MODAL ANALYSIS OF UPRIGHT PIANO SOUNDBOARDS BY COMBINING FINITE ELEMENT ANALYSIS AND COMPUTER-AIDED DESIGN

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Abstract. This study presents a visual model for analyzing the vibration modes of piano soundboards by combining the tools of finite element analysis and computer-aided design. Based on the predicted results from the model, changes of natural frequency and maximum displacement of the soundboard as a function of wood properties, structure, and rib size were discussed. Wood grain direction affected the mode shape of the soundboard. Among the 10 property factors investigated, density presented the greatest impact to the vibration mode of the soundboard followed by Young's modulus, shear modulus, and Poisson's ratio. Increasing the thickness of the resonance board and the use of ribs had positive impacts on the natural frequency of the soundboard. However, the amount of natural frequency was decreased for those that were lower than 100 Hz. Natural frequency increased as the intensity, density, and size of ribs increased. Rib height had a greater effect on the variation of natural frequency than the intensity, density, and rib width. In general, increases in rib intensity, density of wood species, and rib width presented negative effects on the maximum displacement.

Keywords: Modal analysis, piano soundboard, finite element analysis, computer-aided design.

INTRODUCTION

The soundboard is the main radiating component of a piano and is made from wood. Soundboard quality greatly affects the piano's acoustical performance. The main element of a soundboard is the resonance board, which is a large thin wood panel made from glued wood strips, usually spruce, that are 80-100 mm wide. The resonance board cannot be too thick, otherwise it may decrease the sound performance of a piano. However, a minimum soundboard thickness is essential to provide enough stiffness for the soundboard structure. To use a thinner resonance board and maintain sufficient stiffness of the soundboard structure, reinforcement ribs (a set of parallel nearly equidistant stiffeners) are

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usually used, which allows the thickness of the resonance board to be decreased to 6-10 mm. The ribs are also made out of spruce or other wood species glued together. The cross-section of the ribs is usually about 25 mm wide and 10-20 mm (or more) high. The overall shape of the soundboard depends on the type of piano, ie. rectangular for upright pianos and a half round-and-high hat shape for grand pianos (Ege et al 2013). The width of the soundboard is about 1.4 m, in agreement with that of the keyboard. The height or length ranges are from about 0.6 m for small uprights to more than 2 m for some concert grand pianos. Two bridges are glued to the opposite side of the resound board: 1) a short bar, and 2) a long thick bar. The two bridges are slightly curved with strings attached.

The sound of the piano is enriched by the soundboard. The vibration properties of the soundboard are critical to the sound performance of a piano. Suzuki (1986) studied the vibration and sound radiation of the soundboard from a Steinway grand piano. The mode shapes and frequencies of six resonances below 200 Hz were shown, and their relations to intensity patterns were discussed. Giordano (1998) illustrated that the variation in mass distribution of a soundboard could affect its vibration properties and the piano sound quality. Xing et al (2007) showed that an additional mass block greatly affected the energy of the low-frequency mode of a soundboard. Liu et al (2013) measured the vibration properties of resonance boards of eight wood species and evaluated the acoustical properties of the pianos after the resonance boards were incorporated into the pianos. Using experimental methods to study soundboards may have limitations because each measurement can only be made on one specific soundboard. Often, a full-sized soundboard has to be cut into small specimens for property measurements, which can be costly. Many researchers have applied mathematical modeling and computer simulation to study vibration performance and other properties of a soundboard (Giordano 1997; Berthaut et al 2003; Ortiz-Berenguer et al 2008; Boutillon and Ege 2013). Finite element analysis (FEA) has proven to be a viable tool in designing new piano soundboards (Kindel and Wang 1987; Berthaut et al 2003; Ortiz-Berenguer et al 2008). However, most of the previous studies only used two-dimensional models. Also, the effect of wood grain direction was not considered. Wood is an anisotropic material. The mechanical and physical properties, thermal conductivity as well as sound velocity in the longitudinal, tangential, and radial directions can be significantly different. Sound performance can be greatly affected by the grain direction of the wood. A three-dimensional (3-D) FEA model with respect to the directional effect on the sound performance can be beneficial to the structural design of a piano.

In this study, we present a 3-D visual model for modal analysis of upright soundboards using the tools of both FEA and computer-aided design (CAD) and considering the effects of both wood grain direction and species. The goal of modal analysis in structural mechanics is to determine the natural mode shapes and natural frequencies of soundboards. CAD technology can provide a 3-D visualization with flexibility in the model configuration. The natural frequency and maximum displacement (vibration strength) change as a function of structure and rib size are presented and discussed based on the predicted results from the model.

MODELING AND MODAL ANALYSIS

An upright piano soundboard was used for modeling with both CAD and FEA for the modal analysis. Three wood species, spruce, pine, and beech, were used in the modal analysis. Density of the wood species and elastic moduli, shear moduli, and Poisson's ratios in longitudinal (L), tangential (T), and radial (R) directions of the three wood species (Table 1) were used for the model inputs.

Modeling of a Piano Soundboard

Figure 1 depicts the basic approach for modeling an upright piano soundboard. The soundboard is

Table 1. Properties of three wood species used as parameter inputs in finite element analysis.[a]

Species	ρ (kg/m^3)	E_L (MPa)	E_R (MPa)	E_T (MPa)	G_{LT} (MPa)	G_{LR} (MPa)	G_{TR} (MPa)	μ_{RT}	μ_{LR}	μ_{LT}
Spruce	371	11,583	896	496	690	758	39	0.43	0.37	0.47
Pine	550	16,272	1103	573	676	1172	66	0.68	0.42	0.51
Beech	750	13,700	2240	1140	1060	1610	460	0.75	0.45	0.51

[a] *E*, Young's modulus; *G*, shear modulus; μ, Poisson's ratio, ρ is density. The soundboard has been considered as orthotropic, longitudinal (L), tangential (T), and radial (R) (Wang 2007; Liang et al 2009).

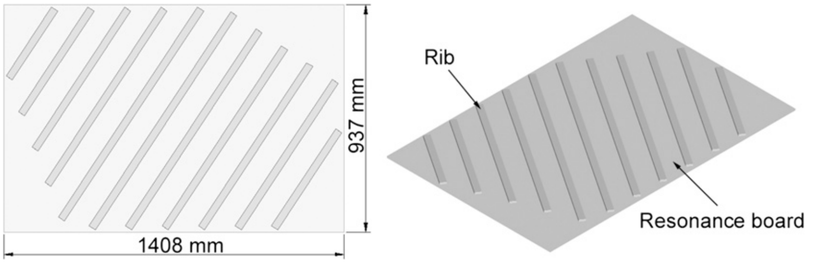

Figure 1. Model of upright piano soundboard with ribs.

composed of two parts: ribs and resonance board. For the convenience of modeling, a rectangular cross-section was assumed for the rib. Dimensions of the soundboard used in the modeling were considered 1.408 m long and 937 mm wide.

Modal Analysis

The structural dynamics approach was used to analyze the natural frequency and the mode shape of a structure. For linear constant system with N degrees of freedom, the vibration equation can be expressed as (Fu 1990)

$$[M]\frac{d^2x(t)}{dt^2} + [C]\frac{dx(t)}{dt} + [K]x(t) = f(t) \qquad (1)$$

where [M] is the mass matrix for the elastic system; [C] is the damping matrix of the elastic system; [K] is the stiffness matrix; $\frac{d^2x(t)}{dt^2}$, $\frac{dx(t)}{dt}$, and x(t) are the acceleration vector, the velocity vector, and the displacement vector; and f(t) is the excitation force vector. Structural stiffness matrix [K] and mass matrix [M] are n by n square matrix, where n is the number of freedom degrees. Through solving this equation, n natural frequencies of the structure and the amplitude value vector of each node can be obtained. Commercial FEA software, ANSYS, was used for the modal analysis of the piano soundboard.

The critical steps and boundary conditions for the modeling are

1) Input the soundboard model generated from the AutoCAD software into the ANSYS software;
2) Define the element types: the element type "SOLID186" in ANSYS was selected for the soundboard simulation;
3) Define the constants of material properties: the linear orthotropic material properties were input based on the selected wood species;
4) Mesh the CAD models into the ANSYS software to convert the solid model into a finite element model (FEM);
5) Define the coordinate directions of the model based on the grain directions for both resonance board and ribs, as shown in Fig 2. The grain direction of ribs was assumed to be perpendicular to that of the resonance board;
6) Define the boundary conditions and loads: constraints were applied to all nodes located where the soundboard is attached to the back of the piano;
7) Choose the analysis type and calculation options in ANSYS: the modal analysis

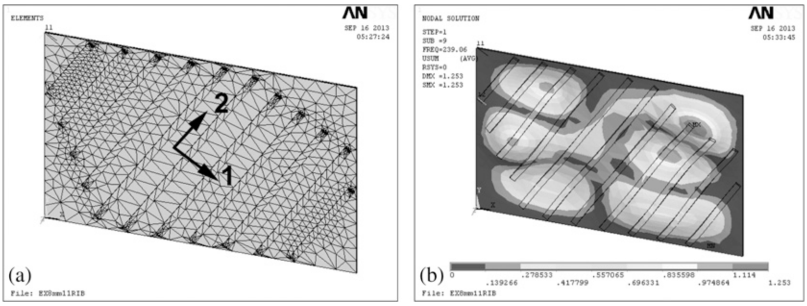

Figure 2. (a) Screen print of a simulated soundboard model. Direction 1 is the longitudinal grain direction of the resonance board; direction 2 is the longitudinal grain direction of the ribs. (b) Typical contour maps of the modal analysis result.

method "Block Lanczos" was selected, and then the frequency range and loading steps were confirmed; and

8) Solve this FEM model: the shapes and frequencies of the 1st to 15th modes were generated.

The modal analysis results were expressed in contour maps. Figure 2 illustrates a screen print of the simulated soundboard.

RESULTS AND DISCUSSION

Wood Properties

Figure 3 shows that the mode shape of a soundboard is affected by the wood grain direction. When no ribs were added (Fig 3a-b), the long axis of the mode shape ellipse was in the same direction as the wood longitudinal grain of the resonance board. Different wood grain directions of the resonance board may generate different mode shapes. When ribs were added, because the longitudinal grain direction of the ribs was perpendicular to that of the resonance board, the ellipse long axis parallel to grain direction of the resonance board was suppressed, and the vibration was distributed across the soundboard (Fig 3c). Wogram (2000) performed an experimental study and showed that adding ribs could decrease the difference in bending stiffness (modulus of elasticity) of the whole soundboard system among different directions and thus decrease the variation of the sound performance in terms of the directions.

To investigate the effect of different property factors on the natural frequency and the maximum displacement of mode, simulations were conducted on the soundboard using the property values of the spruce species as references. We isolated the change of the target property factor (increased by 10%) while keeping the other properties the same. The natural frequency and the maximum displacement of the soundboard were simulated, and the percentage changes of each property factor were calculated. Table 2 shows the average percentage change values in

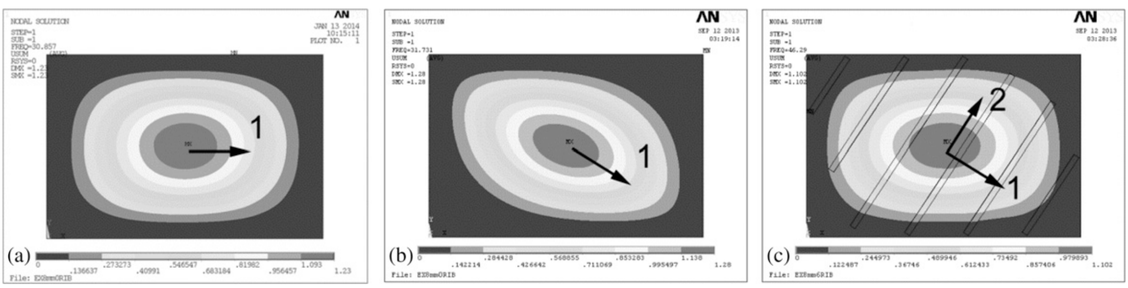

Figure 3. Effect of wood grain direction on mode shape of a resonance board without ribs (a-b) and with six ribs (c). Direction 1 is the longitudinal grain direction of the resonance board. Direction 2 is the longitudinal grain direction of ribs.

Table 2. Percentage changes in natural frequency and maximum displacement of the soundboard affected by the 10 property factors of wood (the values in the table were calculated when the target property factor increased by 10%, whereas other properties remained the same; the values are the average of the changes for all 15 modes; the negative value means as the property increases, the frequency or maximum displacement decrease; rib intensity: 6; resonance board thickness: 8 mm; rib cross-section dimension: 25 mm wide and 15 mm high; the species of the resonance board and rib are the same).

Properties	ρ	E_L	E_R	E_T	G_{LT}	G_{LR}	G_{TR}	μ_{RT}	μ_{LR}	μ_{LT}
Natural frequency (%)	−4.66	1.95	0.61	0.48	0.41	0.21	1.11	0.13	0.07	0.01
Maximum displacement (%)	−4.65	−1.95	0.37	0.51	−0.15	0.48	0.51	−0.27	0.12	0.02

natural frequency and maximum displacement of all 15 modes for each property factor. As shown in Table 2, wood density has the greatest effect (>4.6%) on both the natural frequency and the maximum displacement among the 10 property factors investigated. Increasing the wood density of the resonance board and ribs can decrease the natural frequency and the maximum displacement of the soundboard. Among the property factors of Young's modulus (E), shear modulus (G), and Poisson's ratio (μ), Young's modulus tends to have the greatest effect on sound performance (0.37-1.95%), followed by the shear modulus (0.15-1.11%), and then the Poisson's ratio (0.02-0.27%). For the Young's modulus (E) of a soundboard, the E in the longitudinal direction had a relatively higher impact (1.95%) on the vibration modes compared with the other two directions (0.37-0.61%). Increasing E_L increased the natural frequency and decreased the maximum displacement of the soundboard. For the shear modulus (G), the modulus from the tangential direction to the radial direction (G_{TR}) had a greater effect (1.11% for the frequency and 0.51% for the displacement) on the vibration modes compared with that from G_{LR} and G_{LT} (0.15-0.48%). For the Poisson's ratios, the ratio from longitudinal to tangential (μ_{LT}) presented the lowest effect on the sound performance (only 0.01-0.02%).

Vibration Modes

Figure 4 shows the vibration behaviors of soundboard for the first 10 modes. From modes 1 to 10 and rows a, b, and c in Fig 4, the mode

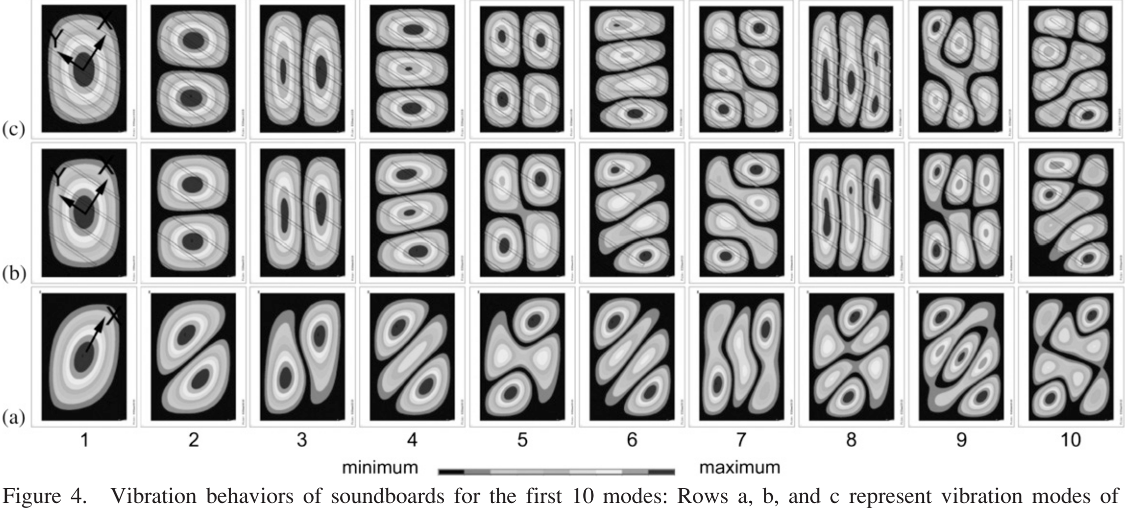

Figure 4. Vibration behaviors of soundboards for the first 10 modes: Rows a, b, and c represent vibration modes of resonance boards, resonance boards with 6 ribs, and resonance boards with 11 ribs, respectively. (Species of resonance boards and ribs, spruce; resonance board thickness, 8 mm; rib cross-section dimension, 30 mm wide and 10 mm high.) In mode 1, the arrow in the X direction indicates the longitudinal grain direction of the resonance boards and the arrow in the Y direction indicates the longitudinal grain direction of the ribs. The other modes have the same grain direction indication as that shown in mode 1.

Table 3. Frequencies of soundboards for the first 10 modes.

Number of modes	Resonance board (Hz)	Resonance board with six ribs (Hz)
1	31.7	46.3
2	52.4	75.6
3	73.0	109.9
4	77.2	119.7
5	102.9	138.6
6	105.9	172.5
7	129.5	189.0
8	139.3	204.8
9	139.8	229.3
10	163.0	232.2

shapes of resonance boards were distributed nearly symmetrically along the grain direction of wood. The vibration distribution of resonance boards with ribs was nearly symmetric in the range of modes 1 to 7. However, as the number of modes increased, the vibration distribution tended to be irregular, especially at higher natural frequencies. The addition of ribs increased the complexity of the soundboard vibration. Adding stiffeners to the soundboard (ribs) improved the sound radiation efficiency, because a stiffer soundboard had less tendency to be subdivided into small vibrating areas as shown in a10, b10, and c10 of Fig 4.

Table 3 shows that at a given number of modes, the natural frequency of resonance boards with ribs was higher than that with no ribs. Four natural frequencies were below 100 Hz for resonance boards with no ribs, whereas only two were below 100 Hz for that with six ribs, indicating that the addition of ribs decreased the resonance capability of soundboards at the lower frequencies.

Rib Intensity

Figure 5a shows that little difference was found in natural frequency between 6 ribs and 11 ribs used in the soundboard. The average natural frequency before mode 5 was improved by about 4 Hz when the number of ribs was increased from 6 to 11. In general, a larger number of ribs may yield higher natural frequencies for the soundboard. Figure 5b shows that a larger number of

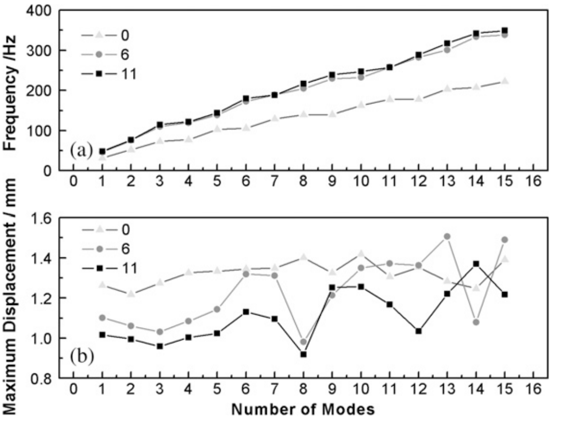

Figure 5. Effect of rib intensity on the vibration modes: (a) natural frequencies; (b) maximum displacement (rib intensity, 0, 6, and 11; species of resonance board and rib, spruce; resonance board thickness, 8 mm; rib cross-section dimension, 30 mm wide and 10 mm high).

ribs had a negative impact on the maximum displacement of the soundboard, especially when the mode was below 10. Similar results were found by Ye (2011). Adding ribs enhances the stiffness of resonance boards. Therefore, vibration capability and maximum displacement were decreased.

Rib Cross-Section Dimension

Figure 6a shows the vibration modes of soundboards at three rib widths, 20, 25, and 30 mm. The numerical results indicated that as rib width increased, the natural frequency of the soundboard increased but only 1.5% compared with 4.0% from the effect of rib height (the average percentage change when width or height was increased by 5 mm). The average natural frequency before mode 5 could be improved about 2 Hz when the rib width increased from 20 to 25 mm. Figure 6b shows that the maximum displacement increased as rib width increased, indicating that a wider cross-section of the rib made the resonance board more difficult to vibrate at a given number of modes.

Figure 7 shows the effect of rib height on the vibration modes of soundboards with six ribs. It was seen from Fig 7a that at the given condition,

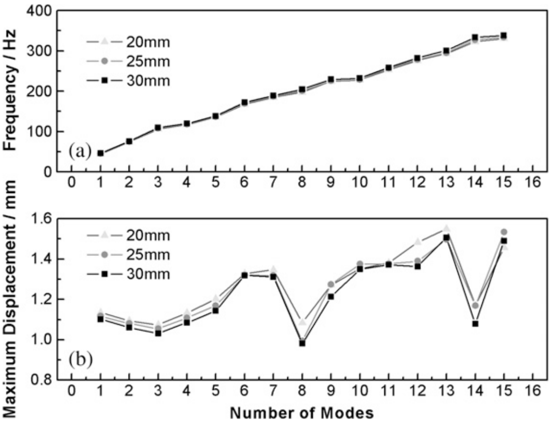

Figure 6. Effect of rib width on the vibration modes of soundboard with 6 ribs: (a) natural frequencies; (b) maximum displacement (rib width, 20, 25, and 30 mm; rib height, 10 mm; species of resonance board and rib, spruce; resonance board thickness, 8 mm).

the natural frequency increased as the rib height and number of modes increased. The average natural frequency before mode 5 was improved by about 8 Hz when rib height increased from 10 to 15 mm. As shown in Fig 7b, as rib height increased from 10 to 15 mm, maximum displacement of the soundboard decreased when the mode was less than 8. No specific trend was found for rib height vs maximum displacement when the mode was greater than 8. However, when rib height was increased to 20 mm, maximum displacement of the soundboard did not show a specific trend. This could be because as rib height increased, the anisotropic nature of the rib might have a more complex effect on soundboard vibration (Giordano 1997).

Among rib intensity, rib height, and rib width, rib height presented the most significant effect on natural frequency. The same conclusion was also presented in Wogram (2000).

Rib Species

Figure 8 shows the effect of rib species on vibration modes of soundboards with six ribs. As shown in Fig 8a, at a given number of modes, the natural frequency of simulated resonance boards was the greatest for beech ribs compared with the other two species, pine and spruce. Table 1 shows that beech presents the greatest density and mechanical properties (except E_L) among the three species used in the study. A higher natural frequency of the soundboard was obtained when beech ribs were used compared with the other two species. As shown in Fig 8b, spruce ribs presented the greatest maximum displacement for the simulated resonance boards, whereas beech showed

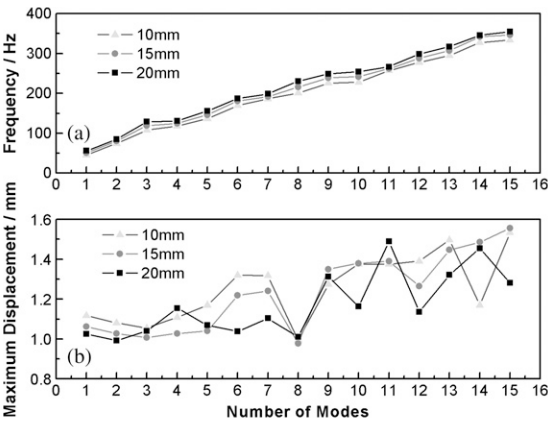

Figure 7. Effect of rib height on the vibration modes of soundboard with 6 ribs: (a) natural frequencies; (b) maximum displacement (rib width, 25 mm; rib height, 10, 15, and 20 mm; species of resonance board and rib, spruce; resonance board thickness, 8 mm).

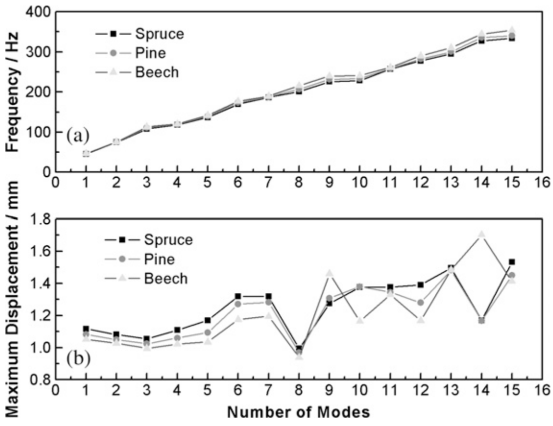

Figure 8. Effect of rib species on the vibration modes of soundboard with 6 ribs: (a) natural frequencies; (b) maximum displacement (rib width, 25 mm; rib height, 10 mm; species of resonance board, spruce; rib species, spruce, pine, and beech, respectively; resonance board thickness, 8 mm).

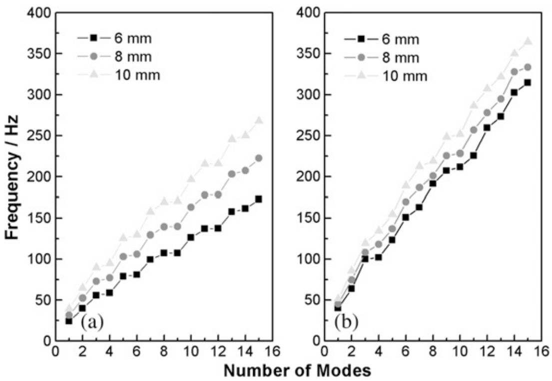

Figure 9. Effect of resonance board thickness on the natural frequency of soundboards: (a) soundboard with no rib; (b) soundboard with 6 ribs (resonance board thickness, 6, 8, and 10 mm; rib cross-section size, 25 mm wide and 10 mm high; species of resonance board and rib, spruce).

the lowest, especially when the mode was less than 8.

Resonance Board Thickness

Figure 9 depicts the effect of resonance board thickness on natural frequency of the soundboard. Both Fig 9a and 9b show that the natural frequency of the soundboards increased as the resonance board thickness and number of modes increased. However, increasing the thickness of the resonance board decreased the amount of the natural frequency that was below 100 Hz. As shown in Fig 9b, greater thickness of the resonance board led to higher natural frequencies. The increase in natural frequency as the mode for the soundboard with six ribs was much quicker than that with no ribs. Increasing the thickness of resonance boards and using ribs enhance soundboard stiffness. Therefore, the frequency increase rate was accelerated. These results agree with previous experimental measurements (Ye 2011).

CONCLUSIONS

1. CAD and FEA tools were successfully combined to analyze modal characteristics of piano soundboards. These methods provide a more visual and convenient approach compared with other modeling and simulation methods. The results from our visual and numerical simulation were in general agreement with the experimental results from prior work in the literature.
2. The grain direction of wood affected the mode shape of soundboards. Adding ribs with the grain direction perpendicular to that of the resonance board decreased the effect of wood grain directions on the mode shape of the soundboard. Among the 10 property factors investigated, density presented the greatest impact to the vibration mode of the soundboard, followed by Young's modulus, shear modulus, and Poisson's ratio. E_L was the most important factor among the three Young's moduli (E_L, E_R, and E_T), whereas G_{TR} was the most important among the three shear moduli (G_{TR}, G_{LR}, G_{LT}). The Poisson's ratio from longitudinal to tangential (μ_{LT}) presented the least effect to the vibration mode of the soundboard.
3. Both increasing the thickness of the resonance board and using ribs had a positive impact on the natural frequency of the soundboard. However, increasing the thickness of the resonance board and using ribs decreased the amount of natural frequencies that was below 100 Hz.
4. The natural frequency of the soundboard increased as rib intensity, density, and cross-section dimension of ribs increased. The numerical results showed that rib height had a greater effect on the natural frequency than that of intensity, density, and width of ribs.
5. In general, the increase of rib intensity from 0 to 11, density of wood (among spruce, pine, and beech), and width of ribs from 20 to 30 mm decreased maximum displacement.

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