2 - k_5^2(m_2 p^2 + k_2 + k_3) +}{A_2} \cdots \right.$$
$$\left. \cdots \frac{+ k_3^2(m_1 p^2 + k_1 + k_2 + k_5) - 2k_2 k_3 k_5}{} \right] \tilde{y}_3. \qquad (19'')$$

Transfer function at a power input Q_3 and an exit $- \tilde{R}_m$ has view:

$$W_{m_3}(p) = \frac{\tilde{R}_{m_3}}{\tilde{Q}_3} = \frac{(k_3 + k_4 + k_5)A_2 -}{A_0} \cdots$$
$$\cdots \frac{- \left[k_5^2(m_2 p^2 + k_2 + k_3) + \right.}{}$$
$$\cdots \frac{\left. + k_3^2(m_1 p^2 + k_1 + k_2 + k_5) - 2k_2 k_3 k_5 \right]}{} W_3(p)$$

where
$$W_3(p) = \frac{1}{A_0}\left[(k_3 + k_4 + k_5)A_2 - k_5^2(m_2 p^2 + \right.$$
$$\left. + k_2 + k_3) - k_3^2(m_1 p^2 + k_1 + k_2 + k_5) - 2k_2 k_3 k_5 \right] \quad (19''')$$

In turn:

$$W_B(p) = \frac{\tilde{R}_{II}}{\tilde{Q}_3} = \frac{k_4 A_2}{A_0}. \qquad (19^{IV})$$

Generally from (19''') and (19^{IV}) follows that peak-frequency characteristics have on three resonances, and also two frequencies on which dynamic responses $\tilde{R}_{m_3}$ will be equal to zero. It corresponds to a mode when the dynamic part of the general response on object of protection m3 will be equal to zero, but there is a loading static, defined by gravity of elements vibroprotection systems and preliminary preloads springs. As to dynamic response to a basic surface *II* according to (19^{IV}) response R_{II} will have also two frequencies «zeroing». We will notice that these modes coincide with modes of dynamic clearing on co-ordinate y_3 at action on object of protection of force $Q_3 \neq 0$ ($Q_2 = 0$, $z_1 = 0$, $z_2 = 0$) that follows from equation (12) or (18).

Special cases are in the present state of affairs possible, for example, k_4 and k_5 can be accepted equal to zero. In this case equation (19) will be transformed to a kind:

$$\tilde{k}'_{c1} = \frac{k_3 A_3 - k_3^2(m_1 p^2 + k_1 + k_2)}{A_3}, \qquad (20)$$

where

$$A_3 = \left(m_1 p^2 + k_1 + k_2 \right)\left(m_2 p^2 + k_3 \right) + $$
$$+ k_2 \left(m_1 p^2 + k_1 \right) \qquad (21)$$

Knowing (20), it is possible to find dynamic response to object of protection m3:

$$\tilde{R}'_{m_3} = \tilde{k}'_{np_1} \tilde{y}'_3, \qquad (22)$$

where $\tilde{y}'_3 = W_3(p)\tilde{Q}_3 \quad (k_4 = 0, \ k_5 = 0)$.

Thus, for object of protection dynamic responses both for contacts to a basic surface, and for the full response representing the sum static and dynamic component can be found. If $k_4 = 0$, $k_5 = 0$, value of response also can be found at use (20).

2. To find dynamic response to intermediate weight m1 VPD, it is necessary to use equation (16):

$$W_1(p) = \frac{\tilde{y}_1}{\tilde{Q}_3} = \frac{\left[k_2 k_3 + k_5(m_2 p^2 + k_2 + k_3) \right]/A_4}{\left(m_1 p^2 + k_1 + k_2 + k_5 \right) - \dfrac{k_5^2(m_2 p^2 +}{A_4}} \cdots$$
$$\frac{+ k_2 + k_3) + \left(m_3 p^2 + k_3 + k_4 + k_5 \right) k_2^2 - 2k_2 k_3 k_5}{}. \qquad (23)$$

where

$$A_4 = \left(m_2 p^2 + k_2 + k_3 \right)\left(m_3 p^2 + k_3 + k_4 + k_5 \right) - k_3^2. \qquad (24)$$

The structural scheme for a case of reduction of the general system to an intermediate element m3 looks like, as is shown in fig. 4.

$$-k_3 - k_4 - k_5 + \frac{k_5^2\left(m_2 p^2 + k_2 + k_3\right) + k_3^2\left(m_1 p^2 + k_1 + k_2 + k_5\right) - 2k_5 k_2 k_3}{A_2}$$

$$(+)$$

$$\widetilde{Q}_3$$
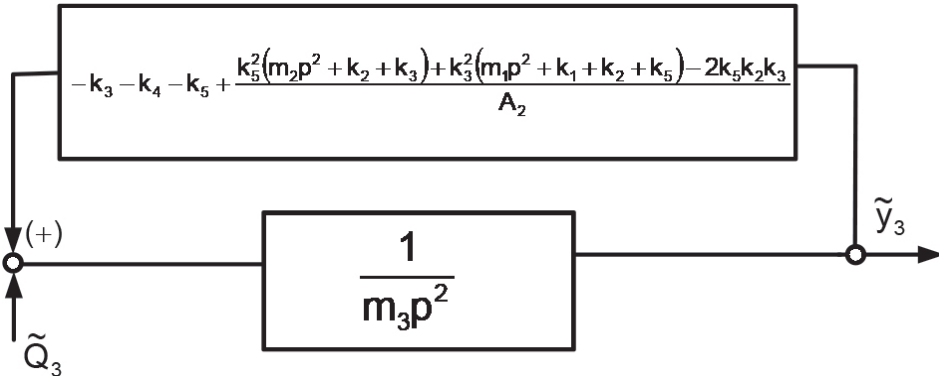
$$\frac{1}{m_3 p^2}$$

$$\widetilde{y}_3$$

Fig. 4. Structural scheme for identification of dynamical response on intermediate element of VPD – m1

Let's notice that at identification of transfer function (23) it is possible to present some external force $\widetilde{Q}_1'$ which looks like on a system input:

$$\widetilde{Q}_1' = \widetilde{Q}_1' \frac{k_2 k_3 + k_5\left(m_2 p^2 + k_2 + k_3\right)}{A_4} \cdot \qquad (25)$$

In this case $\widetilde{Q}_1'$ is the equivalent force of external indignation transferred from object of protection m_3 to intermediate weight VPD - m_1. In turn, if to consider that indignation in system looks like (25) it is possible to define transfer function from $\widetilde{Q}_1'$ on coordinate $\widetilde{y}_3$. The coerced stiffness of the generalized spring, and that is a physical essence of a chain of negative feedback in the structural scheme on fig. 4, is defined (at k5 = 0):

$$\widetilde{k}_c'' = \frac{\left(k_1 + k_2\right)A_5 - k_2^2\left(m_3 p^2 + k_3 + k_4\right)}{A_5}, \qquad (26)$$

where

$$A_5 = \left(m_2 p^2 + k_2 + k_3\right)\left(m_3 p^2 + k_3 + k_4\right) - k_3^2. \qquad (26')$$

As to response identification $\widetilde{R}_I$ it can be found

$$\widetilde{R}_I = k_1 \widetilde{y}_1 = k_1 W_1^*(p)\widetilde{Q}_1, \qquad (27)$$

here $W_1^*(p)$ it is defined from (16) at k5 = 0; that is $W_1^*(p) = W_1(p)_{k_5=0}$.

Dynamic response $\widetilde{R}_I$ is defined by level of interaction with a basic surface I (fig. 1). Response to an intermediate element can be found

$$\tilde{R}_{m_1}(p) = \tilde{k}''_c \tilde{y}_1 = \frac{(k_1 + k_2)\left[\begin{pmatrix} m_2 p^2 + \\ + k_2 + k_3 \end{pmatrix} \cdot \begin{pmatrix} m_3 p^2 + \\ + k_3 + k_4 \end{pmatrix} - k_3^2\right] - k_2^2 \begin{pmatrix} m_3 p^2 + \\ + k_3 + k_4 \end{pmatrix}}{A_5} W_1^*(p)\tilde{Q}'_1. \quad (28)$$

For identification $W_{m_1}(p)$ it is entered with the account $\tilde{Q}'_1$, defined (25), and accepting $k_5 = 0$.

Similarly on dynamic response to a basic surface *I* and an intermediate element m_1 transfer functions from external force $\tilde{Q}_3$ on parameters of responses $\tilde{R}_l$ and $\tilde{R}_{m_1}$, and also

transfer functions on corresponding dynamic responses at power entrance influence $\tilde{Q}_3$ can be received.

3. Let's pass to identification of dynamic response from Q_3 on coordinate y_2, connected with intermediate weight m_2. In this case equation (17) becomes:

$$W_2(p) = \frac{\tilde{y}_2}{\tilde{Q}_3} = \frac{\dfrac{\left[k_5 + k_2 + k_3(m_1 p^2 + k_1 + k_2 + k_5)\right]}{A_6}}{\left(m_2 p^2 + k_2 + k_3\right) - \dfrac{\begin{pmatrix} m_1 p^2 + k_1 + \\ + k_2 + k_5 \end{pmatrix}k_3^2}{A_6} - \dfrac{\begin{pmatrix} m_3 p^2 + k_3 + \\ + k_4 + k_5 \end{pmatrix}k_2^2 + 2k_2 k_3 k_5}{A_6}} \quad (29)$$

where

$$A_6 = \left(m_1 p^2 + k_1 + k_2 + k_5\right)\left(m_3 p^2 + k_2 + k_3\right) - k_5^2 \quad (30)$$

The structural scheme of system for identification of dynamic response from Q_3 on an intermediate element m2 is resulted on fig. 5.
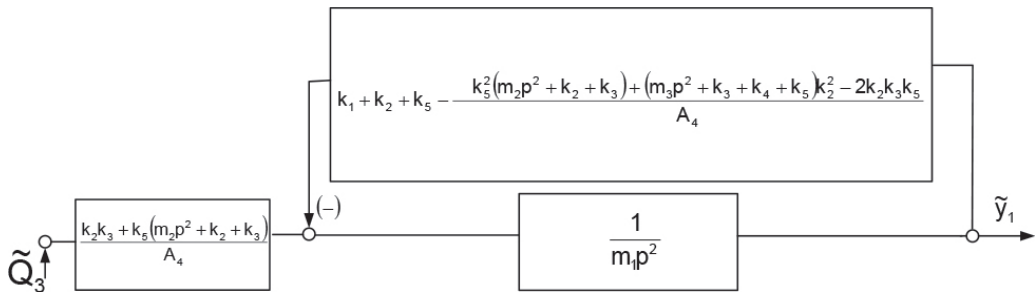

Fig. 5. Coerced structural scheme for identification of dynamicalresponse from $\tilde{Q}_3$ on coordinate $\tilde{y}_2$

The coerced stiffness of the generalized spring is defined under the scheme on fig. 5; at k5 = 0

$$\tilde{k}''_c = \frac{(k_2 + k_3)A_7 - k_3^2(m_1 p^2 + k_1 + k_2) - (m_3 p^2 + k_3 + k_4)k_2^2}{A_7}, \quad (31)$$

where

$$A_7 = \left(m_1 p^2 + k_1 + k_2\right)\left(m_3 p^2 + k_3 + k_4\right). \quad (32)$$

Let's find values $\tilde{k}'_c, \tilde{k}''_c, \tilde{k}'''_c$ directly from the design scheme on fig. 1 at k5 = 0, using a technique stated in [4], and we will receive:

$$\tilde{k}'^*_c = k_4 + \frac{k_3\left[\left(m_1p^2 + k_1\right)k_2 + m_2p^2\left(m_1p^2 + k_1 + k_2\right)\right]}{\left(k_1 + m_1p^2\right)k_2 + m_2p^2\left(m_1p^2 + k_1 + k_2\right) + k_3^2\left(m_1p^2 + k_1 + k_2\right)} \tag{33}$$

$$\tilde{k}''^*_c = k_1 + \frac{k_2\left[\left(m_3p^2 + k_4\right)k_3 + m_2p^2(m_3p^2 + k_3 + k_4)\right]}{\left(k_4 + m_3p^2\right)k_3 + m_2p^2(m_3p^2 + k_3 + k_4) + k_2\left(m_3p^2 + k_3 + k_4\right)} \tag{34}$$

$$\tilde{k}'''^*_c = \frac{k_2\left[\left(m_1p^2 + k_1\right)\left(m_3p^2 + k_3 + k_4\right)\right] + k_3\left(m_1p^2 + k_1 + k_2\right)\left(m_3p^2 + k_4\right)}{\left(k_1 + k_2 + m_1p^2\right)\left(m_3p^2 + k_3 + k_4\right)} \tag{35}$$

III. The method of straight transformations of design scheme. Doing corresponding transformations to equations (19) at k5 = 0 (31), (35), it is possible to show that the equations received on the basis of the offered method and at use of ways of direct convolution, coincide. Design schemes for reception coerced stiffness's are resulted on fig. 6a,b.

In particular, on fig. 6 and the elastic element with stiffness k4 enters into a chain consistently connected elastic and mass-inertial elements between basic surfaces I and II (k1, m1, k2, m2, k3, m3, k4). By transformations the design scheme on fig. 6 and can be transformed to system with one degree of freedom containing (fig. 6 b) the object of protection leaning against the general basic surface I, II. Thus the system is led to a kind of parallel connection from two elastic elements. One of them is a spring with stiffness k4, and the second represents a mechanical chain with the coerced stiffness {{}}.
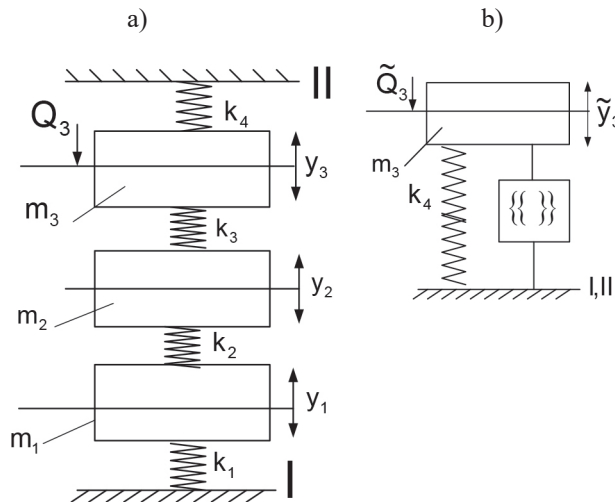
a)   b)

Fig. 6. Principal scheme of formation of coerced stiffness on design scheme (a) at force disturbance $\tilde{Q}_3$ on protection object; b) coerced scheme

The coerced stiffness {{}} in this case is formed in the following sequence of actions:

$$\left(m_1 p^2\right) \rightarrow \left[\frac{\left(m_1 p^2 + k_1\right)k_2}{m_1 p^2 + k_1 + k_2}\right] \rightarrow \left\{\left[\ \ \right] + m_2 p^2\right\} \rightarrow \left\{\left\{\frac{\left\{\ \right\}k_3}{\left\{\ \right\} + k_3}\right\}\right\} \tag{36}$$

which leads to equation (34).

On fig. 7 a, b it is shown, how the coerced stiffness is formed, that is the factor of stiffness of the generalized spring (34) is defined. It is connected with identification of equivalent external force $\tilde{Q}'_1$ under the formula (25) that is resulted on fig. 4.
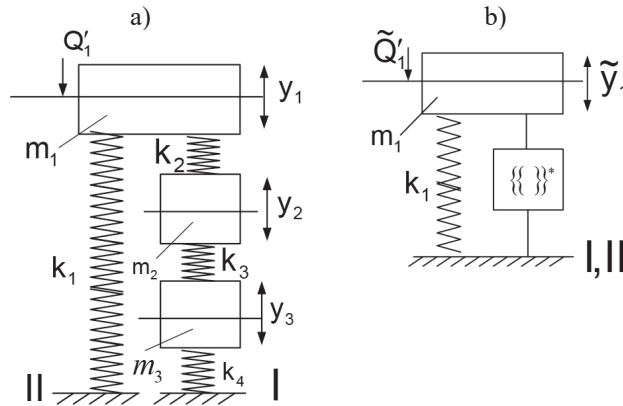

Fig. 7. Principal scheme of formation of coerced stiffness on design scheme (a) at equivalent force disturbance $\tilde{Q}'_1$ on element m_1; b) coerced scheme

At identification of the coerced stiffness of the generalized spring $\{\{\}\}$ ** (fig. 7 b) is used the following sequence of operation of convolution of structure

$$\left(m_3 p^2 + k_4\right) \rightarrow \left[\frac{\left(m_3 p^2 + k_4\right)k_3}{m_3 p^2 + k_3 + k_4}\right] \rightarrow \left\{\left[\ \ \right] + m_2 p^2\right\} \rightarrow \left\{\left\{\frac{\left\{\ \right\}k_2}{\left\{\ \right\}^* + k_2}\right\}\right\}^* \tag{37}$$

which leads to equation (35) that is schematically represented on fig. 7 b. The general stiffness is defined by summation k1 and $\{\{\}\}$ *.

As to identification of the coerced stiffness connected with an intermediate element m2, equivalent force (fig. 5) it is defined by equation

$$\tilde{Q}'_2 = \tilde{Q}_3 \frac{k_2 k_5 + k_3\left(m_1 p^2 + \quad + k_1 + k_2 + k_5\right)}{\left(m_1 p^2 + k_1 + k_2 + k_5\right) \times \left(m_3 p^2 + k_2 + k_3\right) - k_5^2} \tag{38}$$

At the further calculations it is accepted k5 = 0.

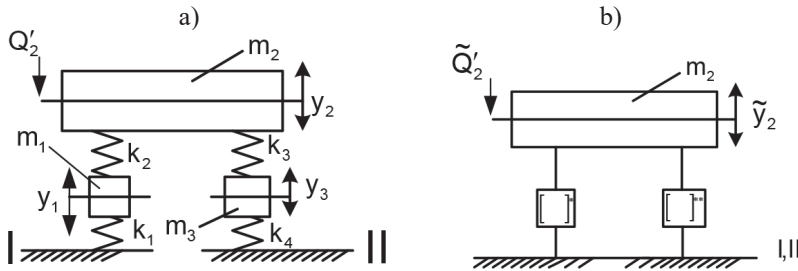The basic design scheme and reduction possibility to the simplified kind are reflected on fig. 8 a, b.

Fig. 8. Principal scheme of formation of coerced stiffness of generalized spring on design scheme (a); b) coerced scheme

In this case equivalent external force $\tilde{Q}'_2$ is defined from equation (36). In turn the general factor of stiffness is formed by the sum of two stiffness's []* and []**(fig. 8 b), on the basis of realization of following sequences of actions:

$$\left(m_1 p^2 + k_1\right) \rightarrow \left[\frac{\left(m_1 p^2 + k_1\right)k_2}{m_1 p^2 + k_1 + k_2}\right]^* \text{ and also } \left(m_3 p^2 + k_4\right) \rightarrow \left[\frac{\left(m_3 p^2 + k_4\right)k_3}{m_3 p^2 + k_3 + k_4}\right]^{**}, \tag{39}$$

that finally, leads to equation (38).

Thus, definition of dynamic responses is connected with an estimation resulted stiffness's the generalized springs which create dynamic the rigidity, depending on frequency of external revolting influences. Two approaches based on the offered method of formation of a chain of return negative communication on allocated mass-inertial an element, and also on procedures of consecutive convolution with use of initial settlement schemes (a Fig. 6a, 7a, 8a) are shown. We will notice that convolution is simple enough only for planar systems, therefore in this case was accepted that.

When non-planarity it is necessary to take into consideration, it is necessary to use a number of preliminary transformations that complicates a technique of definition resulted жесткостей, but leads to similar results.

Conclusions. Thus a basis of a method of definition of dynamic reactions on object of protection, and also on inertial elements vibroprotection devices (ВЗУ) and on basic surfaces I and II is transformation of the block diagramme of system to a kind corresponding to a link of integration of the second order with formation of additional negative feedback which, inherently, and represents dynamic reaction. In system with three degrees of freedom such transformations become in case of need not only for object of protection, but also for intermediate weights. The method is applicable and for systems with a great number of degrees of freedom, however, thus there is an appreciable growth of labour input of calculations. The structure of additional negative feedback can be received more simple way if to use the characteristic equation of system entering into corresponding transfer function. The equation is transformed with allocation necessary partial systems with the subsequent transformations of structure.

At construction of the resulted block diagrams carrying over of the force enclosed initially to object of protection, to an intermediate inertial element can be carried out. Such force is equivalent in the sense that the transfer functions received with use, at carrying over of force to a new point yield that result, as at a usual order. The offered method allows to receive transfer function of dynamic response of an element or a support by multiplication of the corresponding resulted factor of rigidity of the generalized spring (or dynamic rigidity) on corresponding displacement of an element in system of co-ordinates.

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