

MODEL FOR STRESS ANALYSIS AND STRENGTH PREDICTION OF LUMBER¹

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ABSTRACT

A mathematical model has been developed that can predict the elastic and strength behavior of a section of a structural lumber member containing a knot and cross grain. The model, embodied in the computer program KMESH1, accounts for the presence of a knot, the associated grain deviations, and global cross grain, and can define localized stresses and displacements anywhere within the member. These capabilities are illustrated here through an examination of maximum stress concentrations for varying knot locations. The results point out the severe stress concentration that can be caused by an edge knot as opposed to a similar size center knot.

An "effective section technique" is presented as a strength prediction procedure that uses Program KMESH1 and a maximum stress failure theory. Unlike other strength prediction methods, this procedure recognizes that a progressive failure sequence leads to the ultimate member load. Through calculation of stresses and strains, and a predicted progressive failure sequence, the effective section technique was shown to be quite accurate in predicting the strength for two example pieces of lumber.

Keywords: Stress analysis, strength behavior, mathematical models.

A more efficient utilization of the available wood supply is essential to meet the increasing long-term demand for wood products and to insure their economic viability. The realization of an improved utilization of wood for structural applications depends heavily on our abilities to understand more fully and to predict accurately the mechanical behavior of wood members under load.

Inherent variation, inhomogeneous character, and the presence of material discontinuities, all emphasize the complexity of the problem that must be confronted in modeling the stiffness and strength of lumber. Fortunately, high-speed computers and associated numerical solution methods have provided powerful tools to investigate the above problem.

Recent research efforts have resulted in a mathematical analysis model capable of describing the variable and inhomogeneous nature of lumber by modeling knots and the associated grain deviations as well as global cross grain (Cramer 1981; Dabholkar 1980). Initial work has also begun on the modeling of cracks (Cramer 1981). While work continues on refinement of the model, its current usefulness lies in its ability to provide stress and strain values for any location in a piece of

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lumber under axial load. More importantly, prediction of strength and mode of failure of a member are also provided. Prediction of strength requires a knowledge of local stress and strain conditions, especially at the vicinity of defects within a member. Such knowledge can, for the first time, be readily determined through the use of this mathematical model.

MATHEMATICAL MODELING OF WOOD WITH DEFECTS

Although a number of studies have examined the behavior and strength of dimension lumber, they have been mostly experimental in nature, resulting in empirical representations of structural behavior. The analytical model presented here combines an orthotropic finite element analysis with a geometric prediction technique for grain angles around knots. The analytical model is contained in the computer program package entitled KMESH1 (Cramer 1981). Its overall capabilities are discussed below.

Certainly, the major effect of a knot in a piece of lumber is that which is caused by the localized grain deviation associated with the knot. A rational procedure to predict the grain angle at any location is employed in KMESH1. The prediction procedure has been named "flow-grain analogy" (Goodman and Bodig 1980). The flow-grain analogy relates grain lines in the vicinity of a knot to streamlines of laminar fluid flow around an elliptical (or circular) object. The grain angles predicted by this method have been compared to actual measured values from two coniferous species with various knot sizes. From this comparison, the analogy has been shown to be quite accurate (Phillips et al. 1981).

A finite element mesh is fitted to the predicted grain pattern so that it simulates the grain deviation around a knot in the two-dimensional, longitudinal (L)-tangential (T) plane of the wood. Figure 1 shows a typical finite element mesh resulting from use of the flow-grain analogy. This figure illustrates the case of a circular knot located in an off-center position in the piece. The grain angle for each element is calculated from the generated flow-grain lines. For a typical quadrilateral element, as shown in Fig. 1, the grain angle is determined by averaging the angle calculated at the midpoints of the upper and lower sides of the element. This method of grain angle generation can also be used to model specimens containing cross grain. This task is accomplished by simply adding (or subtracting) the cross grain angle (angle of load to the grain in the member) to the localized element angles.

KMESH1 is capable of modeling knots that are located anywhere totally within the cross section, as well as knots that intersect the edge but still have at least half their cross section within the member. Knot location is specified to the program by a dimensionless knot location ratio, R (see Fig. 2). The knot location ratio, R , is defined as the distance from the edge of the member to the knot center line, K , divided by the width of the member, W . Depending on the knot location, the program automatically sets the required boundary conditions. If a line of symmetry exists in the assumed geometry of the member, the program will detect this and analyze only one symmetrical half or quarter of the member.

Existing cracks can be approximated by placing a series of thin elements with near-zero properties at the crack location. In addition, the following procedures are automatically performed in the program:

- 1) Computation of the finite element loading for a given applied tension or bending load,

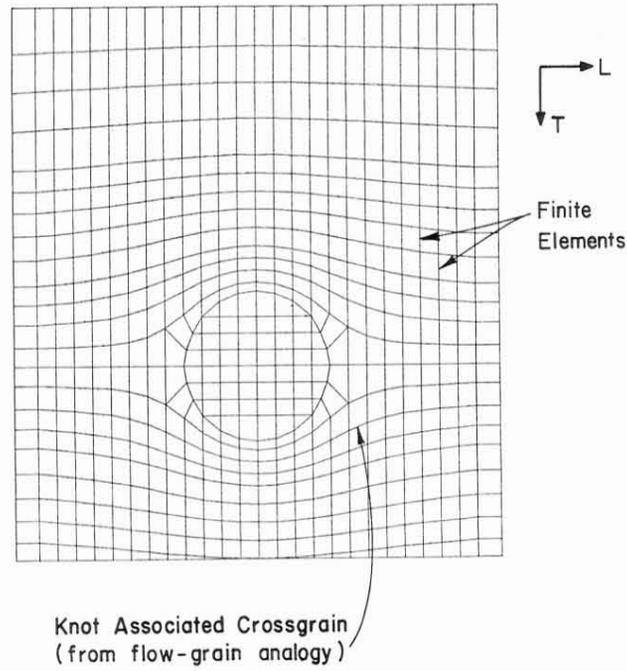


FIG. 1. Simulated grain pattern resulting from the flow-grain analogy.

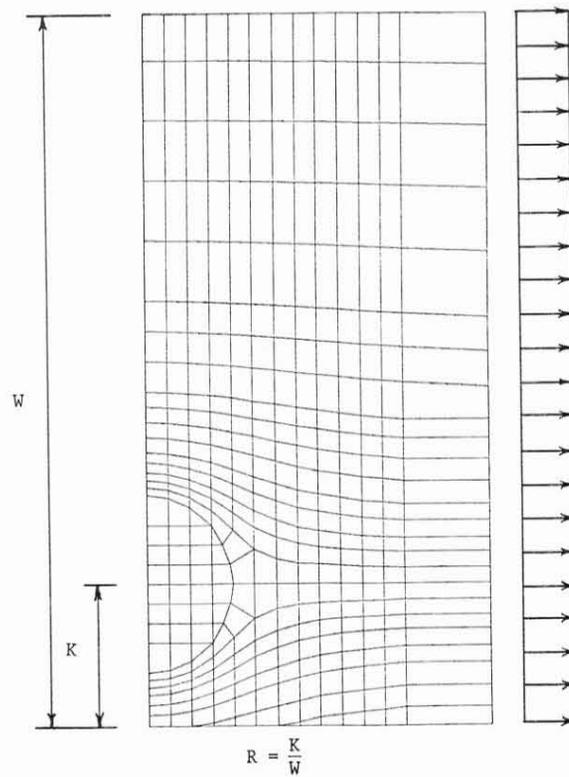


FIG. 2. Example finite element mesh for knot location study and definition of the knot location ratio, R .

- 2) Optional mesh plotting,
- 3) Direct accessing of the finite element calculation routine to perform the stress analysis (Bathe et al. 1974).

Each analysis yields element stresses and corner displacements of each element in the global coordinate directions of the member. Since it is often desirable to examine stress conditions in the element coordinate directions (local parallel- and perpendicular-to-grain directions), a stress transformation is made within the program to provide this information. In addition, the program compares the local coordinate element stresses to clear wood strength values to predict the load necessary to cause a localized failure. Specifically, the element stresses are compared to the clear wood strength in tension perpendicular to grain, tension parallel to grain, and shear parallel to grain. Options exist in the program to plot stress distributions and the deformed shape of the segment of the member if desired.

APPLICATION OF STRESS AND STRAIN ANALYSIS

The usefulness of Program KMESH1 lies in its ability to describe the resulting stress and strain conditions within a member and to predict the ultimate load capacity. A study on the effect of knot location on stress concentration is presented as an example of stress and strain analyses. The member is analyzed as a two-dimensional problem in the longitudinal (L)-transverse (T) plane. The objective of the study was to determine the effect of knot location on the resulting stress field under a uniformly applied tension stress parallel to the long axis of the member.

The member selected was Douglas-fir and the needed material properties, taken from the literature (Bodig and Goodman 1973; Pugel 1980), are listed below:

Wood	$E_L = 2,141,000$ psi	(Bodig and Goodman 1973)
Properties	$E_T = 91,200$ psi	(Bodig and Goodman 1973)
	$E_R = 142,000$ psi	(Bodig and Goodman 1973)
	$G_{LT} = 108,000$ psi	(Bodig and Goodman 1973)
	$\nu_{LT} = 0.47$	(Bodig and Goodman 1973)
Knotwood	$E_L = 2,141,000$ psi	(Bodig and Goodman 1973)
Properties	$E_T = 50,000$ psi	(Pugel 1980)
	$E_R = 50,000$ psi	(Pugel 1980)
	$G_{RT} = 38,000$ psi	(Pugel 1980)
	$\nu_{RT} = 0.42$	(Bodig and Goodman 1973)

A 4 to 1 member width to knot diameter ratio was chosen for the study. A typical finite element mesh for the case of the knot location ratio, R , equal to 0.2 is shown in Fig. 2. As defined previously, a knot location ratio of 0.0 coincides with a half edge knot, while a knot location ratio of 0.5 refers to a center knot. A total of seven knot locations were analyzed.

The resulting maximum stress concentrations (MSC) in the member longitudinal and transverse directions for varying knot locations are shown in Figs. 3 and 4, respectively. Figure 3 indicates that the most severe longitudinal stress concentration occurs when the edge knot is located near the edge of the member ($R \approx 0.13$). Knots that lie totally within the boundaries of the member, but very

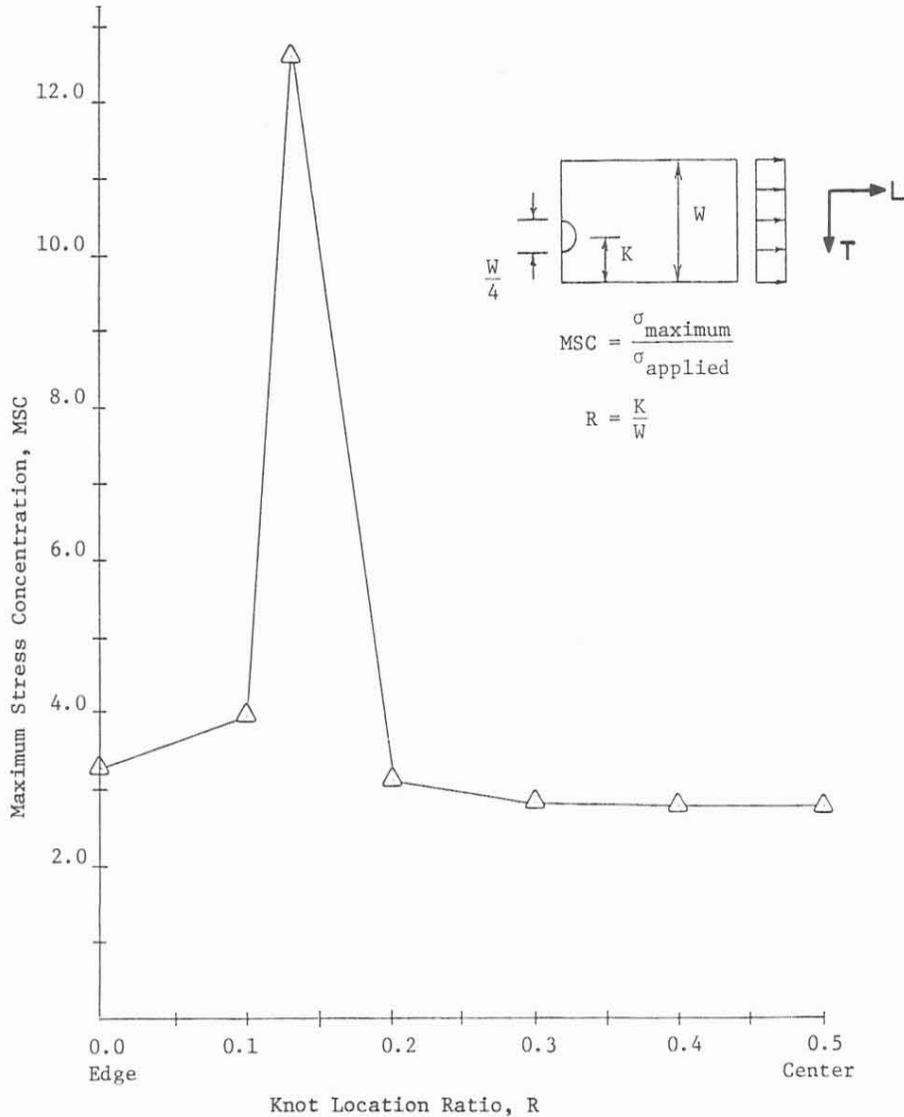


FIG. 3. Longitudinal stress concentration factors for varying knot location.

close to the edge, cause a significantly more severe stress concentration (especially in the longitudinal direction) than other knot locations. The maximum stress concentration decreases as less of the knot is included in the member and as the knot location approaches the center of the member. The lowest stress concentration occurs when the knot is located in the center of the member. It is interesting to note that the center knot condition, with the 4 to 1 member width to knot diameter ratio, has a longitudinal stress concentration value near 3.0. By comparison, a theory of elasticity solution results in a value of 3.0 for the maximum stress concentration in an infinite plate with a small circular hole for isotropic, linear elastic materials.

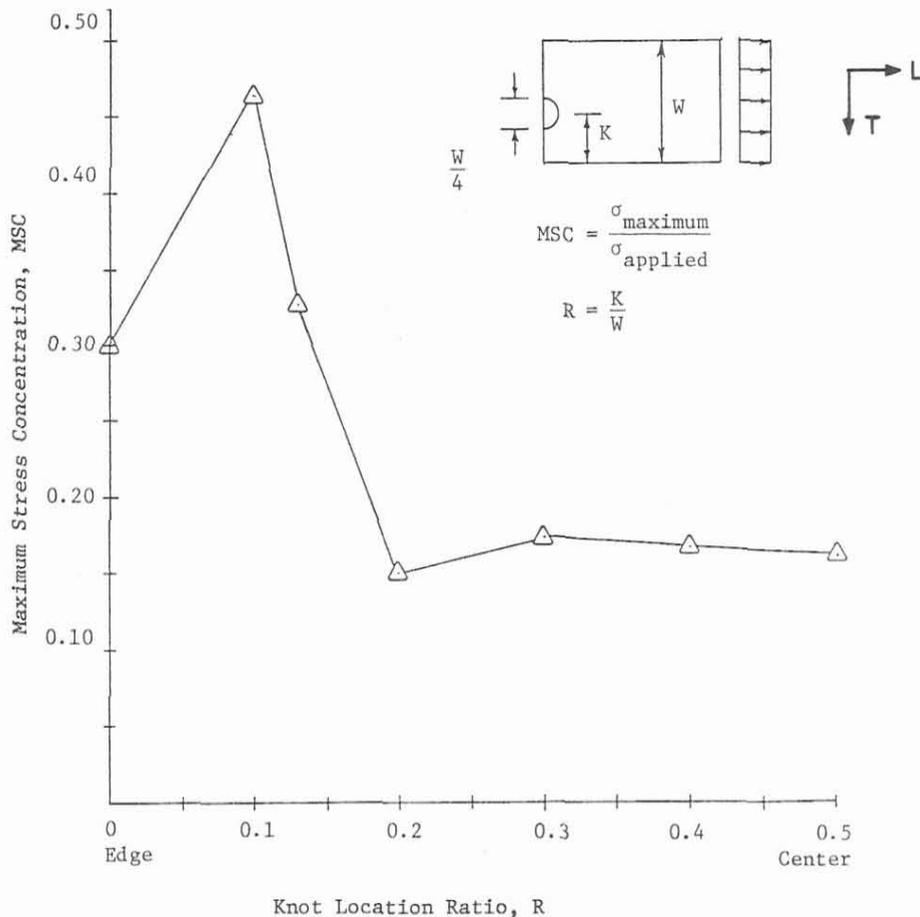


FIG. 4. Transverse stress concentration factors for varying knot location.

In the transverse direction, knots occurring near the edge of a member also cause stresses significantly higher than the stresses resulting from a knot located near the center. As shown in Fig. 4, the resulting transverse stresses are small in comparison to the applied stress. Despite these low values, the transverse stresses can be critical in determining the strength of a member because of the low tensile strength of wood perpendicular to the grain. The perpendicular-to-grain tensile strength of wood may be as low as $1/20$ th to $1/30$ th of the longitudinal tensile strength.

The results of this mathematical study agree with experimental investigations (Kunesh and Johnson 1972; McGowan 1968) that have found edge knots to have a greater detrimental effect on the strength of lumber than center knots. Although this phase of the study did not directly examine the strength of the member, the effect of edge knots on stress concentration indicate that edge knots are likely to produce lower member strength than center knots.

STRENGTH AND FAILURE ANALYSIS APPLICATIONS

The ability to predict accurately the strength of a given material is unquestionably of great importance for engineering applications. Currently, ASTM Standard D245-74 (1980) specifies a procedure to predict the strength of visually graded structural lumber. The prediction procedure is based on the strength ratio concept.

The strength ratio "represents the anticipated proportionate remaining strength" of a specimen with a defect to that which is free of defects (ASTM 1980). The ASTM tensile and bending strength ratios are based on experimental studies. In bending and compression members containing a knot, the strength ratio is defined as the ratio of the moment carrying capacity (or capacity in compression) of the member reduced in cross section by the largest knot to the corresponding capacity of clear wood. Thus, an effective cross-sectional area is defined as the original area minus that of the largest knot. ASTM Standard D245 states that for tension parallel to grain the strength ratios are 55% of the corresponding bending strength ratios. The strength ratio approach does not address directly the effect of the local grain deviation around a knot or the local stress and strain concentrations within the member.

A new method for predicting the strength of lumber is presented here. Through the use of Program KMESH1, this new procedure accounts for the local grain deviation and stress conditions around a knot.

KMESH1 can predict the location where initial failure is likely to occur. This is accomplished by comparing the existing localized stresses to the limiting clear wood failure stresses. The initial failure prediction is based on a maximum stress failure theory. This theory assumes that fracture is initiated when the maximum tensile stress in the parallel- or perpendicular-to-grain direction, or when the maximum shear stress parallel to the grain exceeds the corresponding strength of clear wood.

The prediction of initial failure stress and location provides only limited information on the ultimate load a member can sustain. However, by assuming ultimate failure to be a progressive series of initial failures, a method has been developed by which ultimate failure can be predicted. The method assumes that upon reaching initial failure a crack will form. This crack is likely to propagate, and other cracks may form as the load on the member is further increased. The ultimate failure will occur after a progression of initial failures and crack propagations. A set of assumptions have been employed with KMESH1 to allow efficient modeling of this progressive failure. Currently, it is assumed in the model that upon initial failure a crack will form and propagate along the grain to the end of the segment of the member being analyzed. This assumption seemed reasonable, on the basis of previous stress analysis studies and provides an interim method for assessing the failure capacity of a wood member.

For knots located near the edge of a member, it was assumed that the region below the initial failure crack is ineffective in resisting any further load. Thus a reduced effective section is defined for the next phase of the analysis. Successive analyses are then performed on each progressively smaller effective section. The successive analyses represent a possible failure sequence of the member. The load necessary to cause initial failure in each effective section depends upon the

geometry of the effective section and the controlling failure mode. The attainment of a peak load is an interaction between the strength in the controlling failure mode and the successively reduced cross-sectional area. The peak load achieved from the successive analyses is the predicted strength of the member.

To illustrate the above described effective section technique, two segments, one containing an edge knot and the other a center knot, are analyzed. These example analyses were performed on critical segments of full size lumber that were tested in tension at the U.S. Forest Products Laboratory in Madison, Wisconsin.²

The members were 12-foot-long, 2 × 6 Douglas-fir lumber containing knots and some existing cracks. The test data included knot locations, approximate knot size, modulus of elasticity in the longitudinal direction, moisture content, estimated failure location, and ultimate load capacity. In addition, approximate orientation, location, and size of any existing cracks in the members were recorded.

Segment A contains a knot located near the edge of the member as shown in Fig. 5. The following material properties, computed from the references indicated, were used in the analysis:

$E_L = 2,440,000$ psi	measured
$E_T = 107,000$ psi	(Bodig and Goodman 1973)
$E_R = 154,000$ psi	(Bodig and Goodman 1973)
$G_{L,T} = 118,000$ psi	(Bodig and Goodman 1973)
$\nu_{L,T} = 0.42$	(Bodig and Goodman 1973)

The estimated clear wood tensile strength values (S) for this member are:

$S_L = 15,600$ psi	(U.S. Forest Products Laboratory 1974)
$S_T = 541$ psi	(Pettersen 1981)
$S_{L,T} = 1,290$ psi	(U.S. Forest Products Laboratory 1974)

The value of E_L was taken directly from data provided by the Forest Products Laboratory. The remaining properties were estimated based on their correlation with E_L (Bodig and Goodman 1973). The strength values were determined based on work by Pettersen (Pettersen 1981) and from the *Wood Handbook* (1974). Knot properties were estimated based on work by Pugel (1980) and were assumed the same for both pieces of Douglas-fir lumber. Knot properties used were as follows:

$E_L = E_L$ of the corresponding member
$E_T = 50,000$ psi
$E_R = 50,000$ psi
$G_{RT} = 38,000$ psi
$\nu_{RT} = 0.47$

² Data obtained by U.S. Forest Products Laboratory in cooperation with the American Institute of Timber Construction, which will be published in the report "Tensile Strength and Lumber Properties of AITC 302-24 Grade Tension Laminations," by Catherine M. Marx and James W. Evans.

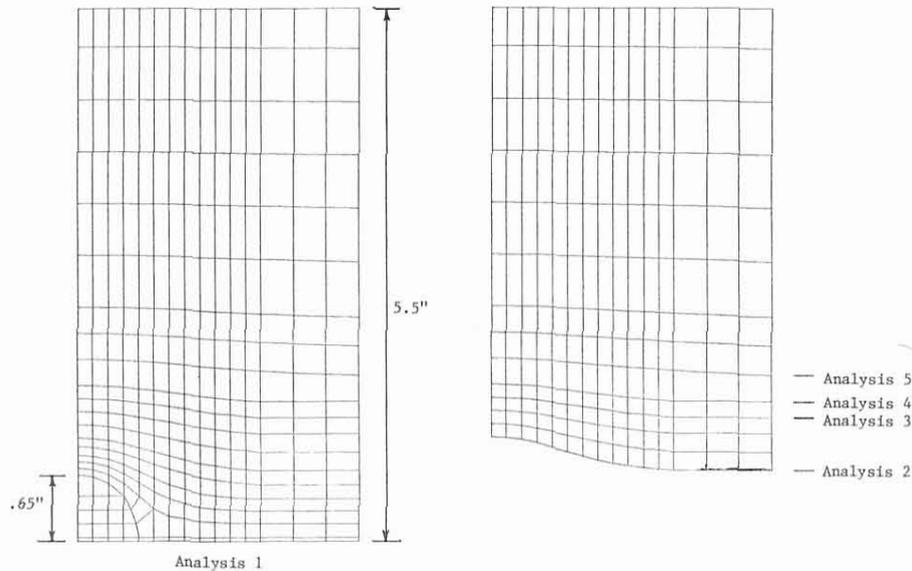


FIG. 5. Finite element mesh for segment A and effective sections for each analysis.

Successive analyses are illustrated by the effective sections as shown in Fig. 5. The results of each of the analyses and the comparisons to the actual failure load are shown in Table 1. As can be seen from these data, the KMESH1 predicted strength (peak load) in this case is very close to the actual test value, while the ASTM predicted load is more conservative.

For critical member segments containing knots located near the center of the member, the modeling technique is essentially the same. For this situation, however, two initial failure locations, one on each side of the knot, must be realized to define a central ineffective region.

Segment B, representing a second example of failure analysis, contains a center

TABLE 1. Analysis results for segment A and the ratio of KMESH1 load to actual load. Perpendicular = failure in tension perpendicular to grain. Parallel = failure in tension parallel to grain.

Segment A			
Analysis	Load (lbs)	Failure mode	Ratio of analysis load to actual load
1	14,690	Perpendicular	0.22
2	32,260	Perpendicular	0.48
3	63,410	Parallel	0.94
4	67,210	Parallel	1.00
5	63,350	Parallel	0.94
ASTM (1)	56,600		0.84
Actual test load	67,400	—	—

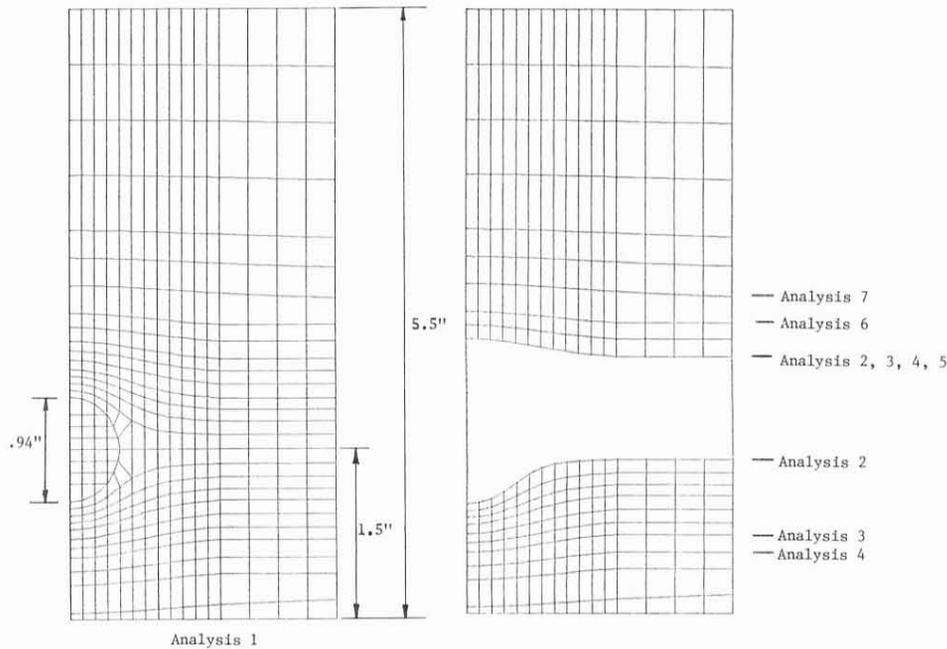


FIG. 6. Finite element mesh for segment B and effective sections for each analysis.

knot as shown in Fig. 6. Material properties for segment B were determined similarly as for segment A and are as follows:

$E_L = 2,010,000$ psi	measured
$E_T = 93,000$ psi	(Bodig and Goodman 1973)
$E_R = 142,000$ psi	(Bodig and Goodman 1973)
$G_{L,T} = 110,000$ psi	(Bodig and Goodman 1973)
$\nu_{L,T} = 0.42$	(Bodig and Goodman 1973)

Estimated strength values (S) were:

$S_L = 15,600$ psi	(U.S. Forest Products Laboratory 1974)
$S_T = 551$ psi	(Petterson 1981)
$S_{L,T} = 1,290$ psi	(U.S. Forest Products Laboratory 1974)

Effective sections are shown in Fig. 6. The results of these analyses are shown in Table 2. The predicted strength in this case was not as close to the actual as in the previous example, but the KMESH1 predicted strength was still closer than the ASTM value. Material properties, as used in the analyses, were only estimates of averages for Douglas-fir species. Significant variability in clear wood strength can be expected, which could affect the results of each analysis. In addition, other defects that occur in the member may also play a role in the ultimate failure.

Unlike other strength prediction methods, the effective section technique presented here recognizes the role of progressive crack formation and propagation in the failure of lumber. One would expect that an accurate modeling of the

TABLE 2. Analysis results for segment B and the ratio of KMESH1 load to actual load. Perpendicular = failure in tension perpendicular to grain. Parallel = failure in tension parallel to grain.

Segment B			
Analysis	Load (lbs)	Failure mode	Ratio of analysis load to actual load
1	31,790	Perpendicular	0.91
2	33,420	Perpendicular	0.96
3	40,580	Parallel	1.17
4	41,300	Parallel	1.19
5	28,700	Perpendicular	0.82
6	31,400	Parallel	0.90
7	29,800	Parallel	0.86
ASTM (1)	60,200		1.73
Actual test load	34,800	—	—

behavior of a piece of lumber during loading to failure would lead to a rational and accurate prediction of strength. In the two example cases considered, the effective section technique predicted the actual strength of the member within .5% for the edge knot example and within 19% for the center knot example. The ASTM predicted strength was conservative by 16% in the edge knot example and overestimated the center knot example strength by 73%.

CONCLUSIONS

A mathematical analysis model has been presented that can predict the elastic and strength behavior of structural lumber containing knots and cross grain.

This model, embodied in the computer program KMESH1, performs stress and strain analyses to quantify mechanical behavior, and predicts strength through the use of an ultimate stress failure theory. The ability of the model to define localized stress and strain behavior has been verified with experimental data. The effect of knot location on the maximum stress concentration within a member was studied. This study demonstrated that KMESH1 can compute stress and strain fields anywhere within a piece of lumber. The severe maximum stress concentration caused by edge knots was demonstrated through this investigation.

The effective section technique proposed here to predict the strength of lumber has been illustrated through two examples. The predicted strength was within .5% of the actual strength in the first example and within 19% in the second example. In both examples, the strength predicted by the effective section technique was significantly closer to the actual strengths than the strengths predicted by the current ASTM(D245) prediction procedure for visually graded lumber. Unlike other strength prediction procedures, the effective section technique acknowledges that ultimate strength is realized through a progressive sequence of failure.

Work continues to expand and further verify the model's capabilities. The method developed provides a rational, mathematical means to analyze the elastic and strength behavior of lumber. Its application, when even further developed, will provide an increased understanding of the behavior of lumber under load, and through better predictions of strength, lead to a more efficient utilization.

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