LOAD DURATION AND PROBABILITY BASED DESIGN OF WOOD STRUCTURAL MEMBERS

Bruce R. Ellingwood

Research Professor
Department of Civil Engineering, The Johns Hopkins University
Baltimore, MD 21218-2699

Erik M. Hendrickson

Research Structural Engineer
Center for Building Technology, National Bureau of Standards
Gaithersburg, MD 20899

and

Joseph F. Murphy

Consulting Engineer
Structural Reliability Consultants
Madison, WI 53705

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ABSTRACT

Methods are presented for calculating limit state probabilities of engineered wood structural members, considering load duration effects due to stochastic dead and snow load. These methods are used to conduct reliability studies of existing wood design criteria. When realistic load processes are considered, it is found that the importance of load duration and gradual damage accumulation has been somewhat overstated. One possible probability-based design method that should be useful in future code development work also is presented.

Keywords: Buildings; damage accumulation; design; duration of load; limit states; loads; probability; reliability; snow; statistics; structural engineering; wood.

INTRODUCTION

Structural design for engineered wood structures is expected to evolve over the next several years from traditional working stress design toward criteria based on concepts of probability-based limit states design (PBLSD) (Ellingwood et al. 1982a; Galambos et al. 1982; Itani and Faherty 1984). In contrast to working stress design, PBLSD provides for consistent levels of performance from one design application to the next. As a naturally occurring construction material, wood presents problems that have not yet been encountered in developing PBLSD for steel and for reinforced concrete construction (ACI 1983; AISC 1986). The strength of wood is highly variable and is dependent on the rate and duration of load (Madsen 1975). Wood member design strength also depends on the grading procedure (visual or mechanical), species, and member size.

The dependence of the strength of wood structural members on load history means that the variation of structural loads in time must be modeled realistically. Moreover, the possibility that failure may occur by progressive accumulation of damage under loads that vary continually in time, rather than by overloading, must be considered. The analysis of damage accumulation in wood structures and

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the development of PBLSD require: (1) descriptions of loads as stochastic processes; (2) probabilistic models of strength; (3) models to evaluate the damage accumulation process; (4) reliability analysis methods to synthesize load and damage accumulation data; and (5) practical formats for risk-consistent limit states design. With these tools, practical PBLSD criteria for wood structures can be developed to be consistent with code performance objectives stated in probabilistic terms. This paper illustrates these concepts for flexural members supporting a roof, where dead and snow loads constitute the significant structural load requirements.

STOCHASTIC MODELS OF DEAD AND SNOW LOAD

A stochastic characterization of the entire load process is required in order to evaluate damage accumulation and limit state probabilities. Stochastic process models describe the spatial and temporal variation of structural loads. The dead and snow loads described here are assumed to be statically equivalent uniformly distributed loads (Ellingwood et al. 1982a; Galambos et al. 1982), in which the spatial variation of the actual load is taken into account.

Dead load

The dead load, D, arises from the weight of permanent construction, attachments, and equipment. The dead load is assumed to be random in intensity but invariant in time, and thus can be modeled by a random variable. The mean value of D, m_D, approximately equals the nominal value of dead load, D_n, in situations where the nominal load is calculated by the designer from the densities and dimensions of permanent construction (ANSI 1982). The coefficient of variation in dead load, V_D, is about 0.10 (Galambos et al. 1982). The dead load can be modeled by a normal probability distribution.

Snow load

Snow loads on roofs depend on the local climate and on the roof exposure, geometry and thermal characteristics. A simple stochastic model of roof snow load, $S_r(t)$, is given by,

$$S_r(t) = C_s(t)\, S_g(t) \tag{1}$$

in which $S_g(t)$ = ground snow load process, which is determined from analyzing basic climatological data (Ellingwood and Redfield 1983), and $C_s(t)$ = ground-to-roof conversion factor, which depends on the characteristics of the roof (O'Rourke et al. 1982).

Records of daily water-equivalents of ground snow are maintained by the National Oceanic and Atmospheric Administration. In contrast to data on snow depth, these water-equivalents can be converted directly to load without considering the density of snowpack, which is highly variable. Moreover, the data reflect the additional weight of rain from occasional winter rainstorms. Daily water-equivalents for the years during which such records have been maintained in Minneapolis, MN (1952–1983) are shown in Fig. 1. The seasonal nature of snow causes the snow load process to equal zero for a relatively predictable period of the year. This aspect of the snow load process is assumed to be deterministic, and the snow loads in Fig. 1 actually are shown for a reduced 182-day year.

The majority of snow load data analyses have been concerned with the prob-

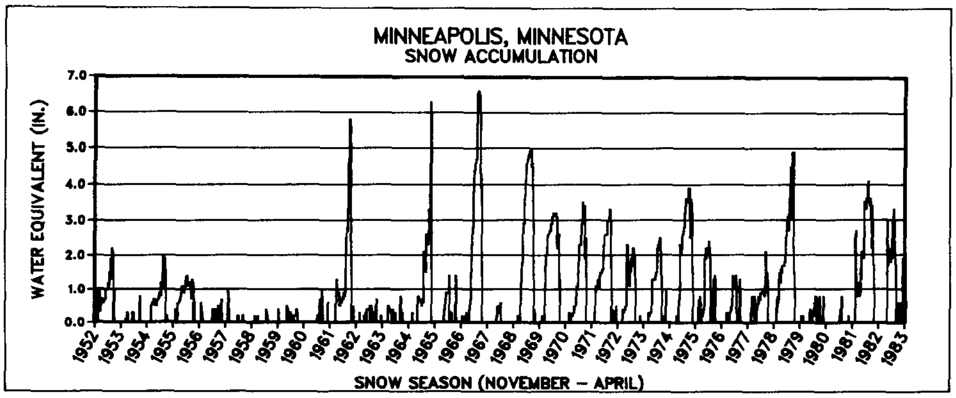

FIG. 1. Ground snow water-equivalents measured at Minneapolis, Minnesota, 1952–1983.

ability distribution of the annual extreme ground snow load, S_{ga}, which is important in structural code development (ANSI 1982). Analyses of these annual extremes (Ellingwood and Redfield 1983) has revealed that the lognormal distribution,

$$F(x) = \Phi\left(\frac{\ln(x) - \lambda}{\zeta}\right); \qquad x > 0 \tag{2}$$

in which λ and ζ = mean and standard deviation of the natural logarithm of the random variable, and Φ = standard normal probability distribution, provides a good fit to the annual extremes at a majority of the stations in the northeast quadrant of the United States.

The annual extreme roof snow load, S_{ra}, is related to annual extreme ground snow load through a conversion factor, C_d, as follows (O'Rourke et al. 1982):

$$S_{ra} = C_d S_{ga} \tag{3}$$

The factor C_d includes, in a rudimentary fashion, the effects on snow load of roof exposure to wind, melting and evaporation, and spatial correlation. Analyses of field measurements of snow accumulation for uniform snow loads on semi-sheltered flat roofs (the standard case in ANSI Standard A58 [1982], for which the design conversion factor equals 0.7) indicated that C_d can be modeled as a lognormal distribution with a median of 0.47 and a coefficient of variation of 0.42 (O'Rourke et al. 1982). Thus, the annual extreme roof load is a lognormal random

TABLE 1. *Statistics of annual extreme roof snow load described by lognormal distribution.*

Station	Ground		Roof					
	λ_g	ζ_g	λ_r	ζ_r	$m_{S_{ra}}$	$V_{S_{ra}}$	S_n*	$m_{S_{ra}}/S_n$
Green Bay, WI	0.410	0.776	−0.345	0.882	5.4	1.09	28	0.19
Rochester, NY	0.815	0.594	0.060	0.727	7.2	0.84	28	0.26
Boston, MA	0.585	0.585	−0.170	0.720	5.7	0.82	21	0.27
Detroit, MI	−0.064	0.630	−0.819	0.757	3.1	0.88	20	0.16
Omaha, NB	−0.062	0.715	−0.817	0.829	3.2	0.99	20	0.16
Cleveland, OH	−0.104	0.581	−0.859	0.717	2.8	0.82	15	0.19

* S_n determined according to ANSI Standard A58.1-1982.

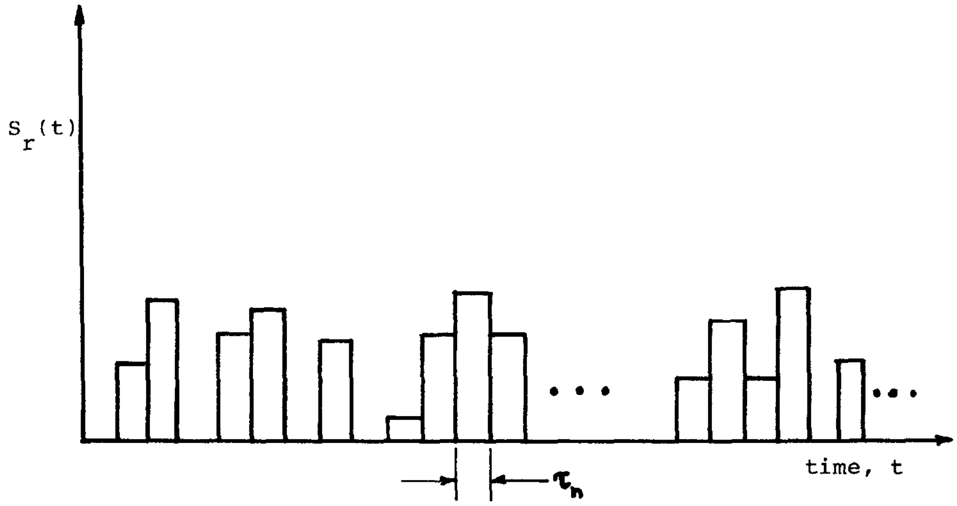

FIG. 2. Bernoulli pulse process model.

variable, since it is the product of two lognormal random variables. Considering the stations in Table 1, the mean of S_{ra} typically is about $0.2S_n$, and its coefficient of variation, $V_{S_{ra}}$, averages about 0.87; S_n = nominal roof snow load defined in Ref. 1.

The appearance of typical snow load records (e.g., Fig. 1) suggests that the temporal variation in snow load can be modeled stochastically, to good approximation, by treating the load as an intermittent pulse process (Turkstra and Madsen 1980). The simplest pulse process model is a Bernoulli pulse process (illustrated in Fig. 2). The duration, τ, of each pulse is assumed constant but nonzero load intensities occur at random during the snow season. The pulse intensities are assumed to be statistically independent and identically distributed random variables, with distribution $F_i(x)$. At any time, the pulse intensity is assumed to be nonzero with probability, p. At Minneapolis, for example, analysis of 31 years of snow data indicates that p = 0.4. The distribution, $F_{max}(x)$, of the maximum load intensity during any period, T, is related to the distribution of pulse intensity by (Turkstra and Madsen 1980),

$$F_{max}(x) = [(1 - p) + pF_i(x)]^{T/\tau} \quad (4)$$

in which T is an integer multiple of τ. Knowledge of $F_i(x)$, p and τ is sufficient to characterize the snow load process in a probabilistic sense.

The distribution of the annual extreme roof snow load is considered to be benchmark information. It is the basis of code-specified snow loads and, as such, is derived from statistical data that have been thoroughly analyzed (Ellingwood and Redfield 1983; O'Rourke et al. 1982). Setting T equal to 1 year, $F_{max}(x)$ in Eq. 4 describes a lognormal random variable, with (on average) $m_{S_{ra}} = 0.2S_n$ and $V_{S_{ra}} = 0.87$ as described above. The distribution of individual pulse intensities can be obtained by fixing p and τ and solving Eq. 4 for $F_i(x)$. A sample function of $S_r(t)$ simulated from $F_i(x)$ so obtained is shown in Fig. 3, assuming that τ = 30 days and p = 0.4. The similarity of Figs. 3 and 1 may be noted. The pulse duration, τ, must be selected to model the variation in time of the snow load

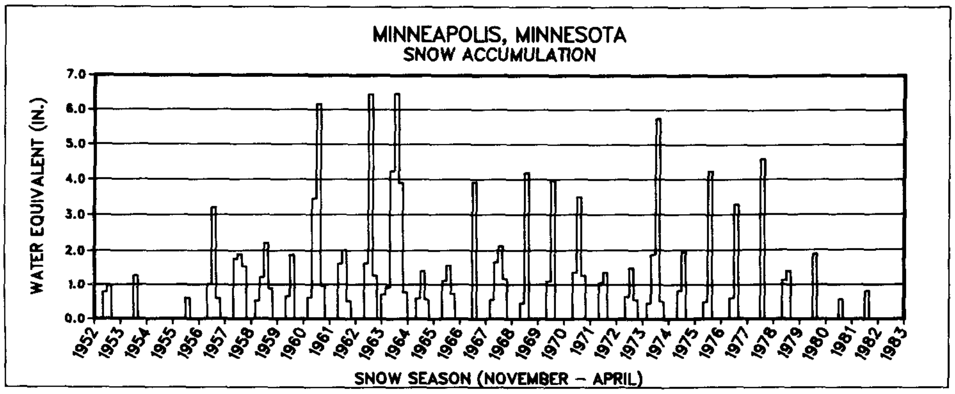

FIG. 3. Simulated snow load pulse process sample function for Minneapolis.

process (e.g., Fig. 1) with sufficient accuracy for damage accumulation analysis. The sensitivity of damage accumulation and limit state probabilities to load pulse parameters p and τ will be examined subsequently.

STRUCTURAL RESISTANCE

Structural members considered in this study are glued-laminated (glulam) beams that are assumed to be part of the main load-bearing system of a roof. Such members are of interest because they are engineered in a manner similar to steel and concrete beams and compete as alternate framing systems. Most of the available data for glulam members are derived from flexural tests of simply supported beams. Since many glulam beams are designed as simply supported, the statistics obtained from these laboratory tests are indicative of beam behavior in a structure. The flexural strength is defined by the modulus of rupture, F_r. The load tests usually are conducted by loading the beams to failure over approximately 5–10 min.

Two glulam beam data sets of flexural strength are used in this paper. The first data set (Data Set MM) is comprised of 73 glulam beams reported by Moody (1977) and 30 beams reported by Marx and Moody (1981), as summarized in Table 2. These data are biased for their structural grades since the 103 beams were specifically fabricated to represent beams with near-minimum load carrying

TABLE 2. *Modulus of rupture tests of 103 glulam beams.*

Beams	F_b
a) From Table IV-2 of Moody (1977).	
A01–A04, A06–A15	2,000
B01–B09, B11–B15	2,200
C01–C15	2,200
41–50	2,400
86–90	1,600
91–95	2,000
96–105	2,400
b) From Marx and Moody (1981).	
F01–F30	2,400

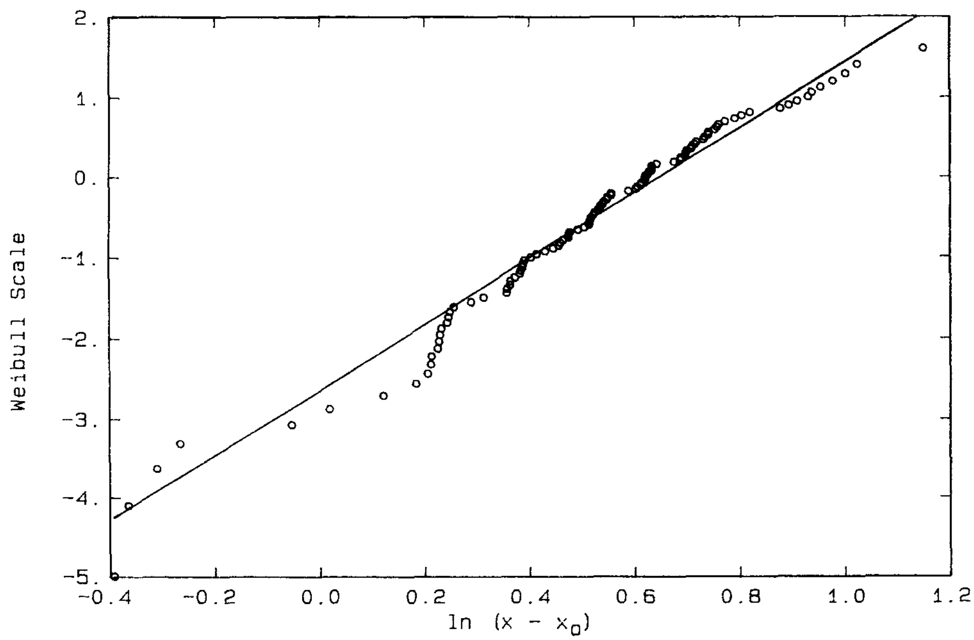

FIG. 4. Three-parameter Weibull probability distribution of modulus of rupture data from data set MM.

capacity. The moduli of rupture of all 103 beams were adjusted to standard conditions: for size, to 12-inch deep beams; for moisture content, to 12%; and for method of loading, to a uniform load and a 21:1 span depth ratio. The test results were normalized by the allowable bending stress, F_b, at standard conditions, and a three-parameter Weibull probability distribution was fit to the data. The three-parameter Weibull cumulative distribution function (cdf) is

$$F(x) = 1 - \exp\left[-\left(\frac{x - x_0}{\eta - x_0}\right)^{\gamma}\right]; \qquad x \geq x_0; \qquad \eta, x_0, \gamma > 0 \tag{5}$$

where γ, η, and x_0 are, respectively, the shape, scale and location parameters of the Weibull distribution. For the above 103 beams, $x_0 = 0.866F_b$, $\eta = 2.776F_b$, and $\gamma = 4.087$. The corresponding mean and coefficient of variation are $2.60F_b$ and 0.18. Figure 4 shows these data on a Weibull probability plot.

An unbiased glulam beam data set (Data Set SF) is obtained from a paper by Sexsmith and Fox (1978). Their data are comprised of 56 glulam beams, the test results of which have been adjusted for this study to the standard conditions noted above. A two-parameter Weibull distribution ($x_0 = 0$) was found to fit their data best, with $\eta = 3.393F_b$, and $\gamma = 6.817$. The corresponding mean and coefficient of variation are $3.17F_b$ and 0.17, respectively. These data are plotted in Fig. 5.

Note that both glulam beam resistance data sets describe short-term flexural strength distributions (i.e., no load duration effects).

LOAD DURATION MODELS

The strength of wood depends on the rate at which load is applied and the time that it is held at a constant intensity. One of the earliest attempts to model load

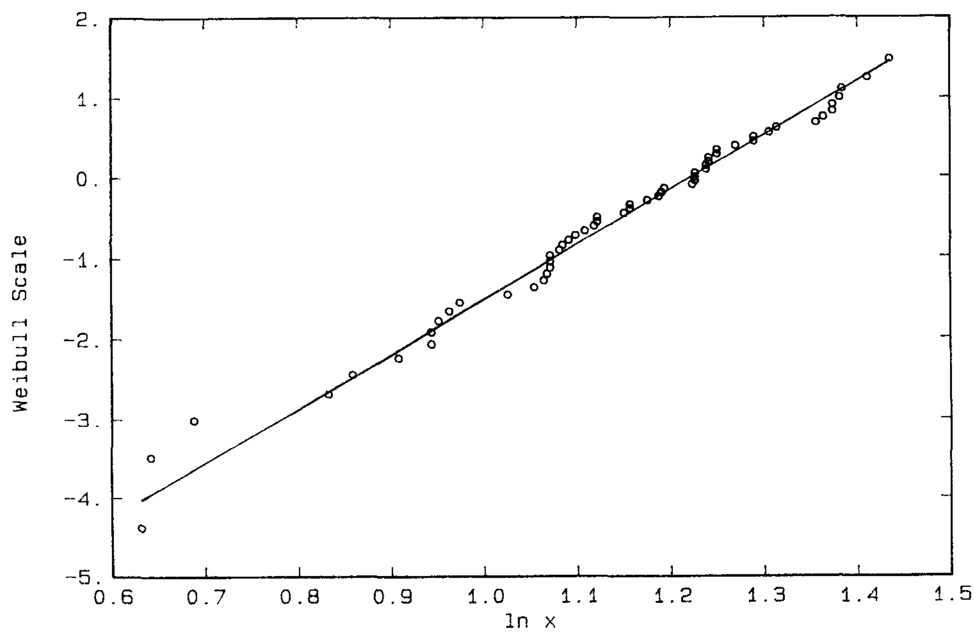

FIG. 5. Two-parameter Weibull probability distribution of modulus of rupture data from data set SF.

duration was the "Madison" curve (Wood 1951), shown in Fig. 6. In Fig. 6, the stress ratio, σ, is the applied stress divided by the stress causing failure in a conventional strength test of 5–10 min duration. The Madison curve is based on tests of small clear specimens subjected to various levels of constant applied stress. The basic allowable stress (e.g., AITC 1980) is based on the assumption that the cumulative duration of stress level corresponding to design live load is 10 yr. Current adjustments to the allowable stress for combinations involving other loads are based on the Madison curve; thus, for example, the 15% increase allowed for load combinations involving snow load corresponds to an assumed cumulative

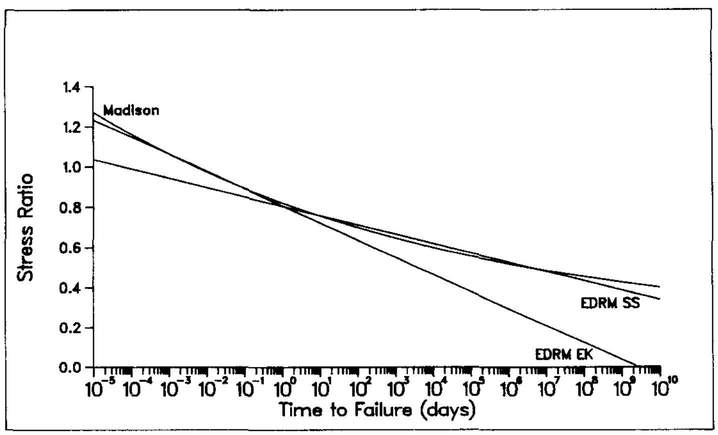

FIG. 6. Damage accumulation models.

duration at design snow load of about 2 months rather than 10 yr. However, it has been observed that the Madison curve may not model load duration effects properly in members of structural size (Barrett and Foschi 1978; Gerhards and Link 1983; Madsen 1975).

Failure of wood structural components appears to occur by progressive accumulation of damage (creep rupture) (Barrett and Foschi 1978; Gerhards and Link 1983). Damage is measured by a state variable, $\alpha(t)$ (Miner 1945) which is assumed to increase monotonically under load. In the undamaged condition, $\alpha(t) = 0$, while failure occurs by definition when $\alpha(t) = 1.0$. The rate at which damage accumulates, $d\alpha/dt$, is postulated to be a function of the applied stress history. For example, the damage rate implicit in the Madison curve, assuming linear damage accumulation, is,

$$\frac{d\alpha}{dt} = A(\sigma - \sigma_0)^B \tag{6}$$

in which A, B and σ_0 = experimental constants determined from test data; $A = 1.5 \times 10^4\ \text{day}^{-1}$, $B = 21.6$ and $\sigma_0 = 0.18$. The parameter σ_0 defines a damage threshold; if σ is less than σ_0 then no damage occurs.

Recent research indicates that the damage rate can be defined as (Gerhards and Link 1983, 1986),

$$\frac{d\alpha}{dt} = \exp(-A + B\sigma) \tag{7}$$

Equation 7, called the exponential damage rate model (EDRM), does not include a damage threshold. Based on tests of Select Structural (denoted as SS) lumber, $A = 40\ \ln(\text{day})$ and $B = 49.75$ (Gerhards and Link 1986); for Douglas-fir 2 × 4's with edge knots on the tension side (denoted as EK), $A = 21.72\ \ln(\text{day})$ and $B = 26.95$ (Gerhards and Link 1983). These two load duration curves are compared to the Madison curve in Fig. 6. Load duration data typically are scattered about models such as those in Fig. 6, indicating some uncertainty associated with the model itself. A portion of this scatter is simply due to variability in F_r. The 'equalrank' assumption (Murphy 1983) made in the sequel ascribes the variability in time to failure under constant stress to variability in F_r. Any variability in time to failure under constant stress due to the model itself is assumed to be small in comparison. Other recent load duration models have postulated the existence of a damage threshold (Barrett and Foschi 1978; Foschi and Barrett 1982). The damage threshold stress ratio has been assumed to lie between about 0.2 and 0.5. The question of whether a damage threshold in wood exists currently is the subject of debate within the wood research community (Itani and Faherty 1984).

Figure 6 shows the extreme sensitivity of the time to failure at constant stress to small changes in applied stress; this sensitivity arises from the magnitude of B in Eqs. 6 and 7. All the damage models in Fig. 6 are fairly close to one another at stress ratios above about 0.75; most load duration data also lie within this range (Barrett and Foschi 1978; Foschi and Barrett 1982; Gerhards and Link 1983, 1986; Wood 1951).

LIMIT STATE PROBABILITIES

As the structural loads and applied stresses vary randomly in time, damage $\alpha(t)$ accumulates stochastically. Linear cumulative damage analysis similar to that used to analyze metallic fatigue can be used to provide a useful (albeit empirical) definition of the limit state for glulam beams. Using this approach,

$$\alpha(t) = \sum_i \Delta\alpha_i \tag{8}$$

in which the increment of damage occurring during time interval Δt_i at stress ratio σ_i is,

$$\Delta\alpha_i = \Delta t_i \exp(-A + B\sigma_i) \tag{9}$$

assuming that the damage rate is defined by Eq. 7. When $t = T_L$, the specified usable life of the structure or some other convenient reference period, the accumulated damage is $\alpha(T_L)$. Failure occurs during the reference period when $\alpha(T_L) = 1$ when $t \leq T_L$.

When a structural member becomes unfit for its intended purpose, it is said to have reached a limit state. Limit state functions (or failure functions) are derived from principles of mechanics and experimental data. The failure condition is given in the form (Ellingwood 1981, 1982a, b),

$$g(X_1, X_2, \ldots, X_k) \leq 0 \tag{10}$$

in which X_i = resistance or load variable. Thus, the cumulative damage limit state for wood members is defined as,

$$g = 1 - \alpha(T_L) = 1 - \sum_i \Delta\alpha_i \tag{11}$$

The limit state probability, P_F, is obtained by integrating the joint probability density of the load and resistance variables over that region of $(X_1, X_2, \ldots, X_k)$ where $g(\mathbf{X}) \leq 0$ (Ellingwood 1981). Because of the complex way that damage in wood structural elements accumulates stochastically under random loads, closed-form solutions for P_F usually cannot be obtained without making untenable simplifying assumptions. Therefore, the limit state probabilities in this study were obtained by Monte Carlo simulation. A reference period of $T_L = 50$ yr was used in all cases. A snow load process sample function was simulated from the distributions described earlier, a glulam beam was selected randomly (i.e., a value of F_r/F_b), and cumulative damage was evaluated using Eqs. 8 and 9. Characteristics of damage accumulation were evaluated, as described subsequently. If $\alpha(T_L) = 1$, a failure was recorded. This process was repeated a large number of times (6,000 or more), and the probability of failure was estimated as the number of failures divided by the number of simulations. Finally, a reliability index was estimated as (Ellingwood et al. 1982b; Hendrickson et al. 1987)

$$\beta = \Phi^{-1}(1 - P_F) \tag{12}$$

in which Φ^{-1} = inverse of the standard normal probability distribution. The index, β, often is used as an alternate to P_F as a measure of reliability.

RELIABILITY ANALYSIS OF EXISTING CRITERIA

The snow load processes, resistance statistics, and damage accumulation models described above are used to evaluate limit state probabilities and reliability indices for individual simply supported glulam beams designed to support a roof in the northern United States. Load sharing is not addressed explicitly, and failure of a beam does not necessarily imply failure of the roof structure. The nominal snow load, S_n, is assumed to be 20 psf (1.0 kPa) and the nominal dead load, D_n, is 5 psf (0.24 kPa).

The stress ratio, σ_i, needed to evaluate damage probabilities for these beams can easily be determined according to whatever design criteria are used. The stress ratio is the applied stress divided by the modulus of rupture, $\sigma_i = F_i/F_r$, where F_i is,

$$F_i = c(D + S_i)/Z \tag{13}$$

in which c = analysis factor, D and S_i = random dead and snow loads, and Z is the section modulus in bending. If the beam is designed according to existing allowable stress procedures, then Z is,

$$Z = c(D_n + S_n)/(1.15F_b) \tag{14}$$

Thus, the stress ratio, σ_i, during an interval of time, Δt_i is,

$$\sigma_i = F_i/F_r = \frac{D + S_i}{D_n + S_n}\frac{1.15}{F_r/F_b} \tag{15}$$

The statistics of D, S_i and F_r have been presented earlier.

Analyses of reliability associated with existing criteria were performed to assess the effects of (1) stochastic snow pulse model characteristics and (2) different resistance data sets and load duration models. Table 3 compares reliability estimates for beams subjected to simulated snow load processes with different pulse durations. Since it is not known exactly what pulse duration is best to use in the Bernoulli pulse model, a range of durations from 2 weeks to 2 months was investigated. The results in Table 3 show that the choice of pulse duration is not critical; thus, for the remaining analyses, the pulse duration was taken to be 1 month. Table 3 also includes an analysis for which the probability, p, that the pulse intensity is nonzero is 1.0 rather than 0.4. Since the reliability estimates are nearly the same for the two values of p, the choice of p also is considered noncritical. The lack of sensitivity of the limit state probabilities (reliabilities) to the temporal parameters p and τ of the snow load process derives from the highly nonlinear characteristics of the damage rate models. The relation of damage increment to stress ratio in forms such as $\Delta\alpha_i \propto \sigma_i^B$ or $\Delta\alpha_i \propto \exp(B\sigma_i)$, with B ranging from 20 to 50 (Foschi and Barrett 1982; Gerhards and Link 1983, 1986; Wood 1951), means that small changes in σ_i cause variations in $\Delta\alpha_i$ of several orders of magnitude, and that $\Delta\alpha_i$ increases from 0 to 1 over a very narrow range of σ_i.

Table 4 compares reliability estimates obtained using the two resistance data sets, and two load duration models, EDRM SS and EDRM EK, described previously. As expected, reliability indices are lower for resistance data set MM (based on near-minimum grade beam specimens) than for data set SF, and are lower using the EDRM EK model (based on bending specimens with edge knots) than when using the EDRM SS model. The failure rate for the minimum quality beams is about four times that for the good quality beams.

The stochastic nature of damage accumulation in the beams during a period $T_L = 50$ yr is illustrated conceptually by the sample functions of $\alpha(t)$ in Fig. 7. In Fig. 7a, $\alpha(t) < 1$, and failure does not occur. Figure 7b illustrates the situation

TABLE 3. *Reliability measures as a function of pulse characteristic.**

Pulse duration	Probability that pulse is on	P_F	β
2 weeks	0.4	0.0156	2.16
1 month	0.4	0.0168	2.13
1 month	1.0	0.0176	2.11
2 months	0.4	0.0186	2.09

* Data set SF and EDRM SS load duration model.

where damage accumulates gradually as a consequence of several load pulses, each with sufficient magnitude to cause measurable damage. Finally, Fig. 7c shows a case where the occurrence of one large load pulse is sufficient to fail the beam. In a sense, the latter case represents an 'overload' failure, although damage accumulates during the single load pulse causing failure.

Gradual damage accumulation (Fig. 7b) is relatively uncommon. The last column of Table 4 gives the percentage of failed beams that failed due to the accumulated effect of two or more snow load pulses. The marginal beams (data set MM, EDRM EK) provided the strongest example of damage accumulation, where 27.1% of the failed beams failed after two or more load pulses; however, only 3.7% of those failing did so after 4 or more pulses, and only 0.3% failed after 7 or more pulses. For the more typical quality beams (data set SF, EDRM SS), only 7.3% of those failing did so under the accumulated effect of two or more snow load pulses. Table 4 shows that in all cases considered, the most common type of failure is that illustrated in Fig. 7c.

The relative unimportance of gradual damage accumulation (Fig. 7b) as a failure mechanism has the same explanation as the insensitivity of P_F (or β) to the temporal characteristics of the snow load process noted above, i.e., the highly nonlinear relation between $d\alpha/dt$ and σ. Thus, it seems that the importance of progressive damage accumulation as a failure mechanism in wood may have been overstated when the behavior of wood beams subjected to realistic snow load processes is considered. The cases studied above reveal that the characteristics of the maximum load pulse to occur in 50 yr determine to a large extent whether or not failure occurs.

PRACTICAL PROBABILITY BASED DESIGN

American National Standard A58 (1982) contains a set of load factors and load combinations for use in limit states design. The loading criteria were derived from statistical modeling and analysis of common structural loads, and are appropriate for all construction materials. Common loading requirements for all construction

TABLE 4. *Reliability Measures for Different Resistance Data Sets and Load Duration Models.**

Resistance data set	Load duration model	P_F	β	Percent of failed members that failed due to two or more pulses
MM	EDRM SS	0.0443	1.70	9.3
MM	EDRM EK	0.0640	1.52	27.1
SF	EDRM SS	0.0168	2.13	7.3
SF	EDRM EK	0.0248	1.97	21.5

* Pulse duration = 1 month, prob (pulse is on) = 0.4.

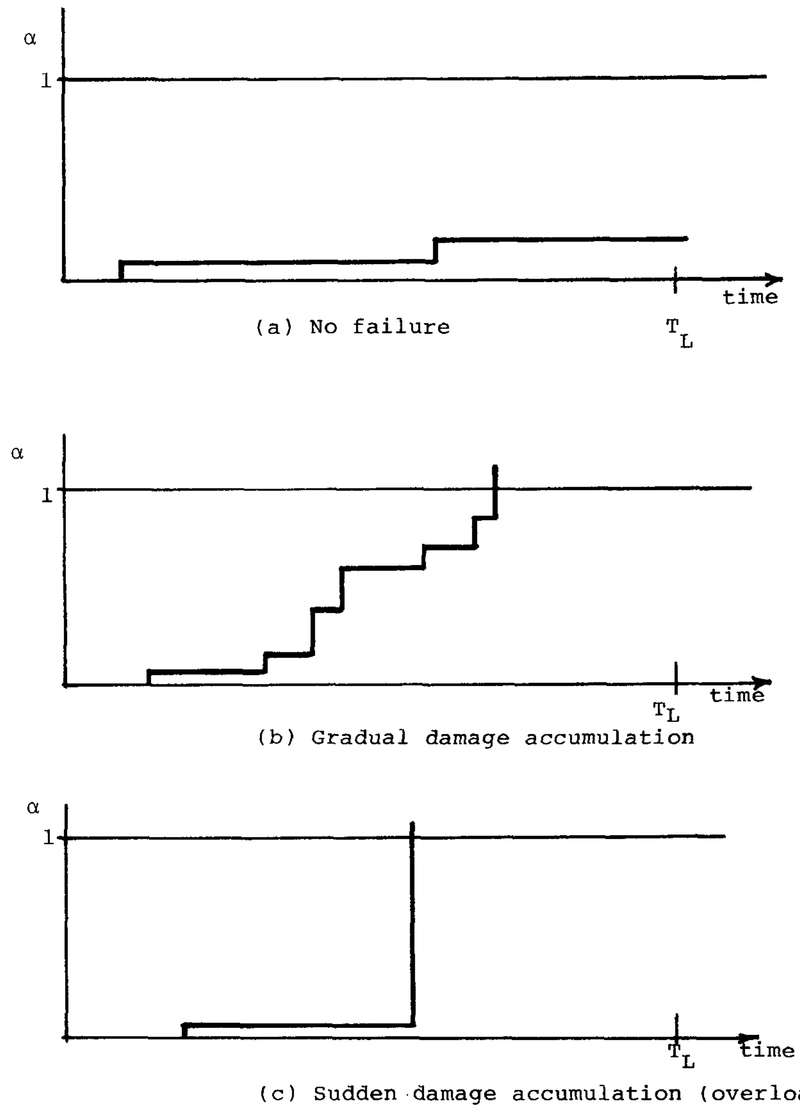

FIG. 7. Classification of damage accumulation mechanisms under stochastic snow load.

materials are an essential feature of the changeover to limit states design. Ultimately, their use will simplify design, will encourage competition among construction technologies, and will facilitate the use of innovative structural systems where construction materials may be mixed. A partial listing of the A58 load combinations, denoted by U, is

$$U = 1.2D_n + 1.6(L_n \text{ or } S_n) \tag{16a}$$
$$U = 1.2\, D_n + 1.3\, W_n + 0.5\, L_n \tag{16b}$$

in which D_n, L_n, S_n and W_n are nominal dead, live, wind and snow loads (ANSI 1982).

The limit states design requirement using these load criteria would be (Ellingwood 1982b),

$$\text{Design resistance } (R_d) \geq \text{Effect of U} \tag{17}$$

The design resistance must be selected so as to be compatible with Eqs. 16. R_d can be set to yield designs in which different structural components have the level of uniform reliability desired by a wood specification committee.

The design resistance in Eq. 17 is calculated using nominal values of material strength taken from nationally recognized standards (AITC 1980) and dimensions in formulas derived from accepted principles of structural mechanics. Resistance factors are included in the computation of R_d to account for inherent variability and other uncertainties that may cause unfavorable deviations of the actual strengths from the design value. It should be emphasized that the reliability obtained from a particular design resistance depends on both the nominal strength and the resistance factors (Ellingwood 1982b).

The design resistance, R_d, is obtained by multiplying the specified nominal resistance, R_n, by a resistance factor, ϕ, on structural action and a load duration factor, λ:

$$R_d = \lambda\phi R_n \tag{18}$$

The resistance factor, ϕ, depends on the nature of the particular limit state of interest (e.g., flexure, compression) and the consequences to the structure of a member reaching that limit state. The factor, λ, takes into account the fact that adjustments to R_d are necessary if the structure is to have the same reliability for combinations of loads with different temporal characteristics. Equations 16–18, taken together, comprise a "load and resistance factor design" (LRFD) format similar to that already adopted by the steel and concrete industries in the United States for limit states design (ACI 1983; AISC 1986). There is a second format for design resistance, termed the partial material factors approach, in which factors are applied directly to each strength variable (e.g., modulus of rupture) rather than to structural action, R_n. This second approach has advantages in reinforced concrete or masonry construction, where composite action of different materials is important, but has fewer advantages for steel or wood construction (Ellingwood 1982b).

It is suggested that the nominal strength of a wood structural member, R_n, be based on the 5% exclusion limit strength obtained in a conventional strength test (ramp loading to failure in 5–10 min). There are historical reasons for this selection, as the current allowable stress is purported to be based on a 5% exclusion limit. It also is advantageous to base nominal strength on a quantity that can be determined directly from laboratory testing. Resistance factors compatible with this nominal strength result in a set of design requirements that are similar in appearance to those already obtained for steel and reinforced concrete (ACI 1983; AISC 1986).

As an example of the use of Eqs. 16–18, the safety check for designing beams in a roof structure to withstand dead and snow load would be,

$$\lambda_s\phi_f F_{rn} Z \geq 1.2D_n + 1.6S_n \tag{19}$$

in which F_{rn} = 5% exclusion limit of modulus of rupture, adjusted for size, Z = section modulus in bending, ϕ_f = resistance factor for flexure and λ_s = load duration factor for combinations involving snow load. The factors ϕ_f and λ_s must be determined so that beams designed by Eq. 19 have the level of reliability desired

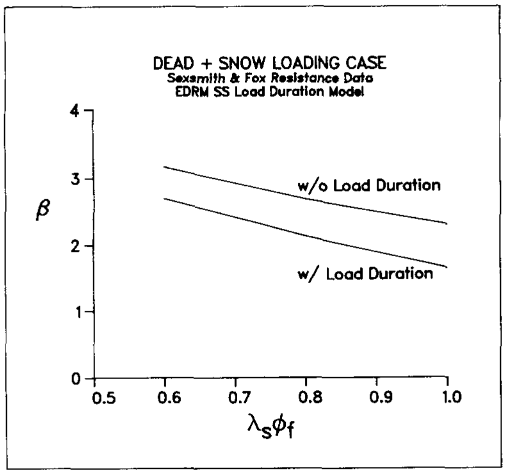

FIG. 8. Variation of β as a function of overall resistance factor using resistance data set SF.

by the code committee. The factor λ_s accounts for snow load duration effects, while ϕ_f reflects inherent variability in short-term flexural strength.

Factors ϕ_f and λ_s can be determined as follows. First, a set of representative beams (different span-depth ratios, S_n/D_n ratios, grade, etc.) is designed as a function of $\phi_f\lambda_s$. The variation in reliability index, β, with $\phi_f\lambda_s$ when creep rupture is taken into account is illustrated in Fig. 8. As expected, β decreases as $\phi_f\lambda_s$ approaches 1. Second, the reliability of the same beams is evaluated ignoring creep rupture. This second evaluation parallels that followed in developing LRFD for steel and reinforced concrete structures (Ellingwood et al. 1982a; Galambos et al. 1982), in which the limit state is reached if the maximum applied moment in 50 yr exceeds the (short-term) modulus of rupture. A second relation between $\phi_f\lambda_s$ and β is obtained, which lies above the curve obtained when creep rupture was considered (see Fig. 8). In the second case, however, $\lambda_s = 1.0$, by definition, since failure by creep rupture is ignored. Thus, if the target reliability is set at β_0, ϕ_f and λ_s can be determined as the ratios of the two curves in Fig. 8. For example, if $\beta_0 = 2.5$, then ϕ_f for overload $= 0.89$, $\lambda_s\phi_f$ for creep rupture $= 0.64$ and so $\lambda_s = 0.64/0.89 = 0.72$.

Factor ϕ in Eq. 18 depends on the limit state and thus would be different from ϕ_f for safety checks involving shear or compression (stability) limit states. Factor λ in Eq. 18 would equal λ_s for all limit states involving snow load as the principal variable load in the load combination. On the other hand, if the principal variable load in the controlling load combination were occupancy live load, then the appropriate value of λ in Eq. 18 would be different from λ_s because live and snow loads have different temporal characteristics. However, the same set of ϕ-values would be used for flexure, shear and compression, regardless of which load combination governed.

SUMMARY AND CONCLUSIONS

Probability-based limit states design criteria can be developed for engineered wood construction. The design criteria are in an LRFD format similar to that used for steel and reinforced concrete construction. The design resistance is checked against the effect of load combinations specified in ANSI A58 (1982). The design resistance is determined as the product of a specified nominal material strength (5% exclusion limit of the distribution of short-term strength), a resistance factor, ϕ, to account for variability in strength (ϕ depends on the limit state being checked), and a factor, λ, to account for the effects of load duration (λ depends on the load combination being considered).

Reliability analyses of existing criteria utilizing a linear cumulative damage model reveal that assumptions regarding the duration and probability of occurrence of load pulses in the stochastic model for snow loads are not critical. Reliability measures are also relatively insensitive to different load duration models. Damage accumulates mainly during the largest snow load pulse, although more load pulses may contribute when either the resistance data or load duration model are based on relatively weak material.

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