ENTHALPY METHOD TO COMPUTE RADIAL HEATING AND THAWING OF LOGS[1]

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ABSTRACT

This paper discusses an enthalpy method to compute transient temperatures of logs. The logs may be initially frozen. It is assumed that the logs are subjected to radial heating in agitated water. The method handles phase change at a distinct temperature, which is an advantage over a previous (temperature) method. Calculations for four test logs were performed by a computerized, explicit finite-difference scheme called LOGHEAT. Model and experiment closely agreed with each other. Simplified "by hand" calculations were also satisfactory.

Keywords: Heat transfer model, thawing model, phase change model, enthalpy method, finite-difference technique, simulation, logheating.

NOTATION

- c = specific heat (J/kgK)
- H = enthalpy (J/kg)
- k = thermal conductivity (W/mK)
- L = latent heat (J/kg)
- MC = moisture content based on dry mass (percent)
- r = radial coordinate (m)
- SG = specific gravity based on dry mass and green volume (−)
- T = temperature (K, or C)
- t = time (s)
- Δ = interval (−)
- ρ = density (kg/m^3)

Subscripts:

- cr = critical
- e = exterior
- i = node 0, 1, 2, ...
- o = initial
- s = phase change
- w = water
- ∞ = ambient

Superscript:

- ν = time level 0, 1, 2, ...

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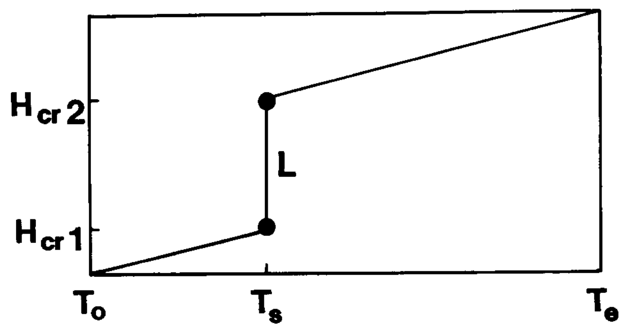

FIG. 1. Typical profile of enthalpy vs. temperature, including a phase change, at any single location within the log.

INTRODUCTION

In a previous article, Steinhagen (1986) discussed a log thawing and heating simulation with potential for process control in veneer and plywood mills. This simulation was based on a temperature method after Bonacina et al. (1973) and used the computer program HEAT by Beckman (1972). This approach produced acceptable results. However, since the method could not handle phase change at a distinct temperature, an arbitrary thawing temperature interval had to be specified. This was not only cumbersome, but also affected the results.

In contrast to the temperature method, the enthalpy (heat content) method after Voller and Cross (1981a and 1981b) is able to handle phase change at a distinct temperature (as well as over a temperature range). Using the enthalphy method in conjunction with an explicit finite-difference scheme, Steinhagen and Lee (1986) and Steinhagen et al. (1987) developed an IBM-PC computer program titled LOGHEAT to determine transient temperatures of frozen and nonfrozen logs subjected to radial heating in agitated water.

This article discusses the mathematical basis of computer program LOGHEAT and also gives simplified "by hand" calculations to estimate thawing times in a quick way.

THEORY

Let us consider the same heating problem as in the previous article (Steinhagen 1986). We will assume that we have a debarked, cylindrical log of a given radius (r_e). The log is "long" (log length greater than $8r_e$, so that only the radial heating is important). The log's initial temperature (T_o) is uniform and is below the freezing/thawing point (T_s) of 0 C. Immediately after submerging the log in the agitated water bath, the log surface temperature (T_e) rises to the level of the bath temperature (T_∞), which is constant throughout the heating cycle, and the log surface begins to thaw. The phase front then moves towards the log center, and the entire log is heated beyond the thawing point. The objective is to compute the heating time (t) necessary for a given radial coordinate (r) at the log's midlength to reach target temperature (T).

Conceptually, the log's thermal conductivity (k), specific heat (c), and density (ρ) may or may not vary with position and temperature, and may change discontinuously with the phase. When a coordinate reaches thawing temperature, the latent heat (L) will be incorporated in the local enthalpy (H), and the local temperature will momentarily be kept constant until all the latent heat needed to complete thawing at this coordinate has been absorbed (Fig. 1, where H_{cr1} and H_{cr2} are the lower and higher critical enthalpies, respectively, during thawing).

The enthalpy method keeps track of the local enthalpies (i.e., the sum of the sensible and latent heats). Temperatures, then, can be determined from the following relationships:

$$T = H/c; \qquad H < cT_s \tag{1a}$$

$$T = T_s; \qquad cT_s \leq H \leq cT_s + L \tag{1b}$$

$$T = (H - L)/c; \qquad H > cT_s + L \tag{1c}$$

For wood thawing, the latent heat is determined by

$$L = L_w\left(\frac{MC - 30\%}{MC + 100\%}\right) \tag{2}$$

where L_w is the latent heat of water fusion (334×10^3 J/kg) and MC is the wood moisture content expressed in percent of the dry mass of wood.

The standard nonlinear equation for radial heat conduction is expressed here in terms of temperature and enthalpy:

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)\right] = \rho\frac{\partial H}{\partial t}; \qquad 0 < r < r_e; \qquad t > 0 \tag{3a}$$

$$\frac{\partial T}{\partial r} = 0; \qquad r = 0; \qquad t > 0 \tag{3b}$$

$$T_e = T_\infty; \qquad r = r_e; \qquad t > 0 \tag{3c}$$

$$T = T_o; \qquad 0 < r < r_e; \qquad t = 0 \tag{3d}$$

Under the given assumptions, Eq. (3a) cannot be solved. We will, therefore, use a finite-difference technique and seek an approximate solution.

FINITE-DIFFERENCE TECHNIQUE

As previously described in more detail (Steinhagen 1986), we may assign a number of equidistant nodes to the log radius. Each node is successively considered the focal node i; adjacent nodes are denoted here as i + 1 and i − 1.

Since the log surface temperature is assumed to be specified, finite-difference equations are needed only for the interior nodes. The explicit (Euler) form of the equations to compute enthalpies (Voller and Cross 1981b) is

$$H_i^{\nu+1} = H_i^\nu + \frac{k\Delta t}{\rho(\Delta r)^2}\left[\left(1 + \frac{1}{2i}\right)T_{i+1}^\nu - 2T_i^\nu + \left(1 - \frac{1}{2i}\right)T_{i-1}^\nu\right];$$
$$i = 1, 2, 3, \ldots, m; \qquad \nu = 0, 1, 2, \ldots, n \tag{4}$$

where Δt is the time step, Δr is the radial distance step, and T_i^ν, $H1_i^\nu$ are the temperature and enthalpy, respectively, at the time $\nu\Delta t$ and coordinate $i\Delta r$. For the center node (r = 0) we have i = 0, and the above scheme becomes

$$H_0^{\nu+1} = H_0^\nu + \frac{4k\Delta t}{\rho(\Delta r)^2}(T_1^\nu - T_0^\nu) \tag{5}$$

Thus the enthalpy ($H_i^{\nu+1}$) at a time $(\nu + 1)\Delta t$ is calculated as the sum of the previous ("old") enthalpy (H_i^ν) and a quantity ($k\Delta t$ . . . .) signifying the enthalpy change (ΔH_i^ν) during the last time increment.

The time step must not exceed the Euler stability limit, i.e.,

$$\Delta t \leq \frac{\rho c(\Delta r)^2}{4k} \tag{6}$$

where we must assign the smallest value possible to the ratio $\rho c/k$.

Equation (4) assumes that $k = k_{i,i+1} = k_{i,i-1}$. This condition being somewhat unrealistic (particularly when the phase front moves through the region $i + 1$, $i - 1$), we determine k for the average temperature of this region, at time $\nu\Delta t$. As the thermal values are gradually adjusted for the "old" temperatures (T^ν), computational errors will accumulate over time.

Once the enthalpy ($H_i^{\nu+1}$) is known, the temperature ($T_i^{\nu+1}$) can be derived using Eqs. (1a) through (1c). If the specific heat changes with time, the temperature may be calculated as the sum of the previous temperature (T_i^ν) and the temperature change (ΔT_i^ν) during the last time increment,

$$T_i^{\nu+1} = T_i^\nu + \Delta T_i^\nu \tag{7}$$

where

$$\Delta T_i^\nu = \Delta H_i^\nu / c_i^\nu \tag{8}$$

COMPUTATIONS

Computer program LOGHEAT (Steinhagen and Lee 1986; Steinhagen et al. 1987) allows for temperature computations of up to 26 nodes and a maximum log radius of 0.3175 m, the spacing between adjacent nodes always being 12.7 mm. LOGHEAT uses a time step of 0.02 hours. Moisture content and density are considered uniform throughout the log and constant throughout the heating cycle. Thermal conductivity and specific heat vary with temperature.

Thermal conductivity (W/mK) in the radial direction is determined by

$$k = (0.096 + 0.0033MC - 0.0008T) \cdot (0.105 + 2.03SG); \quad T < 0\ C \tag{9a}$$

$$k = (0.138 + 0.0019MC + 0.00022T + 0.000011MC\ T)(0.105 + 2.03SG); \quad T > 0\ C \tag{9b}$$

(Wood species with much ray volume may have a 10% larger conductivity value than suggested by Eqs. 9a, 9b.)

Specific heat (J/kgK) is calculated as

$$c = 2280 + 16.6T; \quad T < 0\ C \tag{10a}$$

$$c = 2000 + 8.71MC + 4.98T; \quad T > 0\ C \tag{10b}$$

In the Eqs. (9a) through (10b), specific gravity (SG) is based on dry mass and green volume, moisture content (MC) is expressed in percent of dry mass, and temperature (T) is given in C. These equations were developed from data by

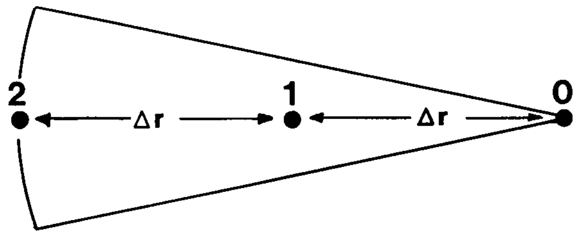

FIG. 2. Positioning of nodes along the log radius.

Kanter and Chudinov (Steinhagen 1977b) for a range of −40 to 100 C, 0.3 to 0.7 in specific gravity, and 30 to 130% moisture content; extrapolations beyond these boundaries may produce significant errors.

Four transient temperature test profiles were generated using LOGHEAT. These profiles were compared with the earlier computations via HEAT (Steinhagen 1986) and with available experimental data (Steinhagen 1977a). We will focus here on a log identified in the above papers as No. 10 (eastern white pine). Input data for the computations were as follows: specific gravity (SG) = 0.32, moisture content (MC) = 97%, exterior radius (r_e) = 0.23 m, initial temperature (T_o) = −22 C, specified surface temperature (T_e) = bath temperature (T_∞) = 54 C, thawing temperature (T_s) = 0 C. Also simplified "by hand" calculations were performed for this log using only three nodes (Fig. 2) positioned at r = 0 (node 0), $r = r_e/2$ (node 1), and $r = r_e$ (node 2), employing a time step of 4 hours. These simplified calculations were performed to examine the validity of estimating log thawing times in a quick way and without a computer.

When comparing results, it should be borne in mind that the calculations via LOGHEAT were performed with an average moisture content for the log, and with temperature-dependent thermal properties. The simplified "by hand" calculations were carried out with constant thermal properties in each phase, based on the average temperature of the phase. The previous calculations with HEAT were based on position- and temperature-dependent thermal properties and an assumed thawing interval of 3 degrees (−1.5 to 1.5 C).

RESULTS AND DISCUSSION

An exemplary temperature history with phase change is given in Fig. 3, showing computational and experimental data for the center of log No. 10 (eastern white pine). The log center temperature offers the most rigorous condition to evaluate the results, because of accumulation of position- and time-dependent (temperature-dependent) errors.

The computations closely agreed with the experimental data. In general, the discrepancy between measured and computed time, for a given temperature, was within the 10% target. This was also observed in the other test cases (logs No. 1, aspen; 4, black cherry; 7, red oak) as shown earlier by Steinhagen et al. (1987). From a practical point of view, this result was very satisfactory.

Overall, the differences among the computed data were minor. However, the enthalpy method was much simpler to use than the temperature method as there were no problems with arbitrary thawing ranges.

At long heating times, when the log was completely unfrozen and the log center

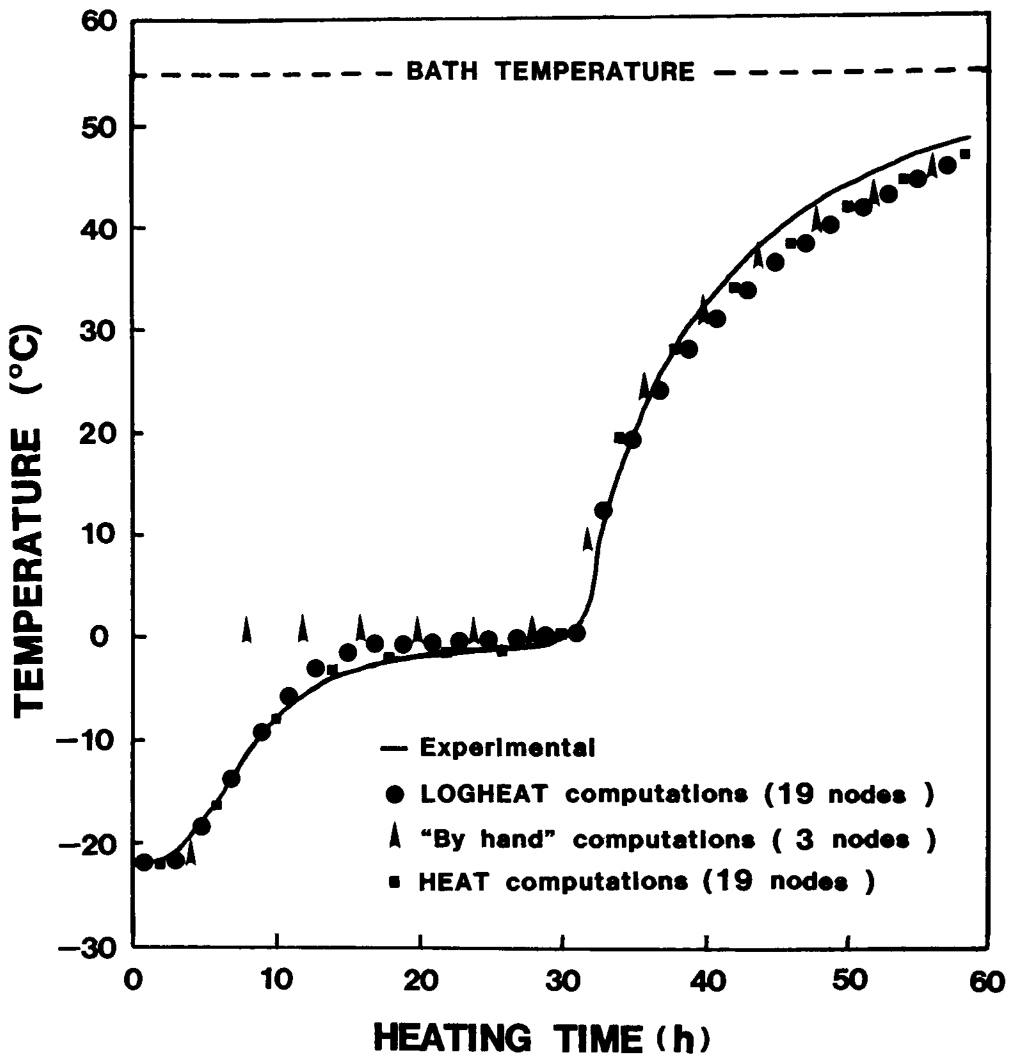

FIG. 3. Temperature vs. heating time at the center of log No. 10 (eastern white pine).

temperature approached the ambient temperature, the computational temperature increment was clearly smaller than the experimental temperature increment. The reason was that the thermal values were adjusted for the "old" temperatures ($T^{\prime\prime}$) in the HEAT and LOGHEAT computations, and for the average temperature in the simplified "by hand" calculations; the thermal diffusivity ($k/\rho c$) which is inversely proportional to the heating time thus did not grow in value as fast as it should have.

CONCLUSIONS

The enthalpy method appears more practical for simulating log heating with phase change than the previously discussed temperature method. This is because problems associated with arbitrary thawing temperature intervals do not exist with the enthalpy method. The test temperature profiles generated by the computerized model LOGHEAT closely agreed with experimental data. The simplified "by hand" calculations were also satisfactory.

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