

# ELASTIC CONSTANTS FOR HARDWOODS MEASURED FROM PLATE AND TENSION TESTS

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## ABSTRACT

Measurements of shear moduli  $G_{LR}$  and  $G_{LT}$ , Young's moduli  $E_L$ ,  $E_R$ , and  $E_T$ , and Poisson's ratios  $\nu_{LT}$  and  $\nu_{LR}$  were made at approximately 12% moisture content from material cut from 18 eastern hardwood logs. Shear moduli calculated from off-axis tension tests with angle of load to the L-axis of  $20^\circ$  were slightly larger than those from plate tests.  $E_R$  values determined from off-axis tensile tests closely approximated those determined from tensile tests in the R direction. Poisson's ratios for basswood, cottonwood, and soft maple were negative (strains parallel and perpendicular to the load direction were both positive) in the LT plane for loadings at  $20^\circ$  to the L direction. Some significant correlations were found between the reciprocals of elastic constants and the reciprocals of density at test, also between the reciprocals of shear moduli and the reciprocals of Young's moduli  $E_R$  and  $E_T$ . There was less variability in measurements made in the LR plane than in the LT plane.

*Keywords:* Density, hardwoods, plate twisting tests, Poisson's ratio, shear modulus, tensile tests, Young's modulus.

## NOTATION

- |   |   |
|---|---|
| 1 = subscript for direction parallel to load axis   | $G_{LT}$ = shear modulus in the LT plane (plate test)                                   |
| 2 = subscript for direction perpendicular to load axis                                      | $(G)_{LR}$ = shear modulus in the LR plane (off-axis tension test)                      |
| i = subscript L, R, or T  | $(G)_{LT}$ = shear modulus in the LT plane (off-axis tension test)                      |
| A = cross-sectional area of specimen  | L, R, T = longitudinal, radial, and tangential axes                                     |
| $D_t$ = density at test moisture content  | $M_A = \epsilon_A/\sigma_1$ , $M_B = \epsilon_B/\sigma_1$ , $M_C = \epsilon_C/\sigma_1$ |
| $E_i$ = Young's modulus in the i direction ( $\sigma_i/\epsilon_i$ )                        | P = axial load parallel to specimen length  |
| $(E)_i$ = Young's modulus in the i direction from off-axis tensile test (Equations 2 and 3) | $\alpha$ = angle in degrees between longitudinal direction L and direction of load P    |
| $G_{LR}$ = shear modulus in the LR plane (plate test)                                       | $\epsilon_i$ = strain in i direction  |
|   | $\epsilon_1, \epsilon_2$ = strains parallel and perpendicular to the load axis          |

- $\epsilon_A, \epsilon_B, \epsilon_C$  = strains at  $0^\circ, 45^\circ,$  and  $90^\circ$  to direction of load P in off-axis tension test
- $\nu_{12} = M_C/M_A$  from off-axis tension test ( $-\epsilon_2/\epsilon_1$ )
- $\nu'_{12}$  = Poisson's ratio calculated from Equation 7
- $\nu_{LR} = -\epsilon_R/\epsilon_L$
- $\nu_{LT} = -\epsilon_T/\epsilon_L$
- $\sigma_i = P/A$
- $\zeta_p$  = angle between the direction of maximum principal strain and the axial load P

### INTRODUCTION

A project investigating the mechanical properties of eastern hardwoods is being conducted at Michigan State University. In previous work, relationships between elastic constants other than those for shear were investigated (Yu 1990; Weigel 1991). The current study looks at two ways to measure Young's moduli and the shear moduli  $G_{LR}$  and  $G_{LT}$ . Equations relating elastic constants to density and to each other are being sought. The testing technique also provides information about the Poisson's ratio  $\nu_{12}$  for specimens loaded at an angle to the L direction in the LR and LT planes. Mature wood as opposed to juvenile wood was used for the experiments to help simplify the analysis. Juvenile wood could be a topic for another study.

Two methods for measuring shear moduli are by the use of plates and by the use of off-axis tension specimens. Plates are frequently tested by the method described in ASTM D3044-76 (American Society for Testing and Materials 1989), which is designated for plywood plate testing but has also been used for solid wood. A major difficulty with the test for solid wood is making the plates. Off-axis tensile specimens are frequently used to measure shear modulus in synthetic composites such as glass filament composites (Greszczuk 1969). The technique has also been used with wood (Ebrahimi and Sliker 1981; Schuldt 1972; Zhang and Sliker 1991). One of the more prac-

tical angles to use between the L direction and the load axis is  $20^\circ$ . According to Zhang and Sliker (1991), this angle produces a value for shear modulus that most closely approximates that from a plate test. Schuldt (1972) arrived at this same angle by finite element analysis. Zhang and Sliker found a tension specimen to be better than a compression specimen for measuring shear moduli. They also found that the maximum strain in off-axis tension specimens occurred somewhere in the range of 20 to 35 degrees with the load axis.

Poisson's ratios can have greatly different values for specimens loaded at an angle to the grain than for those loaded parallel or perpendicular. When a wood specimen is loaded in the L, R, or T directions, the strain perpendicular to the load axis will be opposite in sign to that parallel to it; i.e., for a tension test in the LT plane, Poisson's ratio  $\nu_{LT} = -\epsilon_T/\epsilon_L$  and is treated as being a positive Poisson's ratio. However, it is possible for some woods loaded at an angle to the grain to have a negative Poisson's ratio where the strains parallel and perpendicular to the load axis both have the same sign; i.e.,  $-\nu_{12} = \epsilon_1/\epsilon_2$ . This was shown for spruce loaded at various angles to the grain in the LT plane in a publication by Stavsky and Hoff (1969). In the paper by Zhang and Sliker, there was a basswood sample loaded at an angle to the grain of  $20^\circ$  in the LT plane that had a negative Poisson's ratio.

There are numerous studies showing strong correlations between Young's moduli and also shear moduli with specific gravity or density. Many of the regression equations are of the exponential type, although linear regressions seem to work almost as well according to Bodig and Goodman (1973). Good correlation often depends on having a wide range of densities or specific gravities to work with. A large range of densities can be obtained by using data from species grown all over the world since some tropical woods are either lighter or denser than our native U.S. species. Bodig and Goodman (1973), Guitard (1987), and Kellogg and Ifju (1962) combined data for species from many countries. Correlation coefficients (R) in Bodig

and Goodman's article for equations relating Young's moduli and shear moduli to density are between 0.915 and 0.988, while coefficients of variation (CV) for the regressions are between 11% and 30%. Most researchers have separated hardwoods from softwoods in their analyses.

Elastic constants can also be predictors for other elastic constants. There are some good examples of this in Bodig and Goodman's 1973 paper, which gives equations for  $E_R$ ,  $E_T$ ,  $G_{LR}$ , and  $G_{LT}$  as functions of  $E_L$ . There are also equations in which  $E_R$  and  $E_T$  are the independent variables. Correlation coefficients  $R$  for all of their equations of this type for hardwoods were greater than 0.900. Density was equal to or better as a predictor for elastic constants than were the elastic constants themselves judging by the magnitude of the correlation coefficients and the CVs. None of the previously mentioned publications shows very strong associations between Poisson's ratios and density or the other elastic constants.

The objectives of this research were to compare the shear moduli  $G_{LR}$  and  $G_{LT}$ , the three Young's moduli ( $E_L$ ,  $E_R$ , and  $E_T$ ), and the Poisson's ratios  $\nu_{LR}$  and  $\nu_{LT}$  obtained by different test methods; and to look for relationships between pairs of elastic constants and between elastic constants and density.

#### PROCEDURE

The goal of the testing was to measure shear moduli by means of a standard plate test and then to measure Young's moduli, Poisson's ratios, and shear moduli from tension specimens cut from the plates. Half of the plates were to have their broad surfaces LR planes and half were to have their broad surfaces LT planes. Relationships among the various elastic constants determined by the plate and tension tests would be examined.

The source of the test material was 18 eastern hardwood logs that were 30 or more inches in diameter. Species were selected to provide a range of densities and elastic constants. Selection was also based on a wood's reputation

for straightness of grain; i.e., woods like sycamore and elm were not included because of their interlocking grain. In most instances, each species was represented by two logs from different trees. A list of the species and their densities as taken from test panels is given in Table 1. The 18 logs were squared and then live-sawn into 3½-inch thick boards that were 12 to 24 inches wide and 8 to 12 feet long.

Equilibrium moisture content desired for test material was 12%. Drying of the green or partially air-dried boards took place in a small dry kiln that could handle only a limited amount of material at a time. A slow kiln schedule taking about 30 days was employed so that some boards were air-drying while the others were in the kiln. Fast drying species like cottonwood often were at a low moisture content before there was an opportunity to put them in the kiln. A problem was encountered with some oak boards that were left to air-dry for a long time in that they honeycombed before they could be put into the kiln. This happened with flatsawn boards of white oak A.

As a preliminary step to making 14-inch by 14-inch by ½-inch plates, blanks one-inch thick by 6 to 8 inches in the R or T direction by 18 inches in the L direction were cut from sections of the large dried boards. Frequently, only one blank could be cut from an 18-inch section of the approximately ¾-inch thick kiln-dried boards because of slope in grain. However, all blanks for a given plate came from the same board. In most cases, enough blanks were cut from each log to make a plate with its broad surfaces in the LR plane and one with its broad surfaces in the LT plane. Cutting straight-grained blanks with representative radial and tangential faces was a three-dimensional problem, which could not be met with black cherry B in the LR plane. The flow of a drop of dye was used to find the L direction when it was not obvious. Since the blanks in the LR plane had to come from quartersawn boards and those in the LT plane from flatsawn boards, they were not necessarily well matched as to wood properties such as being from the same growth rings.

TABLE 1. Test species and their densities at 0% and 12% moisture content. Except for black walnut and white oak, specimens were cut from two logs (A and B).

	Log designation	Density*			
		In LR plane		In LT plane	
		@ 0% mc (g/cc)	@ 12% mc (g/cc)	@ 0% mc (g/cc)	@ 12% mc (g/cc)
Basswood ( <i>Tilia americana</i> L.)	A	0.464	0.489	0.470	0.493
	B	0.377	0.407	0.401	0.425
Black cherry ( <i>Prunus serotina</i> Ehrh.)	A	0.712	0.729	0.596	0.635
	B			0.605	0.649
Black walnut ( <i>Juglans nigra</i> L.)	A	0.597	0.638	0.619	0.659
Eastern cottonwood ( <i>Populus deltoides</i> Bartr.)	A	0.470	0.490	0.431	0.446
	B	0.413	0.441	0.438	0.467
Hard maple ( <i>Acer saccharum</i> Marsh.)	A	0.726	0.765	0.740	0.763
	B	0.646	0.686	0.644	0.682
Red oak ( <i>Quercus</i> spp.)	A	0.669	0.708	0.665	0.720
	B	0.631	0.683	0.570	0.617
Soft maple ( <i>Acer saccharinum</i> L. or <i>A. rubrum</i> L.)	A	0.608	0.644	0.515	0.552
	B	0.475	0.510	0.478	0.515
White ash ( <i>Fraxinus americana</i> L.)	A	0.588	0.624	0.633	0.673
	B			0.594	0.629
White oak ( <i>Quercus</i> spp.)	A	0.700	0.750		
Yellow-poplar ( <i>Liriodendron tulipifera</i> L.)	A	0.468	0.499	0.535	0.569
	B	0.500	0.517	0.429	0.461

\* Determined from moisture content and density sample of Fig. 1.

Final manufacture of the plates occurred after the one-inch-thick blanks had come to moisture equilibrium in a room where the temperature was maintained at 68 F and the relative humidity at 65%. Two or three months elapsed between placing the blanks in the conditioning room and laminating them using a polyvinyl adhesive. Three blanks were laminated to make up the 14-inch dimension in the R or T direction with the center lamina being 5 inches wide and the two outer ones 4½ inches wide as shown in Fig. 1. For the LT plate manufacture, the center for growth ring curvature was alternated in adjacent laminae. Final dimensioning of plates to ½ inch in thickness was completed three or more days after laminating.

After the shear modulus had been determined for a given plate, specimens as shown in Fig. 1 were cut from it for the various tension tests and for measuring density and moisture content. An extra piece was kept with the test samples for monitoring any moisture con-

tent changes that might occur during subsequent specimen preparation and testing.

#### TESTING

As previously noted, the plate tests were completed first. Following these tests, tension samples and auxiliary samples for measuring moisture content and density were cut from the plates. Reinforcements were applied to the ends of the tension specimens and strain gages were mounted on them. At least one week elapsed between the mounting of strain gages and the loading of the tension specimens. All conditioning and testing were done in a room maintained at 68 F and 65% relative humidity.

Shear moduli were determined for the 14-inch by 14-inch by ½-inch plates according to the procedures described in ASTM D3044-76 (American Society for Testing and Materials 1989). Rate of loading was 0.12 inches per minute on an Instron Model TTD testing machine. In a given test sequence, loads were

recorded at 0.010-inch intervals of deflection to a maximum of 0.150 inches. After the first loading, the plates were rotated 90° and the loading procedure was repeated. Following this, the plate was turned upside down, and data were taken for two more loadings at 90° to each other. The averages of the four loadings for each plate are listed in Tables 2 and 3. There were no significant differences among the four loadings for any of the plates.

Four different tension specimens were manufactured from each plate as shown in Figs. 1 and 2. All of these except Specimen A were cut from the central lamina of each plate. Reinforcements for gripping the ends were added to each of the tension specimens and strain gages were mounted on them as shown in Fig. 2. Details for making and mounting the strain gages used were given in papers by Sliker (1967, 1989) and Zhang and Sliker (1991). Free-filament gages without backing material were used to minimize reinforcement of the substrate. Identical gages were placed on opposing faces of the specimens to eliminate the effect of bending, if any. Strain gages were mounted parallel and perpendicular to the load axis for specimen Type A for measuring Poisson's ratios  $\nu_{LR}$  in the LR plane and  $\nu_{LT}$  in the LT plane. Specimen Type B, which was loaded at 90° to the L direction, had gages mounted in the load direction to measure  $E_R$  in the LR plane and  $E_T$  in the LT plane. Specimen Type C, which was loaded at an angle of 20° to the grain, had strain gages mounted at 0°, 45°, and 90° to the load axis for measuring shear moduli and Young's moduli. Specimen Type D had strain gages mounted in the load direction, which was also the L direction, for measuring  $E_L$ .

Strain and stress data were taken for each tension specimen as 10 equal weights were applied within one minute to a hanger attached to the bottom of the specimen. The top of the specimen was attached by a universal joint to a supporting frame. Each weight applied to specimens of Type A, C, and D was 10 pounds, while the weights used for specimens of Type B were each 5 pounds. Strains at 0° and 90° to the L direction for Specimen Type A and at

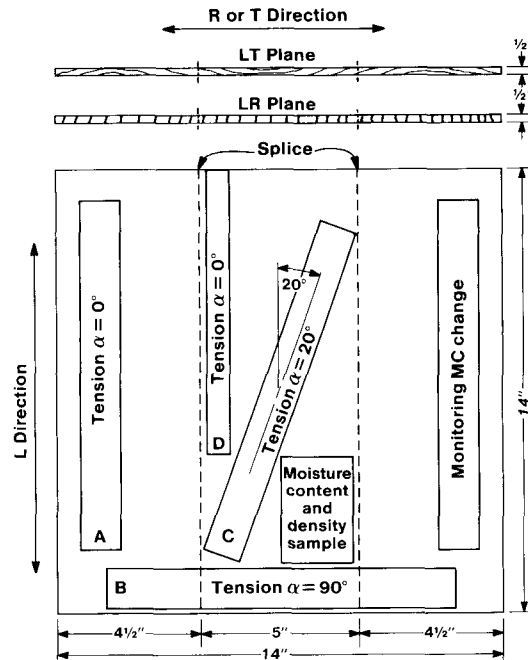


FIG. 1. Plate used for measuring shear moduli showing locations of samples cut for subsequent tests.

90° to the load direction for Specimen Type C were measured with a meter reading strain to  $10^{-7}$  inches per inch. Strains for the other gages were read to  $10^{-6}$  inches per inch.

#### EQUATIONS

Equations for calculating shear moduli and Young's moduli for filamentary composites loaded at an angle to the L direction are given in a publication by Greszczuk (1966).

These are:

$$(G)_{LT} = \frac{1}{2(M_A - M_C) + (2M_B - M_A - M_C) \cdot (\cot \alpha - \tan \alpha)} \quad (1)$$

$$(E)_L = \frac{(1 - \nu_{LT} \tan^2 \alpha)}{M_A + M_C \tan^2 \alpha - (2M_B - M_A - M_C) \tan \alpha} \quad (2)$$

$$(E)_T = \frac{1}{M_A + M_C \cot^2 \alpha + (2M_B - M_A - M_C) \cot \alpha + \nu_{LT} \cot^2 \alpha / E_L} \quad (3)$$

TABLE 2. Values for panels in the LT plane for shear modulus measured from the plate test. Young's moduli measured in tension in the L and T directions, shear moduli and Young's moduli measured from off-axis tension specimens, comparisons of moduli measured by the different methods, and the angle of load to grain for which the maximum strain was calculated.

Specimen	$\zeta p$ (degrees)	$G_{LT}$ from plate test (psi)	$E_L$ $\alpha = 0^\circ$ (psi)	$E_T$ $\alpha = 90^\circ$ (psi)	$\alpha = 20^\circ$			$(G)_{LT}/G_{LT}$	$(E)_L/E_L$	$(E)_T/E_T$
					$(G)_{LT}$ (psi)	$(E)_L$ (psi)	$(E)_T$ (psi)			
BA-A-LT	34.3	$0.897 \times 10^5$	$1.759 \times 10^6$	$0.555 \times 10^5$	$0.995 \times 10^5$	$2.182 \times 10^6$	$0.739 \times 10^5$	1.11	1.24	1.33
BA-B-LT	35.2	0.583	1.462	0.374	0.635	1.040	0.391	1.09	0.71	1.05
BC-A-LT	26.4	1.296	1.479	1.190	1.389	1.607	1.274	1.07	1.09	1.07
BC-B-LT	21.0	1.442	1.356	1.506	1.687	1.307	1.788	1.17	0.96	1.19
BW-A-LT	25.5	1.361	1.562	1.044	1.242	1.363	1.221	0.91	0.87	1.17
COT-A-LT	34.8	0.781	1.462	0.440	0.799	1.730	0.543	1.02	1.18	1.23
COT-B-LT	32.3	0.843	1.689	0.603	0.874	1.237	0.567	1.04	0.73	0.94
HM-A-LT	30.1	1.603	2.575	1.564	1.394	5.662*	1.674	0.87	2.20*	1.07
HM-B-LT	21.5	1.363	2.099	1.449	1.318	0.887	0.991	0.97	0.42	0.68
RO-A-LT	28.3	1.200	1.343	1.031	1.329	2.520	1.528	1.11	1.88	1.48
RO-B-LT	27.9	1.089	1.471	1.219	1.103	1.516	1.087	1.01	1.03	0.89
SM-A-LT	30.8	1.104	1.515	0.862	1.160	1.674	0.869	1.05	1.10	1.01
SM-B-LT	26.5	1.165	1.564	0.738	1.262	1.097	0.899	1.08	0.70	1.22
WA-A-LT	24.9	1.284	1.892	1.695	1.536	1.818	1.758	1.20	0.96	1.04
WA-B-LT	24.0	1.188	1.340	1.154	1.481	1.402	1.351	1.25	1.05	1.17
YP-A-LT	26.6	1.109	1.639	1.150	1.249	1.726	1.354	1.13	1.05	1.18
YP-B-LT	20.3	1.138	1.140	0.710	1.408	0.917	1.225	1.24	0.80	1.73
Average	27.7	$1.144 \times 10^5$	$1.609 \times 10^6$	$1.017 \times 10^5$	$1.228 \times 10^5$	$1.501 \times 10^6$	$1.132 \times 10^5$	1.08	0.99	1.14
CV	17.0%	22.4%	20.9%	39.8%	22.4%	29.6%	37.7%	9.7%	32.2%	20.4%

\* Omitted from calculations.

TABLE 3. Values in the LR plane for shear modulus measured from the plate test, Young's moduli measured in tension in the L and R directions, shear moduli and Young's moduli measured from off-axis tension specimens, comparisons of moduli measured by the different methods, and the angle of load to the grain at which the maximum strain was calculated.

Specimen	$\zeta p$ (degrees)	$G_{LR}$ from plate test (psi)	$E_L$ $\alpha = 0^\circ$ (psi)	$E_R$ $\alpha = 90^\circ$ (psi)	$\alpha = 20^\circ$			$(G)_{LR}/G_{LR}$	$(E)_L/E_L$	$(E)_R/E_R$
					$(G)_{LR}$ (psi)	$(E)_L$ (psi)	$(E)_R$ (psi)			
BA-A-LR	27.0	$1.090 \times 10^5$	$2.167 \times 10^6$	$1.205 \times 10^5$	$1.221 \times 10^5$	$1.475 \times 10^6$	$1.130 \times 10^5$	1.12	0.68	0.94
BA-B-LR	26.6	0.776	1.183	0.872	0.873	1.003	0.842	1.13	0.85	0.97
BC-A-LR	12.3	2.160	1.468	2.664	2.726	1.345	3.257	1.26	0.92	1.22
BW-B-LR	13.8	1.967	1.639	2.636	2.313	1.201	2.316	1.18	0.73	0.88
COT-A-LR	25.3	1.343	1.879	1.724	1.401	1.888	1.733	1.04	1.00	1.01
COT-B-LR	23.0	1.305	1.589	1.983	1.465	1.667	2.218	1.12	1.05	1.12
HM-A-LR	21.9	2.261	1.812	2.927	2.525	2.378	3.218	1.12	1.31	1.10
HM-B-LR	22.7	2.061	2.066	2.476	2.141	2.041	2.538	1.04	0.99	1.03
RO-A-LR	21.5	1.517	1.558	2.703	1.631	1.617	2.747	1.08	1.04	1.02
RO-B-LR	21.7	1.395	1.709	2.413	1.620	1.595	2.314	1.16	0.93	0.96
SM-A-LR	16.0	1.972	1.415	2.703	2.368	1.401	2.575	1.20	0.99	0.95
SM-B-LR	25.1	1.563	1.243	1.719	1.530	1.787	1.613	0.98	1.44	0.94
WA-A-LR	19.5	1.567	1.433	2.410	1.668	1.315	2.181	1.06	0.92	0.90
WO-A-LR	20.3	1.721	2.002	2.513	1.945	1.663	2.774	1.13	0.83	1.10
YP-A-LR	21.9	1.267	1.386	1.920	1.487	1.489	2.089	1.17	1.07	1.09
YP-B-LR	22.3	1.265	1.291	1.718	1.502	1.447	1.878	1.19	1.16	1.09
Average	21.3	$1.577 \times 10^5$	$1.615 \times 10^6$	$2.162 \times 10^5$	$1.776 \times 10^5$	$1.582 \times 10^6$	$2.214 \times 10^5$	1.12	0.99	1.02
CV	19.8%	26.4%	18.6%	27.3%	28.7%	21.1%	30.25%	6.4%	19.4%	9.2%

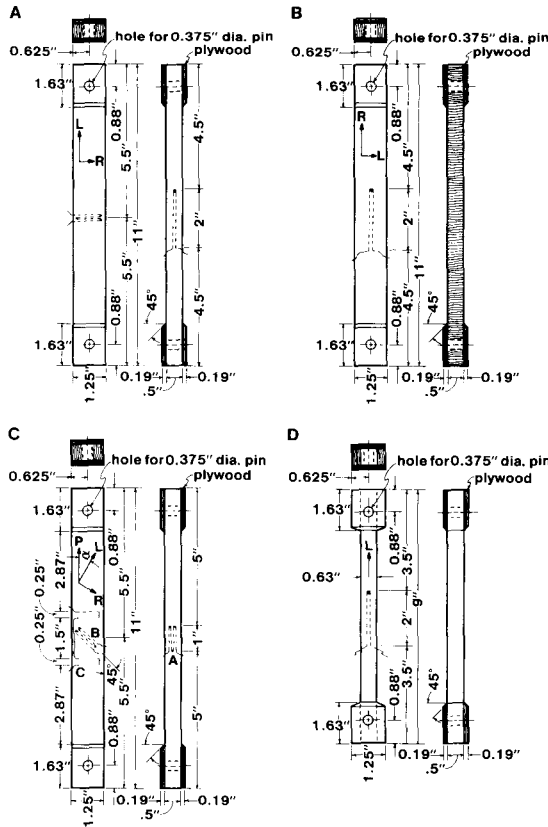


FIG. 2. Tensile specimens cut from plate to measure elastic constants in the LR plane. Specimen A was used for determining Poisson's ratio  $\nu_{LR}$ ; B for determining  $E_R$ ; C for determining  $(G)_{LR}$ ,  $(E)_L$ , and  $(E)_R$ ; D for determining  $E_L$ . Strain gages are shown as dashed lines in midsection of specimens.

In addition

$$\nu_{12} = \frac{M_C}{M_A} \quad (4)$$

The formula for calculating the angle between the load axis and the maximum strain in an off-axis tensile specimen is taken from a book by Perry and Lissner (1962).

$$\zeta_p = \frac{1}{2} \tan^{-1} \left[ \frac{2M_B - M_A - M_C}{M_A - M_C} \right] \quad (5)$$

Equations used for calculating Poisson's ratios for loadings at an angle to the grain when the elastic constants are known for loadings

made parallel and perpendicular to the grain are given in a book chapter by Stavsky and Hoff (1969). These are:

$$\frac{E_L}{E_1} = \cos^4 \alpha + \frac{E_L}{E_T} \sin^4 \alpha + \frac{1}{4} \left( \frac{E_L}{G_{LT}} - 2\nu_{LT} \right) \sin^2 2\alpha \quad (6)$$

$$\nu'_{12} = \frac{E_L}{E_1} \left[ \nu_{LT} - \frac{1}{4} \left( 1 + 2\nu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{G_{LT}} \right) \sin^2 2\alpha \right] \quad (7)$$

Subscript changes for using the above equations in the LR plane can be made by substituting R for T.

## RESULTS

Densities for the specimens cut in the LR and LT planes from the 18 logs of the 10 test species are given in Tables 1, 4 and 5. At 12% moisture content, densities ranged from 0.407 g/cc for a basswood sample to 0.765 g/cc for a hard maple sample. In many cases, density for a specimen in the LR plane differed from that for the specimen in the LT plane from the same log. This was due to LR and LT specimens not being closely matched. LR specimens were cut from quartersawn boards and LT specimens from flatsawn boards. Since the logs were essentially live-sawn, LR and LT specimens were cut from different sections of the log oriented at 90 degrees to each other.

Moduli of elasticity  $E_L$ ,  $E_T$ , and  $E_R$  were determined from tension specimens loaded at 0° and 90° to the grain. Their values can be found in Tables 2 and 3. Similar to the densities noted above and for the same reason,  $E_L$  determined in the LR plane differs from  $E_L$  found in the LT plane for a number of logs. However, the average value for  $E_L$  of all the test material in the LR plane corresponds very closely to that for the LT plane:  $1.615 \times 10^6$  psi in the LR plane versus  $1.609 \times 10^6$  psi in the LT plane. The coefficient of variation (CV) for all values of  $E_L$  in the LR plane is 18.6% while



TABLE 4. Values for panels in the LT plane for moisture content, density at test moisture content, Poisson's ratio, and angle of load to grain and strain per unit stress values measured from off-axis tension specimens.

Specimen	Moisture content (%)	Density @ test MC (g/cc)	$\alpha$ (degrees)	$M_L$ (1/psi)	$M_R$ (1/psi)	$M_C$ (1/psi)	$\nu_{LT}$
BA-A-LT	10.4	0.490	20.75	$1.645 \times 10^{-6}$	$2.652 \times 10^{-6}$	$0.354 \times 10^{-6}$	0.220
BA-B-LT	10.4	0.422	19.5	2.574	4.171	0.814	0.332
BC-A-LT	11.5	0.633	21.5	1.360	1.596	-0.133	0.588
BC-B-LT	10.1	0.642	21.0	1.259	1.186	-0.222	0.455
BW-A-LT	12.3	0.660	18.0	1.318	1.490	-0.177	0.415
COT-A-LT	10.8	0.445	15.75	1.421	2.402	0.261	0.366
COT-B-LT	11.5	0.466	17.75	1.716	2.515	0.268	0.480
HM-A-LT	12.5	0.764	22.25	1.118	1.600	-0.174	0.362
HM-B-LT	12.0	0.682	18.25	1.599	1.542	-0.0799	0.432
RO-A-LT	12.2	0.721	20.5	1.177	1.529	-0.196	0.428
RO-B-LT	12.2	0.617	19.0	1.441	1.811	-0.131	0.387
SM-A-LT	11.9	0.552	19.0	1.367	1.911	0.0869	0.503
SM-B-LT	12.0	0.514	19.5	1.556	1.803	0.0557	0.481
WA-A-LT	11.2	0.671	21.0	1.185	1.313	-0.223	0.451
WA-B-LT	11.9	0.628	19.0	1.222	1.298	-0.173	0.523
YP-A-LT	11.1	0.567	22.0	1.476	1.758	-0.203	0.451
YP-B-LT	10.4	0.456	20.0	1.596	1.471	-0.160	0.442
Average		0.584					0.430
CV		17.8%					19.3%

that for  $E_L$  in the LT plane is 20.9%.  $E_R$  from tests in the LR plane had an average value of  $2.162 \times 10^5$  psi with a CV of 27.3% while  $E_T$  from tests in the LT plane had an average value of  $1.017 \times 10^5$  psi and a coefficient of variation of 39.8%. These test values were at moisture contents close to 12%. Test values, moisture contents, and density at test are listed in Tables 2, 3, 4, and 5. Some of the differences in moisture content at time of test between LR and LT samples of the same log can be attributed to differences in drying techniques and to slight changes in the conditions in the controlled temperature and humidity room over the many months at which conditioning and testing took place.

Shear moduli were determined by two methods: the plate shear test and the off-axis tensile test with an angle of load to grain of approximately 20 degrees. In the LT plane, the shear moduli  $(G)_{LT}$  calculated from the off-axis tensile test ranged from 0.87 to 1.25 times the shear moduli  $G_{LT}$  from the plate test and were an average of 8% greater than those from the plate test. In the LR plane, the shear moduli

$(G)_{LR}$  from the off-axis tensile test ranged from 0.98 to 1.26 times the shear moduli  $G_{LR}$  from the plate test and were an average of 12% greater than those from the plate test. The CVs were smaller for ratios of  $(G)_{LR}/G_{LR}$  than they were for  $(G)_{LT}/G_{LT}$ , showing that there was greater accuracy of measurements in the LR plane. Individual test values for shear moduli are in Tables 2 and 3. Which method gives the more representative shear modulus for design use is difficult to say. An advantage of the off-axis tensile specimen is that it is smaller than the plate test specimen; in the current testing, it was easy to make from a single piece of wood without laminating.

Other elastic constants  $(E)_L$ ,  $(E)_T$ , and  $(E)_R$  were calculated from the 20° off-axis tension specimens using Eqs. (2) and (3) and were compared with the values from the tension tests made at 0° and at 90° to the grain in Tables 2 and 3. The ratio of  $(E)_L$  to  $E_L$  in the LT plane average 1.06. If the unusually large calculated value of  $5.662 \times 10^6$  psi for  $(E)_L$  of HM-A-LT is not used, the ratio of  $(E)_L$  to  $E_L$  in the LT plane is 0.99. This is the same as the ratio of

TABLE 5. Values for panels in the LR plane for moisture content, density at test moisture content, Poisson's ratio, and angle of load to grain and strain per unit stress values measured from off-axis tension specimens.

Specimen	Moisture content (%)	Density @ test MC (g/cc)	$\alpha$ (degrees)	$M_A$ (1/psi)	$M_B$ (1/psi)	$M_C$ (1/psi)	$\nu_{LR}$
BA-A-LR	11.2	0.488	19.0	$1.359 \times 10^{-6}$	$1.640 \times 10^{-6}$	$-0.121 \times 10^{-6}$	0.451
BA-B-LR	10.9	0.404	21.0	2.145	2.537	-0.156	0.406
BC-A-LR	12.5	0.729	20.0	0.930	0.608	-0.257	0.461
BW-A-LR	12.0	0.638	18.38	1.038	0.735	-0.233	0.449
COT-A-LR	11.5	0.489	20.0	1.179	1.335	-0.280	0.460
COT-B-LR	11.3	0.439	18.5	1.111	1.136	-0.327	0.349
HM-A-LR	13.3	0.769	19.5	0.728	0.709	-0.179	0.430
HM-B-LR	12.0	0.686	20.0	0.878	0.885	-0.180	0.399
RO-A-LR	11.5	0.706	17.5	1.018	0.975	-0.295	0.273
RO-B-LR	12.4	0.684	19.0	1.096	1.058	-0.300	0.285
SM-A-LR	11.2	0.641	19.25	0.959	0.736	-0.225	0.454
SM-B-LR	11.6	0.509	19.0	1.083	1.211	-0.207	0.503
WA-A-LR	11.5	0.622	19.0	1.172	1.033	-0.299	0.385
WO-A-LR	12.9	0.753	20.0	1.005	0.914	-0.267	0.357
YP-A-LR	10.5	0.495	19.0	1.178	1.145	-0.334	0.393
YP-B-LR	11.0	0.516	19.0	1.191	1.178	-0.284	0.398
Average		0.598					0.403
CV		19.9%					15.7%

$(E)_L$  to  $E_L$  in the LR plane. There was quite a bit of variation in the ratios of  $(E)_L$  to  $E_L$ ; they ranged from 0.42 to 1.88. There was better matching of  $(E)_L$  to  $E_L$  in the LR plane than in the LT plane as evidenced by the CV of 19.4% for  $(E)_L/E_L$  values in the LR plane, compared to a CV of 32.2% for  $(E)_L/E_L$  values in the LT plane even with the numbers for HM-A-LT removed. Greszczuk (1966) found much greater variation between calculated and observed values of  $E_L$  for fiberglass laminates loaded at various angles to the filament axis than he did for calculated and observed values of shear moduli and Young's moduli perpendicular to the fiber axis.

Values of  $(E)_R$  calculated from the 20° off-axis specimens in the LR plane corresponded fairly closely to the values of  $E_R$  obtained from loadings made in the R direction: the values for  $(E)_R$  averaged 2% greater than the values of  $E_R$ , and the ratios of  $(E)_R$  to  $E_R$  had a CV of only 9.2%. Individual values of  $(E)_R/E_R$  ranged from 0.88 to 1.22.

Values of  $(E)_T$  calculated from the 20° off-axis specimens averaged 14% greater than the values of  $E_T$  measured on the specimens load-

ed in the T direction. Ratios of  $(E)_T$  to  $E_T$  ranged from 0.68 to 1.48, and the ratios of  $(E)_T/E_T$  had a CV of 20.4%. The greater CV values for shear moduli and Young's moduli in the LT plane than in the LR plane suggest that some wood factors contribute to the greater variabilities in the LT plane. One such factor is that the opposing broad surfaces of a specimen in the LR plane are more nearly alike in physical properties than are the opposing broad faces of a specimen in the LT plane. In addition, growth ring curvature would have a greater effect on measurements in the LT plane than in the LR plane. Also, measurements of the angle between the L direction and the load direction were easier to make on the radial surface than on the tangential surface.

Poisson's ratios  $\nu_{12}$  for the specimens loaded at an angle  $\alpha = 20^\circ$  to the L direction can be calculated from the test data and from theoretical equations. The quantity  $\nu_{12}$  represents the strain perpendicular to the load axis divided by the strain parallel to the load axis ( $-\epsilon_2/\epsilon_1$ ), which also equals  $M_C/M_A$ . Values of  $\nu_{12}$  for the off-axis tensile specimens are given in Table 6. The Poisson's ratios in the LT plane

TABLE 6. Poisson's ratios which were measured and those which were calculated for angle of loading  $\alpha$  in off-axis tension specimens; calculated angle of loading to grain at which Poisson's ratio for off-axis tensile specimens became negative in LT plane.

Log number	LT plane				LR plane		
	Angle $\alpha$ at test (degrees)	$\nu_{12}$ at angle $\alpha$		Angle at which $\nu'_{12}$ becomes negative (degrees)	Angle $\alpha$ at test (degrees)	$\nu_{12}$ at angle $\alpha$	
		$\nu_{12} = M_r/M_A$	$\nu'_{12}$			$\nu_{12} = M_r/M_A$	$\nu'_{12}$
BA-A	20.75	-0.215	-0.375	7.4	19.0	0.089	0.161
BA-B	19.5	-0.316	-0.331	8.5	21.0	0.073	0.150
BC-A	21.5	0.098	0.100	29.6	20.0	0.276	0.270
BC-B	21.0	0.176	0.153				
BW-A	18.0	0.134	-0.024	17.0	18.38	0.224	0.305
COT-A	15.75	-0.184	-0.328	8.7	20.0	0.237	0.255
COT-B	17.75	-0.156	-0.134	13.0	18.5	0.294	0.297
HM-A	22.25	0.156	0.034	27.8	19.5	0.246	0.271
HM-B	18.25	0.050	0.155		20.0	0.205	0.209
RO-A	20.5	0.167	0.015	21.5	17.5	0.290	0.308
RO-B	19.0	0.091	0.166		19.0	0.274	0.315
SM-A	19.0	-0.064	-0.024	17.9	19.25	0.235	0.312
SM-B	19.5	-0.036	-0.211	13.2	19.0	0.191	0.248
WA-A	21.0	0.188	0.256		19.0	0.255	0.314
WA-B	19.0	0.142	0.157	34.9			
WO-A					20.0	0.266	0.277
YP-A	22.0	0.138	0.106		19.0	0.284	0.313
YP-B	20.0	0.100	-0.193	14.1	19.0	0.238	0.272

for the basswood, cottonwood, and soft maple samples are negative, whereas those for the other species are positive. All specimens in the LR plane have positive Poisson's ratios. A negative Poisson's ratio means that the observed strain in the 2 direction is positive. Since the strain in the 1 direction is also positive, all of the strains in that plane will be positive tensile strains.

Values calculated from Eqs. 6 and 7 for  $\nu'_{12}$  can be found in Table 6. The quantities  $G_{LR}$  and  $G_{LT}$  used in the equations were those from the plate testing;  $E_L$  values were those from testing the small tension specimens in the L direction,  $E_R$  and  $E_T$  were obtained from loadings on the specimens with their main axis in the R and T directions, and  $\nu_{LR}$  and  $\nu_{LT}$  were measured on the larger tensile specimens loaded in the L direction. All of the specimens that had negative values of  $\nu_{12}$  from the off-axis tensile tests also had negative values of  $\nu'_{12}$  as calculated by Eqs. 6 and 7. In addition, there were negative values for  $\nu'_{12}$  for specimens BW-A and YP-B in the LT plane. There was

better correspondence between the calculated  $\nu'_{12}$  and the observed  $\nu_{12}$  in the LR plane than in the LT plane. This was partly because the calculated values  $\nu'_{12}$  changed at a faster rate with angle of loading in the LT plane than in the LR plane, and some of the greater rates of change of  $\nu'_{12}$  with angle of loading occurred in the region where the test measurements were made. Furthermore, none of the Poisson's ratios in the LR plane became negative. For 16 values of calculated Poisson's ratios ( $\nu'_{12}$ ) in the LR plane ranging between 0.150 and 0.315, there were 9 that differed by less than 0.030 from the values of  $\nu_{12}$  at the test angle of 20°. For 17 values of calculated Poisson's ratios ( $\nu'_{12}$ ) in the LT plane ranging between 0.256 and -0.375, there were 5 that differed by less than 0.030 from the values of  $\nu_{12}$  at the test angle of 20°. Since the calculation of  $\nu'_{12}$  depends on more than 6 measured quantities and  $\nu_{12}$  on only two, it would be expected that  $\nu_{12}$  would have the smaller error.

Certain observations can be made by plotting Eq. 7 for specific cases. This has been done

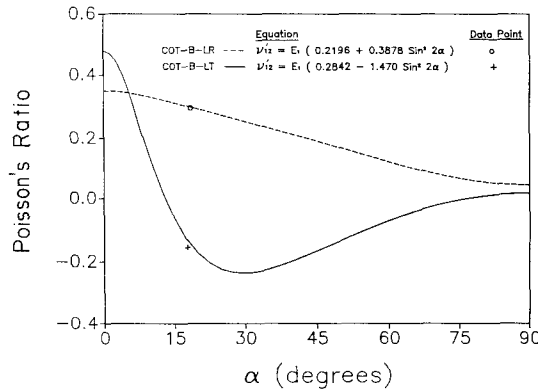


FIG. 3. Plots of Poisson's ratios versus angle of loading to grain using data for COT-B-LR and COT-B-LT specimens in Eq. (7). Data points for tensile specimens loaded at approximately 20° to the grain are shown.

for COT-B-LR and COT-B-LT in Fig. 3. The test values of  $\nu_{12} = M_C/M_A$  have also been inserted as a circle for the one on the LR plane and a cross for the one in LT plane. Both of these points come close to their theoretical plots. However, it is easy to see why the values of  $\nu'_{12}$  and  $\nu_{12}$  might differ more in the LT plane than in the LR plane because of the rapid change in values for  $\nu'_{12}$  in that plane for small angle changes. No negative values were found for Poisson's ratios in the LR plane for any of the specimens. The graph shows almost a linear decrease from 0° to 90° in this plane for COT-B-LR. However, in the case of COT-B-LT, negative Poisson's ratios exist for angles between 13° and 77°. By means of Eqs. (6) and (7), the angles between which values of Pois-

son's ratios for the other test specimens in the LT plane are negative can be found. Negative values are indicated in the LT plane for 12 of the 17 test panels in Table 6. Basswood panels show negative Poisson's ratios for angles  $\alpha$  between 8 and 82°. Other specimens become negative at a larger angle and for a shorter range of angles.

A number of linear regression equations were attempted between pairs of elastic constants and also between elastic constants and density. No significant relationships were found for either of the Poisson's ratios  $\nu_{LR}$  or  $\nu_{LT}$  as functions of density,  $E_L$ ,  $R_R$ , or  $E_T$ . This agrees with Bodig and Goodman's (1973) conclusions. Bodig and Goodman's average values for  $\nu_{LR}$  and  $\nu_{LT}$  for hardwoods were 0.37 and 0.50, respectively. CVs for these numbers are 29.7% and 23.4%, respectively. Yu (1990) and Weigel (1991) found Poisson's ratios using compression specimens made from wood from the same logs as used in this experiment. Combining the results of both these publications, average values were  $\nu_{LR} = 0.376$  and  $\nu_{LT} = 0.450$ ; CV was 11.3% for both ratios. Averages and CVs for the test values of  $\nu_{LR}$  and  $\nu_{LT}$  of this report at 0.403 (CV = 15.7%) and 0.430 (CV = 19.3%), respectively. There is no statistically significant difference between the Poisson's ratios from Yu's and Weigel's compression samples and the Poisson's ratios from tension samples in this report at the 95% confidence level.

No highly significant correlation was found between  $E_L$  and density, nor were  $G_{LT}$ ,  $G_{LR}$ ,

TABLE 7. Equations showing relationships between elastic constants and density at test moisture content, and between shear moduli and Young's moduli in the R and T directions.

Equation	R <sup>2</sup>	CV %
$(1/E_T) = -1.727 \times 10^{-5} + 1.651 \times 10^{-5}(1/D_t)$	0.819	22.2
$(1/E_R) = -0.303 \times 10^{-5} + 0.469 \times 10^{-5}(1/D_t)$	0.668	24.5
$E_T = -0.971 \times 10^5 + 3.402 \times 10^5 D_t$	0.768	19.8
$E_R = -0.470 \times 10^5 + 4.401 \times 10^5 D_t$	0.788	13.1
$(1/G_{LT}) = -0.265 \times 10^{-5} + 0.675 \times 10^{-5}(1/D_t)$	0.697	16.3
$(1/G_{LR}) = -0.152 \times 10^{-5} + 0.480 \times 10^{-5}(1/D_t)$	0.689	17.9
$G_{LT} = -0.088 \times 10^5 + 2.109 \times 10^5 D_t$	0.738	11.8
$G_{LR} = -0.118 \times 10^5 + 2.835 \times 10^5 D_t$	0.657	16.0
$(1/G_{LT}) = 0.435 \times 10^{-5} + 0.414 \times (1/E_T)$	0.872	10.6
$(1/G_{LR}) = 0.203 \times 10^{-5} + 0.935(1/E_R)$	0.861	12.0

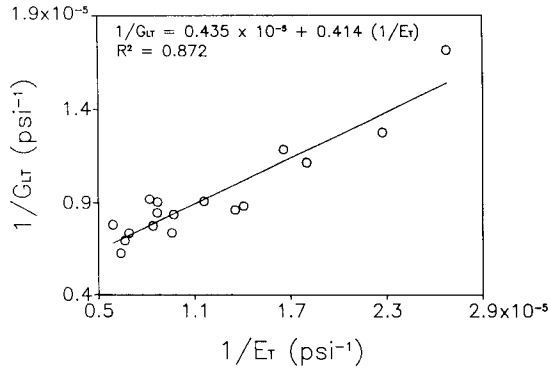


FIG. 4. Relationship between the reciprocal of the shear modulus in the LT plane and the reciprocal of Young's modulus in the T direction.

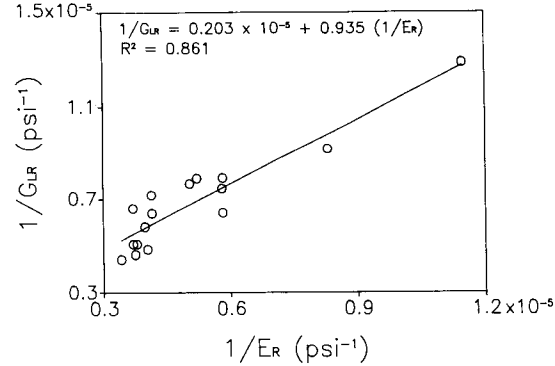


FIG. 5. Relationship between the reciprocal of the shear modulus in the LR plane and the reciprocal of Young's modulus in the R direction.

$E_T$ , and  $E_R$  significantly correlated with  $E_L$ . Other researchers such as Bodig and Goodman (1973) and Guitard and Amri (1987) had found such relationships to be highly significant at the 99% level. However, they worked with a much broader range of densities and values of  $E_L$  by using data for tropical woods. The CVs for their equations were about the same as the CVs for average density and for average  $E_L$  given for the current test data in Tables 2 and 3.

Some significant relationships were found relating  $E_R$ ,  $E_T$ ,  $G_{LT}$ , and  $G_{LR}$  to density;  $G_{LT}$  to  $E_T$ ; and  $G_{LR}$  to  $E_R$ . In many cases, a greater correlation was achieved by using the reciprocal of these quantities as the variables rather than the variables themselves. The better equations relating elastic constants to density at time of test and shear moduli to Young's moduli are given in Table 7. There are particularly strong correlations between  $1/G_{LT}$  and  $1/E_T$  and between  $1/G_{LR}$  and  $1/E_R$ . Figures 4 and 5 are graphs of these data. Reciprocals of shear moduli and Young's moduli are compliances that can be used in finding a strain value when the stress is known.

#### SUMMARY AND CONCLUSIONS

Measurements of shear moduli, Young's moduli parallel and perpendicular to the grain, and Poisson's ratios were made in the LR and LT planes for material at approximately 12%

moisture content from 18 logs. Shear moduli were measured both by the plate method and by an off-axis tensile test with an angle between load and grain axis of 20°. Quantities  $E_L$ ,  $\nu_{LR}$ , and  $\nu_{LT}$  were measured on tension parallel to the grain specimens, while  $E_R$  and  $E_T$  were measured on specimens loaded in tension at 90° to the grain. All tension specimens were cut from the plates used for measuring the shear moduli. Results were as follows:

1. Shear moduli from the off-axis tension tests were compared with those from the plate tests.
  - a. In the LT plane, shear moduli from the off-axis tension tests averaged 8% greater than those from the plate tests.
  - b. In the LR plane, shear moduli from the off-axis tests averaged 12% greater than those from the plate tests.
  - c. There was less variability in the LR plane than in the LT plane between shear moduli calculated from the off-axis tests and those calculated from the plates.
2. Young's moduli ( $E_L$ ), ( $E_T$ ), and ( $E_R$ ) were calculated from the stresses and strains in the 20° off-axis tensile specimens and were compared with these same quantities obtained from tensile tests made at 0° and 90° to the grain. The best matching of these Young's moduli between test methods was for  $E_R$ , where the average ratio of ( $E_R$ ) to

- $E_R$  was 1.02 with a CV of 9.2%; for  $(E)_T/E_T$ , the average was 1.14 with a CV of 20.4%; for  $(E)_L/E_L$  the average was 0.99 in both the LT and LR planes. CVs for  $(E)_L/E_L$  were 32.2% in the LT plane and 19.4% in the LR plane. With some improvement in technique, an off-axis tensile specimen could be used to measure shear moduli and Young's moduli more accurately.
3. Poisson's ratios  $\nu_{12} = M_C/M_A$  for basswood, cottonwood, and soft maple samples loaded at 20° to the L direction in the LT plane were negative, while  $\nu_{12}$  for the other species was positive. Since a negative  $\nu_{12}$  is defined as a positive lateral strain over a positive longitudinal strain, there would be no negative strains in the LT plane at the 20° angle of loading for the basswood, cottonwood, and soft maple. By means of theoretical equations, it was shown that some, but not all, of the other species would have negative Poisson's ratios in the LT plane if the angle of loading to the L direction were increased. None of the specimens in the LR plane had negative Poisson's ratios.
  4. Average values for Poisson's ratios were 0.430 with a CV of 19.3% for  $\nu_{LT}$  and 0.403 with a CV of 15.7% for  $\nu_{LR}$ . These did not appear to be functions of density or Young's moduli.
  5. No highly significant correlation was found between  $E_L$  and density. In addition, no significant correlations were found between any of the other elastic constants and  $E_L$ . In part, this was because the ranges of densities and of  $E_L$  were relatively small.
  6. Some significant linear correlations were found with  $1/E_T$ ,  $1/E_R$ ,  $1/G_{LT}$ , and  $1/G_{LR}$  as functions of  $1/D_i$ , where  $D_i$  is the density at approximately 12% moisture content. Other significant relationships were found between  $1/G_{LT}$  and  $1/E_T$  and between  $1/G_{LR}$  and  $1/E_R$ . In many cases, better equations resulted from using the reciprocals of the elastic constants as variables rather than the numbers themselves.
  7. Measurements made in the LR plane generally had smaller CV values than those measured in the LT plane. Reasons for this could be that parallel faces were more nearly alike in the LR plane than in the LT plane, growth ring curvature was not as pronounced in the LR specimens as in the LT specimens, and angle of loading to L direction was easier to measure in the LR plane. The rate of change of Poisson's ratio with angle of load to the L direction was greater in the LT plane than in the LR plane.

#### ACKNOWLEDGMENTS

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