AN APPLICATION OF THE FINITE ELEMENT METHOD TO THE DRYING OF TIMBER

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(Received 25 April 1979)

ABSTRACT

The finite element method has been used to solve a set of differential equations that govern the transfer of heat and mass in porous bodies. Computed results are presented and are compared with the results of previously published experimental work for timber drying.

Keywords: Drying, numerical analysis, finite element method, moisture content, temperature, experimental results.

INTRODUCTION

The drying of timber, from its natural green state to the required moisture content, is an important part of the production cycle. During this stage, nonuniform temperature and moisture distributions are induced, which cause shrinkage stresses resulting in possible damage to the final product. The process is usually governed by an empirical drying schedule that is based on experience gained over many years of operating drying kilns. As energy costs rise, the optimization of these lengthy drying schedules is becoming a matter of concern.

The analysis of drying-induced stresses in timber can be considered in two distinct stages. Firstly, the distribution of heat and mass transfer potentials in the body has to be calculated. These are then expressed in the form of equivalent stresses by means of suitable shrinkage (or expansion) coefficients together with an appropriate constitutive relationship. A simple linear elastic law may be used (Lewis et al. 1977), or more complex models may be considered (Lewis et al. 1979). However, this paper is concerned solely with the first part of the analysis, i.e. the heat and mass transfer cycle.

The transfer of heat and mass in a capillary porous body is a coupled phenomenon, which can be described by means of a system of partial differential equations (Luikov 1966). An analytical solution of these equations is very difficult to achieve, and consequently solutions are given for only simple geometries and boundary conditions. In most cases numerical techniques have to be used. One such technique, the finite element method, has been successfully used to solve these equations (Lewis et al. 1975; Comini and Lewis 1976), producing results that correlate to within 1% of the analytical solutions in the case of one-dimensional problems.

The aim of this paper is to compare the numerically calculated heat and mass transfer potentials with previously published experimental results and hence show that this model can be applied to the problem of timber drying.

NUMERICAL ANALYSIS

Theory

The basic differential equations (Luikov 1966) governing the transfer of heat and moisture may be written as follows:

$$\rho c_q \frac{\partial t}{\partial \theta} = k_q \nabla^2 t + \epsilon\lambda\rho c_m \frac{\partial u}{\partial \theta}$$

$$\rho c_m \frac{\partial u}{\partial \theta} = k_m \delta \nabla^2 t + k_m \nabla^2 u \qquad (1)$$

where u is the moisture potential, t is the temperature, θ the time, k_q and k_m the thermal and moisture conductivity coefficients, respectively, ρc_q and ρc_m heat and moisture capacity respectively, δ the thermo-gradient coefficient, λ the heat of phase change and ϵ the ratio of the vapor diffusion coefficient to the coefficient of total diffusion of moisture.[1] It can be seen that the equations are two-dimensional and that the thickness of the timber in the directions of flow is implicitly taken into account.

A general set of boundary conditions may be given as:

$$t = t_w \qquad (2)$$

on Γ_1, the portion of the boundary with a constant temperature,

$$k_q \frac{\partial t}{\partial n} + j_q + \alpha_q(t - t_a) + (1 - \epsilon)\lambda\alpha_m(u - u_a) = 0 \qquad (3)$$

on Γ_2, the part of the boundary subjected to heat flux,

$$u = u_w \qquad (4)$$

on Γ_3, the portion of the boundary with a constant moisture potential and

$$k_m \frac{\partial u}{\partial n} + j_m + k_m\delta \frac{\partial t}{\partial n} + \alpha_m(u - u_a) = 0 \qquad (5)$$

on Γ_4, which is the part of the boundary subjected to a moisture flux.

The variables α_q and α_m are convective heat and mass transfer coefficients, respectively, and j_q and j_m are direct fluxes.

In the finite element analysis, the region Ω over which the solution is required is discretized into a number of eight-noded isoparametric elements. The variation of the temperature and the moisture content throughout Ω is then approximated in terms of the corresponding nodal values t_r and u_r as

$$t \simeq \hat{t} = \sum_{r=1}^{n} t_r(\theta) N_r(x, y)$$

$$u \simeq \hat{u} = \sum_{r=1}^{n} u_r(\theta) N_r(x, y) \qquad (6)$$

where N_r denotes the standard piecewise defined quadratic shape functions (Zienkiewicz 1977) and n is the total number of nodes introduced by the discretization process.

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[1] Moisture potential and moisture capacity are defined fully in Luikov (1966) pp. 248–255, but for reasons of conciseness, the full explanation cannot be repeated here. However, it can simply be stated that by analogy with heat transfer, moisture content and heat content are considered similar properties, whereas moisture potential is analogous to heat potential (temperature) and moisture capacity analogous to heat capacity. The thermo-gradient coefficient δ is obtained from experiment as the ratio of the moisture content gradient to the temperature gradient in the absence of mass transfer (Luikov 1966, p. 242).

Substitution of these approximations into the governing equation, using Galerkin's approach,[2] results in a system of differential equations that may be written in matrix form as

$$\boldsymbol{C}\frac{\mathrm{d}\boldsymbol{\phi}}{\mathrm{d}\theta} + \boldsymbol{K\phi} + \boldsymbol{J} = 0 \qquad (7)$$

where $\boldsymbol{K}$ and $\boldsymbol{C}$ are symmetric matrices, $\boldsymbol{\phi}$ is a vector of nodal temperatures and moistures, and J is derived from the applied boundary conditions (Comini and Lewis 1976).

The numerical solution of equation (7) is accomplished by the use of Lees (1966) three level time stepping scheme in which the differential equation is replaced by the recurrence relationship

$$\boldsymbol{\phi}^{n+1} = -[\boldsymbol{K}/3 + \boldsymbol{C}/2\Delta\theta]^{-1} \times [\boldsymbol{K\phi}^{n}/3 + \boldsymbol{K\phi}^{n-1}/3 - \boldsymbol{C\phi}^{n-1}/2\Delta\theta + \boldsymbol{J}^{n}] \qquad (8)$$

where $\Delta\theta$ is a time step and superscript n denotes an evaluation at time $n\Delta\theta$.

The scheme requires two starting values of $\boldsymbol{\phi}$ and stationary values are normally assumed. The repeated use of equation (8) then enables the distribution of the temperature and moisture content to be obtained at various stages of the drying process.

Example

In the calculations, the following values for the thermophysical properties of spruce were used (J. Johnson 1978):

Coefficient of thermal conductivity	= 0.65 W/m K
Heat capacity	= 2500 J/kg K
Coefficient of moisture conductivity	= 2.2×10^{-8} kg/m.s. °M
Moisture capacity	= 0.01 $kg_{moisture}/kg_{dry\ body}$ °M
Density	= 370 kg/m^3
Thermo-gradient coefficient	= 2.0°M K^{-1}
The ratio of the vapor diffusion coefficient to the coefficient of the total diffusion of moisture	= 0.3

It has previously been established (Robins 1978) that variations in the values of ϵ and δ do not have a significant effect on the temperature and moisture distributions obtained.

Boundary conditions of heat flux and moisture flux were used, as given by equations (3) and (5), and the following values were assumed:

Convective heat transfer coefficient = 22.5 W/m^2 °K

Convective mass transfer coefficient = 2.5×10^{-6} kg/m^2 sec

The direct fluxes were taken to be equal to zero.

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[2] Galerkin's approach states that the residuals obtained by substituting the approximation in equation (6) into the governing equations are minimized by ensuring that the integral of the weighted errors over the domain is equal to zero, with the shape functions N_r being used as weighting functions.

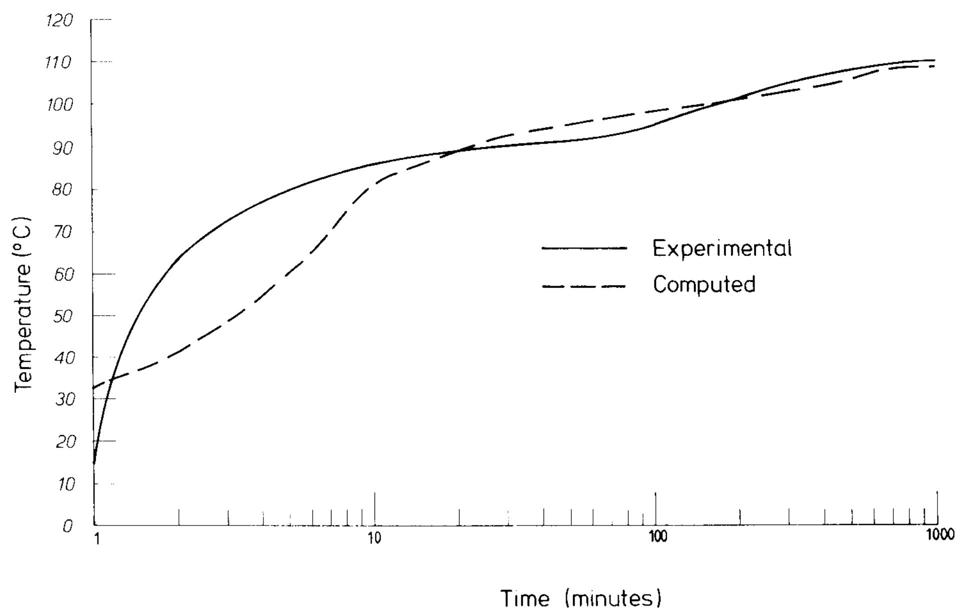

FIG. 1. Temperature at the surface.

EXPERIMENTAL RESULTS

The experimental results used are found in a paper by R. Keylwerth (1952). A specimen of spruce, 24-mm thick in the radial direction, is used for the experiment and drying is allowed to take place only in the radial direction. The initial and final conditions are given as follows:

i) Initial temperature = 10 C
ii) Final temperature = 110 C
iii) Initial moisture content = 86%
iv) Final moisture content = 8%
v) Wet bulb temperature = 87 C.

Values of temperature at the surface, temperature at the center, moisture content at the surface, and moisture content at the center were recorded during the course of the experiment and the results are shown in Figs. 1 to 4.

The curve of moisture content against time (Fig. 3) shows the moisture content at the surface remaining constant during the first ten minutes. The reason for this is that the partial vapor pressure of water in the wood is lower than the partial vapor pressure of the air. Only warming of the wood takes place until the vapor pressure of the wet wood exceeds the vapor pressure of the air. Therefore the experiment can be considered in two distinct stages; (a) in the initial stage temperature is the only variable, (b) after ten minutes temperature and moisture both vary. The numerical work was therefore also modelled in the same way.

DISCUSSION OF RESULTS

The curve of moisture content at the surface against time (Fig. 3) shows a very sharp fall in moisture content between 40 and 100 min followed by a rise in value

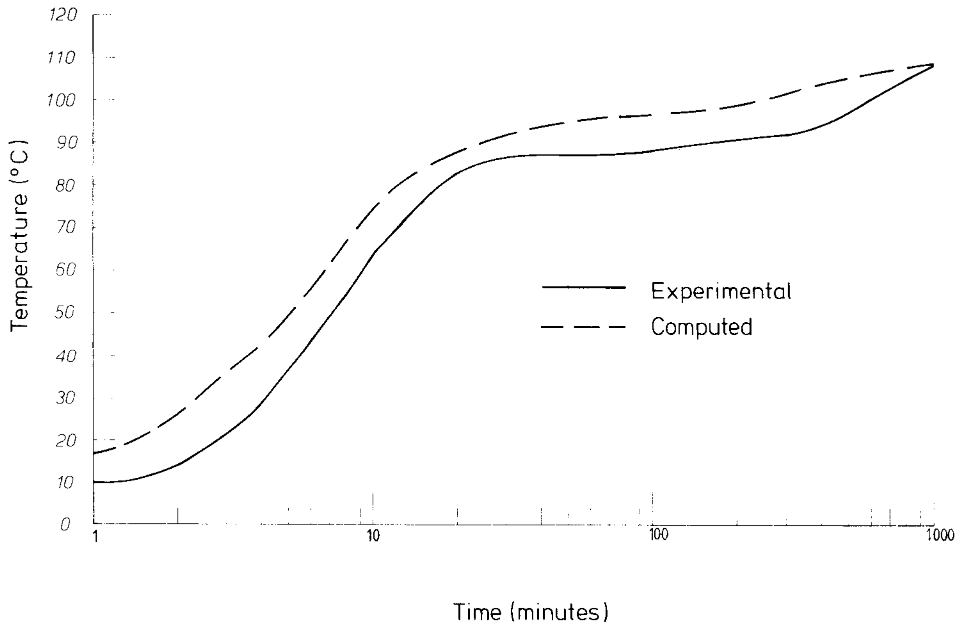

FIG. 2. Temperature at the center.

during the next 100 min. This feature is not shown for the moisture content at the center or for the values for the board as a whole. No explanation is given by the author for this behavior, and the experimental values in this region are therefore considered to be suspect. It can be seen that the largest differences between experimental and computed results occur in this area.

During the drying of wood, the moisture moves both by diffusion and bulk flow. Above the fiber saturation point, bulk flow begins to play an important part,

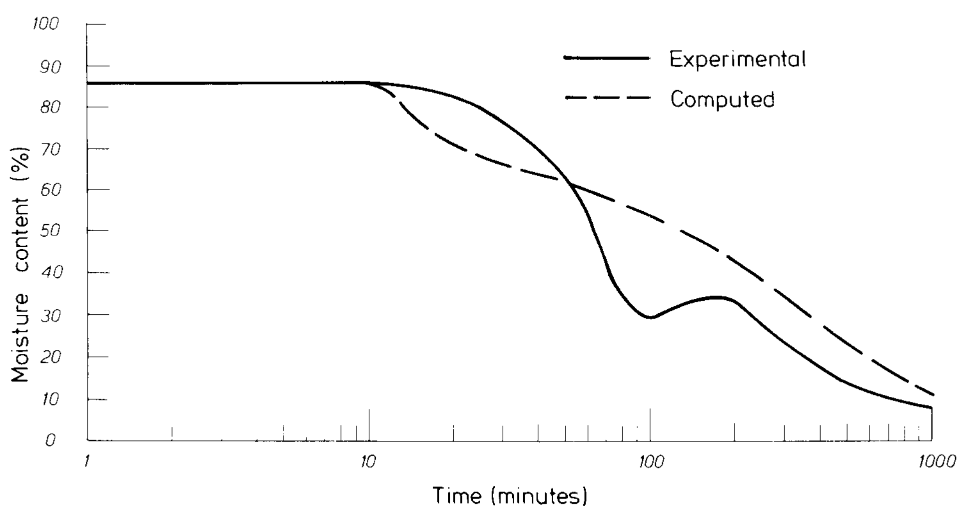

FIG. 3. Moisture content at the surface.

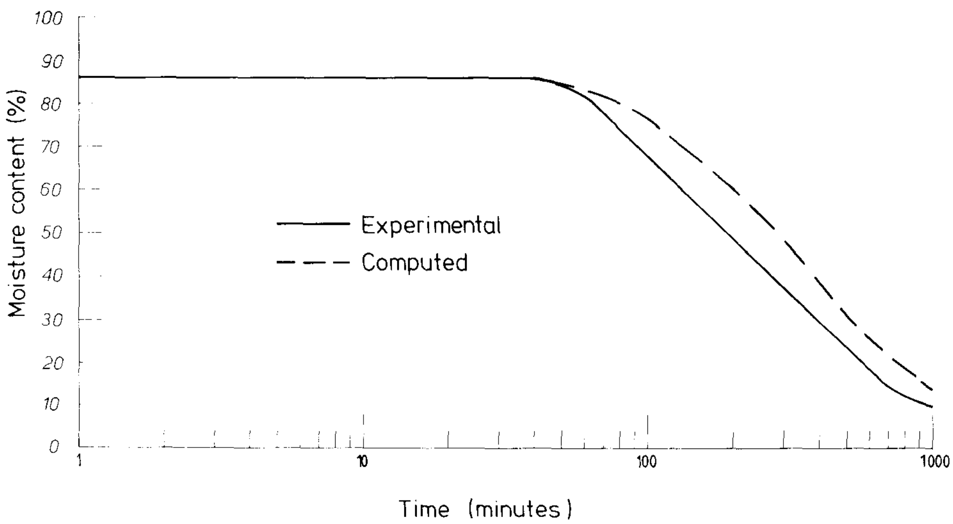

FIG. 4. Moisture content at the center.

whereas below the fiber saturation point diffusion is the main driving force. However, the relative contributions are not well defined. The numerical model used does not take account of bulk flow of water; therefore the values obtained at the higher moisture contents may have been slightly affected by this.

The values of the thermophysical parameters used are known to vary with changing moisture content, but as a first assumption they were considered to be constant during the analysis. This could explain why initially the computed moisture content at the surface is lower than the experimental values, but at the later stages of drying the reverse occurs with experimental results lower than computed.

The finite element method, when applied to solve Eq. 1, has obtained results that compare reasonably well with experimental values.

ACKNOWLEDGMENTS

The authors wish to acknowledge the financial assistance given by the Science Research Council, which enabled H. R. Thomas to pursue a course of full time research.

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