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On the Successful Prediction of Radioactive Decay

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Abstract

We describe an experimental setup involving a single radioactive atom which decays in unit time with probability ½, but such that it is possible to successfully predict whether the atom decays with probability greater than ½. We also describe a strategy which correctly predicts with probability greater than ½ which of two radioactive atoms is more likely to decay in unit time, given that we can only observe one of the two species. These ideas are mathematical in nature but are very likely to have applications in physics as well as other areas.

***Keywords*:** radioactive decay, prediction, random variable, probability

I. Introduction

In a previous paper [1], we described an experiment in which a radioactive atom in a box decayed in unit time with probability equal to ½, but such that the probability of a correct prediction of whether or not the atom decays is greater than ½. This was done by using two radioactive atoms, one with a probability of decay of 1/3 and one with a probability of decay of 2/3, each being chosen for the experiment with probability ½. To the observer, it would appear that the radioactive atom had a decay probability of ½, but to some extent this was 'sleight of hand', as what the experimenter could not observe was that the experiment used two different radioactive atoms with two different decay probabilities.

In the first section, we construct an experiment which eliminates the "subterfuge" described above by using a **single** radioactive atom, but embeds the experiment in a compound Bernoulli trial as described in ([2]). Although it would have been possible to entitle this paper *Predicting the Fate of Schrodinger's Cat II*, it simplifies the discussion to remove the cat, and its possible demise, from the experiment.

The author is not a physicist, but the phenomenon of squeezed light [3] seems similar to the experiment in [1]. In both cases a presumed limitation (quantum uncertainty in [3], probability of successful prediction in [1]) can be overcome through the judicious use of probability distributions. In [1] it is a distribution of decay probabilities, and in [3] it is a distribution of states.

II. Predicting the Decay of a Single Radioactive Atom

A train runs on an east-west railroad track (east is to the right in Fig. 1) with a finite number of stations. The train carries a box with a single radioactive atom, which decays in unit time with probability 1/2. The motion of the train is determined by whether or not the radioactive atom decays in unit time; if it decays, the train moves one station east, and if it does not decay, it moves one station west.

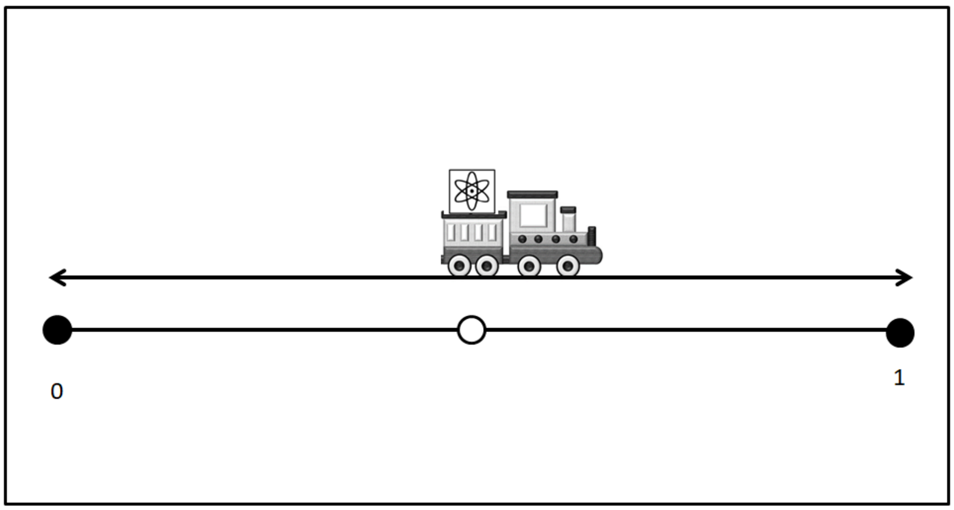

Fig. 1 – Initial Configuration

We assume that the train is initially stopped at a station, but at neither the easternmost nor the westernmost stations. The stations at the end of the line are assumed to be located at 0 and 1 respectively on the unit interval, depicted for convenience below the railroad track in Fig. 1. We assume further that the train has an equal probability at being at one of these non-terminal stations, and is currently stopped at Middletown, whose location on the unit interval is depicted by the white dot. We denote this location by m, $0 < m < 1$.

The experimenter, who is (not unsurprisingly) a passenger on the train, now chooses a random number x on the unit interval from a distribution which is non-zero on any open interval. If $x > m$, (or x lies to the east of the train's current location), he predicts that the radioactive atom will decay; if $x < m$, (or x lies to the west of the train's current location), he predicts that the atom will not decay. The experimenter does not need to know the value of m (where Middletown is located on the unit line) in order to predict which direction the train will proceed; the selection of random number could be done by computer. Alternatively, he could simply look out the train window and use the location of the first bird he sees as the random number; if it is to the east of the train he predicts that the atom will decay, etc.

The radioactive atom now decays – or not – with probability ½. The conductor subsequently announces, “Next stop – Willoughby.” The experimenter, who does not know the location of any of the stops on the line, does know that he is either one stop east or one stop west of Willoughby, each with probability ½.

A Stop at Willoughby is the title of an episode from Season 1 of the classic TV series, *The Twilight Zone.* In the episode, the town Willoughby could be envisioned as a superposition of real and imagined states.

Let A denote the station one stop east of Willoughby, and let p_A denote the probability that x lies to the east of A. Let B denote the station one stop west of Willoughby, and let p_B denote the probability that x lies to the west of B. Middletown is either station A, with probability ½, or station B, also with probability ½.

If the train is at station A, the train must go west to get to Willoughby. The probability of correctly predicting that it goes west is $1-p_A$. Similarly, if the train is at station B, the probability of correctly predicting that it goes east is $1–p_B$. The probability of correctly predicting its direction of travel is therefore

$$½(1-p_A) + ½(1-p_B) = 1 – ½(p_A + p_B)$$

By assumption, the random variable was chosen from a distribution which is non-zero on any open interval, so the term on the right in the above equation is greater than ½. But a successful prediction of the direction of travel is equivalent to a successful prediction of whether or not the radioactive atom will decay. Moreover, this prediction was made **before** the atom actually decayed (or not).

Notice that, before the atom actually decays (or not), the prediction will only have a probability of ½ of being successful. If q denotes the probability that x lies to the east of Middletown, the probability of a successful prediction at this stage is ½ q + ½ (1-q) = ½. As soon as the atom decays, there is a destination – and the train is equally likely to be one stop east or west of it. Before that, the destination is not a fixed location, but a probability distribution of possible destinations. It is only when we know that a destination has been determined that we can improve our probability of guessing

This somewhat surprising result can be explained from the standpoint of compound Bernoulli trials, which are discussed in ([2]). The result presented in this paper can be obtained in a variety of settings – there are essentially two linked variables, a discrete one (the decay of the radioactive atom) and a continuous one (position on the unit interval), although the requirement of continuity can be weakened somewhat.

III. Predicting Which of Two Radioactive Atoms Is More Likely to Decay

Assume we have two different species of radioactive atoms with differing probabilities $S < L$ of decaying in unit time. Assume we are only allowed to observe one of the species. We will examine two separate cases: when we can only make a single observation of whether a particular atom decays (or not) in unit time, and when we can make multiple observations of atoms of the same species. In each case, we begin by selectingwhich of these two species to observe by flipping a fair coin.

The Single Observation Strategy

Having selected the species we are to observe, we now conduct a single trial of that experiment, observing whether the selected atom decays in unit time. We will use that result to guess whether we are looking at the species with the smaller decay probability S or the larger decay probability L by employing the following straightforward strategy. If the result of the single trial is that the observed atom decays, guess that we are looking at the species with decay probability L; if the result is that the atom fails to decay, guess that we are looking at the species with decay probability S.

We compute the probability of correctly guessing which species we have observed. With probability ½ we have selected the species with decay probability L. We will guess successfully that we have selected this species if the outcome of the trial is a decay; the combined probability of selecting that species and the single trial resulting in decay is ½ L. Similarly, with probability ½ we have selected the species with decay probability S. We will guess successfully that we have selected this species if the outcome of the experiment is that the atom fails to decay; the combined probability of selecting that species and the experimental outcome resulting in failure to decay is½ (1-S). Our probability of correctly guessing which species we are observing is therefore½ L + ½ (1-S) = ½ + ½ (L – S). If one selects two random numbers from the uniform distribution on [0,1], it is a good problem for a calculus class to show that the average value of the difference between the larger and smaller numbers is 1/3, and in this case the probability of guessing successfully is 2/3.

There is an obvious application of this idea. A manufacturer has two potential suppliers of a component for which the success probability is a critical issue. He obtains one from one of the suppliers and tests it. If either time or budgetary considerations allow for only one test, this criterion could prove useful.

But what if the manufacturer has both time and budget to make more than a single test? We address this question in the following section.

Repeated Observations

One might think that conducting additional observations of the selected species might enable us to improve the probability of correctly guessing whether we are looking at the species with the larger or smaller decay probability. In order to do that, we have to select a guessing strategy. If we were to conduct two trials of the same species, the obvious strategy is to guess that we are looking at the species with the larger decay probability if both trials result in a decay, and at the species with the smaller decay probability if both trials result in a failure to decay. Were we to conduct two-trial samples until we either achieved two successes or two failures, we would correctly guess in the ratio $L^2:S^2$. However, if we are limited to precisely two trials, we must have a decision principle for guessing which species we are looking at if the two trials result in one decay and one failure to decay. The obvious strategy is simply to flip a fair coin and use it to make the decision.

Using this strategy, once again the probability of selecting the species with the larger success probability is ½. If so, we will guess correctly if the outcomes of both trials are decays, and we will guess correctly half the time if the trials result in one decay and one failure to decay. The probability of selecting the species with the larger decay probability and guessing correctly is therefore $\frac{1}{2}(L^2 + \frac{1}{2}\,2L(1-L)) = \frac{1}{2}L$. Similarly, the probability of selecting the other species and guessing correctly is $\frac{1}{2}((1-S)^2 + \frac{1}{2}\,2S(1-S)) = \frac{1}{2}(1-S)$. Once again, the probability of guessing correctly is $\frac{1}{2} + \frac{1}{2}(L-S)$. It may be somewhat surprising to learn that the second guess is of no value.

An Odd Number of Observations

There is a simple extension of the strategy for a single trial to an odd number of trials. If more decays occur than failures to decay, guess that the selected species has the larger decay probability, and if more failures to decay occur than decays, guess that the selected species has the smaller decay probability.

Let $N = 2p+1$ be the number of trials of the selected species. Using the well-known formula for the probability of k successes in n trials of a Bernoulli trial, the probability of selecting the species with the larger decay probability and having more decays than failures to decay is

$$\frac{1}{2}\sum_{k=p+1}^{N}\frac{N!}{k!\,(N-k)!}\left(L^k(1-L)^{N-k}\right)$$

Similarly, the probability of selecting the species with the smaller decay probability and having more failures to decay than decays is

$$\frac{1}{2}\sum_{k=p+1}^{N}\frac{N!}{k!\,(N-k)!}\left((1-S)^k S^{N-k}\right)$$

The sum of these two is the probability of guessing whether the tested species has the larger or smaller decay probability. That sum is

$$\frac{1}{2}\sum_{k=p+1}^{N}\frac{N!}{k!\,(N-k)!}\left(L^k(1-L)^{N-k} + (1-S)^k S^{N-k}\right)$$

We can show that this expression is always greater than ½. If $0 \le S \le \frac{1}{2} \le L \le 1$ and $S < L$, then the probability that the species with the larger decay probability has more decays than failures to decay is greater than or equal to ½. Similarly, the probability that the species with the smaller success probability has more failures to decay than decays is also greater than or equal to ½. At most one of these two probabilities will be equal to ½, and thus the above sum will be greater than ½ in this case.

That leaves two cases: $0 \leq S < L \leq ½$ and $1/2 \leq S < L \leq 1$. Notice that, since N=2p+1, the above expression is equal to

$$\frac{1}{2}\sum_{k=p+1}^{N}\frac{N!}{k!(N-k)!}\left(L^{k}(1-L)^{N-k}\right)+\frac{1}{2}\sum_{k=0}^{p}\frac{N!}{k!(N-k)!}\left(S^{k}(1-S)^{N-k}\right)$$

This sum can be compared to the binomial expansion for x + (1-x) = 1. In order to do this, we need to analyze the function $f(x) = x^k (1-x)^{N-k}$. If k=0, this expression reduces to $(1-x)^N$; a decreasing function of x, and if k=N, it reduces to x^N, an increasing function of x. If 0 < k < N, the derivative is $f'(x) = x^{k-1} (1-x)^{N-k-1} (k – Nx)$.

In the case $0 \leq S < L \leq ½$, if k > N/2 the sign of the derivative is positive, and the function f(x) is increasing. The first of the two summands will become smaller if L is replaced by S, and the expression then becomes the binomial expansion of $½ (S + (1-S))^N = ½$. If $1/2 \leq S < L \leq 1$, and if k < N/2, the sign of the derivative is negative, and the function f(x) is decreasing. The second of the two summands will become smaller if S is replaced by L, and the expression then becomes the binomial expansion of $½ (L + (1-L))^N = ½$. So the probability of guessing correctly using this strategy is greater than ½.

In the limit, as N approaches∞, if L > ½ the probability of more successes than failures approaches 1. Similarly, if S < ½, the probability of more failures than successes also approaches 1. These limits approach ½ when the success probability of the Bernoulli trial is precisely ½. Therefore, in the limit, the probability of guessing correctly using this strategy is either 1 (when S < ½ and L > ½), ½ (when both S and L lie on the same side of ½), or ¾ (if either S or L is ½).

There are, however, some surprises for small values of N, which can be seen from the following table, in which the values for the probability of guessing correctly are computed for odd values of N from 1 to 11.

S	L	N=1	N=3	N=5	N=7	N=9	N=11
0.2	0.4	0.6000	0.6240	0.6297	0.6282	0.6234	0.6174
0.1	0.3	0.6000	0.5940	0.5772	0.5616	0.5489	0.5389

Notice that in the second line of the table the probability of guessing correctly using this strategy decreases when we do additional tests – not only does it cost more and take more time, but it is actually counterproductive! In the first line of the table, we see that we will guess correctly more often if we do 3 tests rather than 1, or 5 tests rather than 3 – but our probability of guessing correctly declines thereafter.

IV. Possible Applications

The results of Section II have potential applications to physics and other subjects as well. In physics, there are instances in which one can only measure one of several variables, so it is potentially valuable to be able to draw conclusions about probabilities associated with the other variables.

We have already mentioned component testing as one possible application. It is amusing that the methods of this section can be applied to completely unrelated phenomena. Suppose one wishes to determine which percentage is larger: the percentage of people in New York City with brown eyes, or the percentage of people in Cleveland, Ohio who have Type O blood. Select one of the two cities at random, and select a single individual from that city, make the appropriate observation, and guess accordingly.

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[1] James D. Stein, “Predicting the Fate of Schrodinger’s Cat”, *Journal for Foundations and Applications of Physic*s, vol. **5**No. 1, pp. 31-34

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[3] https://en.wikipedia.org/wiki/Squeezed_states_of_light