Science Front Publishers *Journal for Foundations and Applications of Physics*, vol. 3, No. 1 (2016)
(sciencefront.org) ISSN 2394-3688

The Cosmological Redshift Manifests the Curvature and Interpreted as a Degree of Hyperbolicity of the Spacetime

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(Received 30 March 2016, Published 19 May 2016)

Abstract

Hubble's law describes a uniformly expanding flat universe. Hubble's law doesn`t explain why distant objects were receding fastest. There is an approximately linear relationship between redshift and distance at small scales for all the FLRW models, and departures from linearity at larger scales can be used to measure *spatial curvature*. Locally the spacetime is flat. For distant objects, the imprint of the curvature is significant, where the spacetime does no longer remain flat. The redshifts from such distant objects increase according to the increase in the curvature of the hyperbolic spacetime. The cosmological (gravitational) redshift can be interpreted as a degree of the hyperbolicity of the curved spacetime. The Universe is globally hyperbolic as we did prove mathematically [S. A. Mabkhout, Phys. Essays, **25**, 112 (2012)]. Such a solution predicts the equation of state of cosmology $P = -\boldsymbol{\rho}$. The hyperbolic structure of the spacetime–not dark energy- causes the accelerated expansion of the universe. Thus, in our non-existing dark energy hyperbolic universe, the increase in the cosmological redshift can only account for the increase in curvature that causes such an accelerated expansion relative to the observer. We developed [S. A. Mabkhout, Phys. Essays, **26**,422 (2013)] the equation of motion in the hyperbolic spacetime: $V = \mathrm{e}^{-\mu/r}\sqrt{(2\mu/r - \mu/a)}$, that describes the speed up motion in the hyperbolic spacetime and predicts the flat rotation curve. In the hyperbolic spacetime, the free fall due to the curvature, causes the non-decreasing speed of the galaxies for large *r*. Thus, the Doppler redshift manifests such curvature. As an object is far distant apart, as much the spacetime appears relatively hyperbolic curved with a high redshift. Its velocity relatively appears to exceed the speed of light "*c*" due to the assumption of flat spacetime.

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***Keywords*:** Hyperbolic spacetime, cosmological redshift, Superluminous speed, Accelerating expansion of the universe, Flat curve.

1. Introduction

Doppler effect occurs when two clocks move away from one another. An example is the red shift of light that we receive from nearby galaxies. If a source of the light is moving away from an observer, then redshift ($z > 0$) occurs; if the source moves towards the observer, then blueshift ($z < 0$) occurs. This is true for all electromagnetic waves and is explained by the Doppler effect. Consequently, this type of redshift is called the *Doppler redshift*. If the source moves away from the observer with velocity *v*, then, ignoring relativistic effects, the redshift is given by [1],

$$z \approx \frac{v}{c}$$

where *c* is the speed of light. In the classical Doppler effect, the frequency of the source is not modified, but the recessional motion causes the illusion of a lower frequency.

Relativistic Doppler effect: A more complete treatment of the Doppler redshift requires considering relativistic effects associated with motion of sources close to the speed of light. A complete derivation of the effect can be found in the article on the relativistic Doppler effect. In brief, objects moving close to the speed of light will experience deviations from the above formula due to the Time dilation of special relativity which can be corrected for by introducing the Lorentz factor γ into the classical Doppler formula as follows:

$$1 + z = \left(1 + \frac{v}{c}\right)\gamma$$

Gravitational redshift: In the theory of General relativity, there is time dilation within a gravitational well. This is known as the gravitational redshift or *Einstein Shift*. The theoretical derivation of this effect follows from the Schwarzschild solution of the Einstein equations which yields the following formula for redshift associated with a photon travelling in the gravitational field of an uncharged, nonrotating, spherically symmetric mass:

$$1 + z = \frac{1}{\sqrt{1 - \left(\frac{2GM}{rc^2}\right)}}$$

This gravitational redshift results can be derived from the assumptions of special relativity and the Equivalence principle; the full theory of General relativity is not required. The effect is very small but measurable on Earth using the Mossbauer effect and was first observed in the Pound-Rebka experiment. However, it is significant near a black hole, and as an object approaches the Event horizon the red shift becomes infinite. It is also the dominant cause of large angular-scale temperature fluctuations in the cosmic microwave background radiation.

The Cosmological Redshift: In the early part of the twentieth century, Slipher, Hubble and others made the first measurements of the redshifts and blueshifts of galaxies beyond the Milky Way. They initially *interpreted* these redshifts and blueshifts as due solely to the Doppler effect, but later Hubble discovered a rough correlation between the increasing redshifts and the increasing distance of galaxies. Theorists almost immediately realized that these observations could be explained by a different mechanism for producing redshifts. Hubble's law of the correlation between redshifts and distances is required by models of cosmology derived from general relativity that have a metric expansion of space. As a result, photons propagating through the expanding space are stretched, creating the Cosmological redshift. This differs from the Doppler effect redshifts described above because the velocity boost (i.e. the Lorentz transformation) between the source and observer is not due to classical momentum and energy transfer, but instead the photons increase in wavelength and redshift as the space through which they are travelling expands. This effect is prescribed by the current cosmological model as an observable manifestation of the time-dependent cosmic scale factor (*a*) in the following way:

$$1 + z = \frac{a_{\text{now}}}{a_{\text{then}}}.$$

This type of redshift is called the Cosmological redshift or Hubble redshift. These galaxies are not receding simply by means of a physical velocity in the direction away from the observer; instead, the intervening space is stretching, which accounts for the large-scale isotropy of the effect demanded by the cosmological principle. For cosmological redshifts of $z < 0.1$ the effects of spacetime expansion are minimal and observed redshifts dominated by the peculiar motions of the galaxies relative to one another that cause additional Doppler redshifts and blueshifts. In particular, Doppler redshift is bound by special relativity; thus $v > c$ is impossible while, in contrast, $v > c$ is possible for cosmological redshift because the space which separates the objects (e.g., a quasar from the Earth) can expand faster than the speed of light. The cosmological redshift, occurs between two clocks that are at different radii and both at rest with respect to the black hole or other center of gravitational attraction. Visible light with the longest period is red. The remote observer see light emitted by the close-in clock to be redder –that is of longer period- than it was at the point of emission. A photon moves through the spacetime, its wavelength is influenced by the expansion of the universe, as if the photon being attached to the expanding fabric spacetime. The cosmological (gravitational) redshift is a consequence of the changing size of the universe; it is not related to velocity at all. The gravitational redshift in curved expanding spacetime is a generalization of the Doppler shift in flat spacetime to curved expanding spacetime, is the reddening of light from distant galaxies as the universe expands. .

The farther the galaxy is, the faster it appears to be moving away from you. We would like to explain what causes farther galaxies to appear moving away faster.

2. The Doppler shift and cosmology

In the widely accepted cosmological model based on General relativity, redshift is mainly a result of the expansion of space: this means that the farther away a galaxy is from us, the more the space has expanded in the time since the light left that galaxy, so the more the light has been stretched, the more redshifted the light is, and so the faster it appears to be moving away from us. Hubble's law follows in part from the Copernican principle. Because it is usually not known how luminous objects are, measuring the redshift is easier than more direct distance measurements, so redshift is sometimes in practice converted to a crude distance measurement using Hubble's law. Big bang cosmology is based on Einstein's general theory of relativity. It is a theory transcending both Newton's mechanics and Einstein's special theory of relativity. The cosmological redshift occurs because the curvature of spacetime was smaller in the past when the universe was younger than it is now. Light waves become stretched in route between the time they were emitted long ago, and the time they are detected by us today. It is tempting to refer to cosmological redshifts as Doppler shifts. By referring to cosmological redshifts as Doppler shifts, we are insisting that our Newtonian intuition about motion still applies without significant change to the cosmological arena. A result of this thinking is that quasars now being detected at redshifts of $Z = 4.0$ would have to be interpreted as traveling at speeds of more than $V = Z \times c$ or 4 times the speed of light. This is, of course, quite absurd, because we all know that no physical object may travel faster than the speed of light. To avoid such apparently nonsensical speeds, many popularizers use the special relativistic Doppler formula to show that quasars are really not moving faster than light. The argument being that for large velocities, special relativity replaces Newtonian physics as the correct framework for interpreting the world. By using a special relativistic velocity addition formula the quasar we just discussed has a velocity of 92 percent the speed of light. Although we now have a feeling that Reason has returned to our description of the universe, in fact, we have only replaced one incomplete explanation for another. The calculation of the quasar's speed now presupposes that special relativity (a theory of flat spacetime) is applicable even at cosmological scales where general relativity predicts that spacetime curvature becomes important. The special relativistic Doppler formula is introduced to show how quasars are moving slower than the speed of light! It is also common for popularizers of cosmology to describe how 'space itself stretches' yet continue to describe the expansion of the universe as motion governed by the restrictions of special relativity. By adopting general relativity as the proper guide, such contradictions are eliminated. General relativity leads us to several powerful conclusions about our cosmos[2]**:**

1) special relativity is inapplicable for describing the larger universe;
2) the concepts of distance and motion are not absolutely defined and
3) Preexisting spacetime is undefined.

General relativity must replace special relativity in cosmology because it denies a special role to observers moving at constant velocity, extending special relativity into the arena of accelerated observers. It also denies a special significance to special relativity's flat spacetime by relegating it to only a microscopic domain within a larger geometric possibility. Just as Newtonian physics gave way to special relativity for describing high speed motion, so too does special relativity give way to general relativity. This means that the special relativistic Doppler formula should not, in fact cannot, be used to quantify the velocity of distant quasars. We have no choice in this matter if we want to maintain the logical integrity of both theories. The instantaneous physical distance is not itself observable. Cosmological 'motion' cannot be directly observed. It can only be inferred from observations of the cosmological redshift, which general relativity then tells us that the universe is expanding.

3. The cosmological redshift and the Hyperbolic spacetime

One of the most remarkable discoveries in twentieth century astronomy was Hubble's (1929) observation that the redshifts of spectral lines in galaxies increase linearly with their distance. Hubble took this to show that the universe is *expanding uniformly,* and this effect can be given a straightforward qualitative explanation in the FLRW models. The FLRW models predict a change in frequency of light from distant objects that depends directly on R(t). There is an approximately linear relationship between *redshift* and distance at small scales for all the FLRW models, and *departures* from linearity at larger scales can be used to *measure spatial curvature*. Locally the spacetime is flat. For distant objects, the imprint of the curvature is significant, where the spacetime does no longer remain flat. The redshifts of such distant objects increase according to the curvature of the hyperbolic spacetime. The cosmological (gravitational) redshift can be interpreted as a degree of the hyperbolicity of the curved spacetime. Hubble's law (v_{rec} = HD: recession velocity = Hubble's constant × distance) describes the situation: farthest objects receding fastest. It didn't explain why? Hubble himself was not entirely happy with his distance–velocity formula, which decisively contributed to the inflationary model of the universe. In the paper, jointly with Tolman, he wrote "*The possibility that the redshift may be due to some other cause connected with the long time or distance involved in the passage of light from nebulae to observer, should not be prematurely neglected*"[3]. "The Hubble velocity distance rule is an interesting example how two independently correct facts, i.e. the common Doppler shift and Hubble's experimental distance vs redshift law when "married" together resulted in an unfortunate conclusion. This happened because the only cause of redshift that Hubble was aware, was the common Doppler shift, and thus he obtained a distance–velocity plot [4]. In a general setting and from a logical point of view, the existence of relative velocity is a necessary but not sufficient condition to record a wavelength shift. In Euclidean geometry e.g. wavelength shift uniquely implies existence of a relative velocity while in hyperbolic geometry it does not have a unique implication. Thus while the existence of relative velocity always results in a wavelength shift, the presence of a shift may or may not imply the existence of a relative velocity. Euclidean geometry cannot induce changes in wavelength of electromagnetic radiation. The case of $K = 0$. In Euclidean space geodesics do not deviate. This is the case of hyperbolic space. Geodesics deviate at an exponential rate"[4]. Halton Arp, has collected the evidence over many years and —personally as well as professionally— maintains that extragalactic redshifts are *not caused by an expanding universe*.

4. Why all Quasars are redshifted ?

Quasars are believed to be objects ejected from the centers of the Galaxies (or Black holes). Do all of them blow outwards in opposite direction to us in order to agree all of them with such high redshifts ? Note that the motion of galaxies is random! While, even no one Quasar exhibits a blueshift !. Moreover, according to their high redshift all of the Quasars are very distant away. But the universe is isotropic, so our position is not preferred. Hence why we didn`t observe any Quasar nearby? According to the isotropy, a distant observer should observe the Quasars very distant with respect to him, that is they should be nearby to us!. Contradiction. According to Hubble`s law, if the object is bright then its nearby and the distant objects are faint. The Quasars are very bright, why shouldn`t they nearby? Why we just accept one part from Hubble`s law, that is: the high redshift of the Quasar indicates that its distant and ignored the other part, that is: the brightness of the Quasar indicates they are nearby?!. Finally, Why our Galaxy and many other nearby Galaxies didn`t eject Quasars from their centers? Why this jop is exclusive for distant Galaxies? Because our Galaxy and many other nearby Galaxies are inactive, said astronomers. Why they are the inactive among the active distant Galaxies? False justification. It is clear such Paradigm is not satisfactory and insufficient, it depends on many unjustified reasons, many contradictions and inconsistent. The paradigm must be reconsidered and readjusted. Brighter the Quasar is, higher the redshift and the distance are. The bright the Galaxy, the low redshift the nearby it is. Brightest Galaxies associated with brightest Quasars, but faint Galaxies not. So, if Quasars agree in their brightness they disagree with their redshifts. Yes, the scenario concerning the Quasars no more than speculations and guesses to fabricate suitable explanation to current observations. The problem relies on the similarity of the cosmological redshift to the Doppler redshift that both of them cause recession speed. The first by the expansion of the spacetime and other by receding within the spacetime. If the high redshift of the Quasar is due to the cosmological redshift of the expanding spacetime, why shouldn't agree and coincide with the redshift of the hosting Galaxy. The cosmological redshift must be interpreted in a different way, as I do, as manifests the curvature of the hyperbolic spacetime. Astronomers have found many galaxy pairs and galaxy groups in which the members are evidently close to each other —even interacting— yet have redshifts that are radically at odds! Their redshifts don't make sense: If two galaxies are roughly in the same place then their measured redshifts should agree with each other, since redshift is supposed to be a measure of their distance (although the redshift may include a relatively minor Doppler component due to local motion). The observational fact that they don't is considered anomalous. The mystery is in the cause, and also why some of the anomalies are so extreme. Locally the spacetime is flat, there is no cosmological redshift. For example, observations tell us that space within galaxies, which are rather diffuse objects, do not expand. Thus, where is the "border line" in space which divides expanding space from non expanding space? Two galaxies within our Local Group, including Andromeda, and a few galaxies in the Virgo Cluster display blueshifts and so are moving toward us, but this results from their local motion (peculiar velocity). Why nearby galaxies exhibit blue-shift? Because its peculiar velocity is greater than its recession velocity! The answer is more convenient if we say: *Locally the spacetime is flat through which the curvature is negligible* (no cosmological redshift), where the random peculiar velocity dominates. If cosmological redshift has nothing to do with the Doppler effect, how do we know that galaxies that are very far away are also receding from us ? How to compare between two unrelated concepts, the Doppler redshift and the cosmological redshift ? Andromeda galaxy is blueshifted because its sufficiently nearby where the spacetime is approximately flat and special relativity works. It is blueshifted according to the Doppler effect in flat spacetime.

5. Redshift and Dark Energy.

Although perspective for nearby objects in hyperbolic space is very nearly identical to Euclidean space (i.e. the Universe locally is approximately flat consistent with local observations), the apparent angular size of

distant objects falls off much more rapidly, in fact exponentially. The Universe is globally hyperbolic as we did prove mathematically. Such a solution predicts the equation of state of cosmology, $P = -\rho$. The hyperbolic structure of the space causes the accelerated expansion of the universe equivalent to its negative pressure. "Supernovae (SN) are extremely luminous explosions of dying stars. This makes them directly observable even at very far distances. Classification of SN was originally done spectroscopically, but even this simple identification tells a lot of information about the star's evolution and final explosion. The most important type and the one relevant for cosmological measurements are supernovae Ia. These explosions happen from collapsing white dwarfs in close binary star systems. The explosion is triggered when white dwarf reaches the Chandrasekhar mass limit in process of accretion from the binary companion. Since the limit has little varying value, all SN Ia are considered to have quasi-equal peak absolute magnitude. They are successfully identified both by spectrum and by the light curve. Luckily SN *Ia* is the most luminous and the most frequent type of supernovae explosions in the universe. By measuring their apparent magnitude, and joint with the redshift, this becomes a powerful method for sampling luminosity distance versus redshift relation. That function depends on cosmological model and can be used to constrain cosmological density parameters. Under standard model, the measurements reveal accelerated expansion of the universe that is explained by contribution of dark energy" [5].
The Type *1a* supernova as standard candles can be used to measure the expansion history of the universe i.e. the plot of the scale factor of the universe when the supernova light was emitted versus the time in the past when the supernova explosion occurred. This is done by measuring the apparent brightness L of the supernova and its redshift z. Comparing the apparent brightness to the presumably known intrinsic brightness of the supernova determines its distance and from the distance the time in the past when the supernova exploded can be inferred knowing the velocity of light. The redshift gives the scale factor of the universe at the time of the supernova explosion via $a(t) = 1/(1+z)$. Each supernova then yields a point in the $a(t)$ versus t plot, and a large sample of supernovae thus measure the expansion history of the universe [6]. .

The luminosity L of a nearby star can be determined from its apparent brightness f and distance determined by triangulation using the flat universe inverse square law, gives a predicted connection between the flux f and the redshift z

$$f = \frac{L}{4p\,d^2}$$

$$\because\ z = H_0 d$$

$$\frac{f}{L} = \frac{H_0^2}{4p\,z^2} \qquad (1)$$

Those are appropriate approximations for nearby sources from which light takes only a short time to reach us, traveling a distance that is small compared to that over which space might be curved and small enough that Hubble`s law holds. However, for standard candles that are further away, deviation from Eq.(1) can be expected, arising from *the spatial curvature* of the universe. Deviations can also be expected if the light from a standard candle travels to us over a time during which *the expansion of the universe* is significant [6]. In the 1990's two teams of astronomers, the Supernova Cosmology Project (Lawrence Berkeley National Laboratory) and the High-*Z* Supernova Search (international) were looking for distant Type Ia supernovae in order to measure the expansion rate of the Universe with time. They expected that the expansion would be slowing, which would be indicated by the supernovae being brighter than their redshifts would indicate. Instead, they found the supernovae to be fainter than expected from a uniformly expanding universe. Hence, the expansion of the Universe was accelerating!

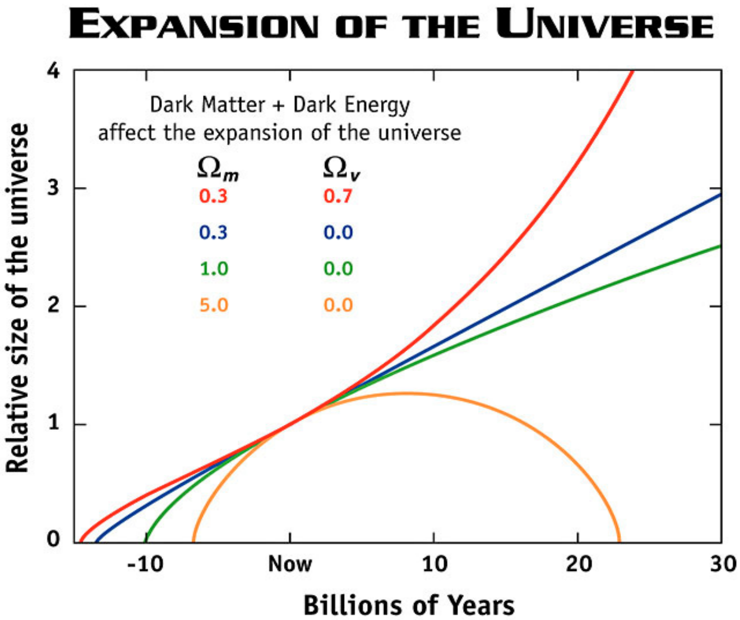

Fig.1. Scientists used to think that the universe was described by the yellow, green, or blue curves. But surprise, it's actually the red *curve* instead [7].

In addition, measurements of the cosmic microwave background indicate that the Universe has a flat geometry on large scales. Because there is not enough matter in the Universe either ordinary or dark matter to produce this flatness, the difference must be attributed to a "dark energy". This same dark energy causes the acceleration of the expansion of the Universe. Some astronomers identify dark energy with Einstein's Cosmological Constant. In the context of dark energy, the cosmological constant is a reservoir which stores energy. Its energy scales as the Universe expands. Applied to the supernova data, it would distinguish effects due to the matter in the Universe from those due to the dark energy. Another explanation for how space acquires energy comes from the quantum theory of matter. In this theory, "empty space" is actually full of temporary ("virtual") particles that continually form and then disappear. But when physicists tried to calculate how much energy this would give empty space, the answer came out wrong - wrong by a lot. The number came out 10^{120} times too big. The cosmological constant is estimated by cosmologists to be on the order of 10^{-29}g/cm^3, or about 10^{-120} in reduced Planck units. Particle physics predicts a natural value of 1 in reduced Planck units, leading to a large discrepancy. It's hard to get an answer that bad. More recently, the WMAP seven-year analysis gave an estimate of 72.8% dark energy, 22.7% dark matter and 4.6% ordinary matter. The cosmological constant has negative pressure equal to its energy density and so causes the expansion of the universe to accelerate. The reason why a cosmological constant has negative pressure can be seen from classical thermodynamics; Energy must be lost from inside a container to do work on the container. A change in volume dV requires work done equal to a change of energy $-P\,dV$, where P is the pressure. But the amount of energy in a container full of vacuum actually increases when the volume increases (dV is positive), because the energy is equal to ρV, where ρ (rho) is the energy density of the cosmological constant. Therefore, P is negative and, in fact, $P = -\rho$. .

We did prove mathematically that the universe is hyperbolic [8]. Such a solution predicts the equation of state of cosmology $P = -\rho$. The hyperbolic structure of the spacetime causes the accelerated expansion of the universe. To obtain the dynamical equation of cosmology, we should combine Einstein field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

with the isotropic homogeneous Robertson- Walker's line-element:

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

to get Friedmann`s equations:

$$\dot{R}^2 + k = (8\pi/3)\rho R^2 \qquad (2)$$

$$2R\ddot{R} + \dot{R}^2 + k = -8\pi p \qquad (3)$$

Where p is the pressure and ρ is the energy density of the cosmological fluid and k is the curvature.

Now we shall solve the differential equation (2) by separating the variables. We assume the Big Bang Model as an initial condition (i.e. R=0 when t=0).

$$\dot{R}^2 + k = (8\pi/3)\rho R^2$$

$$\dot{R}^2 = (8\pi/3)\rho R^2 - k$$

$$\dot{R} = \sqrt{(8\pi\rho/3)R^2 - k}$$

$$dR/\sqrt{(8\pi\rho/3)R^2 - k} = dt$$

$$dR/\sqrt{R^2 - 3k/8\pi\rho} = \sqrt{8\pi\rho/3}\,dt$$

Differential equation (2) allows one to deal with ρ as a parameter since it's not an explicit function of t, so Eq. (2) can be solved for any chosen fixed value , ρ_j , from the stream of the various values of the parameter ρ,

$$\rho_1, \rho_2, \ldots, \rho_{planck}, \ldots, \rho_j, \ldots, \rho_{now}$$

By means of the mean value theorem, we assume approximately that ρ_j evolves to the fixed physical value ρ_j exactly simultaneously associated to the state (t_j, R_j)since ρ_j is not defined and not continuous at the point of singularity t = 0, put

$$\rho(c) = \rho_j \;: 0 < c \le t$$

$$\int_0^R dr/\sqrt{r^2 - 3k/8\pi\rho_j} = \sqrt{8\pi\rho_j/3}\int_0^t d\tau$$

We get,

$$\cosh^{-1}(R/\sqrt{3k/8\pi\rho_j}) - \cosh^{-1}0 = t\sqrt{8\pi\rho_j/3} \qquad (4)$$

(i) Now we use complex analysis as follows: Substitute the first value $\cosh^{-1}0 = i\pi/2$ in equation (4), we get:

$$R(t)=\sqrt{3k/8\pi\rho_j}.\cosh(t\sqrt{8\pi\rho_j/3}+\pi i/2)$$
$$R(t)=\sqrt{3k/8\pi\rho_j}.(\cosh t\sqrt{8\pi\rho_j/3}.\cosh(\pi i/2)+\sinh(\pi i/2).\sinh t\sqrt{8\pi\rho_j/3})$$
$$R(t)=\sqrt{3k/8\pi\rho_j}.(\cosh t\sqrt{8\pi\rho_j/3}.\cos(\pi/2)+i\sin(\pi/2).\sinh t\sqrt{8\pi\rho_j/3})$$
$$R(t)=i\sqrt{3k/8\pi\rho_j}.\sinh t\sqrt{8\pi\rho_j/3}$$

Since the function $\rho(t)$ is always positive, so is any chosen fixed value ρ_j. A simple analysis shows that the R(t) scale solution represented in the last equation is complex if k is positive, negative if k is negative and vanishes if k is zero. So the first value $\cosh^{-1}0=i\pi/2$ is rejected.

Substitute the other value $\cosh^{-1}0=3i\pi/2$ in equation (4) , we get

$$R(t)=\sqrt{3k/8\pi\rho_j}.\cosh(t\sqrt{8\pi\rho_j/3}+3\pi i/2)$$
$$R(t)=\sqrt{3k/8\pi\rho_j}(\cosh t\sqrt{8\pi\rho_j/3}.\cosh(3\pi i/2)+\sinh(3\pi i/2)\sinh t\sqrt{8\pi\rho_j/3})$$
$$R(t)=\sqrt{3k/8\pi\rho_j}(\cosh t\sqrt{8\pi\rho_j/3}.\cos(3\pi/2)+i\sin(3\pi/2)\sinh t\sqrt{8\pi\rho_j/3})$$
$$R(t)=-i\sqrt{3k/8\pi\rho_j}\sinh t\sqrt{8\pi\rho_j/3}$$

The R(t) scale solution in the last equation is real, positive and non-vanishing if and only if k is negative. Since k is normalized, substitute k = -1, in the last equation, we get:

$$R(t)=-i\sqrt{3k/8\pi\rho_j}\sinh t\sqrt{8\pi\rho_j/3}$$
$$R(t)=-i\sqrt{-3/8\pi\rho_j}\sinh t\sqrt{8\pi\rho_j/3}$$
$$R(t)=-i.i\sqrt{3/8\pi\rho_j}\sinh t\sqrt{8\pi\rho_j/3}$$
$$R(t)=\sqrt{3/8\pi\rho_j}\sinh t\sqrt{8\pi\rho_j/3} \qquad (5)$$

Which mean that $R(t)$ either vanishes if k = 0 or complex if k =1.Thus, the curvature k must be negative and consequently the universe must be hyperbolic and open.

Note that the solution represented by Eq. (5) is evaluated only for the values simultaneously associated with ρ_j namely (R_j, t_j)

$$R_j=\sqrt{3/8\pi\rho_j}\sinh t_j\sqrt{8\pi\rho_j/3}$$

Verification: The above scale factor can be verified even at Planck scale as follows

Let us apply equation (5) at Planck scale. To do this we substitute a given Planck time and Planck density in equation (5), while we assume Planck length is unknown.

Note that in geometrical units:-

The speed of light c = 1

1 sec = 2.997×10^{10}cm

1 gram = 7.425×10^{-29}cm

1 eV = 1.324×10^{-61}cm.

And we have Planck scale from the following

Planck time = 5.4×10^{-44}s
Planck length = 1.6×10^{-33}cm
Planck mass = 1.2×10^{25}eV/c^2
Planck density = M/V = M/L^3 = $1.2\times10^{25}(\text{eV}/c^2)/(1.6\times10^{-33})^3$
$=1.2\times10^{25}(1.324\times10^{-61}\text{cm})/(1.6\times10^{-33})^3=3.8789\times10^{62}$cm
Recall equation (13) and put k = -1 we get:-

$$R_p = \sqrt{3/8\pi\rho_p}\,\sinh\sqrt{8\pi\rho_p/3}\,t_p$$

$$R_p = \sqrt{3/8\pi\times3.8789\times10^{62}}$$

$$\times\sinh\sqrt{8\pi\times3.8789\times10^{62}/3}\times5.4\times10^{-44}\times2.997\times10^{10}$$

$= 0.175423\times10^{-31}\times\sinh 0.092255888$
$= 0.175423\times10^{-31}\times0.092386811$
$= 1.62\times10^{-33}$ cm = L_p = Planck length.

(ii) We shall see that the solution of equation (2) satisfies the second order differential equation (3) in order to be consistent. We have from the solution of Eq. (2) for any chosen value ρ_j

$$R = \sqrt{\frac{3}{8\pi\rho_j}}\sinh t\sqrt{\frac{8\pi\rho_j}{3}}$$

$$\dot{R} = \cosh t\sqrt{\frac{8\pi\rho_j}{3}}$$

$$\ddot{R} = \sqrt{\frac{8\pi\rho_j}{3}}\sinh t\sqrt{\frac{8\pi\rho_j}{3}} = \frac{8\pi\rho_j}{3}R$$

Substitute these values in Eq. (3), and k = -1, yields:

$$2R\ddot{R}+\dot{R}^2-1=-8\pi pR^2$$

$$2R\left(\frac{8\pi\rho_j}{3}R\right)+\cosh^2\sqrt{\frac{8\pi\rho_j}{3}}-1=-8\pi pR^2$$

$$2R^2\left(\frac{8\pi\rho_j}{3}\right)+\sinh^2\sqrt{\frac{8\pi\rho_j}{3}}=-8\pi pR^2$$

$$2R^2\left(\frac{8\pi\rho_j}{3}\right)+\frac{8\pi\rho_j}{3}R^2=-8\pi pR^2$$

$$8\pi\rho_j R^2=-8\pi pR^2$$

$$p=-\rho_j$$

The last equation is known as the equation of state of cosmology. The argument of the solution predicts the equation of state of cosmology $p=-\rho_i$. Since the energy density is always positive, the negative pressure implies an accelerated expansion of the universe. Hence equations (2) and (3) are consistent for any chosen fixed value ρ_j of the parameter ρ. The argument of the solution predicts the equation of state $p=-\rho_i$. We exhibit the hyperbolic structure of the universe that explains the accelerating expansion of the universe without needs for an additional components, dark energy. One explanation for dark energy is that it is a property of space. The simplest explanation for dark energy is that it is simply the "cost of having space": that is, a volume of space has some intrinsic, fundamental energy. Just the ordinary energy density state ρ_j remains in the Hyperbolic Universe to derive the accelerating expansion equivalent to its negative pressure. Hyperbolic Universe involves zero [9] cosmological constant (the vacuum energy). The negative pressure $p = -\rho_i$ is the property of the hyperbolic structure of the Universe. Flat universe dominated by matter is modeled as a zero pressure-dust universe model, and the expansion of the universe would be slowing due to the gravity attraction. Which is incorrect, as we shall see below: .

Einstein postulates [10] that the matter dominated universe could be modeled as dust with zero pressure in order to simplify and solves Friedmann`s equations .

$$\dot{R}^2 + k = (8\pi/3)\rho R^2$$
$$2R\ddot{R} + \dot{R}^2 + k = 0$$
$$2\ddot{R}/R + (8\pi/3)\rho = 0$$
$$\ddot{R} = -(8\pi/3)\rho R/2 < 0$$
$$\therefore \ddot{R} < 0$$

The pressure less form of Eq. (2) describes a decelerating expansion state of the universe which is described by the energy tensor of matter for dust where $p = 0$. We solved the second dynamical equation of cosmology, the space-space component; in it is pressure less form : .

$$2R\ddot{R} + \dot{R}^2 + k = 0$$
$$\because k = -1$$
$$2R\ddot{R} + \dot{R}^2 - 1 = 0$$

to be $t = R$, which satisfies the last differential equation.

Substitute $t = R, k = -1$ in the first dynamical equation (2)

$$\dot{R}^2 + k = (8\pi/3)\rho R^2$$
$$(1)^2 - 1 = (8\pi/3)\rho R^2$$
$$0 = (8\pi/3)\rho R^2$$
$$\therefore \rho = 0$$

Hence the zero pressure does not lead to a dusty universe. In fact zero pressure Universe is an empty space, since $\rho = 0$ In the presence of pressure, from Esq. (2) and (3) we can obtain.

$$\ddot{R} = -\frac{4\pi}{3}(\rho + 3p)R$$
$$\because p = -\rho$$
$$\therefore \ddot{R} = \frac{8\pi}{3}R > 0$$
.

which guarantees an accelerating expansion of the universe.

Newton first law states that the body keep moving with a uniform velocity in straight line. Similarly, the free fall of an object in a flat spacetime is uniform. An accelerated motion is described by a curve. For large structure, the curvature of the spacetime can't be ignored. *It is clear from Fig (1) the expansion of the universe is described by a hyperbolic curve*. The distant objects - e.g. supernovae - were influenced under the curvature of the spacetime. They possess an accelerating free fall due to the curvature of the hyperbolic spacetime, that manifests itself by the equation of the state $p=-\rho$ which is the property of the hyperbolic structure of the Universe. The universe is not flat. We did prove that, the universe globally is hyperbolic. The hyperbolic universe doesn't need dark energy to account for the accelerating expansion. The equation of the state $p=-\rho$ associated with the hyperbolic universe, derives such an accelerated expansion.

Astronomers found the supernovae to be fainter than their redshifts would indicate, assuming *flat uniformly expanding universe*. Thus, in our non-existing dark energy hyperbolic universe, the increase in the cosmological redshift can only accounts for the increase of the curvature causes such an accelerated expansion relative to the observer.

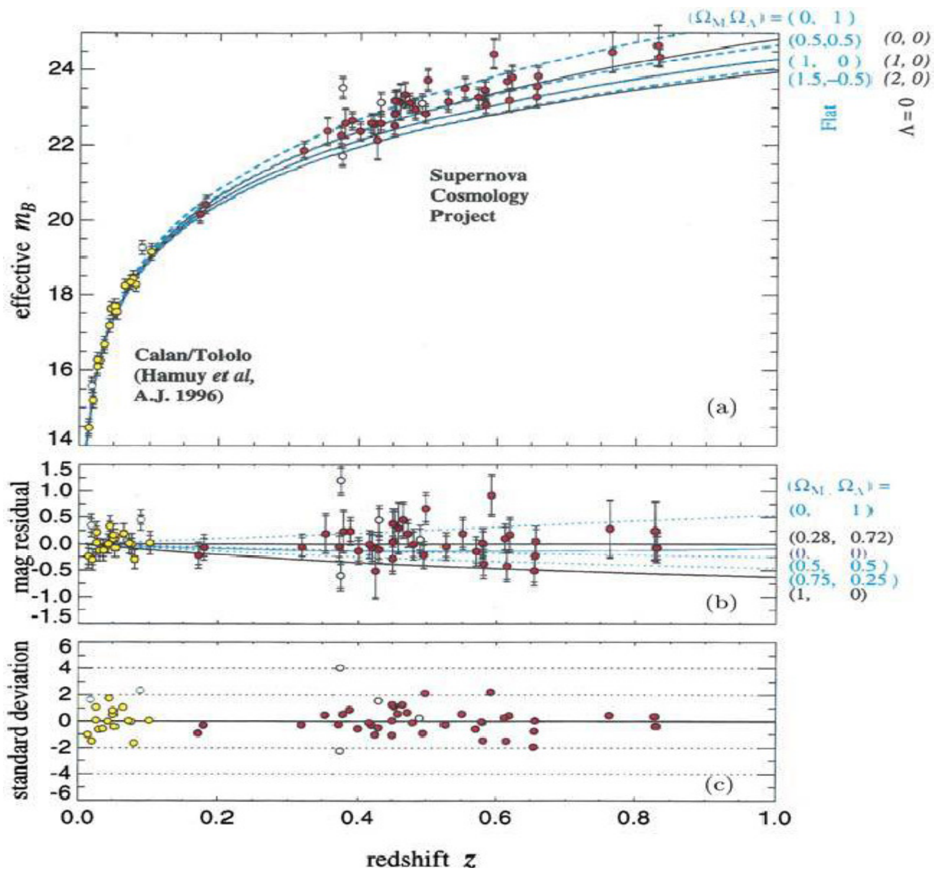

Fig. 2 The Hubble diagram for Type 1a Supernovae, from Perlmutter *et al* [11]. The different curves represent different values of the cosmological parameters Ω_m and Ω_Λ, as labeled. The solid horizontal black line in the center of the lowest panel represents the best fit with $\Omega_m = 0.28$ and $\Omega_\Lambda = 0.72$.

Moreover the hyperbolic evolution of the Universe predicts the large structure of the observable Universe 10^{28}cm associated with 14×10^9 *yr* as shown in Eq.(6), below. Hyperbolic Universe inflates, through the hyperbolic time evolution equation (5), legitimately to 10^{28} cm, very consistent with the current observable universe. Note that in geometrical units: . .

1 sec = 2.997×10^{10}cm

1 gram = 7.425×10^{-29}cm

1 yr = 3.16×10^{7} s

The energy density now $r_{now}=10^{-31}$ g/cm^{3} $=7.425\times10^{-60}$cm^{-2}

The age of the Universe $t_{now}=14\times10^{9}$ yr=1.32587´ 10^{28} cm

Substitute the above data in the hyperbolic time evolution equation of the Universe, yields

$$R_j=\sqrt{3/8\pi\rho_j}\,\sinh\left[t_j\sqrt{8\pi\rho_j/3}\right]$$

$$R_{now}=\sqrt{3/8\pi\rho_{now}}\,\sinh\left[t_{now}\sqrt{8\pi\rho_{now}/3}\right]$$

$$R_{now}=\sqrt{3/\left(8\pi\times7.425\times10^{-60}\right)}\times$$

$$\sinh\left[1.32587\times10^{28}\times\sqrt{8\pi\times7.425\times10^{-60}/3}\right]$$

$$R_{now}=1.6\times10^{29}\times\sinh 0.08287$$

$$R_{now}=1.3\times10^{28}cm \qquad (6$$

6. Redshift and Dark matter

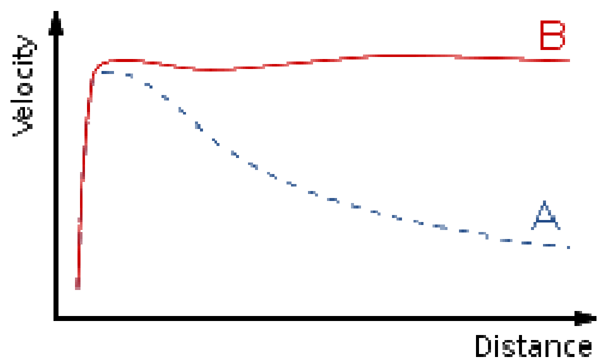

Figure.3. Rotation curve of a typical spiral galaxy: predicted (A) and observed (B). The discrepancy between the curves can be accounted for by adding a dark matter component to the galaxy [12]

Redshifts have been used to make the first measurements of the rotation rates of planets, velocities of interstellar clouds, the rotation of galaxies, and the dynamics of accretion onto neutron stars and black holes which exhibit both Doppler and gravitational redshifts. Additionally, the temperatures of various emitting and absorbing objects can be obtained by measuring Doppler broadening - effectively redshifts and blueshifts over a single emission or absorption line. By mapping the Doppler shift of the 21-cm line of the hydrogen, the velocity of rotation of stars in the galaxy can be approximately determined as a function of distance r from its center. By measuring the broadening and shifts of the 21-centimeter hydrogen line in different directions, astronomers have been able to measure the recessional velocities of interstellar gas, which in turn reveals the *rotation curve* of our Milky Way. This rotation curve, like that of many other galaxies, doesn't fall off for large r. The rotation curve of a galaxy (also called a velocity curve) is the plot of the orbital speed (in km/s) of the stars or gas in the galaxy on the *y*-axis against the distance from the center of the galaxy on the *x*-axis. A general observation of galaxy rotation can be stated as: galaxies with a central bulge in their disk have a rotation curve which is flat from near the centre to the edge (line *B* in illustration), i.e. stars are observed to revolve around the centre of these galaxies at a constant speed over a large range of distances from the centre of the galaxy. However, it was expected that these galaxies would have a rotation curve that slopes down from the centre to the edge (dotted line A in illustration), in the same way as other systems with most of their mass in the centre, such as the Solar System of planets or the Jovian System of moons following the prediction of Kepler's Laws. Something else is needed to account for the dynamics of galaxies besides a simple application of the laws of gravity to the observed matter. It is also observed that galaxies with a uniform distribution of luminous matter have a rotation curve sloping up from center to edge. Most low surface brightness galaxies (LSB galaxies) rotate with a rotation curve that slopes up from the center, indicating little core bulge.

The galaxy rotation problem is the discrepancy between observed galaxy rotation curves and the ones predicted assuming a centrally-dominated mass that follows the luminous material observed. If masses of galaxies are derived solely from the luminosities and the mass-to-light ratios in the disk and core portions of spiral galaxies are assumed to be close to that of stars, the masses derived from the kinematics of the observed rotation do not match. This discrepancy can be accounted for if there exists a large amount of dark matter that permeates the galaxy and extends into the galaxy's halo. Many physicists are nowadays convinced that some form of dark matter has to exist to explain for instance the discrepancy between the flat rotation curves of stars within a galaxy and the rotation curves expected from Kepler's third law. Assuming *flat space* and *circular orbit*, astronomers use the *Virial* theorem to determine the masses of the galaxies [13]: $M = V^2 R / G$, where M is the mass of the galaxy. V is the speed of the galaxy. R is the distance of the galactic center. G is the gravitational constant.

***Rotational Velocity*:** Using the power of the Doppler Shift, scientists can learn much about the motions of galaxies. They know that galaxies rotate because, when viewed edge-on, the light from one side of the galaxy is blue shifted and the light from the other side is red shifted. One side is moving toward the Earth, the other is moving away. They can also determine the speed at which the galaxy is rotating from how far the light is shifted. Knowing how fast the galaxy is rotating, they can then figure out the mass of the galaxy mathematically. As scientists look closer at the speeds of galactic rotation, they find something strange. The individual stars in a galaxy should act like the planets in our solar system, the farther away from the center, the slower they should move. But the Doppler Shift reveals that the stars in many galaxies do not slow down at farther distances. And on top of that, the stars move at speeds that should rip the galaxy apart; there is not enough measured mass to supply the gravity needed to hold the galaxy together. These high rotational speeds suggest that the galaxy contains more mass than was calculated. Scientists theorize that, if the galaxy was surrounded by a halo of unseen matter, the galaxy could remain stable at such high rotational speeds. *Kepler's third law and consequently Virial theorem does no longer hold for Non-Euclidian space. Equation of the radial motion in the galaxy`s hyperbolic spacetime* [14]**:** To seek completeness, it remains to develop an equation describes the speed up motion in the hyperbolic spacetime and predicts the flat curve. To do this, I will follow the following strategy

1- Seek for an equation $v = f(r)$ such that $v = \lim_{r \to 0} f(r) = 0$

2- $v = f(r) \xrightarrow{\text{large } r} \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} \xrightarrow{r \to \infty} \sqrt{-\frac{\mu}{a}}$

3- I guess the required equation –that fits the data- should be

$$v = f(r) = e^{-\frac{1}{r}} \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}.,$$

4- The final step in the mathematical problem solving method is to prove the conjecture

$$v = f(r) = e^{-\frac{1}{r}} \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}.$$

To find such an equation of the radial motion in the galaxy`s hyperbolic space-time, we proceed as follows: The required modified Schwarzschild spherically symmetric metric will be,

$$d\tau^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2 d\Omega^2$$

$$d\tau^2 = \left(1+\nu+(1/2)\nu^2+...\right)dt^2 - \left(1+\lambda+(1/2)\lambda^2+...\right)dr^2 - r^2 d\Omega^2$$

$$d\tau^2 \approx (1+\nu)dt^2 - (1+\lambda)dr^2 - r^2 d\Omega^2$$

The Ricci tensor,

$$0 = R_{tt} = -(1/2)e^{\nu-\lambda}\left(\nu'' + \nu'^2/2 - \nu'\lambda'/2 + 2\nu'/r\right), ...(i)$$

$$0 = R_{rr} = (1/2)\left(\nu'' + \nu'^2/2 - \nu'\lambda'/2 + 2\lambda'/r\right), ...(ii)$$

$$0 = R_{\theta\theta} = -\left\{1 - \left(e^{-\lambda}r\right)' + e^{-\lambda}r\left(\frac{\nu'+\lambda'}{2}\right)\right\}, ...(iii)$$

From $R_{tt} = R_{\theta\theta} = 0$ web have $\nu' + \lambda' = 0$, so $\nu + \lambda = k(constant)$

Write simply

$\lambda = -\nu + \log k$. Equation (i) is now just

$$\left(e^{\nu}r\right)'' = 0$$

$$e^{\nu}r = -\alpha + \beta r$$

Equation (iii) is

$$\left(e^{-\lambda}r\right)' = 1$$

$$\left(e^{\nu}r\right)' = k$$

$$\therefore \beta = k$$

We have now the complete solution

$$e^{\lambda} = \left(1 - 2\mu/kr\right)^{-1} \approx \left(e^{-2\mu/kr}\right)^{-1} = e^{2\mu/kr}$$

$$e^{\nu} = k\left(1 - \alpha/kr\right) = k\left(1 - 2\mu/kr\right) = \left(k - 2\mu/r\right)$$

For radial motion, $d\Omega^2 = 0$ .The Schwarzschild metric will be

$$d\tau^2 = e^{\nu}dt^2 - e^{\lambda}dr^2$$

$$d\tau^2 = \left(k - 2\mu/r\right)dt^2 - e^{2\mu/kr}dr^2$$

The free fall from rest of a star (of mass m and energy E) far from the center possesses [15],

$$\frac{E}{m}=\left(1-\frac{2\mu}{r}\right)\frac{dt}{d\tau}=1$$
$$\left(\frac{d\tau}{dt}\right)^2=\left(1-\frac{2\mu}{r}\right)^2$$
$$\left(\frac{d\tau}{dt}\right)^2=(k-2\mu/r)-e^{2\mu/kr}\left(\frac{dr}{dt}\right)^2$$
$$(1-2\mu/r)^2=(k-2\mu/r)-e^{2\mu/kr}\left(\frac{dr}{dt}\right)^2$$

To our purpose for the hyperbolic space- time ,the velocity far away from the center would be $V_\infty=\sqrt{-\mu/a}$ and consequently $k=1-\mu/a$

$$\left(1-4\mu/r+(2\mu/r)^2\right)=(1-2\mu/r-\mu/a)-e^{2\mu/[(1-\mu/a)r]}V^2$$

neglect the term $(2\mu/r)^2$ and rearrange

$$(1-4\mu/r)=(1-2\mu/r-\mu/a)-e^{2\mu/[(1-\mu/a)r]}V^2$$
$$e^{2\mu/[(1-\mu/a)r]}V^2=(2\mu/r-\mu/a)$$
$$V=e^{-\mu/[(1-\mu/a)r]}\sqrt{2\mu/r-\mu/a}$$

$$V=e^{-a\mu/[(a-\mu)r]}\sqrt{2\mu/r-\mu/a}$$
$$\because -a>>\mu$$
$$\therefore a-\mu\approx a$$
$$(1-\mu/a)(km/s)\approx 1(km/s)$$
$$V=e^{-\mu/r}\sqrt{2\mu/r-\mu/a}$$

Example

A typical galaxy of ordinary enclosed mass (Milky way or Andromeda)

$$M=10^{11}M_\circ=10^{11}\times 2\times 10^{30}kg$$
$$\mu=10^{11}\times 2\times 10^{30}\times 7.4\times 10^{-31}km$$

$$\mu=1.5\times 10^{11}km\times(3\times 10^5 km/s)$$
$$\mu=4.5\times 10^{16}(km^2/s)$$

$$210=\sqrt{\mu/-a}$$
$$(210)^2=\mu/-a$$

$$V = e^{\mu/r(kpc)} \sqrt{2\mu/r(kpc) - \mu/a}$$

$$V = e^{-4.5\times 10^{16}(km^2/s)/(1(km/s)\times r(3.1\times 10^{16}\,km))} \times$$

$$\sqrt{9\times 10^{16}(km^2/s)/(1(km/s)\times r(3.1\times 10^{16}\,km)) + 210^2}$$

$$V = e^{-1.45}\sqrt{3/r + 210^2}$$

The curve of the last equation is drown by visual mathematics program:

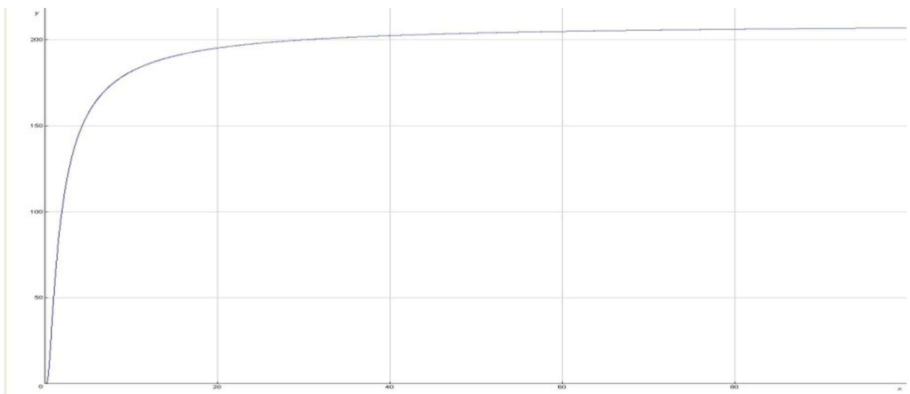

Fig 4. The curve describes the motion of a star in the Milky way (or Andromeda) galaxy. The vertical axis represents the velocity *V*, while the horizontal axis represents the distance from the center of the galaxy

The Doppler Shift reveals that the stars in many galaxies do not slow down at farther distances. The Doppler shift doesn`t explain why? Kepler's third law and consequently Virial theorem does no longer hold for Non-Euclidian space. In the hyperbolic spacetime, the free fall due to the curvature, causes the non-decreasing speed of the galaxies for large *r*, according to our equation of motion in the hyperbolic spacetime $V = e^{-\mu/r}\sqrt{2\mu/r - \mu/a}$. For large distance from the center the spacetime returns flat. The galaxies possessed hyperbolic trajectory, according to Vallado theorem, $V = \sqrt{2\mu/r - \mu/a}$, with constant speed, called the hyperbolic excess velocity, $V_\infty = \sqrt{-\mu/a}$ (describes the flat curve). Dark matter doesn`t exist to account for the mysterious missing mass. Hence, the Doppler Shift accounts for the non-decreasing free fall velocity of the galaxies, caused by the curvature of the hyperbolic spacetime of the cluster. The Doppler Shift, manifests the curvature which causes such high velocity.

7. The Superluminous Speed

Einstein's Equivalence Principle: In small enough regions of spacetime, the laws of physics reduce to those of special relativity in flat spacetime; it is impossible to detect the existence of a gravitational field by means of local experiments. The flat spacetime constraints the domain of applicability of special relativity. The speed of light is the constant "*c*" as long as it is measured locally in the inertial frame of the observer. By locally we mean a sufficient small spacetime that could be regarded approximately flat (Euclidean), through which light propagates in Euclidean straight lines. As objects are separated far distant apart, the spacetime no longer remains flat as well as no longer the velocity of light remains the constant speed "c". In fact the spacetime appears hyperbolic in a large structural scale. Distant objects (e.g., galaxies with $z \geq 1.5$), out of the inertial frame of the observer, relatively recede by velocity greater than the speed of light "*c*". That is due to the hyperbolic spacetime where the line cannot be drawn globally Euclidean straight. The velocity of light depends on its path through the spacetime. The speed of light is absolutely "*c*" only in the locally Euclidean flat spacetime; otherwise it varies –relatively- dependently upon how much spacetime is curved. It slows in the spacetime near compact objects like the sun or a black hole (delay of time), while it increases in a distant spacetime where spacetime appears hyperbolic. "The general relativistic (GR) interpretation of the redshifts of distant galaxies, as the expansion of the universe, is widely accepted. However this interpretation leads to several concepts that are widely misunderstood. Since the expansion of the universe is the basis of the big bang model, these misunderstandings are fundamental. Probably the most common misconceptions surround the expansion of the Universe at distances beyond which Hubble's law ($v_{rec} = HD$: recession velocity = Hubble's constant × distance) predicts recession velocities faster than the speed of light. Recession velocities exceed the speed of light in all viable cosmological models for objects with red shifts greater than $z \sim 1.5$. A common misconception is that the expansion of the Universe cannot be faster than the speed of light. Since Hubble's law predicts superluminal recession at large distances ($D > c/H$) it is sometimes stated that Hubble's law needs special relativistic corrections when the recession velocity approaches the speed of light. However, it is well-accepted that general relativity, not special relativity, is necessary to describe cosmological observations. When observables are calculated using special relativity, contradictions with observations quickly arise. Moreover, we know there is no contradiction with special relativity when faster than light motion occurs outside the observer's inertial frame. General relativity was specifically derived to be able to predict motion when global inertial frames were not available. Galaxies that are receding from us superluminally are at rest locally (their peculiar velocity, $v_{pec} = 0$) and motion in their local inertial frames remains well described by special relativity. Rather, the galaxies and the photons are both receding from us at recession velocities greater than the speed of light"[16]. As an object is far distant apart as much the spacetime appears relatively hyperbolic curved. As an object is far distant apart as much appears with a high redshift. Its velocity appears-relative to the observer- to exceed the speed of light "*c*" due to assumption of flat spacetime.

8. Conclusion

1-The instantaneous physical distance is not itself observable. Cosmological 'motion' cannot be directly observed. It can only be inferred from observations of the cosmological redshift, which general relativity then tells us that the universe is expanding. .

2 - Hubble's observation that the redshifts of spectral lines in galaxies increase linearly with their distance. Hubble took this to show that the universe is expanding uniformly. There is an approximately linear relationship between redshift and distance at small scales for all the FLRW models, and departures from linearity at larger scales can be used to *measure spatial curvature.*

3 - Hubble wrote "The possibility that the redshift may be due to some other cause connected with the long time or distance involved in the passage of light from nebulae to observer, should not be prematurely neglected."

4 - However, for standard candles that are further away, deviation from Eq.(1) can be expected, arising from the spatial curvature of the universe. Deviations can also be expected if the light from a standard candle travels to us over a time during which the *expansion of the universe* is significant

5 - The universe is not flat. We did prove that, the universe globally is hyperbolic. The hyperbolic universe doesn't need dark energy to account for the accelerating expansion. The equation of the state $P = -\rho$ associated with the hyperbolic universe, derives such an accelerated expansion. Astronomers found the supernovae to be fainter than their redshifts would indicate, assuming *flat uniformly expanding universe*. Thus, in our non-

existing dark energy hyperbolic universe, the increase in the cosmological redshift can account for the increase of the curvature that causes such an accelerated expansion relative to the observer.

6 - Two galaxies within our Local Group, including Andromeda, and a few galaxies in the Virgo Cluster display blueshifts and so are moving toward us, but this results from their local motion (peculiar velocity). Why nearby galaxies exhibit blue-shift? The answer is more convenient if we say: Locally the spacetime is flat through which the curvature is negligible (no cosmological redshift), where the random peculiar velocity dominates. Andromeda galaxy is blue-shifted because its sufficiently nearby where the spacetime is approximately flat and special relativity works. Its blue-shifted according to the Doppler effect in flat spacetime.

7 - The Doppler Shift reveals that the stars in many galaxies do not slow down at farther distances. The Doppler shift doesn`t explain why? Kepler's third law and consequently Virial theorem does no longer hold for Non-Euclidian space. In the hyperbolic spacetime, the free fall due to the curvature, causes the non-decreasing speed of the galaxies for large r, according to our equation of motion in the hyperbolic spacetime $V = e^{-\mu/r}\sqrt{2\mu/r - \mu/a}$. For large distance from the center the spacetime returns flat. The galaxies possessed hyperbolic trajectory, according to Vallado theorem, $V = \sqrt{2\mu/r - \mu/a}$, with constant speed, called the hyperbolic excess velocity, $V_\infty = \sqrt{-\mu/a}$ (describes the flat curve). Dark matter doesn`t exist to account for the mysterious missing mass. Hence, the Doppler Shift accounts for the non-decreasing free fall velocity of the galaxies, caused by the curvature of the hyperbolic spacetime of the cluster. The Doppler Shift, manifests the curvature –not dark matter- which causes such high velocity and traces the flat rotation curve.

8 - As an object is far distant apart, as much the spacetime appears relatively hyperbolic curved. As an object is far distant apart as much appears to possess a high redshift. Its velocity appears-relative to the observer- to exceed the speed of light "c" due to assumption of flat spacetime.

9- After all, in the hyperbolic spacetime a group of objects would grow apart even when not moving as their worldlines would be divergent.

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