CALCULATION OF WOOD DENSITY VARIATION FROM X-RAY DENSITOMETER DATA

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ABSTRACT

A method is proposed for quantifying the variation in wood density as recorded by X-ray densitometry. This method uses the sum of the squared deviations from the mean density, and so approximates a true statistical variance. This gives greater meaning to variation than has previously been described. The density variation can be computed for a single growth ring, or several growth rings. The calculations are shown using data from 11-year-old *Pinus radiata* trees.

Keywords: X-ray densitometry, wood density, density variation, *Pinus radiata.*

INTRODUCTION

In short rotation forestry, a major concern is wood quality. Of the characteristics most frequently used to determine wood quality, density is probably the single most useful indicator (Elliott 1970). Because of the positive relationships between density and chemical pulp yield on a weight per wood volume basis (Wangaard 1958), and between density and strength of solid wood (Wood Handbook 1974), high density is generally synonymous with high quality wood.

In the past, wood density was measured gravimetrically, but the size of wood chips needed for these measurements meant that detailed analysis of within-ring density was a time-consuming and tedious procedure. Since the introduction of X-ray densitometers (Polge 1963) and Beta-ray densitometers (Cameron et al. 1959), these indirect methods of measuring density have become widely used. Densitometry provides a continuous recording of wood density across the wood sample. The considerable amount of data generated needs to be condensed into a meaningful quantitative form, without loss of valuable micro-densitometrical information.

Wood density variation (within and among rings) is an important microdensitometrical characteristic, and several attempts have been made to quantify this variation within wood samples. Harris (1969) proposed a number of measurements (proportions of the growth ring reaching maximum and minimum density, proportion of the growth ring in transition to latewood, extreme contrast and latewood ratio) that would provide information on within-ring density variation. However, these measures needed to be estimated visually from the traces. Furthermore, a single measure of variation is more useful for most purposes. This was attempted by Echols (1973) in his density distribution index computed by

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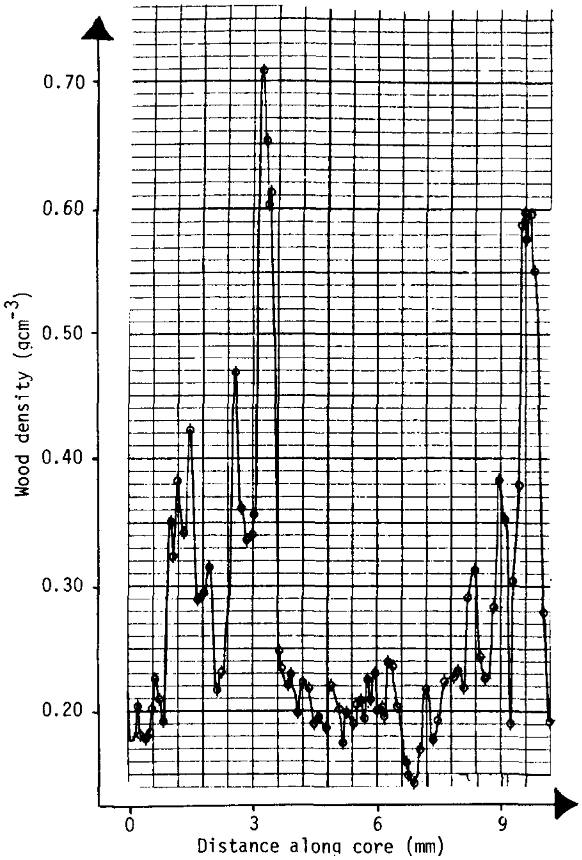

FIG. 1. Chart-paper trace from a densitometer, with circles (O) indicating the points that should be digitized so that the function, composed of straight lines drawn between successive pairs of points, reproduces the density fluctuations of the trace.

weighting the proportion of wood in density classes above and below the mean density. Olson and Arganbright (1977) pointed out that Echols' method was actually based on median density rather than the mean. They also argued that since a single reference value was not used, a single measure of uniformity for the whole tree could not be computed. This would produce confusion where low density distribution indices for each height level in a stem might imply a highly uniform tree, which would not be the case if mean density varied among the height levels. To overcome this confusion, Olson and Arganbright (1977) developed a uniformity factor that related the volume distribution of density to a selected reference base that could be the mean or median specific gravity of a growth ring, an increment core, a whole tree, or some preselected value possibly reflecting a desired value. Although a useful concept, this did not provide a numerical value that could easily be interpreted in relation to the mean specific gravity.

An approximation to a statistical variance was developed by Ferrand (1982), and a value corresponding to the square root of this is in use at Oxford Forestry

Institute (Kanowski 1985). In this approach the sums of squared deviations from the mean density were computed. However, as with most earlier approaches, measurements were taken at preselected discrete positions across the growth ring, therefore failing to take account of variation occurring between these points. A more precise method that we describe here is to measure the sum of squared deviations from the mean continuously along the entire trace. Hence our variation measure fully incorporates the detail of densitometric data that can be used to qualify the mean value. This variation can be easily computed along with other densitometric measures. The variation measure can be expressed for any part of a ring, for whole rings, for whole cores or discs and, with minor modification, can be used to express variation weighted for proportional volume of the rings actually represented in a disc. We present the methods here and, as an example, calculations are made on data collected from 11-year-old *Pinus radiata* trees.

CALCULATION METHODS

Our system of densitometry of increment core samples provides data as chart-paper traces with density along the Y axis and distance along the core on the X axis. This trace is then digitized by collecting the X and Y coordinates at as many of the points at which slope of the density trace changes as possible (Fig. 1). A variable number of points per ring can be digitized because these points do not represent a sample of points but rather they define the function.

Density can be described as a function composed of straight lines drawn between successive pairs of points. Therefore, the density mean is the area under the function divided by the distance along the abscissa, and the variation is the squared deviations of the function from the mean. Theoretical definitions of the density mean ($\bar{\rho}$) and variation (VAR) are as follows:

$$\bar{\rho} = \frac{1}{X_n - X_1} \int_{i=X(1)}^{X(n)} Y \, d(X) \tag{1.1}$$

$$VAR = \frac{1}{X_n - X_1} \int_{i=X(1)}^{X(n)} (Y - \bar{Y})^2 \, d(X) \tag{1.2}$$

where:

$Y = a_i + b_i X$

X takes the values (X_i, X_{i+1}) $i = 1, \ldots, n - 1$

a_i = intercept for segment i

b_i = slope for segment i.

1. Density of growth ring

To calculate the density mean and variation within a ring from the digitized data, other equivalent computational forms are derived from the above theoretical definitions. Equation (1.1) defines the density mean as the area under the function divided by the distance along the abscissa. An easy way to calculate the area under the density function is to sum the midpoints between successive pairs of digitized points and weight them by the proportion of the growth ring occupied by the interval.

Mean density within a ring ($\bar{\rho}r$) then becomes the following:

$$\bar{\rho}r = \sum_{i=1}^{n} \frac{Y_i + Y_{i+1}}{2} \frac{X_{i+1} - X_i}{(X_n - X_1)} \qquad (1.3)$$

a form that can be easily solved in computer algorithms. In the case of density variation within the ring (VARr), substitutions of $Y = a_i + b_iX$ are made to Eq. (1.2) and simplifications lead to Eq. (1.4).

$$VARr = \frac{1}{X_n - X_1} \sum_{i=1}^{n-1} (a_i - \bar{Y})^2(X_{i+1} - X_i) + b_i(a_i - \bar{Y})(X^2_{i+1} - X_i^2) + \frac{b_i^2}{3}(X^3_{i+1} - X_i^3) \qquad (1.4)$$

The details of the substitutions and simplifications are shown in Appendix 1.

2. Density of stem cross-section

These measures of mean density and density variation could be useful for certain analyses, but they do not exploit the relationship among rings, nor do they consider that outer rings contribute more than inner rings to tree density. Mean density of each ring from a core sample can be weighted according to its contribution to the cross-sectional area of the stem from which it came. These weights, derived from a circle of area πr^2, are

$$W_j = \frac{(X_j^2 - X^2_{j-1})}{X_m^2} \qquad (2.1)$$

where:

W_j = weight of ring j
X_j = distance from the pith to the outside of ring j
X_m = core radius.

The cross-sectional mean density ($\bar{\rho}$sect) is then:

$$\bar{\rho}sect = \frac{1}{m} \sum_{j=1}^{m} W_j\bar{\rho}r(j) \qquad (2.2)$$

where:

m = number of rings per core radius
W_j = weight of ring (j) Eq. (2.1)
$\bar{\rho}r(j)$ = mean density of ring (j) Eq. (1.3).

Likewise, cross-sectional density variation (VARsect) can be calculated by a weighted average of each within-ring density variation, added to the weighted among-ring density variation.

$$VARsect = \sum_{j=1}^{m} W_jV_{ring(j)} + \sum_{j=1}^{m} W_j(\bar{\rho}r(j) - \bar{\rho}sect)^2 \qquad (2.3)$$

where:

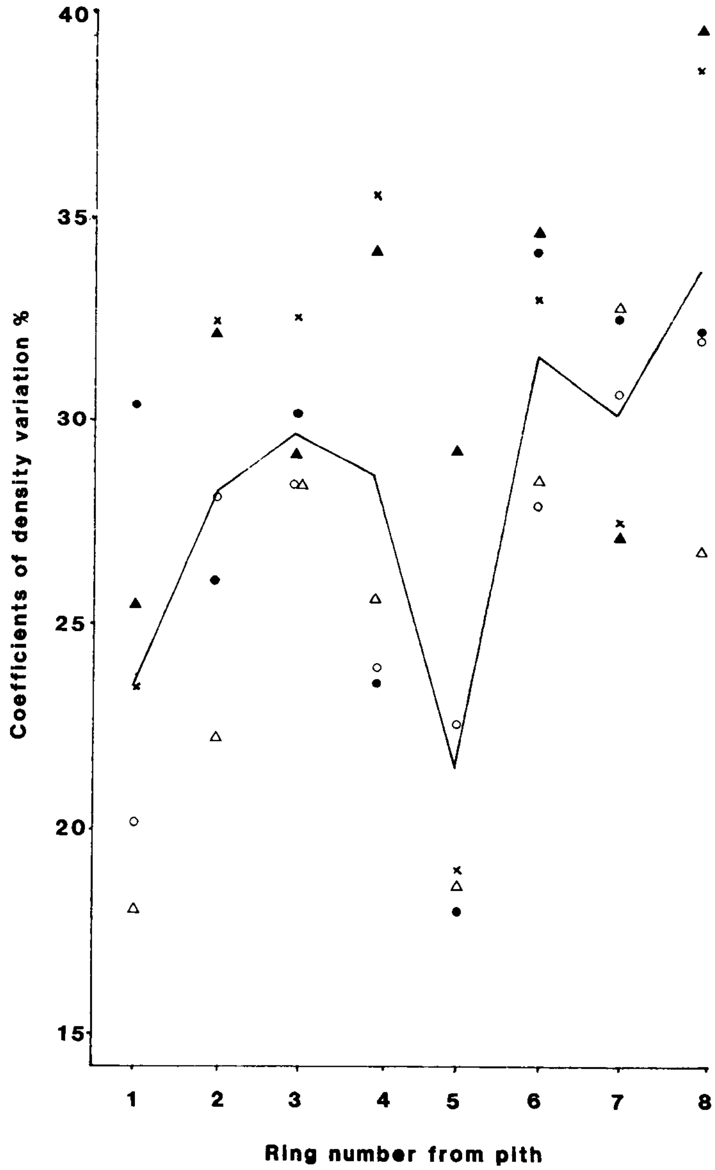

Fig. 2. Coefficients of density variation $\frac{\sqrt{VARr}}{\bar{\rho}r}$ for 5 clones of the Año Nuevo population of *Pinus radiata*. ● A32, x A46, ○ A61, △ A76, ▲ A86.

m = number of rings per core radius
VARr(j) = density variation within ring (j) Eq. (1.4).

APPLICATION OF THE PROCEDURE

To demonstrate the calculation of density variation within and among trees, a portion of the data from a study of wood density of *Pinus radiata* (Walker 1984)

TABLE 1. *Individual ring measures of density mean (g cm^{-3}) and variation calculated from digitized points of densitometer traces for 5 clones of the Año Nuevo population of* Pinus radiata.

	Annual rings (counting from the pith)							
	1	2	3	4	5	6	7	8
Clone A32								
Mean	0.267	0.241	0.293	0.308	0.289	0.314	0.329	0.394
Variation	0.006591	0.003928	0.007851	0.005237	0.002663	0.011630	0.011510	0.016230
Clone A46								
Mean	0.237	0.250	0.278	0.284	0.276	0.286	0.297	0.352
Variation	0.003088	0.006605	0.008216	0.010280	0.002721	0.008987	0.006662	0.018620
Clone A61								
Mean	0.192	0.230	0.246	0.249	0.223	0.229	0.254	0.262
Variation	0.001472	0.004186	0.004889	0.003528	0.002510	0.004096	0.006099	0.007073
Clone A76								
Mean	0.286	0.288	0.321	0.337	0.317	0.366	0.367	0.387
Variation	0.005284	0.008590	0.008741	0.013350	0.008638	0.016230	0.009918	0.023560
Clone A86								
Mean	0.199	0.238	0.271	0.273	0.266	0.299	0.319	0.332
Variation	0.001266	0.002777	0.005925	0.004880	0.002434	0.007248	0.011040	0.007946

TABLE 2. *Density mean and variation calculated from weighted individual ring measures for 15 clones.*

Clone	Density mean g cm^{-3}	Density variation
A32	0.323	0.011610
A46	0.293	0.009950
A61	0.237	0.005499
A76	0.339	0.012440
A86	0.292	0.007913
C05	0.354	0.018340
C31	0.307	0.007880
C47	0.286	0.016820
C59	0.301	0.006466
C99	0.253	0.005803
M05	0.303	0.009749
M11	0.324	0.008109
M42	0.292	0.007758
M59	0.365	0.012550
M66	0.320	0.007660

Populations of *Pinus radiata*: A = Año Nuevo, C = Cambria, and M = Monterey.

is presented. Increment cores were collected at breast height from an 11-year-old clonal plantation (Bolstad and Libby 1982). For this demonstration one ramet per clone and 5 clones from each of 3 mainland California populations were selected. The cores were prepared and X-rayed, and the film was passed through a densitometer according to the methods of Olson et al. (1978). From the densitometric traces, points were digitized and the data were processed with computer algorithms based on Eqs. (1.3) and (1.4).

Mean growth ring density and its variation are shown for successive rings from the pith for 5 clones of the Año Nuevo population in Table 1. The variance here provides a meaningful statistic for describing variation about the mean value. From Table 1 mean density increased outwards from the pith, and this increase was associated with increasing density variation. Since density variation was computed as a variance, the variation about the mean can be standardized as a coefficient of variation. This latter statistic (Fig. 2) provides three points of interest from the data in Table 1. First, although there were ring to ring fluctuations, density variation tended to increase outwards from the pith independently of the increase in mean density. Second, seasonal fluctuations in density variation were clearly shown after standardization for mean density. A sharp reduction in the coefficient of variation occurred in the fifth growth ring from the pith formed in 1978. This growth ring was formed in a season of unusually heavy rainfall for this part of California, and relative to other growth rings it was exceptionally wide. Although mean density of this ring was slightly depressed, the coefficient of variation of density showed a proportionally greater reduction. Larson (1972) proposed that in growth rings close to the pith, enhancement of radial growth could result in the formation of cells transitional between early and latewood that would reduce within-ring density variation. Third, among these five clones, trees with greater mean density also had greater coefficients of density variation, suggesting that their greater density was not uniform throughout the growth ring.

It is often desirable to compare variation in density within trees with that among

TABLE 3. *Density means and density variation within and among clones of 3 populations of* Pinus radiata.

Population	Mean density g cm^{-3}	Density variance among clones $\sigma^2/n - 1$	Mean density variation within clones
Año Nuevo	0.297	0.001520	0.009482
Cambria	0.300	0.001343	0.011060
Monterey	0.320	0.0007777	0.009165

trees. This would commonly be achieved by analysis of variance using mean density for individual growth rings, or some other well-defined increments, as the source of within-tree variation. However, this considerably underestimates within-tree variation by not taking account of within-ring density variation. In Table 2 the mean density and density variation have been computed for all eight growth rings weighted for the proportion of a cross-sectional disc occupied by each growth ring according to the method described earlier. Data are presented for five clones for each of three populations. As a comparison of variation among and within clones of a population, the statistical variance ($\sigma^2/n - 1$) of mean density of the five clones is compared with the mean density variation of the five clones (Table 3). Although these two measures of variance are not strictly comparable, they serve to show the importance of within-ring density variation in assessing density variation within clones. Density variation ranged from 6 to 12 times greater within than among clones (Table 3).

CONCLUSIONS

We have presented formulae, and demonstrated the methods to calculate mean density and density variation from densitometric traces. The measure of density variation is based on the sums of squared deviations from the mean. This measure is similar to a statistical variance, and can be expressed as variation within the growth ring, within a stem cross-section, or within a whole tree. It incorporates the detail of the densitometer trace into a quantitative measure of variability. Although in our example we had to first digitize densitometric traces, the methods could equally be applied to systems of direct computer input.

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APPENDIX 1.

Derivation of an equation to calculate within-ring density variation (VARr) from a function composed of straight lines drawn between successive pairs of density points.

$$\mathrm{VARr} = \frac{1}{X_n - X_1} \int_{x=1}^{n} (Y - \bar{Y})^2 \, d(X)$$

where: $Y = a_i + b_i X$

X takes the values $[X_i, X_{(i+1)}]$

$i = 1, 2, \ldots, n - 1$

$$b_i = \frac{Y_{(i+1)} - Y_i}{X_{(i+1)} - X_i}$$

$$a_i = Y_i - b_i X_i$$

$$= \frac{1}{X_n - X_1} \sum_{i=1}^{n-1} \int_{X_{(i)}}^{X_{(i+1)}} (Y - \bar{Y})^2 \, d(X)$$

$$= \frac{1}{X_n - X_1} \sum_{i=1}^{n-1} \int_{X_{(i)}}^{X_{(i+1)}} (Y^2 - 2Y\bar{Y} + \bar{Y}^2) \, d(X)$$

$$= \frac{1}{X_n - X_1} \sum_{i=1}^{n-1} \int_{X_{(i)}}^{X_{(i+1)}} [a_i^2 + 2a_i b_i X + b_i^2 X^2 - 2(a_i + b_i X)\bar{Y} + \bar{Y}^2] \, d(X)$$

$$= \frac{1}{X_n - X_1} \sum_{i=1}^{n-1} \Big[a_i^2(X_{i+1} - X_i) + a_i b_i (X^2_{i+1} - X_i^2) + \frac{b_i^2}{3}(X^3_{i+1} - X_i^3)$$
$$- 2\bar{Y}a_i(X_{i+1} - X_i) - b_i\bar{Y}(X^2_{i+1} - X_i^2) + \bar{Y}^2(X_{i+1} - X_i) \Big]$$

$$= \frac{1}{X_n - X_1} \sum_{i=1}^{n-1} \left[(a_i - \bar{Y})^2(X_{i+1} - X_i) + b_i(a_i - \bar{Y})(X^2_{i+1} - X_i^2) + \frac{b_i^2}{3}(X^3_{i+1} - X_i^3) \right].$$