COMPARISON OF ELASTIC CONSTANTS OF WOOD DETERMINED BY ULTRASONIC WAVE PROPAGATION AND STATIC COMPRESSION TESTING

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Abstract. In Brazil, most reports on the elastic properties of wood include only the elastic modulus in the longitudinal direction. This is because of the difficulty in determining other properties through static testing. The purpose of this study was to evaluate the methodology for determining the three Young's moduli (E_L, E_R, E_T), the three shear moduli (G_{LR}, G_{LT}, G_{RT}), and the six Poisson ratios (ν_{LR}, ν_{LT}, ν_{RL}, ν_{RT}, ν_{TL}, ν_{TR}) using ultrasonic technology. For testing, we used specimens in the form of cubic prisms from the following species: Garapeira (*Apuleia leiocarpa*), Cupiuba (*Goupia glabra*), and Sydney Blue gum (*Eucalyptus saligna*). The ultrasonic tests were performed with plane transducers of longitudinal and transverse waves, both with a 1-MHz frequency. For comparison, the same specimens were tested by static compression. Based on the confidence intervals of the means, the results of the ultrasonic test produced values of longitudinal elasticity moduli (E_L, E_R, and E_T) and shear moduli (G_{TR}, G_{TL}, and G_{LR}) statistically equivalent to those obtained with static compression. In the case of the Poisson ratio, the results, using the confidence intervals, indicated that ν_{RL}, ν_{LR}, and ν_{LT} were not statistically equivalent to those obtained in static tests for any of the species. Conversely, ν_{TL}, ν_{TR}, and ν_{RT} were statistically equivalent to those obtained in static tests for all the species. In conclusion, the ultrasonic test for determining the Young's and shear moduli of wood was found to be simpler and less expensive than the static compression test, and the results are equally useful.

Keywords: Young's modulus, shear modulus, Poisson ratios.

INTRODUCTION

In Brazil, tables of information regarding the elastic properties of wood commonly show only values for the longitudinal elastic modulus. This is partly because this property is the most commonly used in determining dimensions, but also because the determination of the shear modulus in the three planes of symmetry and the longitudinal elastic modulus in the radial and tangential directions is more difficult when using standard static tests. Similarly, almost no information about the Poisson ratio is available. Nevertheless, as structural calculation software has become increasingly common and available for use by engineers and architects, the need for knowledge of properties along all major axes of wood has become more evident.

Keunecke et al (2007) have reported that the use of ultrasonic wave propagation in determining elastic properties is now well accepted and frequently used. These authors suggested that one positive feature of the method is that it is nondestructive, permitting multiple tests to be per-

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formed on the same specimen. Additionally, they reported that the use of ultrasound permits the test to be performed on small specimens such as needed for small-diameter trees, reducing the influence of growth ring curvature. Bucur and Archer (1984) have emphasized that the best way to obtain results that approximate the orthotropic condition of the wood is by using small specimens in which the curvature of the growth rings can be neglected.

Keunecke et al (2007) have also reported that determining the shear modulus using static testing is complicated and imprecise because it is difficult to generate a pure shear stress and to measure the corresponding strains. This same conclusion was reached by Sinclair and Farshad (1987), who compared a static method (bending) and two wave propagation methods (vibration and ultrasound) to determine the elastic constants of wood (longitudinal and shear moduli).

Bucur (2006) presented the theoretical details of obtaining the elastic parameters of wood by ultrasound. Previous work by Bucur and Archer (1984) showed errors that may arise when determining the velocity in different directions of propagation for longitudinal and transverse waves. According to the authors, the greatest errors are observed in the determinations of velocities outside the symmetry axes. These velocities are used to obtain the off-diagonal terms of the stiffness matrix, which, in turn, are used to obtain the six Poisson ratios.

The objective of this study was to determine the three Young's moduli (E_L, E_R, E_T), the three shear moduli (G_{LR}, G_{LT}, G_{RT}), and the six Poisson ratios (ν_{LR}, ν_{LT}, ν_{RL}, ν_{RT}, ν_{TL}, ν_{TR}) using ultrasound and to compare the results with those obtained using a static compression test.

MATERIALS AND METHODS

Materials

For testing, prismatic specimens of 30 × 30 × 90 mm were taken from boards of three trees, Garapeira (*Apuleia leiocarpa*), Cupiuba (*Goupia glabra*), and Sydney Blue gum (*Eucalyptus saligna*), that were approximately 200 × 200 × 1800 mm. The same specimens were used for ultrasonic and static compression tests. All specimens were conditioned in a climate-controlled chamber to about 12% (±1%) MC.

The ultrasonic tests were performed using a Panametrics-NDT EPOCH4 (Olympus/Panametrics NDT Inc, San Diego, CA) with longitudinal and transverse flat transducers at a frequency of 1 MHz. The transducers had an external diameter of 18 mm, permitting them to fit on the specimen for testing in all directions. Based on the average velocity values obtained for the species studied, the wavelength (λ) for the longitudinal transducer was approximately 5.5 mm longitudinally (L), 2.5 mm radially (R), and 1.5 mm tangentially (T). These values indicate that, regardless of direction, the path length (λ) was at least 12 times the wavelength (λ). Bucur (2006) indicated the importance of having a value for λ that is a few times greater than λ to approach the hypothesis of infinite mode of propagation. Bartholomeu et al (2003) concluded, in a survey conducted using *Eucalyptus* from several sources, that for $\lambda/\lambda > 5$, velocity values become nearly constant and thus are unaffected by the path length.

Bucur (1983) and Trinca and Gonçalves (2009) also highlighted the fact that the specimen must have sufficient length to avoid the field region near the transducer. For the transducer used in this study, the near-field is about 8 mm in L, 17 mm in R, and 28 mm in T directions. Considering the dimensions of the specimen (30 mm × 30 mm × 90 mm), the transducer near-field could be effectively avoided only in the L direction. In other cases, it may have some influence on the readings.

Based on preliminary studies (Trinca et al 2009), Panametrics shear wave couplant (Olympus NDT Inc, Waltham, MA) was used for the ultrasonic tests. This couplant had less signal attenuation than medical gels, starch glucose, corn glucose, 6% carboxy methylcellulose, and 10% carboxy methylcellulose, especially for the shear wave.

To apply the load for the static compression test, a test machine (EMIC DL30000, São José dos Pinhais, Brazil) with a 300-kN capacity was used, and for deformation readings, 120-Ω, 5-mm KFG-5-120-C1-5 strain gauges were used. Using an eight-channel data acquisition system (HBM QuantumX; Hottinger Baldwin Messtechnik, Darmstadt, Germany), it was possible to follow the deformations of the strain gauges (Fig 1) as well as the applied load. Data analysis was performed using Matlab (MathWorks, Inc, Natick, MA).

Methods of Obtaining Specimens

For each replication, three specimens were prepared, one taken from each axis of symmetry (Fig 2), meaning there was a single specimen having the largest dimension in the R, T, or L directions. This procedure was not necessary for ultrasonic testing, because it was possible to obtain all of the necessary velocities from each axis of symmetry to determine the diagonal of the stiffness matrix using a single specimen. However, for static compression tests, this procedure is necessary both for bonding of strain gauges and because it is impossible to ensure that, even if the test is performed only in the elastic range, there would be no structural changes in a specimen with consecutive tests.

Bucur and Archer (1984) and Keunecke et al (2007) used specimens with dimensions much smaller than those used in this work. Bucur and Archer (1984) used cubes with 16-mm sides and Keunecke et al (2007) used 10-mm cubes. According to the authors, the use of such small specimens minimizes the influence of the curvature of growth rings. In our case, the minimum size was 18 mm so that the transducer could fit entirely on the specimen. However, our objective was to compare wave propagation tests with compression tests performed according to the Brazilian standard NBR 7190/97 (NBR 1997), where the proposed specimen size is 50 × 50 × 150 mm. In preliminary testing, these dimensions were not appropriate because of the difficulty in obtaining specimens without excessive growth ring curvature, which has a significant influence on the results. The NBR 7190/97 standard permits specimen reduction as long as the proportions are maintained (height equal to three times the width) and the section is not less than 18 mm long. Trinca (2006) demonstrated that the use of test specimens of 30 × 30 × 90 mm

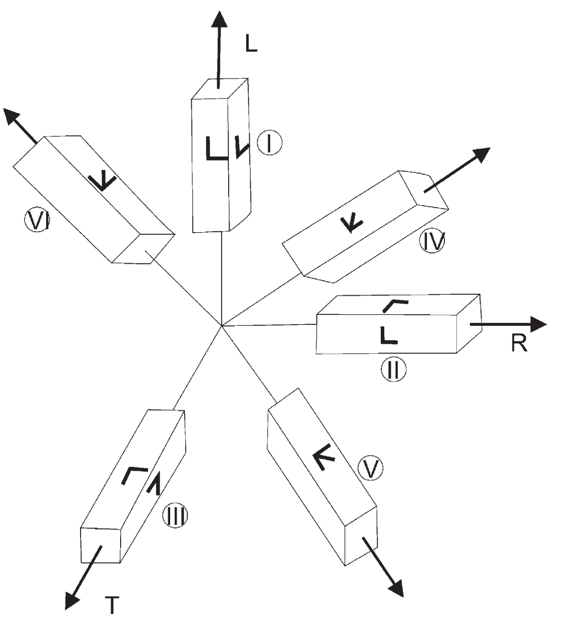

Figure 1. Position of strain gauges on the specimens. Adapted from Mascia (1991).

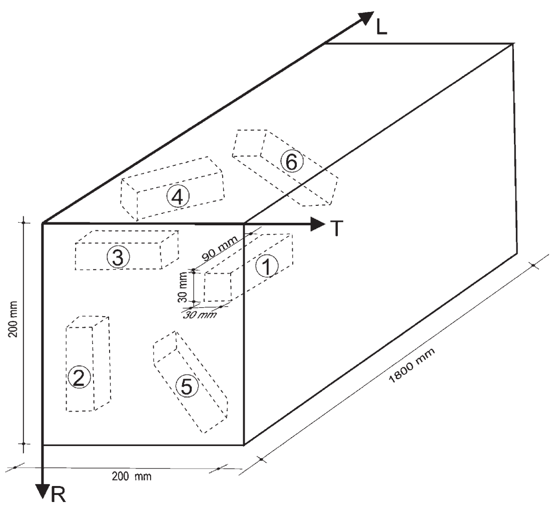

Figure 2. Scheme for obtaining specimens in and out of symmetrical axes. Adapted from Mascia (1991).

permit results that are statistically equivalent to those obtained using the standard dimensions.

To determine the shear modulus in a simple compression test, the specimen must be oriented at an angle to produce distortions, without which the test cannot be conducted. For ultrasonic tests, the specimen must be at an angle to obtain off-diagonal measurements. As such, for replication, another three specimens were taken at an angle of 45° in relation to the three planes of symmetry, LR, LT, and RT (Fig 2). The 45° angle was adopted because the static tests were performed according to the method proposed by Mascia (1991) and Furlani (1995). Results presented by Bucur and Archer (1984) for the C_{13} term of the stiffness matrix obtained at different angles of specimen orientation (15°, 30°, 45°, 105°, 120°, and 135°) showed that the lowest relative error was obtained for 45° (11.9%).

Thus, for each replication, six specimens were prepared (one each obtained along the three axes of symmetry and three obtained at 45° for each plane of symmetry) or 36 specimens per species and a total of 108 (Fig 2).

Ultrasonic Tests

Tests on specimens taken along the axes of symmetry were performed using longitudinal and shear transducers. With the longitudinal transducer positioned in different directions on the test specimens taken from the axes of symmetry, it was possible to determine V_{LL}, V_{RR}, and V_{TT}. Likewise, with the transducer positioned in different horizontal directions on the specimens taken from the axes of symmetry, it was possible to obtain V_{LR} and V_{RL}, V_{LT} and V_{TL}, and V_{RT} and V_{TR}. For those at 45° in the LR, LT, and RT planes, it was possible to obtain V_α for each plane. For these specimens, measurements were taken only with the shear transducer (quasitransverse waves) since Bucur and Archer (1984) indicated that, although it is theoretically possible to use the quasilongitudinal velocity for determining terms outside the diagonal of the stiffness matrix, this practice, almost without exception, generates imaginary values for calculating the off-diagonal terms and therefore has no practical application. Figure 3 illustrates the ultrasonic testing in the specimens.

Specimen Preparation for Compression Testing

After the ultrasonic tests, strain gauges were attached to the faces of the specimens. In the case of specimens taken from the axes of symmetry, for each direction (L, R, and T), two strain gauges were positioned on parallel faces, totaling six gauges per specimen. In the case of specimens at 45° to the planes, two strain gauges were bonded at 45° (parallel faces), and another four were bonded in the directions of the planes, also on parallel faces. For example, for the LT plane, two strain gauges were bonded at 45° on parallel faces, two strain gauges in the L direction (on parallel faces) and two strain gauges in the T direction (on parallel faces) (Fig 1).

Determination of the Stiffness Matrix

The diagonal terms of the stiffness matrix [C] were obtained using Eqs 1-6 from the Christoffel tensor (Bucur 2006):

$$C_{LL} = C_{11} = \rho {V_{LL}}^2 \quad (1)$$

$$C_{RR} = C_{22} = \rho {V_{RR}}^2 \quad (2)$$

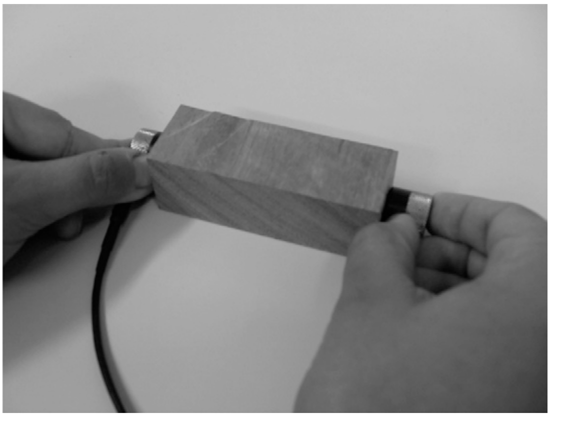

Figure 3. Example of time measurement of ultrasonic wave propagation in the specimen.

$$C_{TT} = C_{33} = \rho V_{TT}^2 \qquad (3)$$

$$C_{RT} = C_{44} = \rho(V_{RT} + V_{TR})/2)^2 \qquad (4)$$

$$C_{LT} = C_{55} = \rho(V_{LT} + V_{TL})/2)^2 \qquad (5)$$

$$C_{LR} = C_{66} = \rho(V_{LR} + V_{RL})/2)^2 \qquad (6)$$

where ρ is the density of the wood; V_{LL}, V_{RR}, and V_{TT} are the longitudinal velocities; and V_{RT}, V_{TR}, V_{LT}, V_{TL}, V_{LR}, and V_{RL} are the transverse velocities obtained according to the details given previously.

In the case of tests performed on the faces at an angle, the off-diagonal terms of the stiffness matrix were obtained using Eqs 7-9, also from the Christoffel tensor (Bucur 2006):

$$(C_{12} + C_{66})n_1n_2 = [(C_{11}n_1^2 + C_{66}n_2^2 - \rho V_\alpha^2)(C_{66}n_1^2 + C_{22}n_2^2 - \rho V_\alpha^2)^{1/2} \qquad (7)$$

$$(C_{23} + C_{44})n_2n_3 = [(C_{22}n_2^2 + C_{44}n_3^2 - \rho V_\alpha^2)(C_{44}n_2^2 + C_{33}n_3^2 - \rho V_\alpha^2)^{1/2} \qquad (8)$$

$$(C_{13} + C_{55})n_1n_3 = [(C_{11}n_1^2 + C_{55}n_3^2 - \rho V_\alpha^2)(C_{55}n_1^2 + C_{33}n_3^2 - \rho V_\alpha^2)^{1/2} \qquad (9)$$

where α is the angle of ultrasonic wave propagation (outside the axes of symmetry), which in this case is 45°; $n_1 = \cos\alpha$, $n_2 = \sin\alpha$, and $n_3 = 0$ when α is taken in relation to axis 1 (Plane 12); $n_1 = \cos\alpha$, $n_3 = \sin\alpha$, and $n_2 = 0$ when α is taken in relation to axis 1 (Plane 13); and $n_2 = \cos\alpha$, $n_3 = \sin\alpha$, and $n_1 = 0$ when α is taken in relation to axis 2 (Plane 23).

If all terms of [C] are known, the calculation of the flexibility matrix [S] can be performed using the inverse matrix $[C]^{-1}$. With the [S] matrix, it is possible to determine the three Young's moduli (E_L, E_R, E_T), the three shear moduli (G_{LR}, G_{LT}, G_{RT}), and the six Poisson ratios (ν_{LR}, ν_{LT}, ν_{RL}, ν_{RT}, ν_{TL}, ν_{TR}).

$$[S] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Compression Tests

For the compression test (Fig 4), seven channels of the data acquisition system were used: six for the strain gauges and one for the load cell. Using the Matlab program to compile the data from the seven channels, it was possible to obtain the stresses (σ) and specific strain (ε) for each direction. Eqs 10-18 show the relationships used to calculate the elasticity modulus and the Poisson ratio.

$$E_L = \sigma_L/\varepsilon_L \qquad (10)$$

Figure 4. Compression test in the testing machine.

$$\upsilon_{LT} = \varepsilon_T/\varepsilon_L \tag{11}$$

$$\upsilon_{LR} = \varepsilon_R/\varepsilon_L \tag{12}$$

$$E_R = \sigma_R/\varepsilon_R \tag{13}$$

$$\upsilon_{RT} = \varepsilon_T/\varepsilon_R \tag{14}$$

$$\upsilon_{RL} = \varepsilon_L/\varepsilon_R \tag{15}$$

$$E_T = \sigma_T/\varepsilon_T \tag{16}$$

$$\upsilon_{TR} = \varepsilon_R/\varepsilon_T \tag{17}$$

$$\upsilon_{TL} = \varepsilon_L/\varepsilon_T \tag{18}$$

For the determination of transverse modulus G, Eqs 19, 20, and 21 were used:

$$\varepsilon'_{11} = \varepsilon_{11}\cos^2\alpha + \varepsilon_{33}\sin^2\alpha + \gamma_{13}\sin\alpha\cos\alpha \tag{19}$$

$$\varepsilon'_{22} = \varepsilon_{22}\cos^2\alpha + \varepsilon_{33}\sin^2\alpha + \gamma_{23}\sin\alpha\cos\alpha \tag{20}$$

$$\varepsilon'_{11} = \varepsilon_{11}\cos^2\alpha + \varepsilon_{22}\sin^2\alpha + \gamma_{12}\sin\alpha\cos\alpha \tag{21}$$

where ε_{ii}□ is the strain toward the inclined plane, ε_{ii} is the strain in the direction of the axes that correspond to the planes in analysis, and γ_{ij} is the tangential strain in the plane being considered.

With the strain gauges positioned along the inclined direction and the principal direction, ε□$_{ii}$ and ε_{ii} can be determined by a simple compression test so that with a known angle α, the only unknown variable in the expression, γ_{ij}, can be determined. Using γ_{ij}, the tangential strain in the specified plane (γ□$_{ij}$) is determined by Eq 22.

$$\gamma'_{ij} = 2(\varepsilon_{jj} - \varepsilon_{ii})\sin\alpha\cos\alpha + \gamma_{ij}(\cos^2\alpha - \sin^2\alpha) \tag{22}$$

The shear stress in the inclined plane is given by Eq 23 and the shear modulus by Eq 24.

$$\tau'_{ij} = 2(\sigma_{jj} - \sigma_{ii})\sin\alpha\cos\alpha + \tau_{ij}(\cos^2\alpha - \sin^2\alpha) \tag{23}$$

$$G'_{ij} = \frac{\tau'_{ji}}{\gamma_{ji}'} \tag{24}$$

For α = 45°, and considering $\gamma_{ij} = 0$ for simple compression, the longitudinal and tangential strains can be obtained using Eqs 25 and 26, respectively, and using coordinate transformations, the stress in the inclined direction is obtained using Eq 27.

$$\varepsilon'_{ii} = \frac{\varepsilon_{ii} + \varepsilon_{jj}}{2} \tag{25}$$

$$\gamma'_{ij} = (\varepsilon_{jj} - \varepsilon_{ii}) \tag{26}$$

$$\sigma'_{ii} = \frac{\sigma_i + \sigma_j}{2} + \frac{\sigma_i - \sigma_j}{2}\cos 2\alpha + \tau_{ij}\sin 2\alpha \tag{27}$$

For α = 45°, and considering that $\tau_{ij} = 0$ and $\sigma_j = 0$, the stress in the inclined direction can be simplified (Eq 28), and the shear modulus can finally be calculated using Eq 29.

$$\sigma'_{ii} = \frac{\sigma_i}{2} \tag{28}$$

$$G'_{ji} = \frac{\sigma_i}{2(\varepsilon_{jj} - \varepsilon_{ii})} \tag{29}$$

Calculations of the shear moduli in the three planes were performed in Matlab using the terms detailed previously.

RESULTS AND DISCUSSION

Table 1 summarizes the results for the ultrasonic wave velocities along different propagation and polarization directions for the three species. For each specimen, at least six repeated measurements were performed in the same direction, which allowed for the determination of absolute (velocity) and relative (percentage) errors in the measurements. These errors occur from the method itself and are associated with the influences of growth ring curvature and fiber inclination, both of which cause wave dispersion. The variability of results for different specimens was calculated using the coefficient of variation

Table 1. Average velocity results obtained with ultrasonic tests.

Parameter	Symbol	Average (ms^{-1})	CV (%)	Error ($m.s^{-1}$)	Error (%)
	Apuleia leiocarpa				
Longitudinal velocities in axes	V_{11}	5408	2.85	48.7	0.90
	V_{22}	2203	0.46	22.1	1.00
	V_{33}	1765	1.57	13.9	0.78
Transverse velocities in axes	V_{44}	818	2.97	9.82	1.20
	V_{55}	1305	2.32	15.2	1.16
	V_{66}	1480	2.60	14.9	1.00
Transverse velocities in principal directions	V_{45} (LT)	1127	6.00	19.1	1.69
	V_{45} (RT)	1167	4.3	15.0	1.29
	V_{45} (LR)	865	2.0	8.0	0.93
	Goupia glabra				
Longitudinal velocities in axes	V_{11}	5152	1.50	45.0	1.00
	V_{22}	2223	2.34	23.3	1.05
	V_{33}	1638	1.16	11.0	0.67
Transverse velocities in axes	V_{44}	887	3.13	9.84	1.11
	V_{55}	1094	12.3	12.4	1.13
	V_{66}	1551	1.14	10.0	0.69
Transverse velocities in principal directions	V_{45} (LT)	1220	0.67	15.9	1.30
	V_{45} (RT)	1034	3.09	12.0	1.17
	V_{45} (LR)	919	6.5	10.3	1.12
	Eucalyptus saligna				
Longitudinal velocities in axes	V_{11}	5752	1.98	27.0	0.7
	V_{22}	3187	1.82	22.0	0.69
	V_{33}	1891	1.72	16.1	0.85
Transverse velocities in axes	V_{44}	1000	4.00	10.0	1.00
	V_{55}	1203	8.5	7.2	0.6
	V_{66}	1706	1.00	13.1	0.77
Transverse velocities in principal directions	V_{45} (LT)	1342	1.00	19.5	1.45
	V_{45} (RT)	1140	3.5	14.0	1.23
	V_{45} (LR)	1045	15.7	14.5	1.39

CV, coefficient of variation; L, longitudinal; T, tangential; R, radial.

(CV) obtained from the average of six specimens for each species.

The average density for Garapeira was 812 kg/m^{-3}, with 5.4% CV; for Cupiuba, 850 kg/m^{-3}, with 4.5% CV; and Sydney Blue gum, 850 kg/m^{-3}, with 12.1% CV.

Using the velocities and densities, the coefficient results for the stiffness matrix were calculated (Table 2).

Table 3 shows the average results of elastic parameters obtained by inversion of the stiffness matrix and of the static compression test for the three species. To facilitate comparisons, the results are presented with the range of variability (confidence interval).

Bucur (2006) presented results of velocity measurement errors along the axis of symmetry for the beech and Douglas fir, which ranged from 0.7% (V_{55}) to 0.9% (V_{11}). It appears that the error values in our study were, in general, slightly greater than those obtained by Bucur (2006), although always smaller than the variability (CV) of the material. These higher values of error may be associated with the larger sizes of the specimens used in this research, making it difficult to obtain usable specimens without fiber inclination or growth ring curvature. In the case of Bucur (2006), tests were performed on 16-mm cubes.

For the variability of velocity (CV), Bucur (2006) obtained values of 2.81% (V_{55}) and 7.51% (V_{33}), and Keunecke et al (2007) obtained values of 2.6% (V_{44}) and 9.8% (V_{33})

Table 2. Average stiffness coefficients obtained with ultrasonic tests.

Species	C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}	C_{12}	C_{13}	C_{23}
Apuleia leiocarpa	23,988 (3.54)	3978 (0.64)	2555 (2.39)	549 (4.46)	1427 (22.94)	1850 (27.93)	5499 (5.8)	3773 (9.16)	1340 (34.26)
Goupia glabra	22,551 (2.19)	4200 (3.06)	2279 (1.61)	668 (4.39)	1028 (22.02)	2044 (1.51)	5955 (2.24)	1954 (12.39)	936 (14.42)
Eucalyptus saligna	28,122 (2.05)	8633 (2.94)	3041 (2.64)	851 (5.18)	1235 (12.86)	2486 (13.16)	10,931 (0.41)	3111 (26.08)	2171 (32.71)

Units are in MPa. Values in parentheses are the coefficient of variation (%).

Table 3. Average elastic parameters obtained by inversion of the stiffness matrix (ultrasonic test) and by the static compression test.

E_L	E_R	E_T	G_{TR}	G_{TL}	G_{LR}	υ_{RL}	υ_{TL}	υ_{LR}	υ_{TR}	υ_{LT}	υ_{RT}
						Apuleia leiocarpa					
						Ultrasonic test					
14,529 ± 1429	2515 ± 481	1816 ± 308	549 ± 28	1427 ± 471	1850 ± 721	0.189 ± 0.011	0.062 ± 0.036	1.065 ± 0.027	0.270 ± 0.124	0.885 ± 0.246	0.365 ± 0.149
						Static test					
14,333 ± 4410	2323 ± 1041	1452 ± 690	536 ± 73	1489 ± 1172	1865 ± 2062	0.040 ± 0.011	0.078 ± 0.040	0.180 ± 0.099	0.330 ± 0.066	0.250 ± 0.124	0.790 ± 0.424
						Goupia glabra					
						Ultrasonic test					
13,859 ± 185	2528 ± 187	2024 ± 75	668 ± 35	1028 ± 249	2044 ± 38	0.247 ± 0.009	0.062 ± 0.034	1.358 ± 0.046	0.238 ± 0.033	0.480 ± 0.082	0.288 ± 0.045
						Static test					
13,583 ± 658	2113 ± 468	1813 ± 406	642 ± 81	892 ± 449	1950 ± 124	0.045 ± 0.021	0.075 ± 0.019	0.222 ± 0.093	0.320 ± 0.066	0.280 ± 0.066	0.830 ± 0.269
						Eucalyptus saligna					
						Ultrasonic test					
14,199 ± 1289	3987 ± 578	2426 ± 404	851 ± 54	1235 ± 185	2486 ± 403	0.344 ± 0.018	0.024 ± 0.027	1.231 ± 0.032	0.222 ± 0.099	0.271 ± 0.015	0.459 ± 0.028
						Static test					
13,617 ± 1573	3680 ± 465	2180 ± 269	829 ± 76	1172 ± 312	2360 ± 646	0.038 ± 0.009	0.060 ± 0.011	0.333 ± 0.087	0.300 ± 0.066	0.780 ± 0.090	0.420 ± 0.108

Units are in MPa.

for cubic specimens (10-mm sides) of yew and spruce. Those values are within the same range as in this study. Similarly, the CVs for stiffness were within the range obtained by Bucur (2006), 2.81-18.22%, and by Keunecke et al (2007), 9.7-23.4%.

For the three species, $V_{11} > V_{22} > V_{33} > V_{66} > V_{55} > V_{44}$, as expected, which coincided with the results presented by Bucur (1983), Bucur and Archer (1984), Bucur (2006), and Keunecke et al (2007). This order is the result of anisotropy and the orthotropic acoustic and mechanical characteristics of wood described by Bucur (2006) and Keunecke et al (2007). In the longitudinal direction, the wave meets fibers or tracheids, structures which have large length-to-diameter ratios and that behave like tubes. In the radial direction, the wave meets the rings, which still guide the direction of the wave. In the tangential direction, there is not a conductive structure for the wave. Bucur (2006) presented a series of references to analogies in the use of ultrasound to determine the elastic constants of wood and highlighted some theories that explain the numerical differences in the results of static tests. Static tests represent an isothermal process, while dynamic tests involve adiabatic processes. In an isothermal process, the internal energy of the material neither increases nor decreases, while in an adiabatic process, there is an increase in the internal energy of the material.

Sinclair and Farshad (1987) highlighted that, for all tests (static, ultrasonic, and vibration), the same amount of difficulty exists in applying the methodology to specimens but that ultrasound produced more accurate measurements. The results provided by these authors demonstrated that the ultrasonic test produced values 73% higher for the longitudinal elasticity modulus than the static test. The authors attributed these differences to the fact that E_L was calculated directly by Eq 1 (C_{LL}) and not by the complete expression that would involve Poisson ratios. Initially, the authors assumed that the influence of the Poisson ratio would be small, but discussions since then have indicated that this hypothesis is not correct.

Bucur (2006) presented results for Douglas fir (same species used by Sinclair and Farshad 1987) in which C_{LL} was 22% higher than E_L obtained by the static testing. The same author presented results for sitka spruce in which E_L was obtained through the complete stiffness matrix, and in this case, the E_L value was only 5% higher than that obtained by the static test. Considering that only the mean values were presented, it is not possible to analyze whether the results could be considered statistically equivalent. In the case of our research, the values for the stiffness constants (C_{LL}) were even greater than those for the static modulus (E_L). However, when the values were corrected by the Poisson ratios, the differences were reduced, and considering the mean, the elastic parameters obtained by ultrasound showed values only slightly higher than those obtained statically.

Considering the confidence intervals, the values of the Young's moduli (E_L, E_R, and E_T) and shear moduli (G_{TR}, G_{TL}, and G_{LR}) obtained by ultrasound and compression are statistically equal. Taking averages as the reference, the numerical differences for Young's moduli were 11.3% for Garapeira, 11.3% for Cupiuba, and 7.7% for Sydney blue gum, and for shear moduli 2% for Garapeira, 8% for Cupiuba, and 4.3% for Sydney blue gum. The greatest differences were always in the T or R directions, because of the influence of the growth ring curvature.

In the case of the Poisson ratio, the comparison results, using the confidence intervals, indicated that ν_{RL}, ν_{LR}, and ν_{LT} were not statistically equivalent to those obtained in the static test for any of the species. On the other hand, ν_{TL}, ν_{TR}, and ν_{RT} are statistically equivalent to those obtained in the static test for all species. The results of static tests (using the confidence interval) were consistent with the average Poisson's ratios proposed by Bodig and Jayne (1982) only for ν_{RL}, ν_{TR}, and ν_{RT}.

Values greater than 1.00 were not expected for isotropic solids but for crystals, some composites, and materials with honeycomb structure, Poisson's ratios can be $\nu_{ij} > 1$ and $\nu_{ji} < 1$. Bucur and Archer (1984) and Bucur (2006) also presented values of Poisson's ratio greater than 1.00 in the LR, LT, and RT planes. Bucur (2006), quoting various authors, presented a theoretical reasoning that explains why the Poisson ratio can have a value greater than 1.0 for anisotropic solids. The author emphasized the possibility that wood, although idealized as orthotropic, may have a real condition that is far from the ideal when there are other variation-causing parameters involved, such as the curvature of growth rings or fiber inclination. In such cases, Poisson ratios greater than 1.0 are not impossible.

Consistency of our data (dynamical and static tests) requires that

$(1 - \nu_{ij}\ \nu_{ji}) > 0$ and if $(1 - \nu_{LR}\ \nu_{RL} - \nu_{RT}\ \nu_{TR} - \nu_{LT}\ \nu_{TL} - 2\ \nu_{TR}\ \nu_{RL}\ \nu_{TL}) > 0$. In all cases these relationships were confirmed. It is important also to verify that C matrix and the inverse S matrix are positive, and the elastic constants E and G are positive, which is true for our data.

The terms of the stiffness matrix for the static test were calculated directly. Thus, if the behavior of the wood is effectively orthotropic, the flexibility matrix condition $-\nu_{rq}/E_r = -\nu_{qr}/E_q$ will be satisfied. Table 4 presents these relations for the three species using both methods and an average of 11 species presented by Bodig and Jayne (1982). The results in this table indicate that there are many deviations from orthotropic theory.

In the case of our research, these deviations may be related to the growth ring curvature, as the dimensions of the specimens were not so small as to completely avoid such conditions. For the Bodig and Jayne (1982) values, information regarding the density and size of the specimens or how the test was conducted is not provided. Keunecke et al (2007) also discussed this issue and stated that only the ratios $-\nu_{RT}.E_R^{-1}$ and $-\nu_{TR}.E_T^{-1}$ are really comparable.

In the case of ultrasonic testing, orthotropy is assumed, meaning $C_{12} = C_{21}$, $C_{13} = C_{31}$, and $C_{23} = C_{32}$, so when taking the inversion, the theoretical condition of the matrix [S] will be induced. Table 4 shows the obtained ratios for the symmetric terms.

Results of Poisson ratios obtained by ultrasound, presented by Bucur and Archer (1984), François (1995), and Bucur (2006), indicated values of the same order of magnitude as those obtained in this research. The highest values (sometimes up to 1.0) were obtained for ν_{LR} and ν_{LT}. The ν_{TL} and ν_{RL} values in static testing are the smallest; however, in the ultrasonic test, they do not always behave in the same way both for the results of other authors and in our work.

Table 4. Relationship of the flexibility matrix terms (10^{-5}) obtained by the compression and ultrasonic test.

	Compression test					
Species	$\frac{\vartheta_{RL}}{E_R}$	$\frac{\vartheta_{LR}}{E_L}$	$\frac{\vartheta_{TL}}{E_T}$	$\frac{\vartheta_{LT}}{E_L}$	$\frac{\vartheta_{TR}}{E_T}$	$\frac{\vartheta_{RT}}{E_R}$
Apuleia Leiocarpa	1.72	1.25	5.37	1.74	22.73	34.01
Difference (%)	37.6		209		49.6	
Goupia glabra	2.13	1.63	4.14	2.06	17.65	39.28
Difference (%)	30.7		100		123	
Eucalyptus saligna	1.03	2.44	2.75	5.72	13.76	11.41
Difference (%)	137		108		21	
	Ultrasonic test					
Apuleia Leiocarta	7.67	7.67	6.08	6.08	12.50	12.50
Goupia glabra	9.80	9.80	3.44	3.44	12.10	12.10
Eucalyptus saligna	8.73	8.73	2.00	2.00	12.60	12.60
	Bodig and Jayne (1982)					
Range for 11 species	1.34-7.95		1.59-6.48		17.2-128.0	
Difference average for 11 species (%)	0-75		2-38		2-157	

Bodig and Jayne (1982) commented that the measurement of very small Poisson ratios is complicated, because it requires high-precision equipment to measure deformations. As an alternative, researchers could use large specimens, but the impossibility of obtaining well-directed and straight growth rings would be even more unfavorable.

Bodig and Jayne (1982) also presented some relations between the longitudinal and shear elasticity moduli. According to these authors, the relationships vary greatly across species, but overall, the magnitudes of these relationships are approximately $E_L:E_R:E_T \approx 20:1.6:1.0$, $G_{LR}:G_{LT}:G_{RT} \approx 10:9.4:1.0$, and $E_L:G_{LR} \approx 14:1.0$.

Table 5 summarizes the ratios obtained for the three species using the two test methods. It is noted that the E_L/E_T ratios are much smaller than the values suggested by Bodig and Jayne (1982) for all species and types of tests (static or ultrasonic). Similar results were obtained by Keunecke et al (2007) in comparison with ratios obtained from Halász and Scheer (1986). Keunecke et al (2007) observed ratios of E_L/E_T = 7.26 and 11.8 for the spruce and yew species. These authors argued that the lower the microfibril angle in the S2 cell wall layer, the larger the longitudinal stiffness, and in the case of radial and tangential stiffness, the higher the density, the higher these values will be. Consequently, denser species tend to produce larger tangential and radial stiffness and, therefore, smaller differences in the longitudinal direction. Cupiuba and Sydney blue gum had equal densities, which were greater than those of Garapeira. Those species also had lower E_L/E_T ratios. In the case of E_R/E_T, both our results and results from Keunecke et al (2007) were similar to values suggested by Bodig and Jayne (1982). For relations between Young's moduli in longitudinal and transverse directions, the ratios were also much lower than those suggested by Bodig and Jayne (1982). This result suggests that the orthotropy of the tested species in the present study was smaller than that expected by Bodig and Jayne (1982), who reported that an E_L/E_T ratio near 20 would make the wood the most orthotropic material known. In general, the values of the ratios obtained in the static test were similar to those obtained ultrasonically.

Mascia (1991) obtained, from static compression testing, GLR/GRT and GLT/GRT ratios close to 8.0. Likewise, Bucur (2006) presented results for 11 species whose GLR/GRT ratios ranged from 3.3 to 20.8, GLT/GRT between 1.9 and 21.4, and EL/GLR between 4.1 and 21.2. Bucur and Archer (1984) presented results for six species and showed GLR/GRT ratios between 2.89 and 16.9, GLT/GRT between 2.6 and 13.1, and EL/GLR between 5.6 and 8.8. Keunecke et al (2007) obtained GLR/GRT = 4.7 for yew and 11.6 for spruce, GLT/GRT = 4.5 for yew and 11.1 for spruce, and EL/GLR = 9.6 for yew and 22.4 for spruce. These values demonstrate the great variability of results for these wood parameters.

CONCLUSIONS

The values of longitudinal and shear moduli obtained by ultrasound were statistically equivalent to those obtained by static compression. The determination of the shear modulus by ultrasound was much simpler. The Poisson ratios obtained by ultrasound show results that conflict with those expected, mainly for the LT and LR

Table 5. Relationship between elastic parameters in different axis or planes.

Species	Test	E_L/E_T	E_R/E_T	G_{LR}/G_{RT}	G_{LT}/G_{RT}	E_L/G_{LR}
Apuleia Leiocarpa	Ultrasonic	8.0	1.4	3.4	2.6	7.9
	Static	9.9	1.6	3.5	2.8	7.7
Goupia glabra	Ultrasonic	6.8	1.2	3.1	1.5	6.8
	Static	7.5	1.2	3.0	1.4	7.0
Eucalyptus saligna	Ultrasonic	5.9	1.6	2.9	1.5	5.7
	Static	6.2	1.7	2.8	1.4	5.8

planes, including values greater than 1.0 for the LR plane. The compression tests showed values close to those proposed by Bodig and Jayne (1982) for hardwoods only for ν_{RL}, ν_{TR}, and ν_{RT}.

The size of the specimens may have affected the results both in the static test and ultrasonic tests, because it was not possible to guarantee, for all cases, that the growth rings lacked curvature.

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