A MODEL FOR VISCOELASTIC CONSOLIDATION OF WOOD-STRAND MATS. PART I. STRUCTURAL CHARACTERIZATION OF THE MAT VIA MONTE CARLO SIMULATION

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(Received October 1994)

ABSTRACT

A procedure using Monte Carlo simulation was developed to characterize the spatial structure of randomly formed, wood-strand mats. The simulation reproduces the number of strands in the centroids of imaginary strand columns of finite size. The vertical distances between the adjacent strands and the location of the column centroid relative to the constant length of each strand are also simulated. A data base was collected on realistic mats produced from strands of constant size and non-planar geometries (i.e., random bow, cup, and twist). The procedure can be used in a model for predicting the mechanical behavior of random strand mats during consolidation.

Keywords: Monte Carlo simulation, wood-strand mats, consolidation, hot pressing.

INTRODUCTION

During the hot-pressing operation, a loosely formed combination of adhesive and wood material is consolidated into a contiguous composite material. For laminated wood composites such as plywood, LVL, and glulam, the material structure after consolidation is dictated primarily by the geometry of the constituent wood elements. However, for materials produced with discontinuous wood elements such as fibers, particles, and strands, the final material structure is governed by not only the wood elements themselves, but also the forming methods and pressing operation.

With non-veneer wood composites, void space is incorporated into the wood mat as a consequence of the forming operation. The goal of pressing is actually to remove the void space that separates the individual wood elements, thereby providing contact between strands and promoting adhesion. Heat is used in this operation to both soften the wood component and to accelerate the cure of the adhesive. However, the development of material structure, which ultimately will play a large role in determining engineering performance, is often a secondary result of pressing.

Recently, several publications have dealt with a theoretical description of mat forming and modeling the wood mat during consolidation. Steiner and Dai (1994) provided a rationale for model development to describe the spatial structure of wood composites in relation to processing and performance characteristics. A model for simulating the horizontal density distribution in strandboard was published by Suchsland and Xu (1989). The density distribution was related to the internal bond and thickness swell properties of the panels. Harless et al. (1987) published a model to pre-

Wood and Fiber Science, 28(1), 1996, pp. 100–109

dict the density profile of particleboard. Their model predicted the through-the-thickness density gradient of particleboard as a function of manufacturing parameters such as platen temperature and press closing rate.

Focusing on the wood component, the nonlinear compression behavior of wood was studied by Wolcott et al. (1989a). The developed model for structural collapse was based on the theories of cellular materials. To understand the influence of heat and moisture, polymer viscoelasticity was used to qualitatively describe the density gradient formation in strandboard (Wolcott et al. 1990). Experimental data for various density gradients supported the analytical approach presented.

The number of publications focusing on mat consolidation might indicate the importance of hot pressing to the manufacture of wood-composites. There is a considerable potential for managing the final product properties by controlling the wood constituents' behavior during the manufacture. However, a fundamental understanding and comprehensive model are still needed to describe the viscoelastic consolidation of wood-based composites.

OBJECTIVES

While pursuing an overall understanding of wood-composite manufacture, a fundamental model is being developed to describe the mechanical behavior of wood-strand mats during hot pressing. The model is based on theories that incorporate structural nonlinearities in the mat, the viscoelastic behavior of the wood constituents, and time-temperature-moisture interaction during hot pressing. To achieve the ultimate goal of this project, the following specific objectives were pursued in characterizing the mat structure:

1. Develop a conceptual model for mat structure characterization.
2. Experimentally measure relevant parameters in randomly formed wood mats.
3. Develop and statistically validate a simulation procedure to reconstruct the mat structure.
4. Experimentally validate the model.

BACKGROUND AND CONCEPT OF MODEL DEVELOPMENT

To characterize the structure of a randomly formed, wood-strand mat, several parameters of the strand size and geometry, their position, and orientation can be used. Although many of these parameters are either fully or partially controlled in manufacture, they all remain stochastic variables with statistical variability. For this reason, any modeling effort must incorporate methodologies using statistical principles that allow a mat structure to be recreated with computer simulation.

Suchsland (1962) modeled the random strand mat as a system of columns having different numbers of horizontally stacked strands. The strand content of the columns was hypothesized to follow a binomial probability distribution. The mat stress response was computed using the solid wood stress-strain relationship in transverse compression. Most recently Dai and Steiner (1994a, b) developed a mathematical model and computer simulation for describing the structure of randomly formed, single and multilayered strand mats. Based partially on this model, the authors have shown that the formation of the random strand mat can be described by the Poisson distribution of strand centers and strand coverage. This structural characterization of the mat was a basis of a theoretical model to predict the static stress-strain response of the mat in compression (Dai and Steiner 1993). The model applies an infinite summation of the modified Hooke's law proposed by Wolcott et al. (1989a) over the mat area. The number of strands in an infinitesimal column of the mat was determined by the Poisson process. Experimental results, using slender, constant-sized aspen strands with uniform shape (i.e., negligible cup, bow, or twist), showed good agreement with the predicted stress response. However, the model slightly underpredicted the stress response in the early stage of consolidation (i.e.,

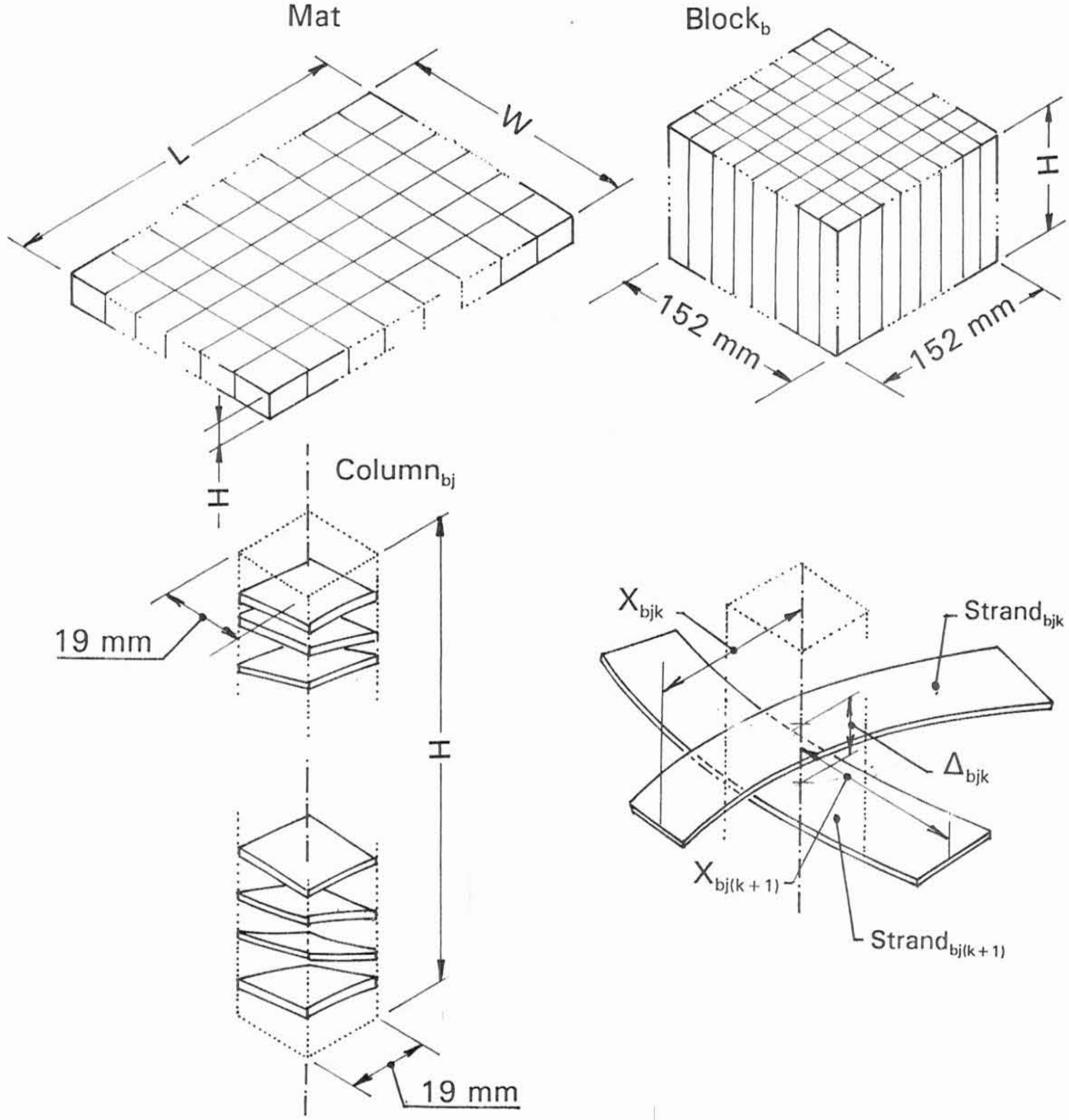

FIG. 1. A schematic of the conceptual model for the static stress-strain behavior of random strand mats. a—an arbitrarily sized mat divided into blocks; b—experimental block, divided into 64 columns; c—the theoretical structure of a column; d—the position of the strands and the interpretation of spatial variables.

$\sigma < 1.5$ MPa). This deviation was attributed to the fact that during the initial stage of compression, stress develops from strand bending instead of transverse compression.

In the research presented here, an attempt is made to further improve these models by structurally characterizing realistic mats. Experiments with yellow-poplar strands showed that the bulk density of the mat can be affected by the magnitude of bow, cup, and twist of the individual strands—namely, the greater the strand curvature, the lower the initial bulk density of the mat. A lower bulk density in the mat results in a larger amount of void space

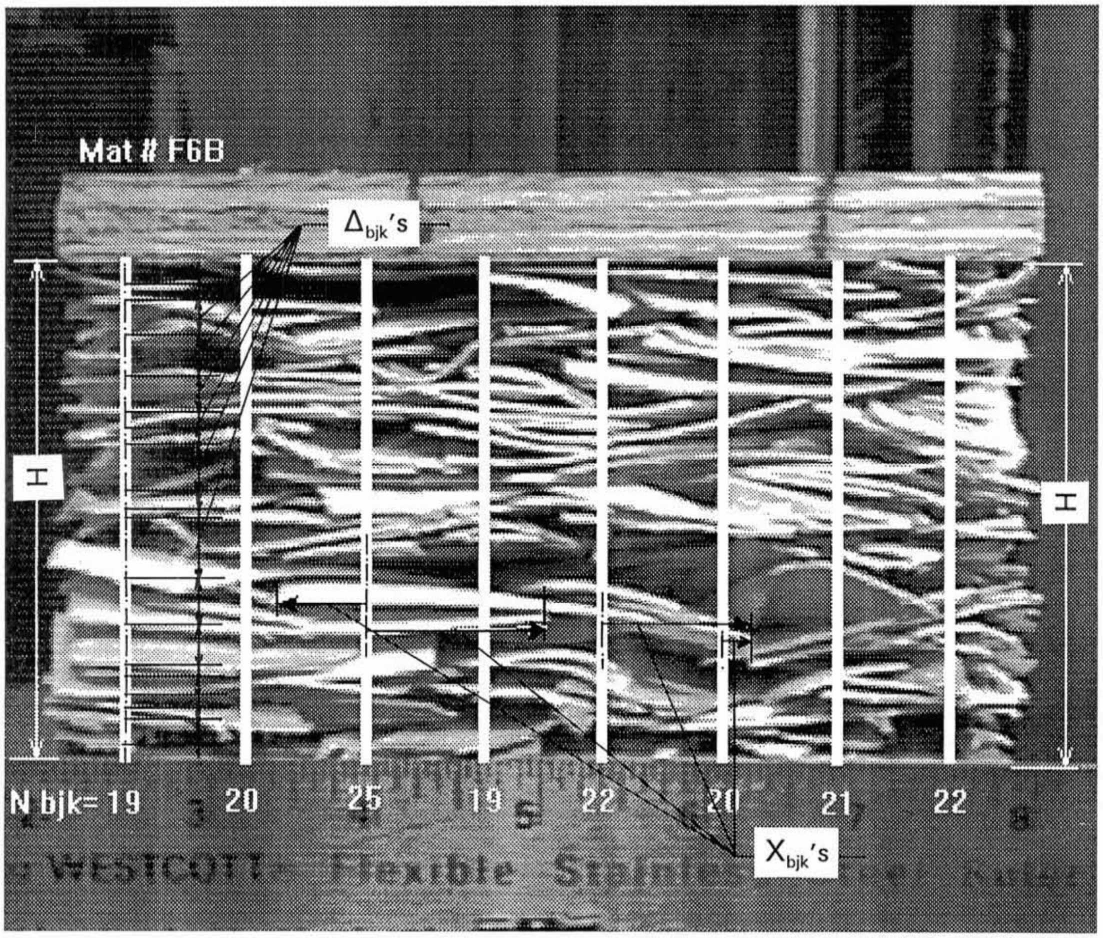

FIG. 2. A digitized image of one side of a sample mat showing the interpretation of the mat characteristics.

within each column. This void space must be removed before the strands can attain contact for proper adhesion. Mechanically, this lower bulk density places a larger contribution on strand bending during the early stage of mat compression. As consolidation continues, overlapping strands form columns of varying height that deform in compression, and the stress development can be described using the modified Hooke's law (Wolcott at al. 1989a; Dai and Steiner 1993).

For model development, the mat area was divided into square sample blocks (152 × 152 mm) (Fig. 1). Each block was further divided into effective columns. The column width was equal to 19 mm (the average strand width). This resulted in 64 columns within a block. The following variables characterized the spatial structure of the mat (Fig. 2):

1. The number of overlapping strands in a column (N_{bj}).
2. The distance between the adjacent strands measured at the centroid of the column or as further referred void height (Δ_{bjk}). This variable was considered as the maximum deflection of that particular strand in bending.
3. The location of the column centroid relative to the strand length (X_{bjk}).
4. The average height of the mat (H).

The subscripts b = 1, . . . , 20, j = 1, . . . , 32 and $k = 1, \ldots, N_{bj}$ denote the block, column, and strand numbers, respectively. Accordingly, measurements of 640 columns from 20 sample blocks were used for data base development. Note that although each block was divided into 64 columns, only 32 column centroids could be measured along the perimeter of the blocks without disturbing the initial mat structure. The average mat height (H) was considered as a control variable because the overlapping strands, with constant thickness (h = 0.8 mm), and the void height between the adjacent strands within a column control the initial height of the mat according to the following:

$$H = \Sigma \Delta_{bjk} + (N_{bj}h) \quad (1)$$

The remainder of this paper presents the description of the mat structure and the development of the simulation routine to generate arrays of the structural mat characteristics (N_{bj}, X_{bjk} and Δ_{bjk}). In general, the simulation should maintain the marginal distributions and preserve any correlation structure of the multiple random variables. Statistical methods used to validate the simulation procedure are also presented.

SIMULATION ROUTINE DEVELOPMENT

Materials and methods of data base assessment

Twenty sample mats (blocks) were manufactured from uniformly sized (0.8- × 19- × 70-mm) strands. Mats were prepared in a 305- × 305-mm forming box. The central section, having a base of 152 × 152 mm, was cut out from the mat after formation using a large paper shear. This technique eliminated the edge effect during formation, and the sample blocks were considered as a section of randomly formed mat. The mat was formed to a target density of 600 kg/m^3 based on 13-mm final panel thickness. From each side of the blocks, a scaled video image was acquired with an image analysis system. On the images, eight column centroids were marked 19 mm apart. For each column, the height and the number of overlapping strands were manually measured and counted (Fig. 2). In addition, one hundred concomitant void height (Δ_{bjk}) and location (X_{bjk}) values were measured from the 80 images. Note that these measurements could only be performed at strands whose lengths were aligned parallel with the image plane. Table 1 summarizes the descriptive statistics of these mat variables. The average height (H) of the sample block was calculated from the 32 measurements, and based on H, the bulk density of the sample mat block was calculated. The average overlapping strand number in the 64 columns of a sample mat was approximated by computing the mean of the 32 counts at the column centroids along the perimeter of the mat.

TABLE 1. *Descriptive statistics of sample mats' variables.*

Statistics	λ_b	N_{bj}	X_{bjk} (mm)	Δ_{bjk} (mm)	H (mm)
Mean (μ)	24.42	24.17	12.69	3.22	73.91
SD (σ)	1.82	3.89	7.86	2.37	3.56
Size (n)	20	640	100	100	20
Min.	21.62	10	2.65	0.71	67.06
Max.	28.38	42	38.72	14.56	80.52
Skewness	0.28	0.12	1.65	2.48	−0.25

λ_b—average overlapping strand numbers in 32 columns of a sample block.

The correlation structure and probability distributions of the original data set

The overlapping strand numbers (N_{bj}) were tested for both lag-1 and lag-2 type serial correlation. The maximum deflection (Δ_{bjk}) and location (X_{bjk}) values are not consecutive measurements; therefore, no serial correlation tests were performed on these data sets. The serial correlation analyses were done for each block having an equal sample size of observation. Of the twenty sample mats, six showed only slight positive lag-1 type serial correlation, while only one lag-2 type correlation was observed. This result seemingly contradicts the fact that Dai and Steiner (1994b) observed spatial correlation between adjacent overlapping strand numbers in randomly formed aspen strand mats. However, the distance between the adjacent count in their study was approx-

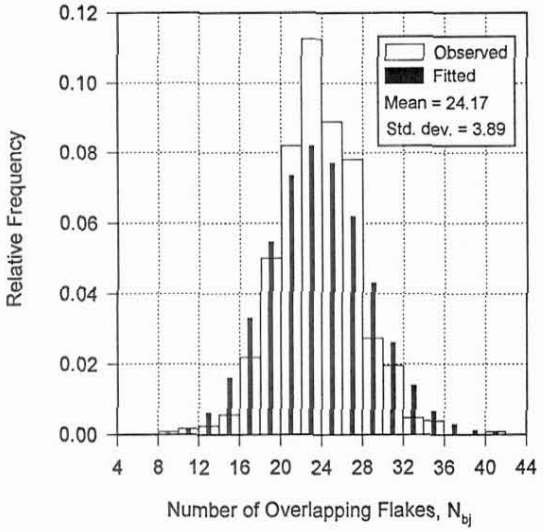

FIG. 3. Frequency histogram of 640 observed overlapping strand numbers. The fitted Poisson (λ) probability mass function, with the parameters of the observation, is overlaid.

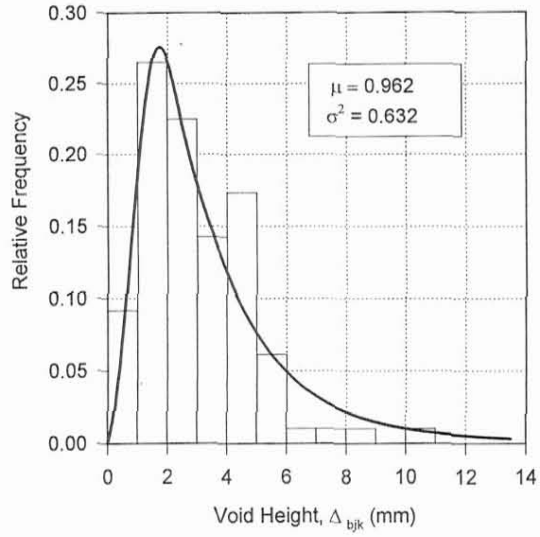

FIG. 4. Frequency histogram of 100 observed void height values. The two-parameter lognormal density function, with the parameters of the natural logarithms listed, is overlaid.

imately $\frac{1}{10}$ of their strand width (i.e., 9.31 mm). Such a high resolution must have serial correlation because a particular strand can be present in ten adjacent imaginary columns. In this study the overlapping strands were counted 19 mm (a strand width) apart from each other, showing no significant serial correlations. The lack of serial correlation simplifies the simulation procedure and indicates that all of the observations in the sample are independent of each other.

In the next step, the correlation between void height (Δ_{bjk}) and location (X_{bjk}) data was investigated. The correlation coefficient of the population was estimated to be $\rho_{\Delta X} = -0.0316$, indicating no significant correlation between these variables.

Finally, theoretical distribution functions were fit to the data to determine the marginal distribution of the variables. The parameters of the probability distributions were estimated using the least-squares method. Visual appraisal and formal goodness-of-fit tests, such as Chi-square (χ^2) and Kolmogorov-Smirnov (KS) tests, helped to assess the probability density functions.

For both location (X_{bjk}) and void height (Δ_{bjk}), the formal tests failed to reject the hypothesized lognormal ($LN(\mu, \sigma^2)$) probability distributions. Figures 3, 4, and 5 show the histograms and the overlaid probability mass and density functions for overlapping strand numbers (N_{bj}), void height (Δ_{bjk}), and location (X_{bjk}) data, respectively. Table 2 summarizes the estimated parameters and the results of the goodness-of-fit tests. Based on the work of Dai and Steiner (1993, 1994a, b), it was hypothesized that the strand deposition in randomly formed mats follows the Poisson (λ) distribution. However, the formal tests rejected the hypothesis that the overlapping strand numbers (N_{bj}), in a finite number of columns, came from a Poisson (λ) distribution where λ is both the mean (μ) and the variance (σ^2) of the data. Notice that the 640 observations of N_{bj} values had a higher mean than variance (Fig. 3, Table 1). Truncated normal and several discrete probability distributions including binomial, discrete uniform distributions were fitted to the N_{bj} data without success. This failure to specify the probability distribution of the overlapping strand numbers might be attributed to

TABLE 2. *Summary of the Goodness-of-Fit test results. All tests were performed with 95% confidence level, $\alpha = 0.05$.*

Mat variables	Fitted distr.	Parameters		D.O.F.[c]	Test statistics and critical values[d]			
		μ	σ^2		χ^2	$c_{1-\alpha}$	KS	$c'_{1-\alpha}$[e]
N_{bj}	Poisson (λ)	24.17	15.13	17	41.46	27.587	—	—
λ_b	$N(\mu, \sigma^2)$	24.42	3.32	9	4.00	16.919	0.320	0.895
Δ_{bjk}	$LN(\mu, \sigma^2)$	0.96[a]	0.63[b]	17	12.40	27.587	0.545	0.895
X_{bjk}	$LN(\mu, \sigma^2)$	2.38[a]	0.56[b]	17	21.60	27.587	0.812	0.895

[a] Mean of natural logarithms.
[b] Variance of natural logarithms.
[c] Degrees of freedom.
[d] Test statistics and critical values were computed according to the relevant literature.
[e] Critical value of KS test when the parameters are unknown.

the low resolution of the strand counts and/or to the fact that the sample blocks were independently formed instead of being sections of a continuous mat.

Thus, a new variable, the mean overlapping strand number (λ_b), was introduced into the simulation procedure. Because λ_b is an average of 32 N_j observations of a sample block, it tends to be normally distributed according to the Central Limit Theorem. The parameters of this normal distribution are as follows:

$$\mu_{\lambda_b} = \mu_{N_{bj}}$$

$$\sigma_{\lambda_b} = \frac{\sigma_{N_{bj}}}{\sqrt{n}} \tag{2}$$

where n is the sample size. Figure 6 shows the frequency histogram of mean strand numbers with the normal probability density function overlaid. Because the Poisson (λ) was still the best estimate of the distribution of N_{bj} data, two key assumptions were made. First, the means of the overlapping strand numbers in the blocks can be generated from a normal probability distribution. Second, using these mean values as different λs for each block to generate Poisson (λ_b) variates, the simulation will better approximate the parameters of the overall N_{bj} observation than generating variates from direct Poisson (λ) using only one mean and variance.

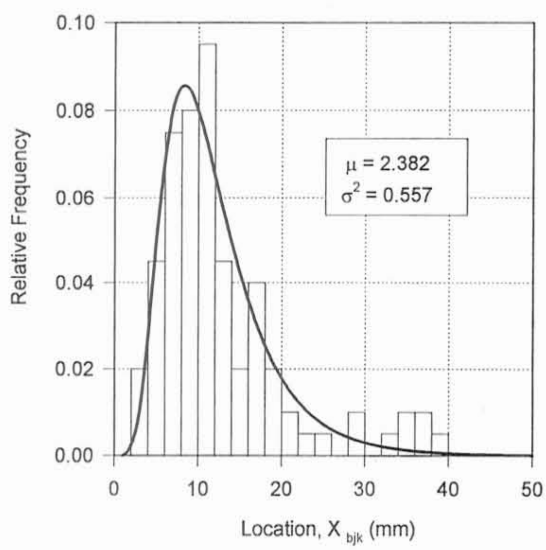

FIG. 5. Frequency histogram of 100 observed location values. The two-parameter lognormal density function, with the parameters of the natural logarithms listed, is overlaid.

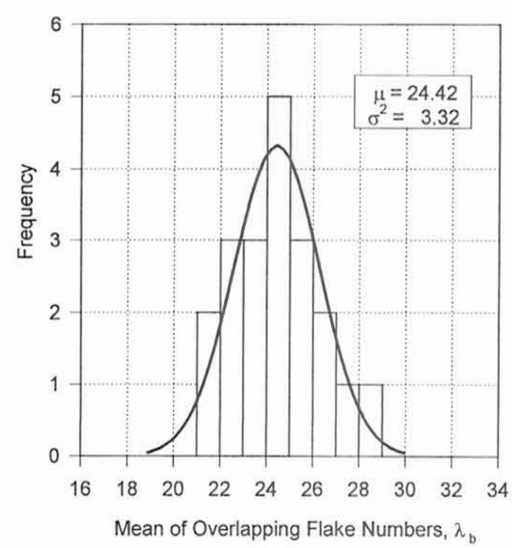

FIG. 6. Frequency histogram of 20 observed average overlapping strand numbers in 64 columns of a sample mat. The normal probability density functions, with parameters listed, is overlaid.

TABLE 3. *Descriptive statistics for five replications of mat characteristics simulation.*

Replication	λ_b			N_{bj}				Δ_{bjk} (mm)			X_{bjk} (mm)			H (mm)		
	μ	σ	n	μ	σ	n	ΣN_j	μ	σ	n	μ	σ	n	μ	σ	n
1	24.12	1.62	20	23.91	4.29	64	1174	3.18	2.16	1174	12.61	7.41	1174	73.4	6.1	64
2	24.95	2.30	20	24.35	5.09	64	1274	3.46	2.49	1274	12.96	8.07	1274	74.2	6.3	64
3	24.98	2.07	20	25.31	4.46	64	1649	3.27	2.21	1649	13.08	8.15	1649	75.0	6.2	64
4	23.52	1.85	20	23.09	4.93	64	1425	3.31	2.40	1425	12.79	8.23	1425	73.9	5.9	64
5	24.31	1.53	20	24.79	5.16	64	1210	3.27	2.31	1210	12.58	7.46	1210	74.3	6.2	64
Average	24.38	1.87	–	24.29	4.79	–	–	3.32	2.36	–	12.84	8.23	–	74.2	6.1	–
Original	24.42	1.82	20	24.17	3.89	640	–	3.22	2.37	100	12.69	7.86	100	73.9	3.6	20

λ_b—average number of overlapping strands in 64 columns in sample block b; N_{bj}—number of overlapping strands in column j of block b; Δ_{bjk}—maximum deflection of the kth strand in the jth column of block b; X_{bjk}—location of the jth column centroid relative to the kth strand's length in block b; and H—height of a column.

The simulation routine

Based on these principles, a simulation procedure was developed. The routine included the following main steps.

1. The mean values of overlapping strands of 64 columns in a sample block were generated from normal probability distribution ($\lambda_b \in N(\mu, \sigma^2)$). The polar method was used to generate normally distributed variates (Law and Kelton 1991).
2. In the next step, using the λ_b values, 64 N_{bj} data for each sample block were generated from the Poisson (λ_b) distribution. The algorithm utilizes the relationship between Poisson (λ) and expo($1/\lambda$) distributions. A FORTRAN routine was written to produce the desired number of deviates from the Poisson distributions. The routine computes the sum of N_{bj} data by $b = 1, \ldots, 20$.
3. For the simulated blocks according to the results of ΣN_{bj} values, the other two mat characteristics, such as location (X_{bjk}) and void heights between adjacent strands (Δ_{bjk}), were generated from the $LN(\mu, \sigma^2)$ probability distributions. This algorithm is based on the fact that if $Y \in N(\mu, \sigma^2)$ then $e^Y \in LN(\mu, \sigma^2)$. Note, that the means and variances are different for the normal and lognormal distributions.

The details and justification of generating random variates from $N(\mu, \sigma^2)$, Poisson (λ) and $LN(\mu, \sigma^2)$ probability distributions are given by Law and Kelton (1991). The routines for generating discrete and continuous random variates included a subroutine for generating standard uniform U(0, 1) random numbers. This FORTRAN subroutine was published by Etter (1987). Tests, recommended by the ASTM D 5124-91 (1991) standard, were performed to confirm the feasibility of this random number generator.

RESULTS AND DISCUSSION

The simulation procedure described above was repeated five times to generate the mat characteristics of N_{bj}, X_{bjk}, and Δ_{bjk}. Table 3 contains the summary statistics of the simulation results. The parameters of the replications and their averages can be compared to the original data. Data from one replication for each variable were used to graphically depict the simulation results. Except for the N_{bj} data, KS goodness-of-fit test was used to test the hypothesis that the simulated data could have come from the previously specified marginal distributions. Each test confirmed the acceptability of the simulation.

Figure 7 shows the frequency histogram of 20 mean overlapping strand numbers. The λ_b values from this replicate were used to generate 1,280 N_{bj} data to confirm the applicability of the Central Limit Theorem for this simulation procedure. Figure 8 shows the frequency histogram of the simulated data and the overlaid Poisson (λ) probability mass function. The two-step simulation produced similar distribution of N_{bj} data to that observed on the twenty real

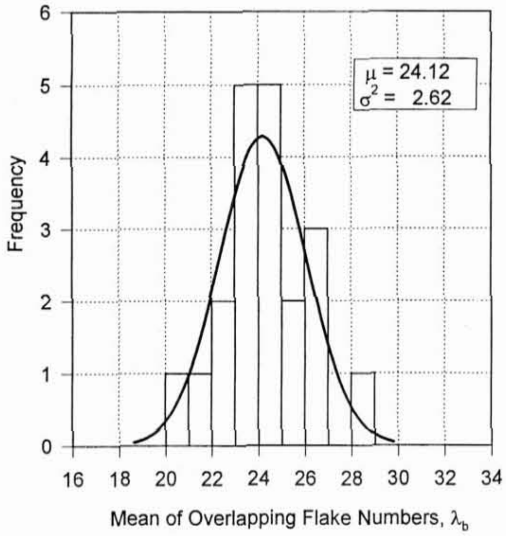

FIG. 7. Frequency histogram of 20 simulated average overlapping strand numbers in 64 columns of a sample mat. The original normal probability density function, with parameters of the simulated data listed, is overlaid.

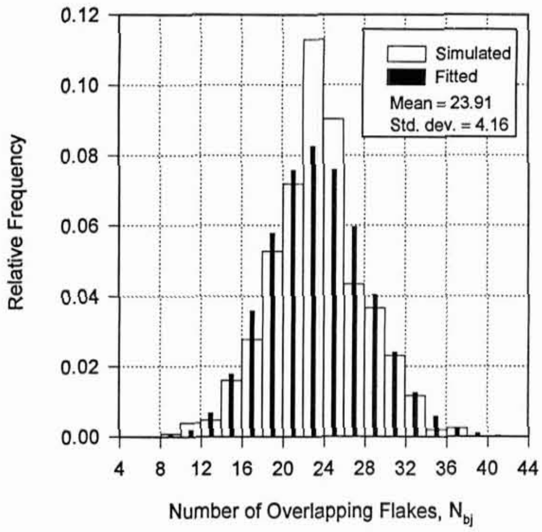

FIG. 8. Frequency histogram of 1,280 simulated overlapping strand numbers. The original Poisson (λ) probability mass function, with the parameter of the simulated data listed, is overlaid.

mats. The means and variances of the simulated and real data are comparable (Figs. 3 and 8). The other two simulated variables were checked visually. The simulation adequately preserved the marginal distributions and resulted in a noncorrelated structure of the data similar to that observed.

From the N_{bj} and the assigned Δ_{bjk} data, the average height of 64 columns in a sample block was computed for each replication using Eq. 1 (Table 3). Notice that the means of 64 column heights are similar to the measured value, but the standard deviations were almost double that of the experimental data. This phenomenon resulted because the original H data were measured from platen to platen. The simulation assigns one Δ value to each strand. However, the number of cavities in a column vary between $N_{bj} \pm 1$ depending upon the shapes of the first and last strands (i.e. convex or concave). Sensitivity analysis of the static stress-strain model revealed that the variability of void height data had no significant effect on the stress prediction. Therefore, this problem was neglected during the simulation routine development.

CONCLUSIONS

The developed Monte Carlo simulation procedure appears to adequately reproduce the marginal distributions of mat spatial characteristics. The simplicity and flexibility of this method present several advantages. With modifications, the described procedure can be used to simulate other wood strand/particle mat structures. Spatial characteristics governed by stochastic variables such as strand orientation, size, and shape of the strands can also be incorporated.

ACKNOWLEDGMENT

This research was funded by the USDA National Research Initiative Competitive Grants Program, Agreement No. 91-37103-6624. The financial support is gratefully acknowledged. The authors wish to express their appreciation to Dr. Frederick Kamke and Christopher Lenth for their contribution to this research at the Department of Wood Science and Forest Products, Virginia Polytechnic Institute and State University, Blacksburg, VA. This manuscript is published as Scientific Article no. 2516 of the West Virginia Agriculture and Forestry Experimental Station.

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