

Information Transfer in Quantum Mechanics

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(Received 14 March 2015, Published 15 April 2015)

Abstract

This paper deals with the most elementary information transmission in Quantum Mechanics. A simple quantum mechanical system under Coulomb-type potential is investigated. Like the classical case, a deep relation between the potential energy and the rate of information transfer is established for quantum-mechanical situation. The corresponding equation is presented. The article shows the circumstance for the free particle, too.

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Keywords: Information transmission, Quantum Mechanics

1. Information in physics literature

In a previous article by the present author [1] the most elementary role of the information is investigated. For the meaning of information in everyday usage, and in Philosophy, a survey is presented in the referred paper, and the articles cited herein [2-7]. Therefore, in the rest of this paper, we restrict ourselves to the physical information. It is shown that information, in its most natural form, has a close relationship with action. We extend the findings of the previous paper, namely the “action-information equivalence” to the quantum mechanics.

1.1 Essential Questions

If we want to investigate the role of information in physics, forgetting the scientific literature for a moment, the following questions arise:

- Is information physical? In other words: can any information originate, be transferred, and received outside the physical world?
- if information is physical, the next question follows automatically: is the information related to a definite, already known physical quantity, (see Ref [8]), perhaps to several quantities depending on the situation, or must be newly defined, as a new notion, as some authors argue (e. g. Ref. [9])?
- Can we derive the laws of physics, based on information as a guiding principle?

As we will see, a wide variety of conceptions can be found even among physicists. Some of these theories are widely accepted, while others are matters of dispute. In the followings, the main tendencies will be shortly outlined.

1.2 Information and Energy

An interesting question is the relation between information and energy. Some authors have the opinion that information and energy are unrelated [10], while others find a close analogy between the two quantities [11]. S.

A. Umpleby sees the energy, matter and information as three main building blocks [12]. One celebrated formula, derived by Hans-Joachim Bremermann refers to the speed of information transfer and states that this speed is 1.36×10^{50} bits per second per Kilogram mass [13, 14]. Bremermann used quantum oscillators in his derivation. He modeled a communication channel, thus it was possible the use Shannon's equations to get bits instead of physical quantities. Maybe a better, but less elegant relation could be constructed, if we replaced the mass with energy. Note that time derivative of the information plays here a role, not the information itself. (A remark: bit is the unit of all type of information. Therefore, the amount of information can be calculated in non-binary decisions, too. The best example is the Shannon entropy, which is rarely an integer in bit units.)

In the latest time, an experiment has been published, which reputedly proves the information-to-energy conversion [15]. However, no further verification is known. As we see, there is no consensus among physicists about the role of energy in information science.

1.3 Information and Entropy

The relation between information and entropy is a well-studied, but even today not a fully solved problem. The first scientist was Ludwig Boltzmann, who discovered the relation between the thermodynamic entropy and the number of microstates within a closed physical system [16]. His formula is the well-known

$$\mathbf{S} = \mathbf{k} \log(\mathbf{W}) \quad (1)$$

where \mathbf{S} is the entropy, \mathbf{k} the Boltzmann's constant, \mathbf{W} is the count of microstates, supposed to be equally probable. There was no reference to information. However, it is easy to insist, if the number of inner microstates is higher, the knowledge (can be called information) of an outer observer on the exact state of the system is less. The logarithm is arising from the required additivity of the information for independent systems vs. the multiplicative property of the probability.

Based on methods of Statistical Mechanics, Gibbs [17] developed a similar formula for microstates with unequal probabilities:

$$\mathbf{S} = -\mathbf{k} \times \sum_i \mathbf{p}_i \times \log(\mathbf{p}_i) \quad (2)$$

where \mathbf{p}_i is the probability of the i -th microstate.

Interestingly, Hartley [18] and later Shannon [19] developed the same two formulas in the same historical order within the theory of communication. Moreover, Shannon, on advice from J. von Neumann, named his own expression „entropy”, seemingly narrowing further the gap between the conceptions of these quantities. Attracted by the apparent similarity, amended with theoretical considerations, some authors presumed a close relation between the two types of entropies (Communication and Thermodynamics). The most direct statement is to affirm the equality of these; two remarks are to be added: - As the thermodynamic entropy may have an indefinite additive constant, the equality could exist in differential form only.

- As the increasing entropy means less information, the entropy and information have an inverse relation.

One of the first scientists representing this theory was G.N. Lewis, who wrote: “gain of entropy eventually is nothing more nor less than loss of information [20]”. Another famous researcher was L. Brillouin, who introduced the term “negentropy” as the negative of entropy [21], which, according to him, is in direct relation to the information. These statements are simple, but suffer from several contradictions, as noticed by some authors, e. g. [22]. The Shannon entropy needs to define a priori signal probabilities, but does not refer to the character of the source or the receiver. In contrary, the Gibbs entropy refers to a property of a definite physical system (although no “entropymeter” is available). Information must exist under non-equilibrium circumstances too, but Gibbs entropy is hard to define in this case [23]. The thermodynamic entropy is closely related to the

temperature by the $dE = T \times dS$ relation, but the temperature is a statistical property. Its applicability for extremely small systems is widely disputed in the scientific literature [24, 25].

As we see, there are three different quantities: the Shannon entropy, the thermodynamic entropy and the physical information. In order to prove the connection among these quantities, several scientific papers have been published. These contain either re-definition of some older theories or try to find new, information-oriented principles. We may remember that even Brillouin “softened” his strict negentropy theory in his later works by writing about “bound” and “general” information [26] Just to mention a few works: Efforts have been made to define the entropy in non-equilibrium state [27]. According to some authors, despite heavy debates, the Shannon entropy must be replaced by other formula for the quantum physics [28]. In some papers the concept of temperature is extended to the case of a single particle, to be able to use the equation of the thermodynamic entropy [29].

In the present paper, I concentrate to define the information in physics in the most general form, independently from the other two quantities.

1.4 Information, Entropy and Action

As mentioned the majority of the papers on physical information are trying to connect the information in Physics to the concept of entropy. However, some authors claim to find a close relationship between the action and information or action and entropy. An interesting paper has been published to prove the action-entropy equality using black hole considerations [30]. However, the author’s aim is not to define physical information, but to find a new, action-based expression for entropy.

A direct comparison between action and information can be found in the paper of John L. Haller Jr., [31], who writes, (in conjunction with the present author): “Over a period of time, the energy of a system acts like an information rate and thus the information need to describe that system for that period of time is equal to the product of the energy and the time divided by the minimum uncertainty.” Unfortunately, J. L. Haller makes an unjustified mathematical step in his derivation. He uses the von Neumann entropy as starting point, but then arbitrarily changes the sign of one of two terms, which normally cancel out each other, getting a time-linear term thereby. *If instead of letting $-i\epsilon t/\hbar$ cancel itself out with its complex conjugate, let’s see what happens if we take the magnitude of both the positive and negative entropy rates add them together.* The aim of Haller’s step is clear, albeit erroneous: to make the von Neumann entropy time-dependent, in order to prove the action-information relationship. Apart from this faulty operation, the article contains some useful and convicting examples. As we shall see below, the action-information relationship is much closer, and easier to derive.

1.5 Information as Building Block

All the papers we have referred to in the previous Sections deduce the physical information from a definite, already defined physical quantity. However, there are also theories, which handle the information as a primary quantity. One of the well-known theories is signed as “It from bit” by John Archibald Wheeler. He says in his biography [32], *“I think of my lifetime in physics as divided into three periods. In the first period, extending from the beginning of my career until the early 1950’s, I was in the grip of the idea that Everything Is Particles. ...I call my second period Everything Is Fields....Now I am in the grip of a new vision, that Everything Is Information”.*

There are some other ultimate information-oriented principles like Computing Universe and so on. According to my opinion, these may be elegant philosophical ideas, or useful tools in computer science, but cannot have an important contribution to the progress of physics. Some authors claim to be able to re-develop the whole Physical Science, using the information as a single guideline. Among them the best known is B. Roy Frieden, with the “Extreme Physical Information (EPI)” [33]. He uses a non-Shannon type (Fisher) information

and an extremalization principle to construct the Lagrange function, and then the basic equations. However, according to some critics, e. g. [34] this work cannot stand a rigorous proof, because of its ad hoc methods and backward derivations.

The goal of the present paper and the future works is not to create new physical principles, but only to derive equations which show the role of the information in different fields of physics.

2. Elementary Information

2.1 Basic Considerations

In the following paragraphs we shall state shortly the considerations presented in Ref. [1]. They are necessary to follow the statements of the present paper.

If we are looking for the basic expressions, which cover the relationship between the physical information and other, well-established physical quantities, we have to follow the principles listed below:

- We must not settle down to any predetermined formula or probability set, we should start with strictly physical considerations, and let the mathematics work thereafter. (We know great many formulas, such like Shannon, Kullback, Rényi entropies, Fisher information etc.). We are not searching only for “useful”, or “valid” information, all kind of information must be included, as Brillouin correctly states [8] p. 12.: *Any notion of human value are totally excluded: moral qualities, and intellectual or artistic values are totally absent. There is actually no human element introduced in this definition and our “information” should never be confused with “science” or “knowledge”.*

- We should treat the information dynamically, i.e. not neglecting its time dependence.

As this approach is quite new in the literature, we have to start with the simplest case, continue with the next step, and look for possible generalization thereafter.

2.2 Simplest Case: Free Particle in Classical Mechanics

Let us start with the thinkable simplest case: a single free point particle, obeying the laws of the non-relativistic classical mechanics.

We could think: the particle moves with a constant velocity, nothing happens. However, the particle produces the space-time. Without it, in a pure vacuum, no time, no space would exist. That is, if we see a definite time interval, something happened, namely, the particle moved (say) from point A to point B. Therefore, we may define the amount of “occurrence”. If somebody asks: “What happened with our particle?” The answer is: “It moved from A to B”. Here, an event happened, which can be an object of a question and an answer. This suggests that a freely moving particle produces information. Because as it moves the particle continues to produce answers to subsequent questions, the information increases with time.

The next question is: how to define a measure for this information? The following considerations (requirements) may help:

- a) The information must be proportional to the particle’s mass: two independent particles with half mass should produce the same information, if they run the same path. More generally, the information must be additive for independent systems. This condition is not new, if we see the known entropy formulas.
- b) The information must be proportional to time (and path length) at constant velocity ($v = \text{const}$). Double path means double amount of question-answer pairs. In this picture, the particle is alone. If we accept Rietdijk’s theory [35], then there is no coordinate system, no reference frame. However, in classical mechanics, it is usual to set up spatial and time coordinates. This means in our case, that the motion of the particle is treated in a reference frame, which can be another particle, can be called “Observer”. So, the information no longer represents the particle alone, but the particle and the reference together. For example, the velocity is not absolute, but referenced to the coordinates. In other words, the information is not

expected to be invariant in Galilean transformations. Because of the symmetry of the space, the information must not depend on the direction; it may be dependent of the absolute value of the velocity only. (We must not forget that all velocities are taken relatively to the frame.) Therefore, it is enough to carry on the calculations for one dimension.

c) Requirement-(b) means that the information consists of a function \mathbf{F} of the velocity, multiplied with the path length (s), the mass \mathbf{m} included:

We may expect that a resting particle produces no information: $\mathbf{F}(\mathbf{v}) = 0$ if $\mathbf{v} = 0$. In order to determine the function \mathbf{F} , imagine a particle \mathbf{P} with mass \mathbf{m} , velocity \mathbf{v} , executes a path length s . The produced information is then,

$$\mathbf{I} = \mathbf{m} \times \mathbf{F}(\mathbf{v}) \times s \quad (3)$$

Including some other, purely physical considerations, we get

$$\mathbf{I} = \mathbf{C} \times \mathbf{A} \quad (4)$$

where \mathbf{A} denotes the action, \mathbf{C} is a scaling constant. (The detailed derivation can be found in Ref. [1])

The constant \mathbf{C} may depend on the boundary conditions, if we want to get the information in *bits*. We see that according to Eq. (4), a free particle produces information, which is proportional to its action. It is interesting to note from Eq. (3) that with increasing velocity a shorter path length needed for producing the same amount of information. The above conditions and the conclusion may seem arbitrary, but there are other authors, who also argue for the action as a measure for “occurrence” or “happening”. A. Faigón, in an interesting paper [29], derived the Lagrange function of the Classical Mechanics by using the $\mathbf{p}\delta\mathbf{q} = \mathbf{const}$ condition only, where \mathbf{p} is the momentum, $\delta\mathbf{q}$ is the uncertainty in the coordinate. We see again the connection between action and occurring. He then extends the theory to the thermodynamics, by making $\delta\mathbf{q}$ dependent of volume of the enclosure. Faigón thereafter tries to define the information of a single particle, but supposes the information being constant instead of increasing. According to some authors, action is the only building block in the physical science. With C.W. Rietdijk [35], *Now we put the hypothesis that the quantum of action, in such world, is simply a universal "atom of occurring", the most elementary process from which all other processes are built up.*

There are other theories, which also handle the action as the basic element. R. R. Sharma traces back all kind of interactions to the so called “Null Action” principle [36]. Another researcher, R. Catania defines the “Boolean Observer”, as cited [37]: *Thus, the maximum information I which can be gained by said observer, whenever an action $A > \hbar$ is involved, is: $I = A/\hbar$ or, the minimum action A required to acquire I bits of information is: $A = I/\hbar$ Relation (1) says that, in the ideal case of maximum (Boolean) observation efficiency, the number of action quanta equals the number of information bits.*

I believe, that by Eq. (4) a straight relationship is established between the action and information, which is more direct than the above examples, it makes it unnecessary to involve entropy, temperature and other factors. For this simple case is true: “information is action”. With our results we can reformulate the well-known “minimal action” rule: “a free particle moves from \mathbf{A} space-time point to \mathbf{B} space-time point so, that it produces minimal information”

A question arises: what is the unit of this information. Action is quantized, unit is the Planck’s constant, and so the information is quantized too. However, one would rather see bits instead of action quanta. We know from mathematics, that all information formulas need a set of probabilities. Roughly speaking, a probability is a quotient: real cases divided by the possible cases. We are now in an inverse situation; we know the information (albeit unscaled yet), and looking for the probabilities. This objective is not hopeless, but an exhaustive investigation exceeds the range of the present paper.

2.3 Coulomb Force

Until this point we treated free particles only. But the world does not consist of free particles. When particles interact, they surely transfer information to each other. When we try to calculate the information transfer quantitatively, we should free ourselves from all preconceptions, all information must be included.

In this Section we investigate the simple, but important type of interactions: the Coulomb-type potential. It covers the electrostatic and gravitational forces. According to present-day science, the interaction is mediated by “virtual” particles. These particles are totally annihilated during the interaction; therefore they are undetectable outside the process. The relationship between the attributes of the virtual particles and the macroscopically measurable forces was investigated by R.C. Harney [38]. His theory was later refined and corrected by C. W. Rietdijk [39], eliminating the unnecessary “borrowing energy from God” hypothesis. We accepted Rietdijk conception, and applied it to the information transfer. The detailed calculation can be read in Ref. [1].

Let the distance between the particles \mathbf{r} , then the flying time ($\Delta\mathbf{t}$) of the photon between the particles is

$$\Delta\mathbf{t} = \frac{\mathbf{r}}{\mathbf{c}} \quad (5)$$

Where \mathbf{c} = velocity of light.

During this time the photon annihilates. Therefore by using the expression $\Delta\mathbf{E} \times \Delta\mathbf{t} = \hbar$, which is valid for photons, the energy of the photon is,

$$\Delta\mathbf{E} = \frac{\hbar}{\Delta\mathbf{t}} = \frac{\hbar\mathbf{c}}{\mathbf{r}} \quad (6)$$

The photon’s momentum is,

$$\Delta\mathbf{p} = \frac{\hbar}{\mathbf{r}} \quad (7)$$

Therefore, if we want to change the particle’s momentum during a time interval of $d\mathbf{t}$, a following equation is valid:

$$\frac{d\mathbf{p}}{d\mathbf{t}} = \frac{d\mathbf{N}}{d\mathbf{t}} \times \frac{\hbar}{\mathbf{r}} \quad (8)$$

where $d\mathbf{N}$ is the count of absorbed photons during $d\mathbf{t}$.

There is a strong reason to see the photon “ticks” as information units. Therefore the $\frac{d\mathbf{N}}{d\mathbf{t}}$ can be called “information per time unit”, signed as $\frac{d\mathbf{I}}{d\mathbf{t}}$.

On the other hand, $\frac{d\mathbf{p}}{d\mathbf{t}}$ is the Coulomb force, which is,

$$\mathbf{F} = \frac{\mathbf{k}_e \times \mathbf{q}_1 \times \mathbf{q}_2}{\mathbf{r}^2} \quad (9)$$

Combining Eqs. (8) and (9) gives

$$\frac{d\mathbf{I}}{d\mathbf{t}} = \frac{\mathbf{k}_e \times \mathbf{q}_1 \times \mathbf{q}_2}{\hbar \times \mathbf{r}} \quad (10)$$

Considering the potential energy:

$$\mathbf{U} = \frac{\mathbf{k}_e \times \mathbf{q}_1 \times \mathbf{q}_2}{\mathbf{r}} \quad (11)$$

With (10) we get:

$$\frac{d\mathbf{I}}{dt} = \frac{\mathbf{U}}{\hbar} \quad (12)$$

This extremely simple expression means that in the case of Coulomb interaction, one of the particles gets information from the other particle. The information per unit time depends on the Coulomb potential, \hbar being a scaling factor. Some remarks:

- Normally, we are used to the fact that the potential energy may contain an arbitrary additive constant. However, we need here the unbiased value, implicitly fixed for $\mathbf{U}=0$ at infinitive distance.
- The formula is valid for both repulsive and attractive forces. Therefore, to get positive information, the factor for the potential energy in Eq. (12) must be replaced for its absolute value.

$$\frac{d\mathbf{I}}{dt} = \frac{|\mathbf{U}|}{\hbar} \quad (13)$$

This means interestingly, that the information may flow in opposite direction to the energy in some cases. This statement has perhaps a deeper philosophical significance. Eq. (13) means, that the information rate is proportional to the absolute value of the static potential. Both of the interacting particles get positive information (there is no negative information). A very useful feature of the formula in Eq. (13) is that it contains the potential energy only, but not the distance between the interacting particles. We will use this fact below.

3. Quantum Mechanics

Based on the above considerations, now we are able to extend our investigations to the quantum region. We restrict our interest to spinless, non-relativistic particles. Spinning particles are the building elements of the standard quantum information theory; the treatment of the connection between the two fields needs a separate article. Referring the relativistic case, some problems must be solved, because some authors found a disagreement between the de Broglie mechanics and the Einsteinian Relativity [15, 46].

We describe two scenarios: free particle and particle moving under scalar potential.

3.1 Free Particle

The expression for the information production of a free classical particle (Eq. (3) and 4)) can easily be extended to the quantum mechanical case: a moving body produces information; the rate of production is proportional to the momentum. The wave function of a free particle is the following:

$$\psi(\mathbf{x}, t) = \mathbf{A} e^{i\mathbf{k}(\mathbf{x}-\mathbf{u}t)} \quad (14)$$

where \mathbf{k} is the wave number $\left(\mathbf{k} = \frac{2\pi}{\lambda}\right)$, \mathbf{u} the phase velocity. The wavelength is inversely proportional to the momentum.

$$\lambda = \frac{2\pi\hbar}{\mathbf{p}(\equiv \mathbf{m} \times \mathbf{v})} \quad (15)$$

The action is simply equal to $\mathbf{A} = \mathbf{m} \times \mathbf{v} \times \mathbf{s}$ just like in the classical case. Let us calculate the necessary path length to produce a given quantity, say $2\pi\hbar$ action (information).

$$\mathbf{m} \times \mathbf{v} \times \mathbf{s} = 2\pi\hbar \Rightarrow \mathbf{s} = \frac{2\pi\hbar}{\mathbf{m} \times \mathbf{v}} = \lambda \quad (16)$$

From Eq. (16) we can conclude the following: *a freely moving particle, whenever it travels a path length of one de Broglie wavelength, it produces action (information) of one Planck constant.*

3.2. Quantum particle with scalar potential

Let us have a non-relativistic quantum particle. For simplicity, let us write the relating Schrödinger equation in one dimension, with the usual notions:

$$\left[\frac{\hbar}{i} \frac{\partial}{\partial t} - \frac{1}{2m} \left(\hbar \frac{\partial}{\partial x} \right)^2 + U(x, t) \right] \psi(x, t) = 0 \quad (17)$$

We have seen in the classical case, that a particle gets information from the potential (Eq. 13). Moreover, the information rate is proportional to the potential energy.

The classical point-like particle has a definite location. However, in quantum mechanics, the uncertainty relation usually forbids to locate the particle exactly. (We do not want to decide, which theory is right: is the particle spread or is it point-like with indefinite location) However, one fact helps us to generalize Eq. (14) to the quantum mechanical case, namely it does contain the position of the particle, *but not that of the source!* Therefore, we can easily calculate the information transfer on a small dV volume:

$$\frac{dI}{dt}(dV) = \frac{1}{\hbar} \psi^*(x, y, z, t) U(x, y, z, t) \psi(x, y, z, t) dV \quad (18)$$

The total received information can be calculated by integrating the expression in Eq. (18):

$$\frac{dI}{dt} = \frac{1}{\hbar} \int_V \psi^*(x, y, z, t) U(x, y, z, t) \psi(x, y, z, t) dV \quad (19)$$

Eq. (19) can be further simplified, considering that the expression under the integral is the Ehrenfest-type average value of the U potential energy [40]. As mentioned above, (see Eq. 13) the received information is always positive, therefore, the absolute value of the potential energy must be taken:

$$\frac{dI}{dt} = \frac{1}{\hbar} |\bar{U}| \quad (20)$$

We can determine the “collected” information by integrating Eq. (19) with respect to time:

$$I_{t_1}^{t_2} = \int_{t_1}^{t_2} dt \frac{1}{\hbar} \int_V \psi^*(x, y, z, t) U(x, y, z, t) \psi(x, y, z, t) dV \quad (21)$$

We think that Eq. (21) is in accordance with the common sense, because the potential energy is the only connection to the outside world. Some remarks are to be listed here:

- Eq. (21) contains a continuous time integral. However, the quanta property of the action (and the information) must be taken into consideration. Strictly speaking, the potential energy itself cannot be a continuous variable, therefore, Eq. (21) has an approximate, semi-classical character, such like the Schrödinger equation itself. This topic needs a deeper investigation in the future. Maybe it will also enable a new step in the dispute about the $\hbar \rightarrow 0$ limit. See Ref. [41] for example.
- The r.h.s. of Eq. (21) can be transformed by using the mathematical identity,

$$\frac{d}{dt} \int \psi^* \hat{A} \psi d^3x = \int \frac{\partial}{\partial t} (\psi^* \hat{A} \psi) d^3x = \int \left[\frac{\partial \psi^*}{\partial t} \hat{A} \psi + \psi^* \frac{\partial \hat{A}}{\partial t} \psi + \psi^* \hat{A} \frac{\partial \psi}{\partial t} \right] d^3x \quad (22)$$

which is valid for any Hermetian operator \hat{A} . This operation enables detailed investigation of the equation's properties. Another possibility arises when we apply the Madelung form $\psi = \sqrt{\rho} e^{is}$ (see Ref. 42). However, these tasks are beyond the goal of the present article.

- We should remember that all the findings listed above are valid for Coulomb type potential only.

4. Conclusion

In the present paper, we extended our ideas regarding Elementary Information in Physics to the field of Quantum Mechanics. We covered two cases: free particle and scalar potential. The following statements are made:

- Free particle produces information, which is proportional to the path length made, in de Broglie wavelength units.
- A particle under scalar potential continuously gets information from the potential source. The time rate of the information is proportional to the Ehrenfest-type average of the potential energy.

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