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# PROTON STRUCTURE FUNCTION CALCULATION BY THERMODYNAMICAL BAG MODEL

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## Abstract

This paper focuses on finding proton structure functions in deep inelastic scattering of leptons on nucleons by MIT Bag model. This model proposed by V. Devanathan and S. Karthiyayini assumes that nucleon is a hot bag containing quarks, which interact with bosons. The nucleon structure function is then expressed in terms of Parton distribution functions where both are functions of Bjorken variable $x$ only. The structure functions calculated by this model are found to be in good agreement with the data obtained from CERN for Bjorken variable $x$ greater than 0.2 only.

***Keywords**:* Fermi and Bose statistical distribution functions, Deep inelastic scattering, Quarks and gluons, Quark distribution functions, Nucleon structure functions.

## 1. INTRODUCTION

In the period of late 1960's, experiment's conducted at Stanford linear accelerator center (SLAC) in which leptons in which leptons i.e. electrons scattered the proton targets with various energy. The results of various such experiments rendered evidence of quarks.

The deep inelastic collision of electron-proton generates a variety of final state hadrons. These hadrons comprises of various types of quarks, antiquarks and gluons which gives strong interactive force; that binds protons inside the nuclei of atoms.

The deep inelastic scattering (DIS) of leptons (electrons or muons) on nucleon; clearly indicate that nucleon is a composite particle known as Partons. Partons are quarks, which interact among themselves by exchange of bosons known as gluons. [10-18]

Parameterization can be done for a particular set of experimental values, but only a model can be universally be used to check with all published experimental values.

It is the purpose of this paper to show that our model (Thermodynamical bag model) explains well the structure of nucleon as our data agrees with the published experimental data. The DIS is related by nucleon structure functions which in turn are expressed in terms of quark distribution functions in the statistical model. The comparison of the results generated from Parton model and experimental results from DIS scattering confirm the thermodynamical bag model explained in this paper.

## 2. THE THERMODYNAMICAL BAG MODEL

The DIS is often characterized by the variables $Q^2 = (-q^2)$ and x which denote the square of the four momentum transfer and Bjorken variable which implies the fraction of nucleon momentum carried by the struck quark. The formula of the variables is given below

$$\upsilon = \frac{q \cdot P}{M} = E - E'$$

$\upsilon$ is the lepton's energy loss in the nucleon rest frame. Here, E and E′ are the initial and final lepton energies in the nucleon rest frame.

$$Q^2 = -q^2 = 2\left(EE' - \vec{k} \cdot \vec{k'}\right) - m_\ell^2 - m_{\ell'}^2$$

$$\approx 4EE' sin^2(\theta/2) \gg m_\ell^2 \; and \; m_{\ell'}^2$$

$$\approx 4EE' sin^2(\theta/2)$$

θ is the lepton's scattering angle with respect to the lepton beam direction.

$$x = \frac{Q^2}{2M\upsilon}$$

In the Parton model, x is the fraction of the nucleon's momentum carried by the struck quark.

This model assumes that the nucleon comprises of three particles, the valance quarks, sea quarks and gluons, confined in a bag of small volume. It is known that quark structure of proton and neutron are as follows

Proton: u u d + quark-antiquark pairs + gluons

(Valance quarks) (Sea quarks)

Neutron: u d d + quark-antiquark pairs + gluons

(Valance quarks) (Sea quarks)

In this statistical model for nucleon, a MIT bag consist of a quark-gluons in which Fermi distribution function depicts the quarks & the Bose distribution function denotes the gluons. The Fermi distribution function gives the number density of up quarks with momentum lying between P and P + dP at temperature T. We assume that quarks are particles of zero rest mass. The equations of state of a proton are:

$$\varepsilon(T)V + BV = W$$

$$n_u - n_{\bar{u}} = 2/V = \mu_u T^2 + \mu_{\bar{u}}^3/\pi^2$$

$$P = (1/3)\varepsilon(T) - B = 0$$

$$n_d - n_d = 2/V = \mu_d T^2 + \mu_d^3/\pi^2$$

The energy density $\varepsilon(T)$ is given by

$$\varepsilon(T) = \varepsilon_u + \varepsilon_{\bar{u}} + \varepsilon_d + \varepsilon_{\bar{d}} + \varepsilon_g$$

$$= \frac{3}{4\pi^2}\left(\mu_u^4 + \mu_d^4\right) + \frac{3}{2}T^2\left(\mu_u^2 + \mu_d^2\right) + \frac{37}{30}\pi^2 T^4$$

It is to be noted that these equations of state are a function of temperature. The greatest advantage of bag model is that it gives the value of the potentials and temperature not only at the ground state (T=0) but also in the excited state at higher temperature. In this manner, it truly describes the thermodynamical bag of nucleon. In the above equations B denotes the bag constant &V indicates the value of the bag. The mass of the nucleon M in the thermodynamical bag model corresponds to T=0 & W denotes the invariant mass of the excited nucleon at some higher temperature T. These equations are solved by Newton's Raphson method to obtain the Parton distribution functions.

ISSN: 2394 – 4854
Pages: 8 – 13

## 3. STATISTICAL DISTRIBUTION FUNCTIONS

The statistical distribution function when transformed to the infinite momentum frame yields the quark distribution function. This has been done by Cleymans and Thews, Mac and Ugaz [3, 4] and Devanathan et al. [5-9]. The distribution functions are given as follows:-

$$u(x) = 2Aln[1 + \exp\left\{\frac{\mu_u - \frac{1}{2}xM}{T}\right\}]$$

$$d(x) = 2Aln[1 + \exp\left\{\frac{\mu_d - \frac{1}{2}xM}{T}\right\}]$$

$$\bar{u}(x) = 2Aln[1 + \exp\left\{\frac{\mu_u - \frac{1}{2}xM}{T}\right\}]$$

$$\bar{d}(x) = 2Aln[1 + \exp\left\{\frac{\mu_d - \frac{1}{2}xM}{T}\right\}]$$

$$g(x) = -\frac{16}{3}Aln[1 - \exp\left\{-\frac{1}{2}\left(\frac{xM}{T}\right)\right\}]$$

$$A = \frac{3M^2VxT}{4\pi^2}$$

These statistical functions depend on parameters like potential of quarks, Temperature and Volume of the bag. It is to be noted that these are not free parameters as invariant mass W is calculated for a given Bjorken variable x by solving equations of state by Newton's Raphson method for an excited nucleon. In statistical functions u quark dominates over d quark whereas for antiquark the opposite is observed. The dominance of anti d quarks over anti u quarks explains the Gottfried Sum Rule (GSR).[19-26] This feature is explained as the $\mu_u$ is greater than $\mu_d$ as a result of 2 valance up quarks and 1 valance down quark for a proton. The figure below denotes the quark distribution function as a function of x.

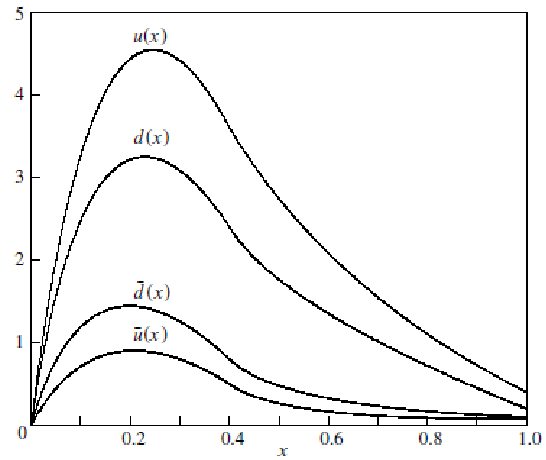



**Figure: 1** The quark distribution functions as a function of x, obtained using the parameters defined

For finding the nucleon structure functions we only consider the light quarks u, d, s and their respective anti quarks. In this model, we assume hadron is in a reference frame where it has an infinite momentum which is valid for high energies. The Parton model is slowed by time dilation due to which the incoming particles are scattered "instantaneously and incoherently" [27-31].

In the Infinite momentum frame (IMF) u(x)dx indicates the probability of finding the u quark carrying the momentum lying between x and x+ dx of the total nucleon momentum. The set of equations below depict's a proton.

$$\int_0^1 [u(x) - \bar{u}(x)]dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)]dx = 1$$

$$\int_0^1 [s(x) - \bar{s}(x)]dx = 0$$

The proton structure, function and the relation between the both structure functions are given below:-

$$F_2^p(x) = x\left[\frac{4}{9}\left(u(x) + \bar{u}(x)\right) + \frac{1}{9}\left(d(x) + \bar{d}(x)\right) + s(x) + \bar{s}(x)\right]$$

$$F_2(x, Q^2) = 2xF_1(x, Q^2)$$

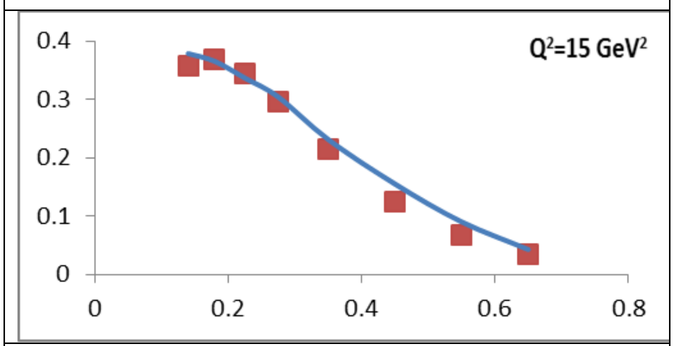
ISSN: 2394 – 4854
Pages: 8 – 13

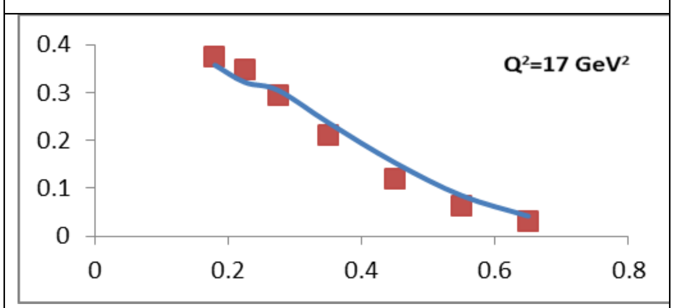
The calculated proton structure functions are then compared with the experimental data published by CERN and tabulated in table 1 for various $Q^2$ values[2]. The corresponding graphs of all tables are shown in graph 1 to 6 respectively.

| $Q^2$ | x | $F_2$ | $F_2$ calculated | Percentage Error |
|---|---|---|---|---|
| ($GeV^2$) | | R=QCD | | |
| 15 | 0.14 | 0.37847 | 0.35701 | -0.05667 |
| 15 | 0.18 | 0.36523 | 0.36750 | 0.00621 |
| 15 | 0.225 | 0.33659 | 0.34430 | 0.02292 |
| 15 | 0.275 | 0.30414 | 0.29607 | -0.02652 |
| 15 | 0.35 | 0.23122 | 0.21388 | -0.07496 |
| 15 | 0.45 | 0.15503 | 0.12409 | -0.19954 |
| 15 | 0.55 | 0.09002 | 0.06715 | -0.25402 |
| 15 | 0.65 | 0.04265 | 0.03522 | -0.17398 |
| 17 | 0.18 | 0.35727 | 0.37458 | 0.04847 |
| 17 | 0.225 | 0.32138 | 0.34752 | 0.08136 |
| 17 | 0.275 | 0.30467 | 0.29553 | -0.02997 |
| 17 | 0.35 | 0.23727 | 0.20988 | -0.11542 |
| 17 | 0.45 | 0.15335 | 0.11900 | -0.22398 |
| 17 | 0.55 | 0.08473 | 0.06291 | -0.25745 |
| 17 | 0.65 | 0.04167 | 0.03223 | -0.22634 |
| 19 | 0.225 | 0.32395 | 0.35045 | 0.08182 |
| 19 | 0.275 | 0.29035 | 0.29511 | 0.01642 |
| 19 | 0.35 | 0.223 | 0.20647 | -0.07412 |
| 19 | 0.45 | 0.15573 | 0.11471 | -0.26338 |
| 19 | 0.55 | 0.08291 | 0.05940 | -0.28348 |
| 19 | 0.65 | 0.04151 | 0.02980 | -0.28205 |
| 21.5 | 0.225 | 0.31436 | 0.35376 | 0.12535 |
| 21.5 | 0.275 | 0.29911 | 0.29469 | -0.01476 |
| 21.5 | 0.35 | 0.23275 | 0.20279 | -0.12869 |
| 21.5 | 0.45 | 0.14488 | 0.11017 | -0.23953 |
| 21.5 | 0.55 | 0.08285 | 0.05576 | -0.32697 |
| 21.5 | 0.65 | 0.04012 | 0.02731 | -0.31914 |
| 24.5 | 0.225 | 0.32498 | 0.35721 | 0.09918 |
| 24.5 | 0.275 | 0.28068 | 0.29424 | 0.04833 |
| 24.5 | 0.35 | 0.22389 | 0.19901 | -0.11108 |
| 24.5 | 0.45 | 0.14465 | 0.10561 | -0.26988 |
| 24.5 | 0.55 | 0.0852 | 0.05216 | -0.38768 |
| 24.5 | 0.65 | 0.04117 | 0.02491 | -0.39475 |
| 24.5 | 0.75 | 0.01357 | 0.01190 | -0.12282 |
| 28 | 0.225 | 0.31917 | 0.36073 | 0.13022 |
| 28 | 0.275 | 0.29223 | 0.29374 | 0.00519 |
| 28 | 0.35 | 0.22548 | 0.19523 | -0.13414 |
| 28 | 0.45 | 0.14024 | 0.10115 | -0.27870 |
| 28 | 0.55 | 0.08108 | 0.04874 | -0.39878 |
| 28 | 0.65 | 0.03871 | 0.02268 | -0.41399 |
| 28 | 0.75 | 0.01366 | 0.01054 | -0.22837 |

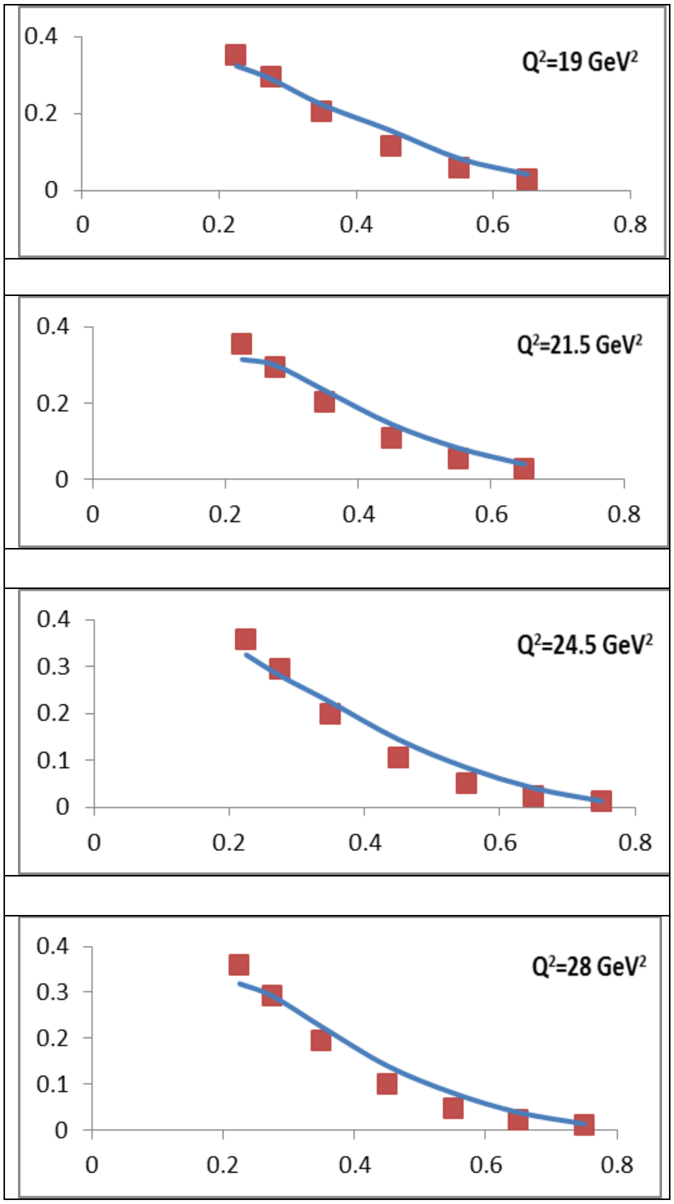
**Table: 1** Table of x and structure function values

**Graph between $F_2$ vs x**
**Blue** –Experimental value And **Red**-Calculated value



ISSN: 2394 – 4854
Pages: 8 – 13




**Graph: 1** Graph between $F_2$ vs x

## 4. CONCLUSION

The thermodynamical bag model discussed in this article does not involve any free parameters. All the parameters have been solved by determining the equations of state of proton considered as an MIT bag consisting of quarks and gluons. These equations are solved in Java language and the results are verified. A good agreement with the published data confirms the validity of the thermodynamical bag model of the nucleons. It is noted that our calculations are agreeing very well for values of Bjorken variables of $x \geq 0.2$. Hence any further analysis is to be extended for low values of x.

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