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# Variation of cross section with special points for $^{16}$O(5/2+) and $^{16}$O(1/2+) states in Alt Grassberger Sandhas version of Faddeev approach

**Anjana Acharya* and Rajib L Nayak**

Department of Physics, Veer Surendra Sai University of Technology,
Burla, Odisha-768018, India

**Corresponding author E-mail*: anjanaachrya1969@gmail.com

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## Abstract
Gauss Legendre special points and weights play a prime role in calculating the cross sections of nuclei in the excited states upto some extent. The inputs taken in these calculations are the separable form of T-matrix and the coupled angular momentum basis. The deuteron is considered to be a mixture of singlet as well as triplet states. The form of the potential is Wood–Saxon type and the parameters are fitted by Reid Soft Core potential. The main objective of our work is to show how the cross section varies with respect to the Gauss Legendre's special points in terms of fermi.



***Keywords***: Special points and weights, Gaussian quadrature method, Stripping reactions, Singularities

## 1. INTRODUCTION
In 1979, two scientists V. N. Koshchev and V. V. Sinitsa had proposed a computational scheme to find out the different parameters associated with the cross section, which are complex in nature. In order to calculate them, they have considered the optimal parameters of the quadrature formulas of the highest degree of accuracy [1]. The scattering amplitude by taking the Gaussian points over an arc in the complex plane are calculated by Paul E Sayler and Dennis C. Suolarski [2]. Matej Batic et al have shown that by applying Gaussian quadrature formula in C, C++ language, the cross section of the ionized atomic collision can be calculated [3]. B. Rennie Kaunda et al have established the applications of a Gaussian quadrature algorithm to cross section construction in bluff displacement studies [4]. Lsslie M. Kerby and Stefan G. Mashnik have found the total reaction cross sections in cascade exciton model by using Monte Carlo N particle transport code at intermediate energies which are having applications in medical physics, accelerator design etc. [5].

Here we started from the AGS version of Faddeev approach as the potential is iterated several times to give the real one [6]. The multi dimensional coupled integral equation then reduced to a one dimensional form [7]. The singularities lying with the reduced form are removed by applying various methods [8-9 ].The the quantum mechanical scattering theory for the cross section are outlined and the correlation between the AGS formalism and quantum mechanical concept to calculate the reaction cross-section are established.

## 2. PRESENT WORK

AGS equation is written as

$$U_{\alpha\beta} = \left(1-\delta_{\alpha\beta}\right)(z-H_0) + \sum_{\gamma=1}^{3}\left(1-\delta_{\alpha\gamma}\right)T_\gamma(z)G_0(z)U_{\gamma\beta}(z) \tag{1}$$

Where $T_\gamma(z)$ is the two-body transition operator in three-body space defined by the Lippmann-Schweinger equation,

$$T_\gamma(z) = V_\gamma + V_\gamma G_0(z)T_\gamma(z) \tag{2}$$

Also

$$G_\gamma(z) = (z-H_\gamma)^{-1} = (z-H_0-V_\gamma)^{-1} \tag{3}$$

Here $V_\gamma$ is the interacting potential for the γth pair of particles , $G_0(z)$ is the resolvant operator for the free state, z represents the total amount of energy and $H_\gamma$is the energy carried by the γth pair of particles.

The matrix element of the AGS operator $U_{\alpha\beta}(z)$ between the asymptotic states viz.

$< \vec{Q}_\alpha d_\alpha \sum_{L_\alpha S_\alpha} \phi^{n_\alpha}_{(L_\alpha S_\alpha)J_\alpha M_\alpha} | U_{\alpha\beta}(z) | \vec{Q}_\beta d_\beta \sum_{L_\beta S_\beta} \phi^{n_\beta}_{(L_\beta S_\beta)J_\beta M_\beta} >$ can be easily related to the cross section of the process $\beta \to \alpha$ as [ 10],

$$\frac{d\sigma_{\beta\to\alpha}}{d\omega} = \frac{h^2}{8\pi^2\mu_\beta Q_\beta}2\pi^4 \left| < \vec{Q}_\alpha d_\alpha \sum_{L_\alpha S_\alpha} \phi^{n_\alpha}_{(L_\alpha S_\alpha)J_\alpha M_\alpha} | U_{\alpha\beta}(z) | \vec{Q}_\beta d_\beta \sum_{L_\beta S_\beta} \phi^{n_\beta}_{(L_\beta S_\beta)J_\beta M_\beta} > \right|^2_{av} \tag{4}$$

Where $Q_\alpha$ and $Q_\beta$are the on-shell momenta of the initial and final particles $Q_\beta^2 = (E + \varepsilon^{n_\beta}_{B_\beta})$ and $Q_\alpha^2 = (E + \varepsilon^{n_\alpha}_{B_\alpha})$. The suffix ‘av’ means that the quantity is to be averaged over the initial and final states.

By choosing an angular momentum basis and applying the orthonormality condition, the AGS equation can be reduced to a one dimensional form as,

$$T_{\alpha\beta}\left(q_\alpha q'_\beta \beta_\alpha \beta_\beta : J\right) =$$

$$\sum_{L_\alpha S_\alpha}\sum_{L_\beta S_\beta}\iint \frac{g^{n_\alpha}_{(L_\alpha S_\alpha)J_\alpha}(p_\alpha)p_\alpha^2 dp_\alpha}{(z-p_\alpha^2-q_\alpha^2)} < p_\alpha q_\alpha (L_\alpha S_\alpha)\beta_\alpha : J | U_{\alpha\beta}(z) | p'_\beta q'_\beta \left(L_\beta S_\beta\right)\beta_\beta : J > \frac{g^{n_\alpha}_{\left(L_\beta S_\beta\right)J_\beta}(p'_\beta){p'_\beta}^2 dp'_\beta}{(z-{p'_\beta}^2-{q'_\beta}^2)} \tag{5}$$

Here the T-matrix is written in the separable form in terms of the form factors.

In quantum scattering, the radial form of the Schrodinger’s equation is

$$\frac{d^2U_l(r)}{dr^2} + \left[k^2 - U(r) - \frac{l(l+1)}{r^2}\right]U_l(r) = 0 \tag{6}$$

$$\Psi_{lm}(r) \equiv \frac{U_l(r)}{r}Y_{lm}(\theta,\emptyset) \tag{7}$$

$$k^2 = \left(\frac{2mE}{\hbar^2}\right)$$

$$U(r) = \left(\frac{2m}{\hbar^2}\right)V(r)$$

$$\sigma_{tot} = \int_0^\infty \frac{4\pi}{k^2}sin^2\delta_l \, d\Omega$$

Where $\delta_l$ is the phase shift [11].

**Our new findings**

It is seen that the cross section values are higher for the even-even nuclei where channel spin is conserved compared to the nuclei not conserving the channel spin even when coulomb interaction is included. In the case of (d, n) reaction on $^{16}O$, the total spin in the initial channel is equal to the one in the final channel. We see that the spin of the stripping particle i.e. that of neutron is equal to ½. So in order to satisfy the principle of channel spin conservation the spin of the residual nuclei should be either equal to ½ or 3/2 so that the cross section value is large. For $^{16}O$ case, we have taken the value of orbital angular momentum $l$ = 2 having spin ½ giving rise to j=5/2. From the experimental data we see that the cross section in the $S_{1/2}$ state of the residual nuclei is higher than that when the residual nucleus is left in the ground state [12].The incident deuteron energy values are 5 Mev and 10 Mev. So the energy ranges are in the intermediate ones. The values of the parameters for $^{16}O$ are shown in the table.

**Table-1**

| Reaction Type | p-Points | p weights | g | Re I | p cot δ | $\sigma$ (in mb) |
|---|---|---|---|---|---|---|
| $^{16}O + n = {}^{17}O$ | 0.11698 | 0.41964 | 0.46394 | 0.0747 | -0.336758184 | 1.10 |
| | 0.58681 | 1.41964 | 0.40826 | 3.37466 | -2.056121918 | 3.06 |
| | 1.31876 | 2.41964 | 0.27106 | 1.5795 | -2.750316471 | 1.50 |
| | 2.14533 | 3.41964 | 0.1604 | 0.73889 | -5.294693297 | 4.28 |
| | 2.87728 | 4.41964 | 0.10524 | 0.40511 | -9.938633344 | 1.31 |
| | 3.34711 | 5.41964 | 0.08263 | 0.3048 | -14.97081464 | 5.94 |
| | 3.5689 | 6.41964 | 0.07725 | 0.31504 | -17.26320975 | 4.50 |
| | 3.90768 | 7.41964 | 0.07959 | 0.38576 | -17.13756466 | 4.52 |
| | 4.21934 | 8.41964 | 0.07211 | 0.35884 | -20.47172606 | 3.20 |
| | 4.45629 | 9.41964 | 0.06936 | 0.3711 | -22.32689355 | 2.70 |
| | 5.18397 | 10.4196 | 0.0609 | 0.31584 | -27.79379143 | 1.75 |
| | 6.95264 | 11.4196 | 0.04697 | 0.2054 | -42.80258854 | 7.43 |
| | 11.3104 | 12.4196 | 0.03004 | 0.0912 | -94.72899543 | 1.54 |
| | 25.418 | 13.4196 | 0 | 0 | - | 0 |
| | 127.504 | 14.4196 | 0 | 0 | - | 0 |

**NOTE:** The p-points are the Gauss Legendre 15 points having unit fermi denoted as K along X-axis as shown in the figure

$p_{weights}$ are the special weights related with the points

g stands for the form factor when coulomb interaction is not included

$g_c$ values are zeroes for that reaction considered

Re I is an abbreviation used to find out the real value of the phase shift applying effective range theory

σ symbolizes the cross section for the reaction measured in mb (along the Y-axis as shown in the figure)

p cot δ term comes from the effective range theory expansion

p = $h$K/ 2π with specific modifications in the units

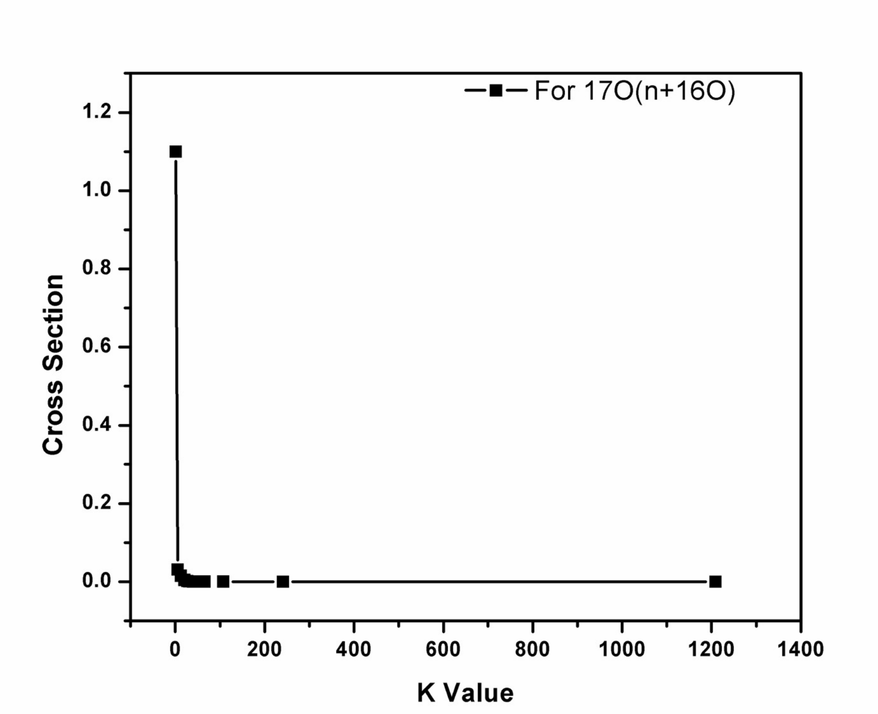


**Figure 1.** Variation of cross section in mb versus momentum for deuteron stripping reaction on $^{16}O$ resulting in $^{17}O$

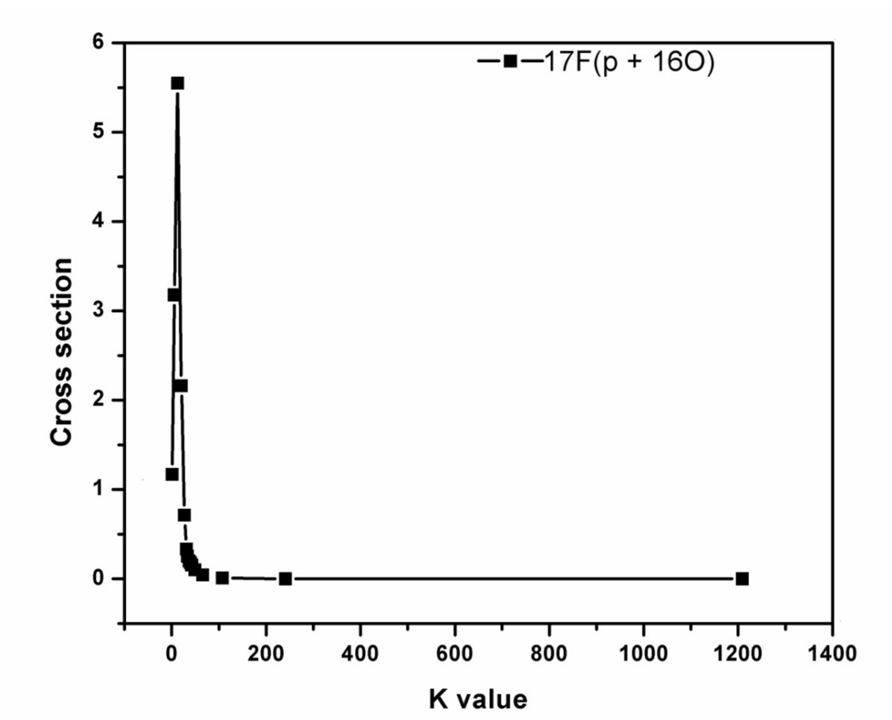


**Figure 2.** Variation of cross section in mb versus momentum for deuteron stripping reaction on $^{16}O$ resulting in $^{17}F$

## 3. CONCLUSION

Referring to the figure 1 with the variation of K values from 0 to 1400 fm, we see that the cross section values varies from 0 to 1.1mb having some clustering effect around the origin.. Figure 2 represents the σ verses K, when some coulomb interaction in Laguerre polynomial term is taken. This occurs due to the increasing value of the Gaussian quadrature points, the form factor tends to decrease. Here, we have used the 15 point quadrature formula for on shell and off shell conditions at different ranges of energy i.e. (i) 0 to $E^{1/2}$ (ii) $E^{1/2}$ to $(E+2\epsilon_B)^{1/2}$ and (iii) $(E+2\epsilon_B)^{1/2}$ to ∞ where $\epsilon_B$ is the binding energy of the residual nucleus with the core. For the isotopes of oxygen which are having applications in nucleosynthesis, the quadrature points can be increased. That is our future work.

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