DIFFUSIVITY AND SURFACE EMISSIVITY IN WOOD DRYING

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ABSTRACT

Edge and end-coated sweetgum (*Liquidambar styraciflua* L.) and redwood (*Sequoia sempervirens* (D. Don.) Endl.) 6 × 10 cm samples, of thicknesses ranging from 0.5 to 3.0 cm in either the longitudinal or tangential directions, were dried, from either the water-soaked condition or from slightly below fiber saturation, to equilibrium in circulated air (300 ft/min) at 100 F and 75% relative humidity.

A constant-rate drying period was observed during the early stages of drying for the initially water-soaked samples but not for those initially below fiber saturation. The apparent diffusion coefficient D', calculated for the samples initially below fiber saturation on the assumption that surface resistance to drying was negligible, that is $D' \approx 0.2\ a^2/t_{0.5}$ (where a is the half-thickness and $t_{0.5}$ is the half-drying time), increased with increasing wood thickness. The true diffusion coefficient D and the surface emission coefficient S (which is inversely proportional to the surface resistance to drying) were calculated from the linear relationships observed between $t_{0.5}/a^2$ (or $0.2/D'$) and $1/a$, or between $t_{0.5}/a$ (or $0.2a/D'$) and a, using Newman's solution to the diffusion equation.

The diffusion coefficient D was higher for longitudinal than for tangential drying for both woods, and higher for sapwood than for heartwood of redwood. The surface emission coefficient S for redwood was found to be 60% greater than for sweetgum, presumably because redwood is less hygroscopic and also less dense than sweetgum.

In a previous paper (Choong and Skaar 1969), a method was described for separating the internal and external resistances to moisture removal in wood drying. The method was based on Newman's (1931) solution to the diffusion equation with constant diffusion coefficient D, in which the surface resistance to evaporation is considered in terms of a surface emission coefficient S. This coefficient is proportional to the rate at which moisture evaporates from a drying surface when the surface moisture content is higher than the equilibrium moisture content of the drying air (eq. 8). The method described in the previous paper is based on an empirical approximation equation to Newman's exact equation that relates the half-drying time $t_{0.5}$, the diffusion coefficient D, the half-thickness a of a drying sample in the direction of movement, and the coefficient H defined as the ratio of S/D. The approximation equation is

$$t_{0.5}(D/a^2) = 0.2 + (0.7/Ha). \qquad (1)$$

The relationship given by eq. 1 is not exact, as is clear from Table 1, in which the values of $t_{0.5}(D/a^2)$ obtained by eq. 1 are compared with those calculated from Newman's exact equation, for several values of $1/Ha$. However, the error is 2% or less over the range of $1/Ha$ covered in the table and is generally less than other errors involved in practical drying measurements.

In the previous paper, eq. 1 was used to calculate D and S (or H) by measuring the half-drying times $t_{0.5}$ for unidirectional drying of two matched wood samples of different thicknesses $2a$. In the present

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Table 1. *Comparison of $t_{0.5}(D/a^2)$ obtained from Newman's Exact Equation and the Approximate Equation (Eq. 1) for various values of $1/Ha$*

$1/Ha$	$t_{0.5}(D/a^2)$	
	Exact equation	Approximate equation
0.00	0.196	0.200
0.10	0.261	0.270
0.25	0.383	0.375
0.50	0.562	0.550
1.00	0.914	0.900
2.00	1.610	1.600

Fig. 1. Plot of a/D_s vs. a from Ogura's data.

paper, eq. 1 is used in a more general form to enable D and S to be calculated from simultaneous drying experiments on samples of several different thicknesses. Equation 1 can be rearranged into either of two forms, as follows:

$$t_{0.5}/a^2 = 0.2/D + 0.7/Sa \qquad (2)$$

or,

$$t_{0.5}/a = 0.2a/D + 0.7/S \qquad (3)$$

Equation 2 predicts a linear relationship between $t_{0.5}/a^2$ and $1/a$, the intercept and slope of which are equal to $0.2/D$ and $0.7/S$. Equation 3 predicts a linear relationship between $t_{0.5}/a$ and a, the intercept and slope of which are equal to $0.7/S$ and $0.2/D$.

In the classical solutions of the diffusion equation that have been applied to wood drying, the surface resistance to drying is generally taken to be negligible compared with the internal resistance. In this case, the measured coefficient may be called the apparent or superficial diffusion coefficient, designated here as D'. It is equal to the value of D obtained for the case where the surface emission coefficient S is taken to be infinite. Equations 2 and 3 then reduce to

$$t_{0.5}/a^2 = 0.2/D' \qquad (4)$$
$$t_{0.5}/a = 0.2a/D' \qquad (5)$$

Combining eq. 2 and 4, and eq. 3 and 5, gives

$$1/D' = 1/D + 3.5/Sa \qquad (6)$$
$$a/D' = a/D + 3.5/S \qquad (7)$$

From eq. 6 and 7, one would expect that the apparent or superficial diffusion coefficient D', calculated by means of eq. 4
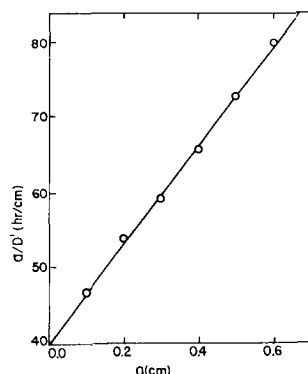
or 5 or from any solution of the diffusion equation in which surface resistance is neglected, should increase with increasing sample thickness $2a$. This appears to be true from a curve of D' against sample thickness given by Ogura (1950) for samples of *Fagus sieboldies* dried at 45 C from the green condition of 70 to 80% to a moisture content of 9% for sample thicknesses $2a$ ranging from 0.2 to 1.2 cm. For example, from his curve the values of D' are 2.2, 3.8, 5.0, 6.1, 6.8 and 7.5×10^{-3} cm^2/hr for $2a = 0.2$, 0.4, 0.6, 0.8, 1.0, and 1.2 cm, respectively. Figure 1 shows a plot of a/D' against half-thickness a. The relationship appears to be linear as anticipated, with a slope of $1/D$ of 60.6 hr/cm^2, and an intercept $3.5/S$ of 41.0 hr/cm. Thus, the true diffusion coefficient D is 0.0165 cm^2/hr (or 4.61×10^{-6} cm^2/sec), and the surface emission coefficient S is 0.0854 cm/hr (or 23.7×10^{-6} cm/sec), and $H = 23.7/4.61 = 5.15$ cm^{-1}.

PROCEDURE

Two species were selected for this study, namely sweetgum (*Liquidambar styraciflua* L.) and redwood (*Sequoia sempervirens* (D. Don) Endl.). One sweetgum sapwood board, and two redwood boards, one sapwood and one heartwood, were obtained in the green condition.[1] From each board,

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[1] The sweetgum board was obtained from a local sawmill. The redwood boards were sent by William Pratt, Arcata Redwood Co., Arcata, Calif., in the green condition.
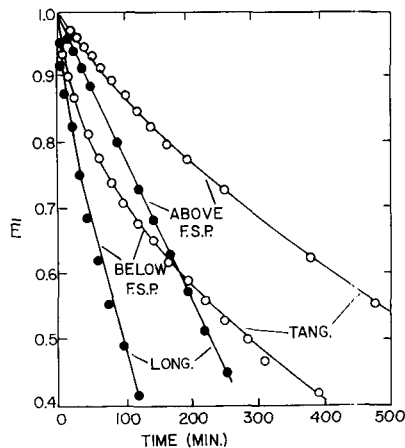
Fig. 2. Relationship of $\bar{E}$ with drying time for sapwood samples of sweetgum approximately 1.0 cm thick.

Fig. 3. Relationship of $\bar{E}$ with drying time for heartwood samples of redwood approximately 3.0 cm thick.

longitudinal and tangential flow samples, measuring approximately 4.0 cm × 6.0 cm in cross section and with nominal thicknesses of 0.5, 0.75, 1.0, 1.75, and 3.0 cm were prepared. Those samples that were to be dried from the nearly water-soaked condition were first impregnated with water using the Lowry process (50 psig for 2 hr); then they were subsequently subjected to periodic alternate vacuum and atmospheric pressure for several months. Those that were to be dried from near fiber saturation point were first partially dried in a conditioning chamber at high humidity, then stored in desiccators over calcium sulfate (nominal 95% RH at room temperature) for at least six months.

A few weeks before the drying experiment, the four edges of each sample were coated with three coats of Dow's Saran F-120 and two coats of Fisher's Sealit in order to provide a unidirectional movement of moisture either longitudinally or tangentially. An experimental drying chamber, as described by Choong and Fogg (1968), was used. The environment of the chamber was controlled at 100 ± 0.2 F and 75% RH, with an air speed of about 300 ft/min. The weight of each sample was measured at periodic intervals during drying until equilibrium was attained.
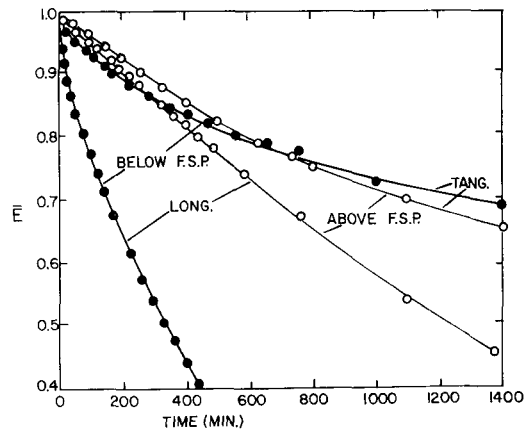
RESULTS

The difference in the drying behavior of the water-soaked (above fiber saturation point) samples compared with those conditioned to a uniform initial moisture content M_i slightly below fiber saturation is clear from Fig. 2 for sweetgum and Fig. 3 for redwood, where the dimensionless variable $\bar{E}$ is plotted against time on a linear scale. The term $\bar{E}$, designated as the fraction of evaporable water remaining in the wood, is defined as $(M - M_e)/(M_i - M_e)$, where M_e is the equilibrium moisture content of the wood in the drying air, and M is the mean wood moisture content at any drying time t.

The initially water-soaked sample is characterized by a constant drying rate (a linear relationship between $\bar{E}$ and time) during the initial stages of drying. As postulated by Sherwood (1929), this constant rate period occurs as long as the surface moisture content is above fiber saturation since the vapor pressure of wood at constant temperature remains essentially constant during this period. There is no constant drying rate period for the samples dried entirely in the hygroscopic range since the surface vapor pressure decreases continually with time as the surface moisture content decreases. Therefore the solutions to the diffusion equations given by Newman, and approximated by Eq. 1 to 3, apply only to
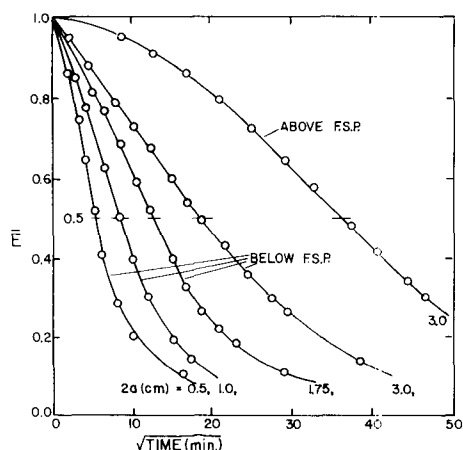
Fig. 4. Relationship of $\bar{E}$ with square root of time for sapwood samples of redwood, showing family of curves for samples of different thicknesses dried in the tangential direction from above and below the fiber saturation point.
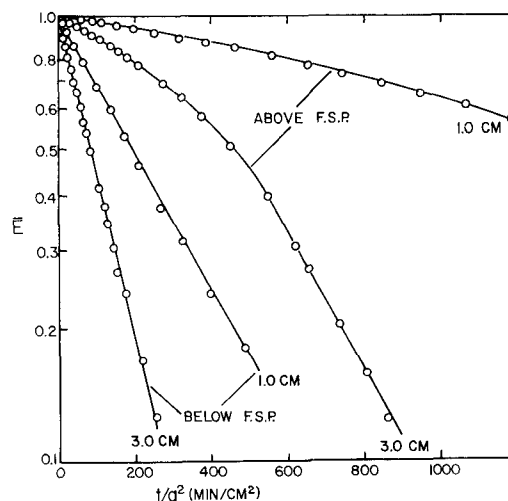

Fig. 5. Plot of Log $\bar{E}$ vs. t/a^2 showing linearity of curves during the latter part of drying for sapwood samples of redwood approximately 1.0 and 3.0 cm thick dried in the longitudinal direction from above and below the fiber saturation point.

the samples dried in the hygroscopic range, under the assumptions of uniform initial moisture content M_i and constant coefficients D and S over the range of drying.

Figure 4 shows the drying curves for redwood samples of varying thicknesses dried in the hygroscopic range, showing $\bar{E}$ as a function of the square root of time. These curves should be linear during the early stages of drying (Stamm 1964), as they appear to be. Also shown in Fig. 4 is the drying curve for a water-soaked redwood sample, which is clearly nonlinear during the early stages of drying.

Figure 5 shows the same drying data plotted as log $\bar{E}$ against t/a^2 for sample thickness of 1.0 cm and 3.0 cm, respectively. Theoretically, these curves should be linear during the latter part of the drying cycle for all values of the surface emission coefficient S or of the dimensionless parameter Ha, according to the Newman solution of the diffusion equation shown in Fig. 6 (taken from Skaar 1954). Also, since the diffusion coefficient D is presumed to be constant, the thicker samples having a larger Ha should have steeper slopes than the thinner samples, as shown in the figure.

Using the half-drying times $t_{0.5}$ obtained from each sample dried entirely in the hygroscopic range, where M_i is definitely less than the fiber saturation point M_f, curves of $t_{0.5}/a^2$ against $1/a$ were plotted as shown in Fig. 7, also for redwood. By use of eq. 2, the true diffusion coefficient D and the surface emission coefficient S were calculated for longitudinal and tangential flow in sapwood and heartwood of redwood, and also for longitudinal and tangential flow in sapwood of sweetgum. The results are summarized in Table 2. Also shown in the table are the superficial diffusion coefficients D' for each wood thickness obtained by use of eq. 4. It is clear that D' increases with increasing wood thickness toward the true value D, in agreement with the results reported by Ogura (1950). A recent sorption study by Mc-Namara and Hart (1971) on yellow poplar (*Liriodendron tulipifera* L.) also confirmed that the diffusion coefficient is directly related to wood thickness.

As anticipated, the diffusion coefficient D is higher for longitudinal than for tangential
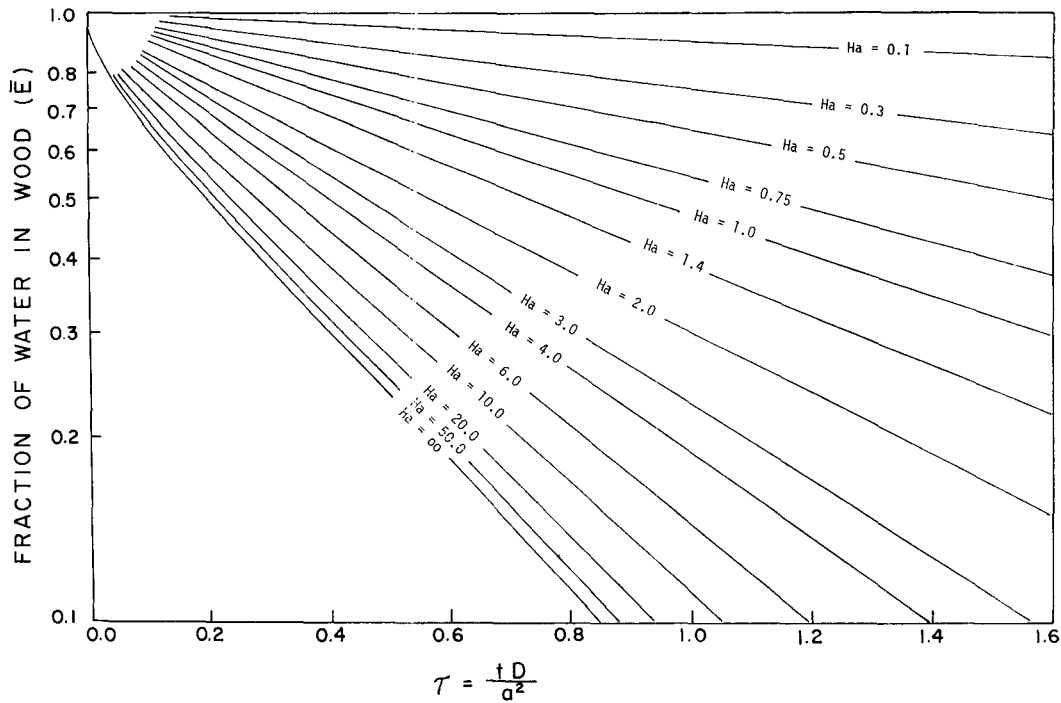
$$\mathcal{T} = \frac{tD}{a^2}$$

Fig. 6. Graphical representation of Newman's solution to the unsteady-state diffusion equation, showing $\bar{E}$ vs. tD/a^2 for various values of Ha.

Table 2. *Calculated coefficients of diffusion (D) and surface emission (S) for sweetgum and redwood samples dried from near fiber saturation to nominal 18% EMC*

| Flow direction and wood type	Diffusion coefficient (D) ($\times 10^6$ cm²/sec)						Emission coefficient (S) ($\times 10^4$ cm/sec)	S/D ratio H (cm⁻¹)
	Apparent value at nominal thickness (cm) of					"True" value		
	½	¾	1	1¾	3			
SWEETGUM								
Longitudinal, sapwood	—	7.8	9.4	14.9	16.2	23.5	1.2	5.1
	4.2	8.1	—	14.6	15.5	25.5	1.1	4.3
Tangential, sapwood	2.3	4.1	5.0	5.4	—	10.4	0.9	8.6
	2.4	4.0	5.2	6.1	—	9.9	0.9	9.1
REDWOOD								
Longitudinal, sapwood	10.7	—	22.8	42.6	63.6	133.4	1.6	1.2
	14.3	—	25.0	36.9	53.0	131.6	1.8	1.2
Longitudinal, heartwood	15.4	—	16.6	23.6	34.7	78.9	1.3	1.6
	14.9	—	17.4	23.3	31.8	66.6	1.7	2.7
Tangential, sapwood	5.5	—	13.6	20.2	27.5	47.0	1.5	3.2
	9.3	—	14.9	18.1	21.5	36.5	1.9	5.2
Tangential, heartwood	1.9	—	1.7	1.9	2.3	2.6	1.8	69.0
	1.9	—	1.8	1.8	2.4	2.2	1.5	68.1
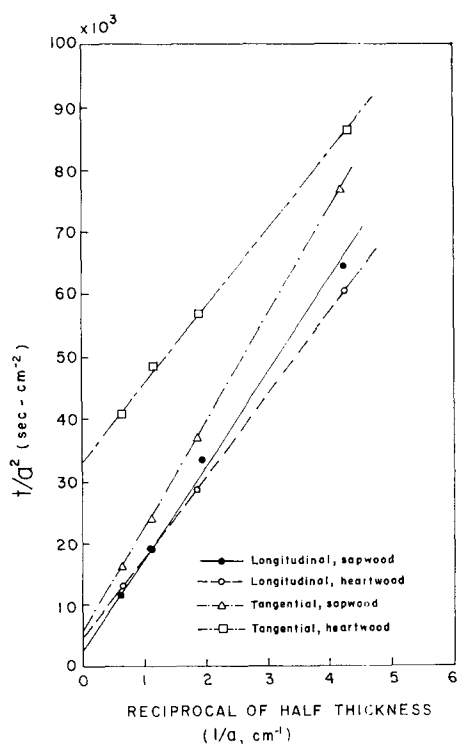
FIG. 7. Relationship of t/a^2 with $1/a$ for redwood samples. (t in this case was at $\bar{E} = 0.5$)

drying in both redwood and sweetgum. It is also higher for sapwood than for heartwood in redwood, but no data are available for this difference in sweetgum.

The mean value for the surface emission coefficient S was significantly higher for redwood, with a range from 1.3 to 1.9 and a mean of 1.64×10^{-4} cm/sec, compared with the sweetgum values, which ranged from 0.9 to 1.2, with a mean of 1.02×10^{-4} cm/sec. The rate of surface evaporation of moisture under given atmospheric conditions, including a fixed air circulation rate, is proportional to the difference in vapor pressure between the wood surface and the drying air. The surface emission coefficient, however, is defined in terms of the differences in moisture concentrations of the wood surface C_a and of the wood in equilibrium with the drying air C_e, as follows,

$$S = F/(C_a - C_e) \tag{8}$$

where F is the moisture flux from the wood surface in gm/cm²-sec, and C_a and C_e are expressed in terms of grams of water per cc of wood, related to the per cent wood moisture content M as follows,

$$C = G_m(M/100), \tag{9}$$

where G_m is the ratio of the oven-dry weight of wood to its volume at moisture content M, numerically equal to specific gravity if cgs units are used.

It can be shown that the surface emission coefficient should be inversely proportional to wood density or specific gravity G_m if it is assumed that the flux F of moisture at the surface is proportional to the vapor pressure difference $p_a - p_e$, where p_a and p_e are the vapor pressures corresponding to moisture concentrations C_a and C_e, respectively. Thus, eq. 8 can be written as,

$$S = A(p_a - p_e)/(C_a - C_e) = A(dp/dC). \tag{10}$$

On the assumption that the variation of G_m with M is negligible, this can be combined with eq. 9 to give

$$S = 100(A/G_m)(dp/dM), \tag{11}$$

where A is a constant of proportionality relating flux F and vapor pressure difference $p_a - p_e$.

The air-dry specific gravity is approximately 0.40 for redwood and 0.52 for sweetgum; hence the surface emission coefficient S for redwood can be expected to be $0.52/0.40 = 1.3$ times the value for sweetgum. The observed ratio is $1.64/1.02 = 1.61$, which compares favorably with theoretical considerations, assuming that the slope of the sorption isotherm dp/dM is identical for the two species. However, redwood is generally less hygroscopic than sweetgum and therefore dp/dM is also expected to be greater for redwood than for sweetgum, thus tending to increase the ratio of S for the two woods. For example, if sweetgum is 1.25 times as hygroscopic as redwood, the ratio of their surface emission coefficient should be $(1.3)(1.25) = 1.62$, compared with the observed ratio of 1.61.

CONCLUSIONS

On the basis of the results of this study, it is concluded that:

1. It is necessary to consider the surface emission coefficient S of wood in predicting the rate of drying.

2. Newman's solution of the diffusion equation with constant diffusion coefficient D and the surface emission coefficient S gives satisfactory and consistent values for both these parameters when applied to redwood and sweetgum dried from slightly below fiber saturation to equilibrium with 75% RH at 100 F.

3. The surface emission coefficient for redwood is about 1.6 times higher than that for sweetgum when they are dried under the conditions given above.

4. Drying of water-soaked redwood and sweetgum samples is characterized by a constant drying rate period during the early stages of drying.

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