THEORETICAL WOOD DENSITOMETRY: III. MEAN DENSITY AND DENSITY VARIATION ON STEM CROSS-SECTIONS

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ABSTRACT

Wood densitometry measures micro-scale densities in specimens that are subsamples drawn from trees receiving various treatments. However, forestry research often requires macro-scale wood density in experimental units for testing for significance of treatment effects. Acquisition of desired macro-scale wood density expressions necessitates mathematical manipulation. To facilitate direct calculation of mean density and density variation in an individual segment or in an arbitrary aggregate of segments and to derive indirect estimation of density mean and variation in annuli and cross-sections, wood density profiles have been modeled as continuous functions whose values are linearly interpolated. Theoretical macro-scale wood density expressions have been derived from applying normalization procedures and both linear and quadratic weights to micro-scale densities. Examples demonstrate the use of newly developed equations to calculate density means and root-mean-squares in single and multiple densitometric data sets.

Keywords: Stem profile, normalization, weighting, latewood, annual ring.

INTRODUCTION

Density in wood is highly variable and is mutable under conditions of changing genetic compositions or growing environments. Because, in the metric system, wood density is numerically equal to the specific gravity of a bulk of oven-dried wood, wood density is used as a synonym of specific gravity that represents a direct measure of ligneous substance in wood. In wood anatomical, wood technological, tree physiological, and plant genetical studies, density is an important physical property for evaluating wood quality. Wood density is traditionally expressed as a ratio of oven-dry weight to saturated volume (Smith 1954), which is commonly referred to as gravimetric, basic, apparent, or relative density. Gravimetric density determination, a standard test method for specific gravity of wood (ASTM 1983), necessitates dissecting an increment core or a strip of thin wood specimen to determine the macro-scale density (the gross density in bulk of wood). The method, although tedious and time-consuming, provides reliable density estimates in bulk of wood (Moura et al. 1987) and has been used to gauge other wood density assessment methods (Villeneuve et al. 1987). Macro-scale density is commonly used in scientific studies (Taylor 1973; Farrington 1980; Turvey and Smethurst 1985; Wilkes 1988) and is invariably used as a control in testing density assessment procedures (Gonzalez 1988) or as a datum in developing new density measuring techniques (Cown and Clement 1983; Hoag and McKimmy 1988).

Radiation wood densitometry measures micro-scale (a point and its vicinity) density on a specimen *in situ.* The technique was initially developed for measuring variation in thickness of paper or board (Bennett 1955) and for assessing density

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as a quality index of timber in bulk (Lakatosh 1956, 1957; Bersenev and Fokina 1958). It was later adapted to analyze the complex anatomical structure of wood (Cameron et al. 1959; Polge 1965) and to trace density gradients in reconstituted wood products (Nearn and Bassett 1968; May et al. 1976). Wood densitometry measures micro-scale densities point by point on a diametral line extending from pith to bark on stem cross-section from which the wood specimen is sampled by increment-core boring or by strip cutting. Detailed micro-scale density values provide a quantitative basis for measuring wood texture (Phillips 1960) which is impracticable from any overall density determinations on sizeable specimens. Texture, in this context, is understood as a representation of density variation in wood.

In radiation wood densitometry literature, the concept of density variation has been termed homogeneity, uniformity, or heterogeneity and has been ascribed to a single observation such as maximum or minimum density, or numerical expressions derived from a set of density values such as range, percentage of latewood, earlywood-latewood ratio, arithmetic mean, standard deviation, density distribution index, or uniformity factor (Ferrand 1982). Using less information than is available, mathematical extremities or ranges in particular specimen(s) are unsuitable expressions and are difficult to use when comparing density variations in different specimens. The biological concept of latewood or its derivatives, when used as a numerical expression of density variation, is vague because its reference datum changes from specimen to specimen, and is in fact undefined because of the absence of precise taxonomic definition of latewood (Elliott and Brook 1967). Besides the mean and standard deviation, other expressions for density variation are either lacking an appropriate physical meaning or missing a definitive unit of measure. Above all, vacillatory use of changeful (in definition) expressions has caused great confusion.

In addition to information about changes in wood texture or density variations in small regions in wood, scientific research often requires descriptive statistics, i.e., summarized wood density information, of logs, timbers, or even stands for subsequent statistical hypothesis testing. These summarized statistics of wood are necessary in anatomical (Heger et al. 1974), technological (Echols 1972), dendrochronological (Fletcher and Hughes 1970; Polge 1970), silvicultural (Rudman and McKinell 1970; Nicholls 1971; Cown 1974), environmental (Lawhon 1973; Ohta 1978), and other studies (Polge & Garros 1971; Conkey 1979) that use wood density as an evaluation criterion and are indispensable in large-scale tree breeding programs (Kanowski 1985) in which considerable amounts of wood specimens are sampled from trees receiving different experimental treatments.

Close scrutiny of the wood densitometry literature reveals that, to meet various information requirements, we need wood density expressions for mean densities and density variations within annual rings, across annual rings, between specimens of the same or different tree heights, between trees of the same species, between trees growing in the same or different stands, and between species (e.g., Olson and Arganbright 1977). It stands to reason that we need to develop macro-scale density expressions from micro-scale density data. Stated differently, the need is to describe the mean and the variation of experimental units from which subsamples are taken for observations. Considering the special data collection method used in wood densitometry, new macro-scale density statistics have to be developed and, at the same time, the physical meaning of quantities to be assessed has to be preserved. In this paper, we report weighted density means and density variations on one or more stem cross-sections. The work represents a continuation of similar studies previously reported (Kanowski 1985, Walker and Dodd 1988).

DENSITY FUNCTIONS IN WOOD DENSITOMETRY

Description of the wood density function

In wood densitometric experiments, a processed increment core or a strip of wood is irradiated by progressively moving it from one end to the other under the radiation source. A particular point on the core can thus be identified by the linear distance from a reference point. The pith is usually used as the reference point, which is also regarded as the center of a hypothetical circular cross-section from which the sample was obtained. When this certain point is irradiated, its corresponding density value is also recorded.

If we designate r as the distance from the reference point to the point of irradiation whose density is ρ (g/cm^3), then $\rho(r)$ is a wood density function of r. This reciprocal one-to-one correspondence between $\rho(r)$ and r can conveniently be displayed by plotting the radial distances as abscissas and the density values as ordinates. By drawing straight lines connecting each adjacent point, the resulting graph is traditionally labeled a wood density profile (Fig. 1), which is intrinsically a sketch of a time series. Although there is no apparent algebraic expression for one variable in terms of the other, $\rho(r)$ is nevertheless a function of r, and the density profiles can serve as a basis for calculating density mean and variation or used for studying wood anatomic features.

Partition of an analysis region

Partition of an analysis region in the specimen is a prerequisite to the formulation of density statistics. Choose arbitrarily two points, R_0 and R_m, on a radial line of a stem cross-section with radius R and designate the distance between them as the analysis or measurement region ΔR (Fig. 2) which is the domain of the density function $\rho(r)$. An analysis region $[R_0, R_m]$ can theoretically be any closed interval so long as the condition $0 \leq R_0 < R_m \leq R$ is satisfied. R_0 is the aforementioned reference point at which $r = 0$. The analysis region can be partitioned into intervals $[R_{i-1}, R_i]$, for $i = 1, 2, \ldots, m$ (≥ 1). In cases where within-ring mean densities are of interest and the measurement region encompasses the entire core sample with m annual rings, R_i denotes a boundary point delineating two consecutive rings except the two terminal points (R_0, R_m) and $0 = R_0 \leq R_m = R$.

Finer division can be obtained by dividing each interval into, say, n_i (≥ 1), subintervals with endpoint R_{ij}, for $j = 0, 1, 2, \ldots, n_i$. Apparently, $R_{i0} = R_{i-1}$ and $R_{in_i} = R_i$ (Fig. 2). To establish a one-to-one correspondence between a density value $\rho(r)$ and its associated distance (r) from the origin, it is necessary to choose R_{ij} as a point of irradiation.

It is important to emphasize that such divisions of a density measurement region are arbitrary and circumstantial; any division can be made as long as its is appropriate for summarizing the information of interest. Undoubtedly, interpretations of densitometric data can best be achieved by taking wood anatomical properties (e.g., annual ring) into consideration.

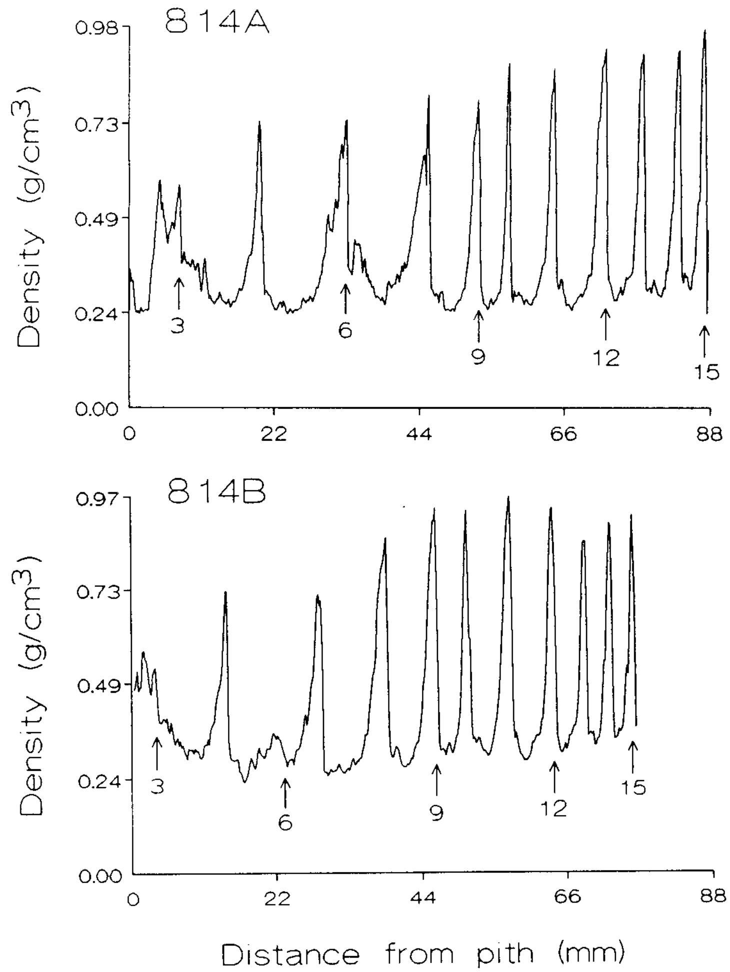

FIG. 1. Wood density profiles of two diametrically opposing lines on a breast-high stem cross-section of a *Pinus patula* tree.

MEAN AND ROOT MEAN SQUARE FROM NORMALIZATION

We have just given the necessary mathematical description and the partition scheme of the domain of a density function. In this section, we will explain the mathematical normalization procedure and introduce the linear and the quadratic weights that will be used for deriving the definition equations for distance- and area-weighted means and variations in bulk of wood. Please note that, in this investigation, macro-scale density variation is measured by the root-mean-square (rms or V).

Normalized mean and RMS

Given $\rho(r)$, a function of r, then, by definition, the first moment about the mean, i.e., the expected value (E) of the function $\rho(r)$, is

$$E\rho(r) = \int_a^b \rho(r)\phi(r)\,dr,$$

and the second moment (MS) about the mean is

$$MS\rho(r) = \int_a^b [\rho(r) - E\rho(r)]^2\,\phi(r)\,dr,$$

where $\phi(r)$ is the frequency function of r and a and b are limits of integration.

Since the expected value is a weighted average of a numerical function over its domain, we may therefore weight the function $\rho(r)$ by choosing a nonnegative function $\psi(r)$ known to be proportional to the frequency function $\phi(r)$ of r, such that

$$\phi(r) = \frac{1}{\kappa}\psi(r) = \frac{\psi(r)}{\int_a^b \psi(r)\,dr}.$$

The integral of $\psi(r)$ over all values of r is known as the normalization factor κ (Korn and Korn 1968). The expected value of $\rho(r)$ now becomes

$$E\rho(r) = \frac{\int_a^b \rho(r)\psi(r)\,dr}{\int_a^b \psi(r)\,dr}. \tag{1}$$

Then, by definition, the root-mean-square of $\rho(r)$ is the positive square root of the second moment about the mean. Note that normalization is a general mathematical procedure whose application is by no means limited to the wood density function $\rho(r)$ that concerns us.

Definitions of distance and area weighted mean and RMS

It is seen that determining the expected value (mean) of a function of a random variable is to define the frequency function of the random variable. We immediately realize that different expected values may be obtained from using different weights. If we note that the selection of the weighting function $\psi(r)$ is at our disposal, it becomes clear that, in order to make the frequency function of the random variable r meaningful, we must necessarily have $\psi(r) = 1$, while attempting to represent densities along a radial line on a cross-section, and have $\psi'(r) = r$, when our interest is focused on quantities associated with a circular cross-section. Note that, in the above and all other expressions in this paper, an apostrophe denotes weighting by a circular area and r is the radius from pith. It follows that the normalization factor assumes the value of

$$\kappa = \int_a^b 1\,dr = b - a \qquad \text{or} \qquad k' = \int_a^b r\,dr = \frac{b^2 - a^2}{2}.$$

Substituting these normalization factors into definition equations previously given (Eq. 1), we have, for distance weighted mean density

$$E\rho(r) = \frac{\int_a^b \rho(r)\psi(r)\,dr}{\int_a^b \psi(r)\,dr} = \frac{1}{b-a}\int_a^b \rho(r)\,dr, \tag{2}$$

and for distance weighted rms of $\rho(r)$ or the distance weighted density variation in the interval [a, b]

$$V\rho(r) = \sqrt{\left[\frac{1}{b-a}\int_a^b [\rho(r) - E\rho(r)]^2\,dr\right]}. \tag{3}$$

For area weighted mean density and density variation, we get, respectively,

$$E'\rho(r) = \frac{\int_a^b \rho(r)\psi'(r)\,dr}{\int_a^b \psi'(r)\,dr} = \frac{2}{b^2-a^2}\int_a^b \rho(r)r\,dr, \tag{4}$$

and

$$V'\rho(r) = \sqrt{\left[\frac{2}{b^2-a^2}\int_a^b [\rho(r) - E'\rho(r)]^2 r\,dr\right]}. \tag{5}$$

DERIVATIONS OF MEAN AND RMS ON A SUBINTERVAL

In the following, we first show that the density value corresponding to any point on a subinterval can be derived by linear interpolation. Second, we demonstrate that, because of the special partition scheme devised for this study, there exist geometric relationships that can be conveniently used to define various weights needed for subsequent development of calculation equations for density means and root-mean-squares. Finally, we show how the various calculation equations can be derived by employing the method of Riemann sum.

The consecutive sum in a definite (Riemann) integral,

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(\xi_i)\,\Delta x_i,$$

is called a Riemann sum. The subintervals into which [a, b] is divided need not have the same length; the point ξ_i in each subinterval $[x_{i-1}, x_i]$ can be any point in the subinterval; f(x) can be positive, negative or zero for points x in [a, b] (Purcell 1965).

Realization of the density function

Densitometric studies often require information about density on an interval or in a plane region (e.g., annual ring or cross-section) rather than the mean and the variance of a set of density readings. Therefore, we shall focus on deriving descriptive statistics about density on different intervals or in different regions. As pointed out previously, there is no explicit mathematical relationship between

$\rho(r)$ and r in a wood density profile although $\rho(r)$ is indeed a function of r. The situation calls for the employment of a mathematical treatment so that the density function can assume a real value for any point on a subinterval. This is accomplished by linear interpolation, in this study.

Considering the case that each of the two endpoints of an arbitrarily small subinterval, $[R_{ij-1}, R_{ij}]$, are irradiated and that no point within this subinterval is irradiated, we define the density value of an arbitrary point on this subinterval as

$$\rho_{ij}(r) = \rho_{ij} + \frac{\rho_{ij} - \rho_{ij-1}}{R_{ij} - R_{ij-1}}(r - R_{ij-1}), \qquad R_{ij-1} \leq r \leq R_{ij}. \tag{6}$$

This is a reasonable approximation because subintervals in wood densitometry are very small; in an order of magnitude from 20 to 200 micrometers (Cown and Clement 1983; Kanowski 1985; Laufenberg 1986; Winistorfer et al. 1986). This method has been used by Walker and Dodd (1988) in their investigation of density variation among *Pinus radiata* clones.

Derivation of density mean and RMS on a subinterval

Based on geometric relationships among subintervals, intervals, and the measurement region graphically described in Fig. 2 and the definitions of normalization factors, κ and κ', we introduce two weights; namely, the linear weight ω and the quadratic weight ω'.

Depicted in Fig. 2 are the geometric relationships: a measurement region, $\Delta R = R_m - R_0 = R - 0$; an interval, $\Delta R_i = R_i - R_{i-1}$; a subinterval, $\Delta R_{ij} = R_{ij} - R_{ij-1}$; and their corresponding disc and annuli; $\Delta A = \pi(R^2_m - R^2_0)$, $\Delta A_i = \pi(R^2_i - R^2_{i-1})$, and $\Delta A_{ij} = \pi(R^2_{ij} - R^2_{ij-1})$. It is apparent that the sum of all subintervals equals the interval and that the sum of all intervals equals the measurement region. Similar relationships hold for annuli and the disc under consideration. Based on these geometric relationships, appropriate weight on various intervals can be assigned by proportionment. Thus,

$$\omega_i = \frac{\Delta R_i}{\Delta R}, \qquad \omega_{ij} = \frac{\Delta R_{ij}}{\Delta R_i}, \qquad \omega_{i \cdot j} = \frac{\Delta R_{ij}}{\Delta R},$$

and

$$\omega'_i = \frac{\Delta A_i}{\Delta A}, \qquad \omega'_{ij} = \frac{\Delta A_{ij}}{\Delta A_i}, \qquad \omega'_{i \cdot j} = \frac{\Delta A_{ij}}{\Delta A}. \tag{7}$$

The above two sets of weights correspond respectively to the two weighting functions, $\psi(r) = 1$ and $\psi(r) = r$, given previously. In addition, these weights satisfy a necessary condition in all weighting procedures—the sum of all weights equals one (e.g., $\Sigma\omega_{ij} = 1$).

Substituting respectively the lower and the upper limit of integration in Eq. (2) by R_{ij-1} and R_{ij} and the integrand by Eq. (6) and then calculating the Rieman sum of the resultant expression, we yield the distance weighted mean density on a subinterval,

$$E\rho_{ij} = \frac{1}{2}[\rho(R_{ij-1}) + \rho(R_{ij})]. \tag{8}$$

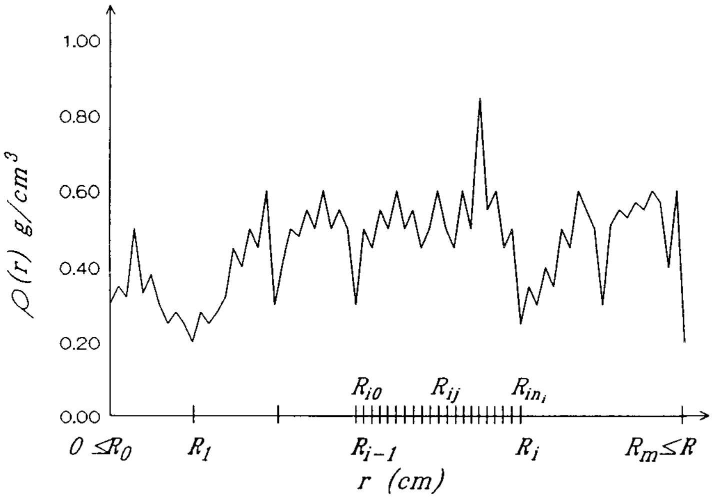

FIG. 2. Partition of a measurement region on a stem cross-section.

By substituting the above result into Eq. (3), changing limits of integration, and then calculating the Riemann sum, we have the distance weighted rms on a subinterval,

$$V\rho_{ij} = \frac{1}{\sqrt{12}}[\rho(R_{ij}) - \rho(R_{ij-1})]. \tag{9}$$

Note that the variable r is omitted in the above equations. Using definition equations (Eqs. 4 and 5) and then following the same argument given above, we obtain the area weighted mean density on a subinterval,

$$E'\rho_{ij} = \frac{1}{3}\left\{\rho_{ij-1} + \rho_{ij} + \frac{\rho_{ij-1}R_{ij-1} + \rho_{ij}R_{ij}}{R_{ij-1} + R_{ij}}\right\} \tag{10}$$

and the area weighted rms on a subinterval,

$$V'\rho_{ij} = \frac{1}{3}(\rho_{ij} - \rho_{ij-1})\sqrt{\left[\frac{1}{2} + \frac{R_{ij-1}R_{ij}}{(R_{ij-1} + R_{ij})^2}\right]}. \tag{12}$$

DERIVATIONS OF MEAN AND RMS ON AN INTERVAL AND IN THE MEASUREMENT REGION

We first obtain the definition equation for the density mean on an interval by substituting the integrand in Eq. (2) by $\rho_i(r)$ and the limits of integration by R_{i-1} and R_i. Second, recall that the sum of subintervals equals the interval, thus, we may write the integral of sum as the sum of integrals,

$$E\rho_i = \sum_{j=1}^{n_i} \frac{1}{R_i - R_{i-1}} \frac{R_{ij} - R_{ij-1}}{1} \frac{1}{R_{ij} - R_{ij-1}} \int_{R_{ij-1}}^{R_{ij}} \rho_{ij}(r)\, dr.$$

Following from Eqs. (2), (7) and (8), we obtain the distance weighted mean density on an interval,

$$E\rho_i = \sum_{j=1}^{n_i} \omega_{ij} E\rho_{ij}. \tag{13}$$

Beginning with the definition given by Eq. (3) and by changing the limits of integration, binomial expansion, and simplification, we yield the calculation equation for the distance weighted rms on an interval,

$$V\rho_i = \sqrt{\left[\sum_{j=1}^{n_i} \omega_{ij}[V^2\rho_{ij} + E^2\rho_{ij}] - E^2\rho_i\right]}. \tag{14}$$

Using a similar argument, we have the following: The area weighted mean density on an interval,

$$E'\rho_i = \sum_{j=1}^{n_i} \omega'_{ij} E'\rho_{ij}; \tag{15}$$

The area weighted rms on an interval,

$$V'\rho_i = \sqrt{\left[\sum_{j=1}^{n_i} \omega'_{ij}[V^{2\prime}\rho_{ij} + E^{2\prime}\rho_{ij}] - E^{2\prime}\rho_i\right]}; \tag{16}$$

The distance weighted mean density in a measurement region,

$$E\rho = \sum_{i=1}^{m} \omega_i E\rho_i = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \omega_{i \cdot j} E\rho_{ij}; \tag{17}$$

The distance weighted rms in a measurement region,

$$\begin{aligned} V\rho &= \sqrt{\left[\sum_{i=1}^{m} \omega_{i \cdot j}[V^2\rho_i + E^2\rho_i] - E^2\rho\right]} \\ &= \sqrt{\left[\sum_{i=1}^{m} \sum_{j=1}^{n_i} \omega_{ij}[V^2\rho_{ij} + E^2\rho_{ij}] - E^2\rho\right]}; \end{aligned} \tag{18}$$

The area weighted mean density in a measurement region,

$$E'\rho = \sum_{i=1}^{m} \omega'_i E'\rho_i = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \omega'_{i \cdot j}\, E'\rho_{ij}; \tag{19}$$

and the area weighted rms in a measurement region,

$$\begin{aligned} V'\rho &= \sqrt{\left[\sum_{i=1}^{m} \omega'_i[V^{2\prime}\rho_i + E^{2\prime}\rho_i] - E^{2\prime}\rho\right]} \\ &= \sqrt{\left[\sum_{i=1}^{m} \sum_{j=1}^{n_i} \omega'_{i \cdot j}[V^{2\prime}\rho_{ij} + E^{2\prime}\rho_{ij}] - E^{2\prime}\rho\right]}. \end{aligned} \tag{20}$$

MEAN DENSITY AND VARIATION IN TWO OR MORE REGIONS

When working with more than one specimen, we denote the measurement region of a cross-section by $[R_{k0}, R_{km}]$ with corresponding radius and area, respectively, as $\Delta R_k = R_{km} - R_{k0}$ and $\Delta A_k = \pi(R^2_{km} - R^2_{k0})$, for $k = 1, 2, \ldots, L$. We then specify the distance and area weights, respectively, as

$$\omega_k = \frac{R_{km} - R_{k0}}{\sum_{k=1}^{L} (R_{km} - R_{k0})} \quad \text{and} \quad \omega'_k = \frac{R^2_{km} - R^2_{k0}}{\sum_{k=1}^{L} (R^2_{km} - R^2_{k0})}$$

It is apparent that summation of ω_k or of ω'_k, k going from 1 to L, equals one. It can be shown that the distance weighted density mean of L cross-sections is

$$E\rho_L = \sum_{k=1}^{L} \omega_k E\rho_k, \tag{21}$$

and the area weighted density mean of L cross-sections is

$$E'\rho_L = \sum_{k=1}^{L} \omega'_k E'\rho_k, \tag{22}$$

Similarly the distance weighted rms can be obtained from

$$V\rho_L = \sqrt{\left[\sum_{k=1}^{L} \omega_k[V^2\rho_k + E^2\rho_k] - E^2\rho_L\right]}, \tag{23}$$

while the area weighted rms,

$$V'\rho_L = \sqrt{\left[\sum_{k=1}^{L} \omega'_k[V^{2'}\rho_k + E^{2'}\rho_k] - E^{2'}\rho_L\right]}. \tag{24}$$

Note that these cross-sections can be samples from different heights of the same tree, the same height of different trees, or different heights of different trees.

EXAMPLES

Two densitometric data sets consisting of micro-scale densities and distances recorded by an X-ray densitometer system (Kanowski 1985) were used for calculations of density means and root mean squares. The data are also graphically presented as density profiles (814A and 814B) with ring numbers identified in Fig. 1. Annual-ring boundaries were delineated by a numerical version of the concept of annual-ring partition pictorially described by Bodner (1983). Subintervals in both data sets are 0.2 mm wide and the measurement region (length of the radius) is 87.6 (0.0–87.6) mm for Specimen 814A and 76.0 (0.4–76.4) mm for Specimen 814B. The two specimens were taken at 1.3 meter above ground from one 15-year-old *Pinus patula* and were diametrically opposite to each other. The sample tree grew on Doleritic soil (sandy clay loam) of good drainage in a plantation at Stapleford, Zimbabwe (Palmer and Ganguli 1988) which is situated at 18°41′S and 32°48′E and is 1,740 m above sea level. The long-term weather records of the plantation show that the mean annual temperature is 15.1 C, mean

TABLE 1. *Distance- and area-weighted and unweighted means and root-mean-square (rms) in g/cm³.*

Data set no.	Ring no.	Distance weighted		Area weighted		Unweighted	
		Mean	rms	Mean	rms	Mean	rms
814A	1	0.31625	0.02808	0.31729	0.02046	0.30360	0.05059
	2	0.36574	0.12269	0.41344	0.11984	0.36558	0.12458
	3	0.48680	0.04587	0.49042	0.04664	0.48836	0.05075
	4	0.36913	0.03331	0.36679	0.03201	0.37305	0.04920
	5	0.35134	0.12152	0.36522	0.12982	0.35241	0.12360
	6	0.35765	0.13857	0.37265	0.14618	0.35845	0.14082
	7	0.39086	0.03142	0.39121	0.03134	0.38800	0.03478
	8	0.40066	0.13144	0.40832	0.13526	0.40114	0.13382
	9	0.36389	0.15840	0.36868	0.16203	0.36936	0.16379
	10	0.38179	0.18022	0.38472	0.18229	0.38896	0.19144
	11	0.38613	0.17609	0.39027	0.17896	0.39126	0.18116
	12	0.42227	0.21108	0.42733	0.21406	0.42513	0.21705
	13	0.43068	0.20098	0.43405	0.20293	0.43321	0.20463
	14	0.43965	0.20052	0.44258	0.20218	0.44429	0.20645
	15	0.49805	0.22203	0.50030	0.22305	0.49305	0.23427
	Total	0.39239	0.16270	0.41323	0.18553	0.39204	0.16578
814B	1	0.49217	0.01416	0.49319	0.01429	0.48600	0.02371
	2	0.52231	0.03555	0.52640	0.03250	0.51733	0.04136
	3	0.48693	0.02689	0.48739	0.02755	0.48325	0.03432
	4	0.38221	0.01472	0.38075	0.01490	0.38239	0.02069
	5	0.37969	0.10793	0.39777	0.11892	0.37879	0.11121
	6	0.29756	0.03278	0.30037	0.03317	0.29837	0.03365
	7	0.41384	0.14065	0.42237	0.14330	0.41018	0.14476
	8	0.39391	0.18769	0.40676	0.19349	0.39244	0.18950
	9	0.44031	0.21347	0.44892	0.21756	0.44029	0.21520
	10	0.45084	0.18884	0.45505	0.19068	0.45052	0.19258
	11	0.47930	0.21696	0.48512	0.21948	0.47652	0.21962
	12	0.44617	0.20403	0.45095	0.20663	0.44685	0.20614
	13	0.45578	0.18261	0.45864	0.18383	0.45142	0.18739
	14	0.49345	0.19020	0.49563	0.19097	0.48648	0.19679
	15	0.52113	0.18424	0.52300	0.18467	0.51147	0.19403
	Total	0.42318	0.17562	0.45002	0.19500	0.42318	0.17780

annual rainfall 1,820 mm, mean annual rainy days 140, and mean annual evaporation 1,100 mm.

For each annual ring (in this case, interval), calculated distance- and area-weighted and unweighted density means and their corresponding root-mean-squares are summarized in Table 1. Included also in Table 1 are density means and root-mean-squares for entire specimens (i.e., measurement region). These descriptive statistics can be used directly for studying changes in ring density due to effects of seed sources, silvicultural treatments, or environmental influences. The grouping of density and distance values in this example is confined within annual-ring boundaries so that derived density statistics are within-ring density means and variations. Pooling of such information can be carried out in any appropriate manner desired. For example, the grouping can be done across ring boundaries to obtain earlywood density mean and variation on a cross-section.

Using area-weighted density means and root-mean-squares of the two specimens (Totals in Table 1), we yielded weighted mean density (0.42912 g/cm³) and density variation (0.19055 g/cm^3) of the entire cross-section (Eqs. 22 and 24). This exercise exemplified a promising procedure for calculating a single density mean and variation from more than one densitometric data set. Quantities so obtained can be used directly for testing for significant differences among groups of trees growing on diverse sites, receiving distinct treatments, or of different species, and so on.

DISCUSSION

The unweighted mean used in our example is the arithmetic mean of a set of observations. It differs from the distance-weighted mean in that the latter is a weighted mean of spatially ordered quantities that have their unique geometric characteristics. Without the partitioning scheme designed specifically for the density function under consideration, it is impossible to apply quadratic weight for the derivation of area-weighted within-ring density means and root-mean-squares.

When subintervals are unequal in width, the weights (Eq. 7) change from one subinterval to another. As a consequence, the magnitudes of weighted means and root-mean-squares (Eqs. 13–20 and 21–24) become unpredictable. In past wood densitometric applications, subintervals are practically identical in width because the wood specimen (or a negative film) is advanced at steps of constant width from one end to another. Even when all subintervals are equal in magnitude, the unweighted and distance-weighted means and root-mean-squares will be different in magnitude for a given interval or the entire measurement region. The former is simply an arithmetic mean while the latter is derived by using the trapezoidal formula of numerical integration.

Some interesting properties exist under the condition of equal subintervals. These mathematical properties and their effects on the calculation of density statistics are discussed below.

As shown in Fig. 3, two subintervals are equal, i.e., $\Delta r_{ab} = \Delta r_{cd}$, and irradiated points are ordered, i.e., $0 = R_0 < r_a < r_b < r_c < r_d < R_m$. Under these conditions, it is easy to deduce:

(1) Two linear weights are not equal, $\omega_{ab} \neq \omega_{cd}$, if and only if the two subintervals are not equal, $\Delta r_{ab} \neq \Delta r_{cd}$. This result imposes a condition on numerical magnitudes of weighted means and variations.
(2) The inner quadratic weight is always smaller than the outer quadratic weight $\omega'_{ab} < \omega'_{cd}$. This is the desired result of this study.
(3) On a certain subinterval:
    (a) the linear weight is numerically greater than the area weight, $\omega_{ab} > \omega'_{ab}$, when this subinterval is in the first half of the measurement region, $r_b < R_m/2$,
    (b) the linear weight is less than the area weight, $\omega_{ab} < \omega'_{ab}$, when the subinterval under consideration is in the second half of the measurement region, $r_a > R_m/2$, and
    (c) the two weights are equal, $\omega_{ab} = \omega'_{ab}$, when the subinterval is at the middle of the measurement region, $(r_a + r_b) = R_m$.

These results are interpreted as the distance-weighted means are not necessarily greater than, equal to, or less than the area-weighted means on a subinterval, on an interval, or in the measurement region.

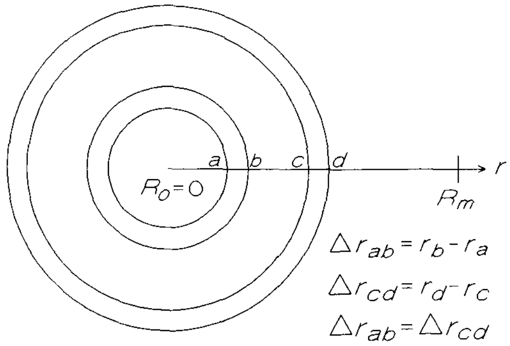

FIG. 3. Relationships between analysis weights and subinterval positions.

Radiation densitometry provides detailed micro-scale density data. Yet, forestry researchers and forest managers felt embarrassed by this technique's overproduction of indigestible information (Dinwoodie 1968; Brazier 1969). The handling of densitometric data is a Herculean task and the benefit to be gained from using this technique would be lost if no readily recognizable format is evolved (Harris 1969). Numerous density expressions have been developed previously but none has gained broad acceptance. This is due, in part, to the complexity of both densitometric data and different research programs. Nevertheless, the need for simple yet flexible density expressions is undoubtedly great. We have, in this study, derived descriptive density statistics that economically summarize densitometric data in concise and versatile formats. Distance-weighted and area-weighted mean and root-mean-squares should satisfy the information need for diversified disciplines and applications.

Knowing that certain previously developed wood density expressions have underutilized, ill-represented, or misinterpreted valuable density data generated from radiation densitometry, Ferrand (1982) enumerated several underlying principles regarding the formulation of density expressions. His ideas were: (1) an expression for density variation must refer only to density and not to anatomical variables, such as ring width; (2) an expression for density variation must have a definition and a definitive unit of measure; (3) density expressions for a set of wood specimens must be developed, each of which should be single-valued and be easily computed from density mean and variations of individual specimens; (4) a variation expression must attach to it an appropriate physical meaning; its value should not change as the physical configuration of the densitometer changes; and (5) density expressions should be able to describe variation in one large region of a core sample without regard to the rings, for parameters so derived are occasionally paralleled with the radiation densitometry method in scientific investigations.

We deliberate that excluding considerations of annual rings is willfully ignoring the anatomical properties of the substance under consideration which will lead to, in many problem areas, speculative and inconclusive results. Furthermore, constructing analysis weights to remove complexities associated with the data would result in unwise underutilization of valuable information already in the densitometric data. By orderly and rationally combining each piece of information in densitometric data sets, we have shown that the inclusion of ring width in the estimations of mean densities and density variations is admissible.

In this study, macro-scale density statistics are rigidly defined, single-valued mathematical expressions that are computable from original densitometric data or from derived density means and root-mean-squares of individual rings, a group of rings, a single segment (part of a ring or parts across annual rings), a combination of segments, or an aggregate of stem cross-sections. These descriptive statistics are measured by the physical unit g/cm^3, comparable to gravimetric densities, and are applicable to most, if not all, of the existing or future densitometers. Additionally, these macro-scale density statistics utilize all measurements in the data sets, interpret geometric meanings of the densitometric data, and refer to an absolute datum (i.e., zero), thus enabling meaningful comparisons of average density or density variation among rings, trees, stands, species, or any other experimental units.

A densitometric data set can be partitioned into proper subsets based on which mean density and density variation of "earlywood" and/or "latewood" can be calculated. These numerical quantities describe physical features that are frequently used in wood anatomical and technological studies. This approach is, of course, dependent on the supposition that there is a rigid taxonomic definition of earlywood/latewood. In the absence of such a generally accepted taxonomic definition and in the presence of newly developed macro-scale density equations, it would be beneficial to conceive a numerical method for classifying earlywood/latewood, instead of pondering the appropriateness of a biologically based earlywood/latewood delineation scheme (e.g., Mork 1928; Klem et al. 1945). Once this is done, we shall be able to calculate other density expressions such as within-ring mean earlywood density directly.

SUMMARIES

Wood densitometry affords an opportunity of measuring micro-scale densities along a diametral line on a stem cross-section which provide a quantitative basis for assessing wood texture that is impracticable from the traditional gravimetric density determination. Unfortunately, the wealth of densitometric information has also created an embarrassment of riches. Wood densitometric data analysis is a more afflictive matter than simple comparison of a few numbers obtained from bulk density determination on sizeable wood specimens. Difficulty in communicating wood density information has been an obstruction to further developments of wood densitometry and a barrier to successful wood technological, forest silvicultural, plant genetical, and dendrochronological research.

To describe mathematically the concept of wood density distribution and to make densitometric data amenable to mathematical modeling and statistical hypothesis testing, we have in this study developed both distance- and area-weighted macro-scale density statistics from micro-scale densitometric data. Derived equations permit calculations of weighted density means and root-mean-squares on subintervals and intervals, and in a density measurement region. These macro-scale density statistics provide direct estimations of average density and density variation within annual rings, across annual rings, between wood specimens of the same or different tree height, between trees of the same species, between trees growing in the same or different environments, and between species. In this treatment, maximum directions for computer programming are covered pedagogically while intricate mathematical derivations are presented in discursive sequence. It is hoped that research scientists and forest managers find these density expressions instrumental.

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REFERENCES

American Society for Testing and Materials. 1983. Standard test methods for specific gravity of wood and wood-base materials. D2395-83.

Bennett, W. E. 1955. Atomics 6(5):134. [Original not seen. See Cameron et al. 1959]

Bersenev, A. P., and A. G. Fokina. 1958. Derev. Prom. (Woodworking Industry) 7(8):9. [Original not seen. See Cameron et al. 1959]

Bodner, J. 1983. Effect of thinning and fertilization on wood properties and intra-ring characteristics in young Douglas-fir. Unpubl. M.S. thesis, Oregon State Univ., Corvallis. 114 pp.

Brazier, J. D. 1969. Some considerations in appraising within-ring density. Proceedings, Second World Consultation on Forest Tree Breeding, Washington, DC. FAO-IUFRO: 381–391.

Cameron, J. F., P. F. Beery, and E. W. J. Phillips. 1959. The determination of wood density using beta rays. Holzforschung 13(3):78–84.

Conkey, L. E. 1979. Response of tree-ring density to climate in Maine, USA. Tree-Ring Bull. 39: 29–38.

Cown, D. J. 1974. Comparison of the effects of two thinning regimes on wood properties of radiata pine. N.Z. J. For. Sci. 4(3):540–551.

———, and B. C. Clement. 1983. A wood densitometer using direct scanning with X-rays. Wood Sci. Technol. 17:91–99.

Dinwoodie, J. M. 1968. Why measure density critically? J. Inst. Wood Sci. 4(3):3–5.

Echols, R. M. 1972. Product suitability of wood . . . determined by density gradients across growth rings. USDA For. Serv. Res. Note PSW-273. 6 pp.

Elliott, G. K., and S. E. G. Brook. 1967. Microphotometric technique for growth-ring analysis. J. Inst. Wood Sci. 18:24–43.

Farrington, A. 1980. Wood and digester factors affecting kraft pulp quality and uniformity. Appita 34(1):40–44.

Ferrand, J.-C. 1982. [Considerations on wood density. Part 2: Calculation of density and of its heterogeneity.] Holzforschung 36(3):153–157.

Fletcher, J. M., and J. F. Hughes. 1970. Uses of X-rays for density determinations and dendrochronology. *In* Tree-ring analysis with special reference to northwest America, Univ. B.C., For. Bull. 7:41–54.

Gonzalez, J. S. 1988. Circumferential variation in the relative density of lodgepole pine. Can. J. For. Res. 19:276–279.

Harris, J. M. 1969. The use of beta rays in determining wood properties—Part 3. Summarizing radial variation in wood density. N.Z. J. Sci. 12:419–431.

HEGER, L., M. L. PARKER, AND R. W. KENNEDY. 1974. X-ray densitometry—A technique and an example of application. Wood Sci. 7(2):140–148.

HOAG, M., AND M. D. MCKIMMY. 1988. Direct scanning X-ray densitometry of thin wood sections. For. Prod. J. 38(1):23–26.

KANOWSKI, P. 1985. Densitometric analysis of a large number of wood samples. J. Inst. Wood Sci. 10(4):145–151.

KLEM, G. G., F. LOSCHBRANDT, AND O. BADE. 1945. Undersoklser av granvirke i forbindelse med slipe-og sulfitkokeforsok. Med. Norske Skogforsoksv 31(9):1–127. [Original not seen. See Elliott and Brook 1967]

KORN, G. A., AND T. M. KORN. 1968. Mathematical handbook for scientists and engineers. McGraw-Hill Book Co., New York. 1130 pp.

LAKATOSH, B. K. 1956. Derev. Prom. (Woodworking Industry) 5(10):13. [Original not seen. See Cameron et al. 1959]

———. 1957. Derev. Prom. (Woodworking Industry) 6(5):9. [Original not seen. See Cameron et al. 1959]

LAUFENBERG, T. L. 1986. Using gamma radiation to measure density gradients in reconstituted wood products. For. Prod. J. 36(2):59–62.

LAWHON, W. T. 1973. Radial growth and wood density of white pine in relation to coal-derived environmental pollutants. Unpubl. Ph.D. dissertation. Univ. Tenn., Knoxville. 109 pp.

MAY, H.-A., H.-P. SCHATZLER, AND W. KUHN. 1976. [Measurement of the density profile of chipboard by means of gamma rays.] Kerntechnik 18(11):494.

MORK, E. 1928. Die Qualitat des Fichtenholzes unter Besonderer Rucksichtenahmauf Scheif- und Papierholz. Papierfabrikant 26(48):741–747.

MOURA, V. P. G., R. D. BARNES, AND J. S. BIRKS. 1987. A comparison of three methods of assessing wood density in provenances of *Eucalyptus camaldulensis* Dehnh. and other *Eucalyptus* species in Brazil. Aust. For. Res. 17:83–90.

NEARN, W. T., AND K. BASSETT. 1968. X-ray determination and use of surface-to-surface density profile in fiberboard. For. Prod. J. 18(1):73–74.

NICHOLLS, J. W. P. 1971. The effect of environmental factors on wood characteristics. I. The influence of irrigation on *Pinus radiata* from South Australia. Silvae Genet. 20(1/2):26–33.

OHTA, S. 1978. [The observation of tree ring structure of soft X-ray densitom I. The effects of air pollution on annual ring structure.] J. Jap. Wood Res. Soc. 24(7):429–434.

OLSON, J. R., AND D. G. ARGANBRIGHT. 1977. The uniformity factor—A proposed method for expressing variations in specific gravity. Wood Fiber Sci. 9(3):202–210.

PALMER, E. R., AND S. GANGULI. 1988. Pulping characteristics of *Pinus patula* grown in Zimbabwe. Overseas Development Natural Resources Institute, Kent, United Kingdom. ODNRI Bull. No. 8. 32 pp.

PHILLIPS, E. W. J. 1960. The beta ray method of determining the density of wood and the proportion of summer wood. J. Inst. Wood Sci. 1(5):16–28.

POLGE, H. 1965. Study of wood density variations by densitometric analysis of X-ray negatives of samples taken with a Pressler auger. Paper presented at the meeting of IUFRO Sec. 41, Oct., Melbourne. 19 pp.

———. 1970. The use of X-ray densitometric methods in dendrochronology. Tree Ring Bull. 30(4): 1–10.

———, AND S. GARROS. 1971. [Influence of defoliation on wood structure of maritime pine.] Ann. Sci. For. 28(2):195–206.

PURCELL, E. J. 1965. Calculus with analytic geometry. Appleton-Century-Crofts, New York. 843 pp.

RUDMAN, P., AND F. H. MCKINNELL. 1970. Effect of fertilizers on wood density of young radiata pine. Aust. For. 34(3):170–178.

SMITH, D. M. 1954. Maximum moisture content method for determining specific gravity of small wood samples. U.S. For. Prod. Lab. Rep. No. 2014.

TAYLOR, F. W. 1973. Variations in the anatomical properties of South African growth *Eucalyptus grandis*. Appita 27(3):171–178.

TURVEY, N. D., AND P. J. SMETHURST. 1985. Variations in wood density of *Pinus radiata* D. Don across soil types. Aust. For. Res. 15:43–49.

VILLENEUVE, M., E. K. MORGENSTERN, AND L. P. SEBASTIAN. 1987. Estimation of wood density in family tests of jack pine and black spruce using the Pilodyn tester. Can. J. For. Res. 17:1147–1149.

WALKER, N. K., AND R. S. DODD. 1988. Calculation of wood density variation from X-ray densitometer data. Wood Fiber Sci. 20(1):35–43.

WILKES, J. 1988. Variation in wood density of *Pinus radiata* in New South Wales, Australia. Can. J. For. Res. 19:289–294.

WINISTORFER, P. M., W. C. DAVIS, AND W. W. MOSCHLER. 1986. A direct scanning densitometer to measure density profiles in wood composite products. For. Prod. J. 36(11/12):82–86.