

# STATISTICAL CONSIDERATIONS FOR REAL-TIME SIZE CONTROL SYSTEMS IN WOOD PRODUCTS MANUFACTURING

*Thomas C. Maness*<sup>†</sup>

Associate Professor

*Christina Staudhammer*

Graduate Research Assistant

and

*Robert A. Kozak*

Assistant Professor

Faculty of Forestry

University of British Columbia

2424 Main Mall

Vancouver, BC

V6T 1Z4 Canada

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## ABSTRACT

Currently, sawmill machinery companies are developing real-time size lumber size control systems using non-contact laser measuring systems. These systems rely on the application of industrial statistics to large quantities of lumber thickness and width data. Because of the sampling intensity and frequent decision making in real-time systems, there is an increased chance of committing Type I or Type II errors when drawing conclusions if statistical methods are incorrectly applied. There is confusion in the industry concerning the appropriate statistical model to use for lumber size control. This survey of the current literature discusses three distinct methods for calculating and partitioning sawing variation, and thereby calculating control limits for control charts. This paper reviews the statistical foundation and current understanding of industrial statistics for implementing real-time SPC systems and makes recommendations for improvement.

*Keywords:* Lumber size control, statistical process control, target sizes, control charts, real-time.

## INTRODUCTION

Lumber production in a modern sawmill occurs at a high rate of speed. In practice, sawing variation is that quantity which measures the variation in width and thickness in sawn boards as a result of inaccurate sawing. This variation is usually caused by movement in the saws or the log hold down mechanisms during the cut (due to a variety of factors), or by movement in the saw or log positioning just prior to the cut. To maximize the value recovery from every log sawn, it is important that the cut be straight with as little side to side

variation as possible. Many studies have shown that profits can be increased substantially by reducing the amount of the sawing variation (Maness and Lin 1995; Wang 1983; Lister 1997).

Measurement plays an important role in helping sawmills improve sawing practices and thereby reduce sawing variation. The basic size control system used by many mills today involves taking 1 or 2 samples per shift from each sawing machine. In each sample, a group of boards is pulled from the production line (usually 4 or 5), and a group of thickness and/or width measurements are taken on each board (usually 4–10 taken from fixed loca-

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<sup>†</sup> Member of SWST.

tions). This results in 16–50 board measurements per sample from which the sample average and variance are calculated, as well as variance components that are used for the purpose of problem solving. In some cases, this information is plotted on statistical control charts for process averages and dispersion.

More recently, sawmill technology companies have focused on the development of real-time size control systems using non-contact laser measuring systems. Although other methods like time series modeling (Cook 1992; Young and Winistorfer 2001) have been proposed, the statistical procedures used in these systems are based largely on procedures developed in the 1970s and 1980s. Because of the sampling intensity and frequent statistical decision making in real-time systems, there is an increased chance of committing Type I or Type II<sup>1</sup> errors when drawing conclusions if the statistical methods are incorrectly applied.

This paper is the introductory article of a series aimed at improving the statistical foundation for implementing real-time statistical process control (SPC) systems. The paper reviews the development of the current understanding of industrial statistics for the purpose of lumber size control. In addition, it points out some assumptions made along the way that may cause problems in real-time systems and makes appropriate suggestions for improvement.

#### A REVIEW OF CURRENT METHODS FOR LUMBER SIZE CONTROL

##### *A statistical approach for determining lumber target sizes*

*Basic methods.*—Warren (1973) first suggested deriving a target board size by randomly selecting boards as they left a machine center and taking a prescribed number of thickness measurements at random locations along

<sup>1</sup> In the context of statistical process control, a Type I error occurs when the manufacturer concludes (falsely) that a correctly performing process is out of control. A Type II error occurs when the manufacturer fails to detect that a poorly performing process is out of control.

the length of the board. Although not explicitly stated, this technique relied on an underlying analysis of variance (ANOVA) model. Warren's design was a completely randomized, one-factor experiment with  $m$  treatments (the number of boards sampled) and  $n$  measurements per treatment (the number of places that each board is measured along its length). The observations from a one-factor completely randomized design (CRD) are described by the model:

$$y_{ij} = \mu_i + \epsilon_{ij} \quad (1)$$

where:  $y_{ij}$  is the  $j$ th observation from the  $i$ th board (treatment),  $\mu_i$  is the  $i$ th board (treatment) effect, and  $\epsilon_{ij}$  is the random error component.

The overall mean of all treatments is  $\mu$ . The model errors are assumed to be independent and normally distributed with a mean of zero and a variance of  $\sigma^2$ , i.e.,  $\epsilon_{ij} \sim \text{NID}(0, \sigma^2)$ . The treatment means are also independent and normally distributed with a mean of  $\mu$  and variance of  $\sigma_\mu^2$ , i.e.,  $\mu_i \sim \text{NID}(\mu, \sigma_\mu^2)$ . The treatments and errors,  $\mu_i$  and  $\epsilon_{ij}$ , are independent, and therefore, the board thicknesses,  $y_{ij}$ , are normally distributed.

The total sums of squares for the model ( $SS_T$ ) are partitioned into sums of squares due to treatment effects ( $SS_{TR}$ ) and sums of squares due to experimental error ( $SS_E$ ). This corresponds to board-to-board differences and within board differences, respectively. The elements of the analysis of variance (ANOVA) table for a one-factor CRD are shown below in Table 1 (Hicks and Turner 1999).

Warren computed the within- and between-board sample variances,  $S_w^2$  and  $S_b^2$ , as:

$$S_w^2 = MS_E \quad (2)$$

$$S_b^2 = \frac{MS_{TR} - MS_E}{n} \quad (3)$$

$S_w^2$  and  $S_b^2$  are the unbiased estimates of the sample variances within and between boards,

TABLE 1. Completely randomized one-factor ANOVA table.

Source of variation	Sums of squares	Degrees of freedom	Mean squares	F
Between boards	$SS_{TR} = n \sum_i (\bar{y}_i - \bar{y}_{..})^2$	$m - 1$	$MS_{TR} = SS_{TR}/(m - 1)$	$F = MS_{TR}/MS_E$
Error (within boards)	$SS_E = \sum_i (y_{ij} - \bar{y}_i)^2$	$m(n - 1)$	$MS_E = SS_E/[m(n - 1)]$	
Total	$SS_T = \sum_i \sum_j (y_{ij} - \bar{y})^2$	$mn - 1$		

where:  $\bar{y}_i$  is the mean thickness of the  $i^{\text{th}}$  board and  $\bar{y}_{..}$  is the overall mean thickness of all boards.

$\hat{\sigma}^2$  and  $\hat{\sigma}_{\mu}^2$ , respectively.<sup>2</sup> The total variation  $S_t^2$  is equal to the sum of the within- and between-board variances:

$$S_t^2 = S_b^2 + S_w^2 = \frac{MS_{TR} - MS_E}{n} + MS_E$$

$$= \frac{MS_{TR} + (n - 1)MS_E}{n} \quad (4)$$

Warren gave the target board thickness,  $T$ , based on  $S_w$  and  $S_b$ , by means of a lookup table. The underlying values were calculated as:

$$T = y_F + Z_{\alpha} S_t = y_F + Z_{\alpha} \sqrt{S_b^2 + S_w^2}$$

$$= y_F + Z_{\alpha} \sqrt{\frac{MS_{TR} + (n - 1)MS_E}{n}} \quad (5)$$

where:  $y_F$  is the finished lumber thickness,  $Z_{\alpha}$  is from the standard normal table, and other terms are defined above.

In subsequent publications, Brown (1982, 1986) calculates  $S_w$ ,  $S_b$  and the target board thickness using the same methods as Warren (1973). Neither Warren nor Brown included a system for monitoring lumber size or methods for constructing confidence limits that could be used in a statistical process control system.

This approach formalized a structure for using industrial statistics to determine the appropriate lumber target size and correctly identified the importance of partitioning the within- and between-board variance to calculate the total variation. A weakness of the method for

calculating target sizes is that it assumed that the variation in lumber sizes was due solely to sawing variation. In fact, boards in a sawmill are incorrectly sized for a number of reasons in addition to sawing variation (e.g., variation in the drying and planing process). These factors should also be included in target size calculations. However, the Warren method provided a solid statistical foundation for further development in this area.

Smithies (1991) outlined a different method for computing the target board thickness. In this method, the between- and within-board variances were computed as Eqs. (6) and (7) below:

$$S_{b[S]}^2 = \frac{SS_{TR}}{n(m - 1)} = \frac{MS_{TR}}{n} \quad (6)$$

$$S_{w[S]}^2 = \frac{SS_E}{m(n - 1)} = MS_E \quad (7)$$

$$S_{t[S]}^2 = \frac{(m - 1)nS_{b[S]}^2 + m(n - 1)S_{w[S]}^2}{nm - 1}$$

$$= \frac{SS_{TR} + SS_E}{nm - 1} = \frac{SS_t}{nm - 1} \quad (8)$$

The within-board variation (Eq. [6]) matches the result of Warren (1973). However, the between-board and total variation are calculated differently. In Eq. (6), the between-board variation is simply the variance of the average board thicknesses. However, the variance of the average board thicknesses is partially composed of the within-board variation, since by the central limit theorem the variance of a group of averages is  $\sigma^2/n$  even if there is no "between-board" effect. Total variation (Eq. [8]) is the variance of all observations, taken without regard to the different boards from which the observations were taken. This un-

<sup>2</sup> While the sample variance,  $S^2$ , is an unbiased estimator of the population variance,  $\sigma^2$ , the sample standard deviation,  $S$ , is not an unbiased estimator of the population standard deviation. Instead,  $S$  estimates  $c_4\sigma$ , where  $c_4$  is a constant that depends on the sample size. See Montgomery (2001) for a discussion.

grouped or “overall variance,” however, gives a biased estimate of the total variance when  $\hat{\sigma}_\mu^2 \neq 0$  (Hicks and Turner 1999). When  $\hat{\sigma}_\mu^2 > 0$ , this estimate will always be less than the total variance, as computed with  $S_b$  and  $S_w$ . Unless there is no board-to-board variation, the variance computed with this method will underestimate the total variation present. This can cause problems when this estimated variance is then used either for determining target sizes or as the basis of a statistical process control system.

Substituting the mean squares into Eq. (8) yields:

$$S_{t|S}^2 = \frac{(m - 1)MS_{TR} + m(n - 1)MS_E}{nm - 1} \quad (9)$$

Comparison of (9) with (4) demonstrates the differences in the two approaches with regards to calculating total sawing variation.

*Considering distinct sources of within-board variation.*—Wang (1984) also conducted a study to develop equations for calculating target sizes. In addition to separating the total sawing variation,  $S_p$ , into  $S_w$  and  $S_b$ , Wang partitioned  $S_w$  into two independent components:  $S_{wb}$  within variation along the board length, and  $S_{ww}$  within variation across the width of the board. Thus,

$$S_w^2 = S_{wl}^2 + S_{ww}^2 \quad (10)$$

Furthermore, the author attempted to consider the effect of sawing variation on both surfaces of the board (top and bottom). He then assumed that the total within-board variation,  $S_w^2$ , was comprised of the within-board variation on each surface.

$$S_{ww}^2 = S_{ww_{top}}^2 + S_{ww_{bot}}^2 \quad (11)$$

Assuming that the top and bottom variation are equal yields:

$$S_{ww}^2 = 2S_{ww_{top}}^2 \quad (12)$$

The resulting target size equation given was:

$$T = y_F + 3(S_{ww}/\sqrt{2} + \sqrt{S_w^2 - S_{ww}^2/2} + S_b) \quad (13)$$

The mathematical derivation of (13) and this result are questionable given the partitioning of variances that underlie the ANOVA model (Eq. [4]). However, the approach is of interest in that it involves several innovations. First, it accounted for two distinct sources of within-board variation. Second, it considered the impact of sawing variation on both the top and bottom faces of the board. The result would have been more successful mathematically had it started with an appropriate ANOVA model incorporating the innovations, and then derived the correct partitioning of the variances.

*Summary of target size methods.*—The work reviewed above has shown that the target size is a function of the sawing variation. Warren (1973) developed the basic statistical approach of Eq. (5) and used the ANOVA model to estimate the total sawing variation and correctly partition it into components. As will be seen in the next section, this foundation was a significant contribution to the understanding and development of lumber size control systems.

*Extension of the target size approach to statistical process control (SPC)*

*Background for SPC.*—The target size determination methods outlined above identified the concepts of between- and within-board variation and set the stage for applying those concepts to an SPC system. W. A. Shewhart (1931) developed the basics of SPC in which a control chart was used to signal the presence of assignable causes in the underlying process.

The basic SPC system can be described as follows. Output from a manufacturing process is sampled by selected subgroups of size  $m$  items at regular intervals. The quality characteristic (e.g., dimension) is measured on each of the  $m$  items in each subgroup, and the averages and sample standard deviations are calculated in the usual manner. A comparison over time of the sample mean and/or standard deviation against a statistical confidence interval based on a known standard determines

whether the process is performing correctly. In the case of the average, the confidence interval is based on the Student's *t*-distribution (or a standard normal distribution if the sample size exceeds 30). In the case of the standard deviation, the confidence interval is based on the Chi-square distribution. Various constants have been developed and tabulated to simplify the method for use in practical applications and to correct for sampling bias when using the sample standard deviation instead of the sample variance in the comparisons.

To simplify use in a manufacturing environment, a graphical control chart is created. While there are many types of control charts, two in particular are used most frequently: the X-Bar chart for process averages and the S (or R) chart for process dispersion. While both must be used in combination to be effective, practitioners often use the X-Bar chart alone. A scatter plot is created in which the X-axis represents the sample number and the Y-axis represents the variable of interest. The variable of interest in the case of the X-Bar chart would be average board thickness, while in the case of the S Chart, it would be the thickness standard deviation. In both charts, the underlying process standard deviation must be known and the process should be stable (or under statistical control).

In each case, an upper and lower control limit is calculated, which is typically three standard deviations (corrected for bias) above and below the mean. For each sample, the variable of interest is plotted on the chart and, if it is outside the control limits, the presence of assignable causes is indicated and remedial action should be taken. These three sigma limits yield a probability of false rejection ( $\alpha$ ) of 0.135% in each tail. Thus, the control chart should have a false alarm rate of approximately 1 sample out of 740 on each control limit. This is generally considered acceptable in production.

From this description, it can be seen that the correct estimation of the mean and standard deviation is crucial. In most cases, the practitioner assumes that the sampled items come

from a normal distribution. However, this becomes complicated in the case of lumber size control. The thickness of a piece of lumber varies all along its length. Thus, it is impossible to measure *m* items—instead the thickness of each item must be measured at *n* locations. This presents interesting possibilities. For example, the process variation is partitioned into *components* that help lead to problem resolution. However, herein lies the source of the difficulty in applying the standard SPC techniques. The practitioner must be very careful to estimate the partitioned variances correctly or the method can lead to erroneous results.

The practitioner could ignore the presence of the two distinct sources of variation (within- and between-board) and calculate only the sample average and total sawing variation, as is suggested by most standard quality control textbooks. The total sawing variation would then be the standard deviation of all the measurements taken in the sample (as was done by Smithies in [8]). This is, in fact, done in practice at many sawmills. However, as shown above, that would lead to a biased result if between-board variation is non-zero, which is likely. This is an important result in itself, and should be noted by system developers.

*Development of the lumber size SPC system.*—Whitehead (1978) extended the statistical methods developed by Warren to a statistical process control procedure. In the proposed procedure, *a* subgroups were randomly selected at time intervals. Each subgroup consisted of *m* boards (systematically selected), and each board was measured at *n* locations. In ANOVA terms, it was a nested model similar the CRD model of Equation (1).

Whitehead extended the methodology developed by Ryan (1989) for making control charts of measurements in subgroupings. In this method, the variation within subgroups is isolated and the variation between subgroups is not considered as subsequent monitoring is conducted with one subgroup at a time.

The analysis was conducted using the average range, which was converted via approx-

imation into the standard deviation—a practice widely accepted at the time. The average range per subgroup,  $\bar{R}_{w_i}$ , and overall average range,  $\bar{\bar{R}}_w$ , were calculated as:

$$\bar{R}_{w_i} = \frac{1}{b} \sum_j R_{w_{ij}} \quad \text{and} \quad \bar{\bar{R}}_w = \frac{1}{a} \sum_i \bar{R}_{w_i} \quad (14)$$

where: a is the number of boards in the sample subgroup, b is the number of subgroups in the overall sample, and  $R_{w_{ij}}$  is the largest width measurement minus the smallest width measurement for each individual board in the subgroup.

The within-board standard deviation was calculated as:

$$S_w = \bar{\bar{R}}_w/d_2 \quad (15)$$

where:  $d_2$  is the statistical constant developed to convert ranges to standard deviation based on the distribution of the relative range,  $(R/\sigma)$ . (See Montgomery [2001])

Sample ranges are converted to sample variances throughout this paper for simplicity in understanding and for ease in comparing the different statistical methods discussed. It can be shown that  $S_w$  in Eq. (15) approximately corresponds to the error variance,  $\hat{\sigma} = MS_E$ , within the ANOVA framework.

The average thickness of the  $j$ th board from the  $i$ th subgroup is  $\bar{y}_{ij}$ . The average range by subgroup,  $\bar{R}_{b_i}$ , and the overall average,  $\bar{\bar{R}}_b$ , were calculated as:

$$\begin{aligned} \bar{R}_{b_i} &= \max_j(\bar{y}_{ij}) - \min_j(\bar{y}_{ij}) \quad \text{and} \\ \bar{\bar{R}}_b &= \frac{1}{a} \sum_i \bar{R}_{b_i} \end{aligned} \quad (16)$$

The between-board standard deviation was then defined as:

$$S_b = \sqrt{(\bar{\bar{R}}_b/d_2)^2 - S_w^2/n} \quad (17)$$

Using ANOVA terms and algebra, it can be shown that Eq. (17) reduces to:

$$S_b = \sqrt{\frac{MS_{B(A)} - MS_E}{n}} \quad (18)$$

Using Eq. (18),  $S_b$  is the component of vari-

ance associated with boards within a subgroup,  $\hat{\sigma}_{\beta_i}$ .

As in Warren (1973), Whitehead computed the total variation as the sum of the within- and between-board variances (Eq. [4]) and the target board thickness as in Eq. (5). Whitehead used these estimates of process variation to develop a SPC system for lumber size using randomly selected groups of boards and control charts for the average board size, the within-board size variation, and the between-board size variation. The 3-sigma control limits for the Target Size, T, are:

$$T - A_2\bar{\bar{R}}_b \leq \mu \leq T + A_2\bar{\bar{R}}_b \quad (19)$$

This corresponds to the following equation using standard deviation instead of range<sup>3</sup>:

$$T - 3S_b/\sqrt{m} \leq \mu \leq T + 3S_b/\sqrt{m} \quad (20)$$

The upper control limits for the range within-boards and between-boards were calculated as  $D_4\bar{\bar{R}}_w$  and  $D_4\bar{\bar{R}}_b$ , respectively. The lower control limit was zero because, based on the sample size, it would otherwise be negative. These control limits can be expressed in terms of standard deviation by using the estimates of  $S_w$  (Eq. [15]) and  $S_b$  (Eq. [18]) as  $B_4S_w$  and  $B_4S_b$ , respectively.

The work by Whitehead admirably extended the basic target size concepts developed by Warren into an SPC system. The basic concepts were correct; however, there is an issue of concern with respect to the estimation of statistical parameters. The standard error term in Eq. (20) does not account for all of the variation in the board thickness as it should correspond to the standard error of the mean of a subgroup of boards,  $\hat{\sigma}_{y_i}$ .

*SPC considering measurement location.*—Brown (1979) used restricted randomization in

<sup>3</sup> The standard error term in (20) should use the total standard error of the mean of a subgroup, which includes the within and between components of variation. Whitehead partially corrected this mistake in a subsequent paper in which  $S_b$  was replaced by  $S$ , in Eq. (20). All other aspects of the SPC procedures were unchanged from the earlier paper. See Neter et al. (1996) for a complete discussion of partitioning the variances.

his experimental design and calculated target size and control limits for lumber thicknesses using the average range instead of the standard deviation. However, Brown calculated the variation around the measurement locations within each group of boards, indicating that the location of the thickness measurements on the board was important. In terms of experimental design, measurement location was used as an experimental block. Thus, it seems reasonable to assume the appropriate experimental design would be a replicated randomized complete block (RRCB).

The total variance,  $S_t^2$ , was computed as the sum of the squared within- and between-standard deviations, as in Eq. (4), and the target thickness was computed as in Eq. (5). However, under an RRCB, this computation ignored components of variance corresponding to the variation in average thickness between boards, and the possible interaction of boards and measurement locations. Control charts for average board thickness and the upper control limits for the range between-boards and within boards are calculated as in Whitehead (1978). Again, the appropriate method would have been to start with the correct ANOVA model (RRCB) and derive the appropriate partitioned variances, rather than applying the results of the one-factor ANOVA model of Warren (1973) to this more complex design.

The method itself, however, raises an important point. The method treated the different measurement locations along the boards as experimental blocks, whereas in the other methods, any consistent differences due to the location of the measurement were not separated and were therefore included in the experimental error (the within-board variance). If the experimenter believed that there was a consistent difference between thickness measurements at locations along the board, blocking would have been appropriate and would have created more information for problem solving likely causes. This may have been an attempt to identify consistent sawing shapes, such as taper and wedge, which are known by practitioners to be important in diagnosing sawing

problems. These important concepts have not been followed up in the literature, but present interesting opportunities for future research.

#### DISCUSSION

The SPC procedures used in practice take sequential samples consisting of several boards and several thickness (or width) measurements per board in the manner proposed by Whitehead (1978). Typically, these data are collected manually with a pair of digital calipers. Perhaps 100 measurements per sample and usually no more than 2 samples per shift are taken.

Control charts correctly designed with three-sigma limits have a probability of a Type I error (producer's risk) of 0.27%. This results in a false-alarm Average Run Length (ARL) of 370, meaning that a false alarm would be indicated once every 185 shifts, if set correctly.<sup>4</sup> This is nearly undetectable. If errors are made in estimating the variance, the false-alarm ARL could shift meaningfully. However, with manual sampling, the false-alarm rate would still be nearly undetectable even with a serious error. For example, suppose the producer's risk is changed by a factor of 10 (from 0.27% to 2.7%) due to errors in estimating the variance. The ARL would change to 37. While this is probably unacceptable from the producer's point of view, a false alarm would still be indicated only once every 18 shifts. This is still undetectable because QC staff would probably assume the presence of assignable causes. Consequently, errors present in the SPC system have gone largely unnoticed.

That said, real-time systems monitor continuously. These systems are capable of hundreds of samples per shift. For example, if 100 samples are taken per 8-hour shift, a correctly set up 3-sigma control charts would indicate a false alarm every 3.7 shifts. If the same error in the paragraph above were made, a false alarm would be indicated every 3 hours. This is clearly unacceptable. In order to develop a workable real-time SPC system in an indus-

<sup>4</sup> See Montgomery (2001) for a discussion of Producer's Risk Average Run Length for control charts.

TABLE 2. Random variables identified and the possible control charts for each.

Random variable	Control chart
$\bar{Y}$ —sample average	X-Bar chart
$S_t^2$ —sample standard error	S (or R) chart for total sawing variation
$S_w^2$ —variation within board	S (or R) chart for within sawing variation
$S_b^2$ —between board effect	S (or R) chart for between sawing variation

trial setting, we need a more complete understanding of the statistical distributions underlying control charts for lumber size control.

This review of the literature has identified four random variables with respect to lumber size control that could be monitored with a statistical process system. These random variables, and the respective control charts that could be applied to them, are shown in Table 2 above.

To correctly apply these control charts to lumber size data, the practitioner is required to know the distribution of each of the four random variables. Each control chart would then represent a set of sequential hypothesis tests to determine if the estimated statistic

from each sample is significantly different from the expected population parameter. If so, it can be concluded that assignable causes are present that should be identified and corrected.

This study of the available literature has also yielded several distinct methods for partitioning variation, and thereby estimating each of these random variables. Ignoring for the moment complicating factors such as measurement location, multiple board surfaces and mathematical error, the methods found in the literature can be interpreted in two different ways. These are summarized in Table 3.

Based on the personal experience of the authors, the control charts currently used in practice are based on one of these two methods.

TABLE 3. Description of two different methods for calculating SPC chart control limits based on a study of the literature<sup>1</sup>.

Method one		
Type of control chart	Variance estimator	Control limits
X bar chart for sample average	Standard error of the mean derived from $\sigma_t^2$ as in Equation (4) divided by $\sqrt{nm}$	UCL: $\hat{\mu} + 3\hat{\sigma}_t/\sqrt{nm}$ LCL: $\hat{\mu} - 3\hat{\sigma}_t/\sqrt{nm}$
S chart for total board variation	$\sigma_t^2$ as in Equation (4)	UCL: $B_4\hat{\sigma}_t$ LCL: $B_3\hat{\sigma}_t$
S chart for within board variation	$\sigma_w^2$ as in Equation (2)	UCL: $B_4\hat{\sigma}_w$ LCL: $B_3\hat{\sigma}_w$
S chart for between board variation	$\sigma_b^2$ as in Equation (3)	UCL: $B_4\hat{\sigma}_b$ LCL: $B_3\hat{\sigma}_b$
Method two		
Type of control chart	Variance estimator	Control limits
X bar chart for sample average	Standard error of the mean derived from $\sigma_t^2$ as in Eq. [8] divided by $\sqrt{nm}$	UCL: $\hat{\mu} + 3\hat{\sigma}_t^2/\sqrt{nm}$ LCL: $\hat{\mu} - 3\hat{\sigma}_t^2/\sqrt{nm}$
S chart for total board variation	$\sigma_t^2$ as in Equation (8)	UCL: $B_4\hat{\sigma}_t$ LCL: $B_3\hat{\sigma}_t$
S chart for within board variation	$\sigma_w^2$ as in Equation (7)	UCL: $B_4\hat{\sigma}_w$ LCL: $B_3\hat{\sigma}_w$
S chart for between board variation	$\sigma_b^2$ as in Equation (6)	UCL: $B_4\hat{\sigma}_b$ LCL: $B_3\hat{\sigma}_b$

<sup>1</sup> The statistical constants  $B_3$  and  $B_4$ , that are used in this table are commonly used in control chart applications and can be found in Montgomery (2001). They are based on confidence limits derived from the chi-square distribution with the appropriate corrections for bias.



At the present time, it is unclear which of these methods, if any, provides the correct procedures for setting up a statistical process control system for lumber size control.

#### CONCLUSIONS

This introductory review has shown that the standard textbook method for applying X-Bar and S charts to lumber size control is mathematically incorrect in the presence of assignable between board variation. The review has also shown that there is confusion about the correct method for incorporating between board variation into the statistical model for use in lumber size control work. Further research will be presented in subsequent articles in the following areas:

- (1) Developing a method to test the actual false alarm rate of control charts and test control charts based on Methods One and Two.
- (2) Developing new statistical models and a corresponding SPC system for lumber size control.
- (3) Empirically studying all components of size variation in dried lumber ready for planing, including drying variation.

Work in these areas will provide only the foundation for further work in the development of real-time lumber size control systems for sawing. The ultimate goal may be to provide a closed-loop expert system that continuously monitors production and automatically makes changes to feed speeds, tool rotation speeds, and alignments of the infeed mechanisms. The development of real-time quality control systems provides an exciting research area that will lead to a new area of investment in forest products manufacturing equipment.

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