# PREDICTING THE EFFECT OF MOISTURE CONTENT ON THE FLEXURAL PROPERTIES OF DOUGLAS-FIR DIMENSION LUMBER

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## ABSTRACT

Current procedures for adjusting estimates of the mechanical properties of lumber for changes in moisture content are based on trends in the observed means. The present study was initiated to develop analytical procedures for adjusting estimates of the flexural properties of 2-inch-thick Douglas-fir dimension lumber that would be applicable to all levels of the flexural properties. Equations are derived for adjusting modulus of rupture (MOR), modulus of elasticity (MOE), moment capacity (RS = MOR × section modulus), and flexural stiffness (EI = MOEX moment of inertia) for changes in moisture content. The best of these equations are found to be significantly more accurate than current procedures for adjusting estimates of strength properties such as MOR and RS. Because MOE and EI are less affected by changes in moisture content, most of the equations work well for these properties.

*Keywords:* Flexural properties, modulus of rupture, modulus of elasticity, flexural stiffness, moment capacity, moisture content, mathematical models.

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## INTRODUCTION

The objective of the present study is to develop and evaluate analytical models that can be used to adjust estimates of the flexural properties of Douglas-fir dimension lumber from one moisture content (MC) to another. Nine models are presented, belonging to the six general types of models considered relevant. The models are evaluated and their predictions compared with each other and with the data. Applicability of the results is uncertain beyond the range of the available data.

## BACKGROUND

In the United States and Canada, major research programs have been conducted to evaluate the mechanical properties of full-size structural lumber, visually graded according to current practice (Green 1983; Forintek 1983). The results of these studies provide a basis for the establishment of new values of allowable design stress for visually graded structural lumber. It was not economically feasible in these studies to evaluate the effect on mechanical properties of varying levels of MC. Most of the lumber was tested at the MC at which it was produced, not conditioned to a single MC. Thus, the In-Grade data produce insufficient information on the relationships between MC and lumber properties to adjust mechanical properties of lumber to specific end use MC levels or to interpret the variations of these properties within and between mills.

Current procedures for adjusting estimates of the mechanical properties of dimension lumber for changes in MC are based on adjustment factors presented in American Society for Testing and Materials Standard D 245-81 (ASTM 1986). The adjustment factors are assumed to be independent of lumber quality and applicable to all levels of the cumulative frequency distribution of the lumber property.

When dried from green to a maximum MC of 19% or of 15%, these adjustment factors are:

| | *Maximum MC* | |
|---|---|---|
| *Property* | *19%* | *15%* |
| Bending | 25 | 35 |
| Modulus of elasticity | 14 | 20 |
| Tension parallel to grain | 25 | 35 |
| Compression parallel to grain | 50 | 75 |
| Horizontal shear | 8 | 13 |
| Compression perpendicular to grain | 50 | 50 |

Older versions of D 245, however, cite procedures for adjusting the estimates for changes in MC that are dependent upon lumber quality (Green 1982).

$$SR_{dry} = SR_{green} + \tfrac{1}{2}(SR_{green} - 50) \qquad (1)$$

where

SR = strength ratio, percent—the ratio of the strength of a member containing a defect to the strength of an equivalent defect-free member.

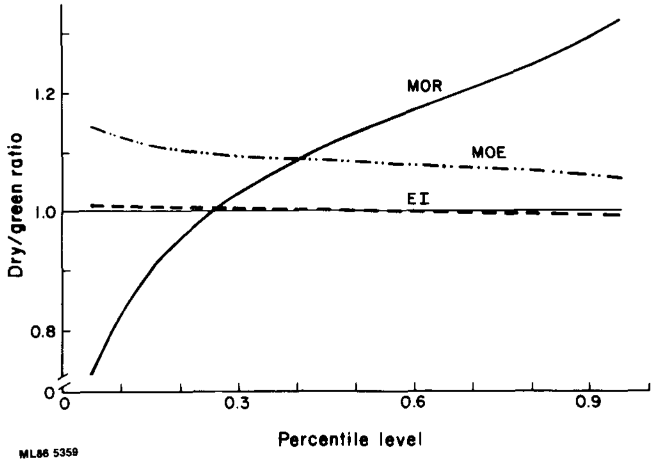

FIG. 1. Effect of MC on the dry:green ratio for three flexural properties of 2 × 6, No. 2 and Better, Douglas-fir dried to an MC of 15% (adapted from Weibull parameter given in Madsen et al. 1980). (ML86 5359)

Thus the adjustment factor ranged from 25% for green lumber having an SR of 100% to no increase for green lumber having an SR of 50%.

Earlier research supports the concept of a quality-dependent adjustment factor. Gerhards (1968, 1970) used matched pairs of 4 × 8 southern pine beams to investigate the effect of seasoning on flexural properties. He concluded that the MC adjustment factor, F, for modulus of rupture (MOR) when green wood was dried to an MC of 12% could be expressed as a function of SR.

$$F = 0.994 + 0.00503\,(SR) + 0.0104\,(DSR - GSR) \qquad (2)$$

where

F = the ratio of MOR at 12% MC to MOR green, and
DSR-GSR = the within-pair difference in the SR of the matched specimens tested dry (DSR) and green (GSR).

The adjustment factor for modulus of elasticity (MOE) was found to be independent of SR. When dried by a mild schedule, the MOE increased about 23% in drying from green to an MC of 12%.

Studies by Madsen (1975) and Madsen et al. (1980) on the flexural properties of Douglas-fir, Hem-Fir, and Spruce-Pine-Fir dimension lumber indicated that the magnitude of the adjustment for MOR or MOE was dependent upon location in the cumulative density function for MOR or MOE. Using Douglas-fir as a typical example (Fig. 1), it was noted that MOR was much more sensitive than MOE to position in the distribution.

## PROCEDURES

In this section we review the experimental procedures briefly before introducing and deriving the analytical models. Finally, we discuss the intersection MC, $M_P$, which is the MC above which properties are assumed to be independent of MC.

### *Experimental procedures*

Green Douglas-fir lumber of three grades (Select Structural, No. 2, and No. 3) and three sizes (2 × 4, 2 × 6, and 2 × 8) was sampled from one sawmill near Vancouver, B.C. The lumber of each size and grade was divided into four equivalent populations in terms of estimated stiffness in the green condition. Each population or group contained approximately 100 pieces. Three of the groups were then equilibrated to 10, 15, and 20% MC. The fourth group was maintained in the green condition prior to testing, by use of a water spray. All pieces were tested to failure on edge in third-point bending using a span-to-depth ratio of 17:1. Flexural properties were calculated using the actual dimensions of the piece at the time of test. The MOE values were not corrected for deflection caused by shear stresses. Additional details concerning the experimental procedures and the data analysis we employed are given in Aplin et al. (1986).

### *Analytical models*

We considered six types of models that may be used to make MC adjustments of the four flexural properties, MOR, MOE, RS (moment capacity), and EI (flexural stiffness).[4] Several variations of each type of model may be produced by making different assumptions concerning the form of the analytical expression or by using various subsets of the data. Coefficients of the individual models are given in the appendix.

*Zero adjustment model.*—The simplest model is one in which no adjustment of estimated properties is made for changes in MC; the MC adjustment factor, F, is unity. This model is useful primarily as a baseline against which to compare the performance of the more complicated models.

*Constant percentage adjustment models.*—These models adjust estimates of a given property by a constant percentage when the MC is changed from one level to another. All current standard adjustment procedures for dimension lumber and for clear wood are of this type. Several variations are possible with this type of model (Green et al. 1986). We present only a linear adjustment formula (models 1, 2, and 3 in Table 1) by which, when adjusting from a property value of $P_1$ at an MC of $M_1$ percent, the value of the property $P_2$ at an MC of $M_2$ percent is given by

$$P_2 = P_1 * \left[\frac{a + bM_2}{a + bM_1}\right] \tag{3}$$

where b represents a change in property per percent change in MC. In obtaining a value for b, MC values greater than the intersection MC, $M_p$, were replaced by the assumed value of $M_p$.

*Surface model.*—A group of surface models may be obtained by plotting nonparametric estimates of the percentiles of a property against MC. In this study, percentile levels of 2, 5, 10, 15, 20, . . . , 90, 95, 98, were plotted for MCs of 10,

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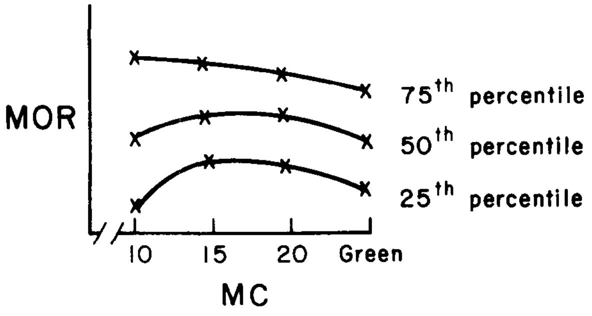
[4] Moment capacity (RS) is the product of modulus of rupture and section modulus, S, where $S = 1/6 \times \text{thickness} \times (\text{width})^2$. Flexural stiffness (EI) is the product of the modulus of elasticity and the moment of inertia, I, where $I = 1/12 \times \text{thickness} \times (\text{width})^3$.



**ML86 5358**

FIG. 2. Contours of percentiles for MOR such as would be used to create a surface model (actual model used 21 percentiles: 2, 5, 10, 15, . . . , 90, 95, 98). (ML86 5358)

15, 20, and green for each grade and size of lumber. Thus, for any grade, size, and percentile level, four values of the property appear on the plot, one for each MC (Fig. 2). These contour lines define a surface that is followed when adjusting from one MC level to another. Contour surfaces were approximated by fitting to each contour line a quadratic curve of the form

$$P = a + bM + cM^2 \tag{4}$$

where a, b, and c are unique for each contour. The contour, in turn, depends upon the value of the property, $P_1$, and the MC, $M_1$. The values of a, b, and c can then be estimated using $P_1$ and $M_1$. Then

$$P_2 = P_1 + b(M_2 - M_1) + c(M_2{}^2 - M_1{}^2). \tag{5}$$

We considered several variations of this type of model. One variation (model 4, Table 1) uses the linear form of Eq. (5).

In a second variation, b and c were modeled using a polynomial function of P and MC. Given values of $P_1$ and $M_1$, the polynomial predicted b and c so that Eq. (5) could be used. This procedure produced what we call a "nonfixed surface." It is nonfixed in the sense that, when predictions of $P_2$ at $M_2$ made from $P_1$ and $M_1$ are used to predict a property value at $M_1$, the value predicted may not be exactly $P_1$. This is because the contour estimate is slightly different when b and c are predicted from $P_2$ and $M_2$ instead of from $P_1$ and $M_1$. Model 4 is a linear version and model 5 a quadratic version of Eq. (5) treated in this way. The nonfixed surface models are easier to compute than the fixed surface model described next. However, care must be taken always to use real data as the starting values $P_1$ and $M_1$.

It is possible to fix the contours so that the original values of $P_1$ and $M_1$ can be obtained from the predicted value of $P_2$ at $M_2$. This is done by fixing a reference MC that is used to estimate b and c. For this paper a reference MC of 15% was chosen, and the P at 15% was found that, when adjusted to an $M_1$ value, gave $P_1$. This $P_{15}$ and M = 15 were then used to predict b and c for use in Eq. (5). This step, added to insure that the contours are fixed, can result in some difficult calculations. In the most useful fixed surface model, a quadratic curve of the form

TABLE 1. *Moisture content adjustment models for flexural properties of Douglas-fir at various moisture contents.*

| | | | | | Average maximum absolute difference[1] | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Mean for MC | | | | 5th Percentile for MC | | | |
| Model number | Model type | Variation | Dependence on grade and size | Number of coefficients | 10% | 15% | 20% | Green | 10% | 15% | 20% | Green |
| | | | | | psi | | | | | | | |
| | | | Modulus of rupture, MOR | | | | | | | | | |
| 0 | Zero adjustment | — | Independent | 0 | 2,277 | 2,277 | 2,277 | 2,277 | 727 | 727 | 727 | 727 |
| 1 | Constant percentage | Linear | By grade and size | 18 | 641 | 577 | 513 | 461 | 715 | 636 | 557 | 493 |
| 2 | | | By grade | 6 | 761 | 686 | 612 | 553 | 713 | 637 | 562 | 501 |
| 3 | | | Independent | 2 | 930 | 833 | 737 | 659 | 654 | 586 | 518 | 464 |
| 4 | Surface | Linear, regression | Function | 10 | 788 | 690 | 592 | 516 | 485 | 427 | 388 | 359 |
| 5 | | Quadratic, regression | Function | 30 | 675 | 523 | 423 | 382 | 472 | 509 | 468 | 357 |
| 6 | | Quadratic, regression (fixed) | Function | 8 | 681 | 546 | 434 | 360 | 503 | 516 | 457 | 357 |
| 7 | Normal | Regression | Function | 25 | 461 | 409 | 314 | 260 | 816 | 724 | 555 | 460 |
| 8 | Weibull | 3-P, regression | Function | 35 | 462 | 416 | 309 | 264 | 506 | 479 | 424 | 324 |
| 9 | | 2-P, regression | Function | 22 | 464 | 411 | 309 | 264 | 589 | 555 | 482 | 403 |
| 10 | Strength ratio | Linear, 0% cutoff | Yes | 2 | 808 | 761 | 715 | 692 | 647 | 532 | 424 | 354 |
| | | | | | $10^6$ psi | | | | | | | |
| | | | Modulus of elasticity, MOE | | | | | | | | | |
| 0 | Zero adjustment | — | Independent | 0 | 0.268 | 0.268 | 0.268 | 0.268 | 0.162 | 0.162 | 0.162 | 0.162 |
| 1 | Constant percentage | Linear | By grade and size | 18 | 0.047 | 0.044 | 0.041 | 0.039 | 0.057 | 0.053 | 0.050 | 0.047 |
| 2 | | | By grade | 6 | 0.053 | 0.050 | 0.046 | 0.043 | 0.061 | 0.057 | 0.053 | 0.050 |
| 3 | | | Independent | 2 | 0.053 | 0.050 | 0.046 | 0.043 | 0.063 | 0.059 | 0.055 | 0.051 |
| 4 | Surface | Linear, regression | Independent | 10 | 0.047 | 0.045 | 0.042 | 0.040 | 0.065 | 0.060 | 0.056 | 0.052 |
| 5 | | Quadratic, regression | Function | 30 | 0.045 | 0.042 | 0.039 | 0.039 | 0.071 | 0.064 | 0.058 | 0.057 |
| 6 | | Quadratic, regression (fixed) | Function | 8 | 0.044 | 0.041 | 0.038 | 0.036 | 0.067 | 0.063 | 0.059 | 0.055 |
| 7 | Normal | Regression | Function | 27 | 0.038 | 0.034 | 0.031 | 0.029 | 0.062 | 0.056 | 0.051 | 0.047 |
| 9 | Weibull | 2-P, regression | Function | 23 | 0.040 | 0.037 | 0.033 | 0.031 | 0.062 | 0.057 | 0.051 | 0.048 |
| 10 | Strength ratio | Linear, 0% cutoff | Yes | 2 | 0.106 | 0.104 | 0.103 | 0.102 | 0.078 | 0.096 | 0.078 | 0.080 |

TABLE 1. *Continued.*

| Model number | Model type | Variation | Dependence on grade and size | Number of coefficients | Average maximum absolute difference[1] | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Mean for MC | | | | 5th Percentile for MC | | | |
| | | | | | 10% | 15% | 20% | Green | 10% | 15% | 20% | Green |
| | | | | | in·lb | | | | | | | |
| | | | Moment capacity, RS | | | | | | | | | |
| 0 | Zero adjustment | — | Independent | 0 | 12,819 | 12,819 | 12,819 | 12,819 | 4,044 | 4,044 | 4,044 | 4,044 |
| 5 | | Quadratic, regression | Function | 30 | 5,006 | 4,444 | 4,048 | 3,835 | 4,822 | 4,587 | 4,268 | 3,969 |
| 6 | Surface | Quadratic, regression (fixed) | Function | 8 | 4,836 | 4,066 | 3,409 | 2,944 | 3,566 | 3,829 | 3,530 | 2,884 |
| 7 | Normal | Regression | Function | 24 | 3,804 | 3,694 | 2,796 | 2,524 | 5,078 | 5,009 | 3,740 | 3,405 |
| 9 | Weibull | 2-P, regression | Function | 24 | 4,274 | 4,087 | 3,088 | 2,767 | 3,816 | 3,851 | 3,387 | 3,001 |
| | | | | | $10^6$ lb·in$^2$ | | | | | | | |
| | | | Flexural stiffness, EI | | | | | | | | | |
| 0 | Zero adjustment | — | Independent | 0 | 2.326 | 2,326 | 2.326 | 2.326 | 1.846 | 1.846 | 1.846 | 1.846 |
| 5 | Surface | Quadratic, regression | Function | 30 | 0.753 | 0.730 | 0.712 | 0.714 | 1.642 | 1.597 | 1.770 | 1.796 |
| 6 | | Quadratic, regression (fixed) | Function | 8 | 0.802 | 0.812 | 0.819 | 0.822 | 1.513 | 1.463 | 1.425 | 1.403 |
| 7 | Normal | Regression | Function | 28 | 0.502 | 0.501 | 0.499 | 0.498 | 1.489 | 1.467 | 1.445 | 1.427 |
| 9 | Weibull | 2-P, regression | Function | 25 | 0.557 | 0.543 | 0.540 | 0.545 | 1.478 | 1.420 | 1.391 | 1.387 |

[1] Based on the average of each of the 9 grade and size combinations at the indicated percentile (or mean level).

of Eq. (4) was fitted to each surface. Then b and c were estimated at 15%, giving $b_{15}$ and $c_{15}$ where

$$b_{15} = D_0 + D_1 P_{15} + D_2 P_{15}^2 + D_3 P_{15}^3 \tag{6a}$$

and

$$c_{15} = E_0 + E_1 P_{15} + E_2 P_{15}^2 + E_3 P_{15}^3 \tag{6b}$$

The parameters $D_0$, $D_1$, $D_2$, $D_3$, and $E_0$, $E_1$, $E_2$, $E_3$ in Table A-5 were estimated from the data. The problem was to find $P_{15}$. We know that going to $M_1$ from $P_{15}$ at $M_{15}$ must give $P_1$. The change in property value is

$$P_{15} - P_1 = b_{15}(15 - M_1) + c_{15}(15^2 - M_1^2). \tag{7}$$

Inserting Eqs. (6a) and (6b) in Eq. (7) gives

$$\begin{aligned} 0 = {} & [P_1 + D_0(15 - M_1) + E_0(15^2 - M_1^2)] + [D_1(15 - M_1) + E_1(15^2 - M_1^2) - 1]P_{15} \\ & + [D_2(15 - M_1) + E_2(15^2 - M_1^2)]P_{15}^2 + [D_3(15 - M_1) \\ & + E_3(15^2 - M_1^2)]\, P_{15}^3 \end{aligned}$$

Because everything except $P_{15}$ is known, we can use a root-finding procedure to find $P_{15}$, then use this value to predict $b_{15}$ and $c_{15}$ in Eqs. (6a) and (6b). Finally, the predicted values of $b_{15}$ and $c_{15}$ are used in Eq. (5) to go from $P_1$ at $M_1$ to any $P_2$ at $M_2$. The resulting contours are fixed. Several versions of this model type were tried, the best of which is the model just described (model 6 in Table 1), the only model of this type reported here.

*Normal model.*—A fourth type of model is obtained by fitting a normal distribution to the data for each combination of grade, size, and MC (Aplin et al. 1986). Given a mean property value $\bar{X}$ and standard deviation S at $M_1$ and at $M_2$, we assumed that if the strength of a particular piece of lumber was the pth percentile at one MC, it would be the pth percentile at any MC. By this assumption, the property $P_2$ at $M_2$ was related to the property $P_1$ at $M_1$ by

$$P_2 = S_2 \frac{P_1 - \bar{X}_1}{S_1} + \bar{X}_2 \tag{8}$$

To generalize this model to grades, sizes, and MC levels not tested in this study, the mean and standard deviation were related to lumber width, grade, and MC through a regression equation. In this study the minimum SR assigned to the grade and the standard dressed dry width were used as the grade and width parameters.[5] The generalized normal model is model 7 in this report.

*Weibull model.*—Another model may be derived in a fashion similar to that used for the normal model except that a Weibull distribution is first fitted to the data. Again, we equated percentile levels in any pair of strength distributions for two specified MC levels. In this instance the relationship between the properties at the two specified MC levels ($M_1$ and $M_2$) is

$$P_2 = \omega_2 \left( \frac{P_1 - \ell_1}{\omega_1}^{m_1/m_2} \right) + \ell_2 \tag{9}$$

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[5] Strength ratios (USDA 1974) are: Select Structural = 65%, No. 2 = 45%, No. 3 = 26%. Standard dressed "dry" widths (U.S. Department of Commerce 1986) are: 4 inches = 3.5 inches, 6 inches = 5.5 inches, 8 inches = 7.25 inches.

where

$m_i$ = the Weibull shape parameter at $M_i$, $i = 1, 2$,
$\omega_i$ = the Weibull scale parameter at $M_i$, $i = 1, 2$, and
$\ell_i$ = the Weibull location parameter at $M_i$, $i = 1, 2$.

The Weibull parameters at each of the four MC levels tested in this study are given in Aplin et al. (1986).

Equation (9) may be used to convert properties from one to another of the four MC levels for a given grade and size combination. To generalize the procedure to MC levels not tested experimentally, a relationship is required that relates each of the Weibull parameters to MC for each grade and size combination. In a previous paper on modeling the effect of MC on the flexural properties of southern pine (Green et al. 1986), a different quadratic function of MC was used for each individual grade and size combination. Although this procedure worked well, it was difficult to generalize the model to sizes and especially to grades not tested.

Alternatively, a regression approach such as was used with the normal model may be used to relate the Weibull parameters given in Eq. (9) to MC, grade, and size. We derived both two-parameter and three-parameter forms of the Weibull regression model (model 8 and 9 of Table 1). The shape and scale parameters for both take the form

$$\text{Parameter} = a_0 + a_1(W) + a_2(SR) + a_3(M) + \ldots + a_{20}(W)^2(M)^2 \qquad (10)$$

The location parameter used for the three-parameter form was determined by observing trends in the minimum MOR values for each combination of grade, size, and MC. The variables W (specimen width), SR (strength ratio), and M (moisture content) used to derive Eq. (10) are defined in Table A-3 of the appendix.

*Strength ratio model.*—Another type of model assumes that the MC adjustment factor depends upon the SR of the individual piece of lumber. The 25% rule (Eq. 1) and Gerhard's model (Eq. 2) are historical models of this type.

To determine the form of the relationship between property and SR, the data were stratified and the results plotted in several subsets of SR. These plots indicated that change in MOR with change in MC increased erratically with increasing SR. Similar plots indicated that change in MOE was virtually independent of change in SR. Because there was no clear-cut trend in the SR relationships, only a linear SR model was investigated.

In this model, the change in property with a given change in MC was assumed equal to unity for SRs between zero and some cutoff value, $y_0$. A linear relationship was used between $y_0$ and F*, where F* is the change in property between MCs for green lumber and lumber with SR = 100. Four values of $y_0$ were assumed: 0, 26, 45, and 50%. Then

$$\begin{aligned} P_2 - P_1 &= 0 && 0 \le SR \le y_0 \\ P_2 - P_1 &= [100 - F^*y_0 + (F^* - 1)SR]/(100 - y_0) && y_0 < SR \le 100 \qquad (11) \end{aligned}$$

The value of F* was obtained using a linear regression between F* and MC for the 1,083 pieces that had an estimated SR of 100%.

TABLE 2. *Estimated intersection moisture content $M_p$ for Douglas-fir dimension lumber.*

| Lumber grade and width | | Moisture content adjustment model | | | |
|---|---|---|---|---|---|
| | | MOR | | MOE | |
| | | Linear | Quadratic | Linear | Quadratic |
| | | % | | | |
| | | Surface model[1] | | | |
| Select structural | 4 | 25.3 | 39.5 | 22.7 | 20.9 |
| | 6 | 22.0 | 56.7 | 22.1 | 23.1 |
| | 8 | 24.0 | 22.0 | 23.8 | 22.7 |
| No. 2 | 4 | 30.3 | 29.5 | 22.0 | 21.1 |
| | 6 | 34.2 | 22.6 | 21.9 | 21.6 |
| | 8 | 34.3 | 20.5 | 22.5 | 31.0 |
| No. 3 | 4 | 34.9 | 23.5 | 23.4 | 22.6 |
| | 6 | 29.9 | 34.0 | 21.2 | 22.6 |
| | 8 | 24.1 | 19.3 | 22.1 | 25.1 |
| All | All | 26.0 | 21.3 | 22.5 | 21.6 |
| | | Constant percentage model[2] | | | |
| All | All | 31.8 | 62.7 | 23.7 | 23.2 |

[1] Linear surface model by regression (model 4 of Table 1), quadratic surface model by regression (model 5).
[2] Linear constant percentage model (model 3). Quadratic model not shown in text.

### *Intersection moisture content*

It is generally assumed that the mechanical properties of small, clear wood specimens decrease with increasing MC up to some MC level. Past this level, properties are assumed to be independent of MC (U.S. Department of Agriculture 1974). The MC above which properties are independent of MC is called the intersection MC, $M_p$ (Wilson 1932).

Traditionally, the value of $M_p$ is chosen as the intersection of two lines on a plot of the logarithm of the property versus MC. The first line describes the linear relationship between log property and MC over a range of MCs for "dry" wood, and the second is a horizontal line representing the property value for green wood. For clear specimens of Douglas-fir, the $M_p$ value is 24% (Wilson 1932).

Estimates of $M_p$ evaluated in this study vary with the property and the MC adjustment model used (Table 2).[6] Because of this variation, there is no solid empirical evidence to reject the clear wood value of $M_p = 24$. This value is used in all modeling.

## EVALUATION AND COMPARISON OF MODELS

In this section the MC adjustment models are compared for each grade and size by adjusting the property estimates at the four MCs to a common MC level. If the model were a perfect predictor of the effect of MC on property change, all four property estimates would be identical. The maximum absolute difference between the estimates is, therefore, an indication of the performance of the model. The maximum absolute difference is the maximum property estimate minus the minimum property estimate after each of the four values has been adjusted to

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[6] Procedures for estimating $M_p$ for the various types of models are given in Green et al. (1986).

the common MC level. Because no trends were observed in the variation of maximum absolute difference with grade and size, the nine values of maximum absolute difference for individual grade and size combinations were averaged and are reported in Tables 1 and 3.

In general, models in which the coefficients are fitted using all the data are preferred to models in which the coefficients are fitted to each grade and size separately. Although the latter tend to fit the experimental data better, the former are more likely to be adequate for grades and sizes not tested. Also, given the limited number of samples tested for any given combination of grade, size, and MC, a model obtained using all the data is likely to be more stable and less likely to overfit the pecularities of the given data set. Coefficients of each model discussed in this section are given in Tables A-1 to A-6 in the appendix.

### *Modulus of rupture*

With the exception of the zero adjustment model, all the models are more accurate when adjusting to green than to dry MC levels (Table 2). However, models that perform well at one MC level also perform well at other MC levels.

As was also observed for southern pine, neither the constant percentage adjustment models (1, 2, and 3) nor the SR model (10) provided a very accurate adjustment of MOR (Table 3). The results are not surprising since the coefficient of determination, $R^2$, between MOR and SR was only 0.19 using all the data.

Of the 11 models evaluated in this paper, the normal model (7) and the two Weibull models (8 and 9) consistently produced the smallest maximum absolute difference at the mean with only minor differences between the three models (Table 1). The fixed and nonfixed surface models (4, 5, and 6) were usually best at the 5th percentile.

Either the two- or three-parameter Weibull model is generally best for adjusting the entire distribution of MOR (Table 3). Although there is little difference between the normal and the Weibull models at the mean, the Weibull models are generally better throughout the rest of the distribution and especially at the 5th percentile (Tables 1 and 3). Except at the 5th percentile, the fixed and unfixed surface models generally produce larger average maximum absolute differences than the Weibull models.

Comparison of the two- and three-parameter Weibull models indicates little difference in performance except at the 5th and 95th percentiles (Tables 1 and 3). Because the location parameter to be used with the three-parameter Weibull MC adjustment model must be selected in a somewhat arbitrary manner, this model may be overfitted to the pecularities of this particular data set. Until more experience is gained with fitting this model to other data sets, the two-parameter model (9) is recommended for the adjustment of MOR distributions.

A direct comparison between the performance of the two-parameter Weibull model and the performance of the other models is shown in Table 4. The numbers in the body of the table indicate the difference between the average maximum absolute value for the two-parameter Weibull and each of the other models. Positive values indicate that the two-parameter Weibull provides the best estimate; negative values otherwise.

Although the maximum absolute difference generally provides the better esti-

TABLE 3. *Comparison of moisture content adjustment models for the properties of Douglas-fir adjusted to a moisture content of 15%.*

| Model number | Model type | Variation | Dependence on grade and size | Number of coefficients | Average maximum absolute difference at indicated point in cumulative frequency distribution[1] | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Mean | 5 | 25 | 50 | 75 | 95 |
| | | | | | psi | | | | | |
| | | | Modulus of rupture, MOR | | | | | | | |
| 0 | Zero adjustment | | Independent | 0 | 2,277 | 727 | 1,662 | 2,201 | 3,182 | 4,075 |
| 1 | Constant percentage | Linear | By grade and size | 18 | 577 | 636 | 627 | 800 | 1,087 | 1,379 |
| 2 | | | By grade | 6 | 686 | 637 | 650 | 896 | 1,137 | 1,563 |
| 3 | | | Independent | 2 | 833 | 586 | 743 | 1,107 | 1,323 | 1,512 |
| 4 | Surface | Linear, regression | Function | 10 | 690 | 427 | 649 | 889 | 985 | 1,159 |
| 5 | | Quadratic, regression | Function | 30 | 523 | 509 | 607 | 667 | 770 | 915 |
| 6 | | Quadratic, regression (fixed) | Function | 8 | 546 | 516 | 605 | 701 | 793 | 989 |
| 7 | Normal | Regression | Function | 25 | 409 | 724 | 557 | 643 | 682 | 1,007 |
| 8 | Weibull | 3-P, regression | Function | 35 | 416 | 479 | 543 | 645 | 693 | 934 |
| 9 | | 2-P, regression | Function | 22 | 411 | 555 | 551 | 635 | 720 | 874 |
| 10 | Strength ratio | Linear, 0% cutoff | Independent | | 761 | 533 | 727 | 932 | 1,250 | 1,506 |
| | | | | | $10^6$ psi | | | | | |
| | | | Modulus of elasticity, MOE | | | | | | | |
| 0 | Zero adjustment | | Independent | 0 | 0.268 | 0.162 | 0.206 | 0.262 | 0.303 | 0.432 |
| 1 | Constant percentage | Linear | By grade and size | 18 | 0.044 | 0.053 | 0.056 | 0.050 | 0.048 | 0.097 |
| 2 | | | By grade | 6 | 0.050 | 0.057 | 0.063 | 0.058 | 0.061 | 0.094 |
| 3 | | | Independent | 2 | 0.050 | 0.059 | 0.065 | 0.057 | 0.065 | 0.097 |
| 4 | Surface | Linear, regression | Function | 10 | 0.045 | 0.060 | 0.065 | 0.055 | 0.064 | 0.083 |
| 5 | | Quadratic, regression | Function | 30 | 0.042 | 0.064 | 0.063 | 0.051 | 0.062 | 0.082 |
| 6 | | Quadratic, regression (fixed) | Function | 8 | 0.041 | 0.063 | 0.062 | 0.051 | 0.068 | 0.083 |
| 7 | Normal | Regression | Function | 27 | 0.034 | 0.056 | 0.052 | 0.050 | 0.053 | 0.074 |
| 9 | Weibull | 2-P, regression | Function | 23 | 0.037 | 0.057 | 0.053 | 0.052 | 0.060 | 0.077 |

TABLE 3. *Continued.*

| Model number | Model type | Variation | Dependence on grade and size | Number of coefficients | Average maximum absolute difference at indicated point in cumulative frequency distribution[1] | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Mean | 5 | 25 | 50 | 75 | 95 |
| | | | | | in·lb | | | | | |
| | | | Moment capacity, RS | | | | | | | |
| 0 | Zero adjustment | | Independent | 0 | 12,819 | 4,044 | 9,192 | 12,225 | 19,938 | 22,607 |
| 5 | Surface | Quadratic, regression | Function | 30 | 4,444 | 4,587 | 5,100 | 6,595 | 7,082 | 8,733 |
| 6 | | Quadratic, regression (fixed) | Function | 8 | 4,066 | 3,829 | 4,513 | 4,986 | 5,891 | 7,750 |
| 7 | Normal | Regression | Function | 24 | 3,694 | 5,009 | 3,977 | 5,568 | 6,342 | 7,933 |
| 9 | Weibull | 2-P, regression | Function | 24 | 4,087 | 3,851 | 3,713 | 5,832 | 6,517 | 7,842 |
| | | | | | $10^6$ lb·in | | | | | |
| | | | Flexural stiffness, EI | | | | | | | |
| 0 | Zero adjustment | | Independent | 0 | 2.326 | 1.846 | 2.497 | 2.432 | 2.766 | 2.919 |
| 5 | Surface | Quadratic, regression | Function | 30 | 0.750 | 1.597 | 1.379 | 0.787 | 1.335 | 1.676 |
| 6 | | Quadratic, regression (fixed) | | 8 | 0.812 | 1.463 | 1.299 | 0.902 | 1.336 | 1.754 |
| 7 | Normal | Regression | Function | 28 | 0.501 | 1.467 | 1.178 | 0.826 | 0.979 | 1.094 |
| 9 | Weibull | 2-P, regression | Function | 25 | 0.543 | 1.420 | 1.216 | 0.837 | 0.903 | 1.165 |

[1] Based on the average of each of the 9 grade and size combinations at the indicated percentile (or mean) level.

TABLE 4. *Performance of moisture content adjustment models compared to the two-parameter Weibull model (model 9) for modulus of rupture adjusted to a moisture content of 15%.*

| Model number | Model type | Variation | Dependence on grade and size | Number of coefficients | Average maximum absolute difference (model X − model 9) at indicated level of cumulative frequency distribution,[1] psi | | | | | | Average of 5 through 95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Mean | 5 | 25 | 50 | 75 | 95 | |
| | | | | | psi | | | | | | |
| | | | Modulus of rupture, MOR | | | | | | | | |
| 0 | Zero adjustment | | Independent | 0 | 1,866 | 172 | 1,111 | 1,566 | 2,462 | 3,201 | 1,702 |
| 1 | Constant percentage | Linear | By grade and size | 18 | 166 | 81 | 76 | 165 | 367 | 505 | 239 |
| 2 | | | By grade | 6 | 275 | 82 | 99 | 261 | 417 | 689 | 310 |
| 3 | | | Independent | 2 | 422 | 31 | 192 | 472 | 603 | 638 | 387 |
| 4 | Surface | Linear, regression | Independent | 10 | 279 | −128 | 98 | 254 | 265 | 285 | 155 |
| 5 | | Quadratic, regression | Independent | 30 | 112 | −47 | 56 | 32 | 50 | 41 | 26 |
| 6 | | Quadratic, regression (fixed) | Independent | 8 | 135 | −39 | 54 | 66 | 73 | 115 | 54 |
| 7 | Normal | Regression | Function | 25 | −2 | 243 | 184 | 240 | 269 | 272 | 144 |
| 8 | Weibull | 3-P, regression | Function | 35 | 5 | −76 | −8 | 10 | −27 | 60 | −8 |
| 10 | Strength ratio | Linear, 0% cutoff | Independent | 15 | 350 | −22 | 176 | 297 | 530 | 632 | 323 |
| | | | | | $10^6$ psi | | | | | | |
| | | | Modulus of elasticity, MOE | | | | | | | | |
| 0 | Zero adjustment | | Independent | 0 | 0.231 | 0.105 | 0.153 | 0.210 | 0.243 | 0.355 | 0.213 |
| 1 | Constant percentage | Linear | By grade and size | 18 | 0.007 | −0.004 | 0.003 | −0.002 | −0.012 | 0.020 | 0.001 |
| 2 | | | By grade | 6 | 0.012 | 0.000 | 0.010 | 0.006 | 0.001 | 0.017 | 0.007 |
| 3 | | | Independent | 2 | 0.013 | 0.002 | 0.012 | 0.005 | 0.005 | 0.020 | 0.009 |
| 4 | Surface | Linear, regression | Function | 10 | 0.008 | 0.003 | 0.012 | 0.003 | 0.004 | 0.006 | 0.006 |
| 5 | | Quadratic, regression | Function | 30 | 0.005 | 0.007 | 0.010 | −0.001 | 0.002 | 0.005 | 0.005 |
| 6 | | Quadratic, regression (fixed) | Function | 8 | 0.004 | 0.006 | 0.009 | −0.001 | 0.008 | 0.006 | 0.006 |
| 7 | Normal | Regression | Function | 27 | −0.003 | −0.001 | −0.001 | −0.002 | −0.007 | −0.003 | −0.003 |
| 10 | Strength ratio | Linear, 0% cutoff | Independent | 15 | 0.067 | 0.018 | 0.043 | 0.052 | 0.051 | 0.104 | 0.054 |

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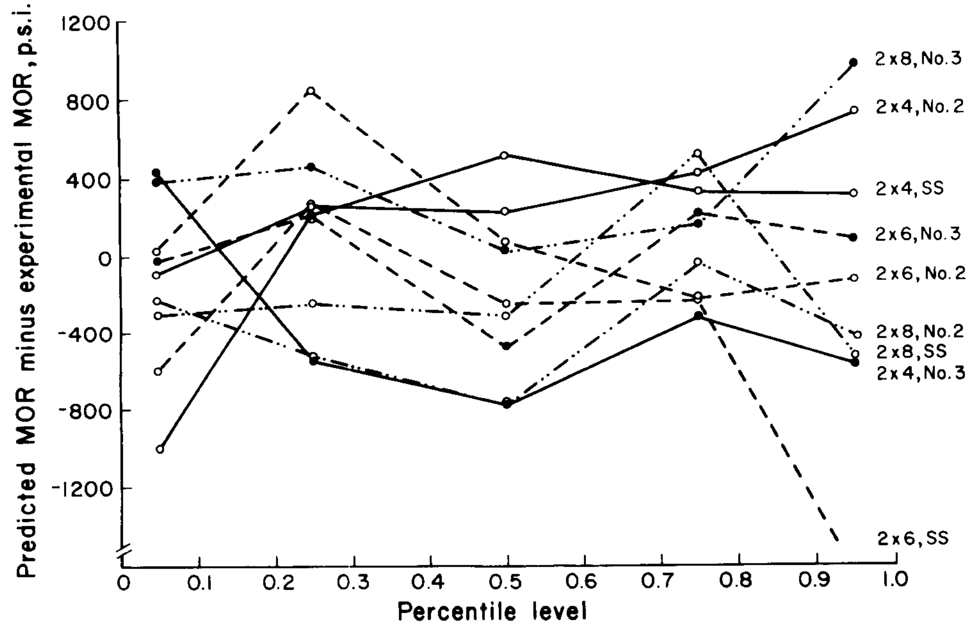
[1] Based on the average of each of the 9 grade and size combinations at the indicated percentile (or mean) level. Positive values indicate that the two-parameter Weibull provides the best estimate; negative values otherwise.



ML86 5357

FIG. 3. Differences at various percentile levels between MOR tested at 10% MC and MOR predicted by adjusting green MORs to 10% MC. (ML86 5357)

mate of performance, a comparison between experimental and predicted MOR values for a given change in MC is also of interest. Such a comparison is shown in Fig. 3 where the MOR's measured in the green condition were adjusted to an MC of 10% using the two-parameter Weibull model (9). The adjusted values, when compared to the MOR's measured at 10%, indicate that the difference is not a function of the position in the MOR cumulative frequency distribution.

In a previous paper, Madsen (1982) suggested that for different MC levels "changes in strength are minor for design purposes and that the same bending stresses should be used for dry and wet service conditions." Figure 4 compares the dry : green ratio predicted by the two-parameter Weibull model (9) for No. 2, 2 × 6, to those obtained by Madsen for Douglas-fir, No. 2 and Better, 2 × 6.[6] From these plots it is obvious that the changes in strength are significant.

### *Modulus of elasticity*

Modulus of elasticity tends to be normally distributed (Aplin et al. 1986; McLain et al. 1984). Also, the effect of MC is not very dependent upon lumber quality (or position in the MOE cumulative frequency distribution) (Green 1982; Green et al. 1986). For these reasons, several of the models gave a satisfactory fit to the data (Tables 1 and 3). The normal model (7) provides the best overall fit, the two-parameter Weibull model (9) being essentially as good (Table 4). Even the simplistic constant percentage adjustment models (1, 2, and 3) provide a reasonable fit to the data.

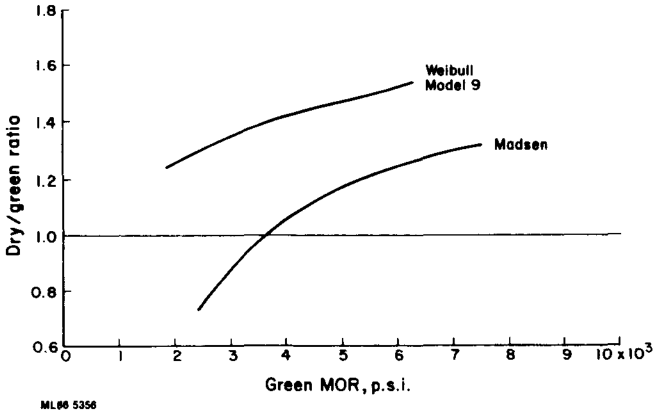


FIG. 4. Dry: green ratio for 2 × 6 Douglas-fir dimension lumber dried to an MC of 15%. (ML86 5356)

### *Moment capacity and stiffness*

As noted with southern pine (Green et al. 1986), the type of model that best fits the MOR/MOE data generally provides the best fit for RS/EI. For this reason, only the two quadratic surface models (5 and 6), the normal model (7), the two-parameter Weibull model (9), and the zero adjustment model (for comparison) are shown in Tables 1 and 3. In general, the normal model tends to give a slightly better fit in the middle of the cumulative frequency distribution, while the two-parameter Weibull fits better in the tails.[7]

## MISUSE OF THE MODELS

The analytical models were developed to predict the effect of changes in MC on the strength of Douglas-fir dimension lumber and are unsuitable for predicting absolute values of properties. A fundamental assumption of the models developed is that changes in properties with change in MC are relatively insensitive to the particular geographic location from which the lumber was sampled. The absolute magnitude of the properties at a given MC may vary considerably from sample to sample.

Care should be exercised when applying the equations to lumber properties or MC levels outside the range of data used to establish the coefficients. It is our experience that a failure to place limits on the use of these equations may lead to unrealistic or even illogical results. We do not recommend that these equations be used for MC less than 8%.

The MOR and MOE limits were established by comparing trends predicted using the models with actual trends observed near the extremes of the data. These limits are given in Table A-7. Applicable limits for EI and RS may be determined by appropriate scaling of the MOR and MOE limits.

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[7] Because of the effect of specimen width on the magnitude of EI and RS, the average values of the maximum absolute difference will be dominated by the values for the wider widths. Therefore, the values of the average maximum absolute error given in Tables 2 and 3 should only be used to compare models.

## CONCLUSIONS

Of the models evaluated in this study we conclude:

1. Neither of the model types used in the past (constant percentage adjustment and SR) provides an acceptable adjustment for the effect of MC on MOR.
2. The analytical model based on the Weibull distribution with parameters determined by a regression procedure (models 8 and 9) provides the best overall fit to the MOR and RS data. Although the three-parameter version (model 8) is slightly more accurate than the two-parameter version (model 9), we lack sufficient experience with similar data sets for other species to justify the use of the location parameters used in this study. Therefore, the two-parameter version is recommended for adjusting lumber strength distributions.
3. The model obtained by fitting a normal distribution to the data and predicting distribution parameters through regression (model 7) provides the best fit to the MOE and EI data. However, all models except the SR model give reasonable adjustments for MOE and EI.

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## APPENDIX

Tables A-1 through A-6 list the coefficients to be used with most of the models presented in this paper. For all the models given in this paper, except the Weibull and normal models given in Tables A-3 and A-4, MOR and RS are expressed in units of 1 psi, MOE and EI as $10^6$ psi. The Weibull and normal models are expressed in $10^3$ psi. Moisture content ($M$, $M_p$) is expressed as a percent. In models 7, 8, and 9 (Tables A-3 and A-4), the specimen width (W) is assumed to be the standard dressed dry width (3.5, 5.5, 7.25, etc.), and the strength ratio (SR) is assumed to be the minimum SR for the structural light-framing grades (26, 45, 55, 65). (The SR for Select Structural 2 × 4s is actually 67 but 65 is assumed in this report.)

TABLE A-1. *Coefficients of the linear constant percentage adjustment model.*[1]

| Model number | Grade | Nominal width | MOR | | MOE | |
|---|---|---|---|---|---|---|
| | | | a | b | a | b |
| 1 | Select structural | 4 | 14,450.3 | −316.33 | 1.9175 | −0.02074 |
| 1 | | 6 | 12,810.2 | −241.09 | 2.1818 | −0.02122 |
| 1 | | 8 | 10,813.8 | −208.81 | 2.0386 | −0.02410 |
| 1 | No. 2 | 4 | 9,404.5 | −179.02 | 1.6421 | −0.02060 |
| 1 | | 6 | 7,596.1 | −138.55 | 1.6020 | −0.01771 |
| 1 | | 8 | 6,303.4 | −85.22 | 1.5339 | −0.01691 |
| 1 | No. 3 | 4 | 6,312.1 | −75.59 | 1.4363 | −0.01889 |
| 1 | | 6 | 5,503.1 | −72.63 | 1.3673 | −0.01462 |
| 1 | | 8 | 5,392.6 | −81.49 | 1.3712 | −0.01639 |
| 2 | Select structural | All | 12,590.38 | −250.4762 | 2.056623 | −0.022753 |
| 2 | No. 2 | All | 7,691.10 | −130.0149 | 1.589974 | −0.018285 |
| 2 | No. 3 | All | 5,595.99 | −68.1797 | 1.390843 | −0.016645 |
| 3 | All | All | 8,601.64 | −148.02 | 1.67839 | −0.01916 |

[1] Equation (3).

TABLE A-2. *Regression coefficients of (nonfixed) quadratic surface models (model 5).*[1]

| Variable | MOR[2] | | MOE[3] | |
|---|---|---|---|---|
| | b | c | b | c |
| Constant | −0.7697463E+04 | 0.1672728E+03 | 0.7563579E+00 | −0.2390278E−01 |
| $M_1$ | 0.1937799E+04 | −0.3922224E+02 | −0.1138130E+00 | 0.4117220E−02 |
| $P_1$ | 0.7071369E+00 | −0.2952671E−01 | −0.8844447E+00 | 0.2106855E−01 |
| $M_1^2$ | −0.1808354E+03 | 0.3448722E+01 | 0.905915E−02 | −0.34826E−03 |
| $P_1^2$ | −0.3206511E−04 | 0.14691064E−05 | 0.9270093E+00 | −0.2376689E−01 |
| $M_1P_1$ | −0.9667900E−01 | 0.417799E−02 | 0.1041533E−01 | 0.5917250E−05 |
| $M_1^3$ | 0.7098496E+01 | −0.1273483E+00 | −0.35143E−03 | 0.1378824E−04 |
| $P_1^3$ | 0.16361676E−08 | −0.5855939E−10 | −0.3623098E+00 | 0.9371550E−02 |
| $M_1^2P_1$ | 0.700956E−02 | −0.26101E−03 | 0.1226970E−02 | −0.4624055E−04 |
| $P_1^2M_1$ | −0.1005114E−05 | −0.4967575E−07 | −0.2322769E−01 | 0.56063E−03 |
| $M_1^4$ | −0.1001912E+00 | 0.168830E−02 | 0.5088906E−05 | −0.2017113E−06 |
| $P_1^4$ | −0.5885839E−13 | 0.1534856E−14 | 0.4914017E−01 | −0.1281570E−02 |
| $M_1P_1^3$ | 0.9710008E−10 | −0.9744899E−12 | 0.5164390E−02 | −0.12762E−03 |
| $M_1^2P_1^2$ | −0.5035050E−07 | 0.2815968E−08 | 0.7217547E−04 | −0.1265619E−05 |
| $M_1^3P_1$ | −0.14130E−03 | 0.4870561E−05 | −0.2843562E−04 | 0.991323E−06 |
| Constant | −0.2339686E+02 | 0.3193140E+00 | 0.4613937E+05 | −0.1098555E+04 |
| $M_1$ | 0.5973324E+01 | −0.8235865E−01 | −0.1204662E+05 | 0.2923694E+03 |
| $P_1$ | −0.6011880E−02 | 0.67865E−03 | 0.3690677E+00 | −0.1413041E−01 |
| $M_1^2$ | −0.5425985E+00 | 0.7438140E−02 | 0.1157441E+04 | −0.2822527E+02 |
| $P_1^2$ | 0.1193750E−02 | −0.3411746E−04 | −0.2882066E−05 | 0.1394021E−06 |
| $M_1P_1$ | −0.1050260E−01 | 0.17790E−03 | −0.5580503E−01 | 0.1839680E−02 |
| $M_1^3$ | 0.2110891E−01 | −0.28691E−03 | −0.4752492E+02 | 0.1158631E+01 |
| $P_1^3$ | −0.1141844E−04 | 0.3525323E−06 | 0.7590174E−11 | −0.7075964E−12 |
| $M_1^2P_1$ | 0.80272E−03 | −0.1410402E−04 | 0.2609380E−02 | −0.8345374E−04 |
| $P_1^2M_1$ | −0.5836668E−04 | 0.1239208E−05 | 0.3229176E−06 | −0.1095333E−07 |
| $M_1^4$ | −0.298730E−03 | 0.40192248E−05 | 0.7065035E+00 | −0.1719949E−01 |
| $P_1^4$ | 0.5946199E−07 | −0.1830510E−08 | −0.1212617E−16 | 0.1675731E−17 |
| $M_1P_1^3$ | 0.2747526E−06 | −0.6371371E−08 | −0.5211465E−12 | 0.2212559E−13 |
| $M_1^2P_1^2$ | 0.6522413E−06 | −0.1404858E−07 | −0.7733530E−08 | 0.2266034E−09 |
| $M_1^3P_1$ | −0.1690938E−04 | 0.30244268E−06 | −0.3856020E−04 | 0.1239378E−05 |

[1] Equation (5) of text.

[1] In psi.

[3] In $10^6$ psi.

TABLE A-3. *Regression coefficients to be used with normal and Weibull models for MOR and MOE.*

| | Two-parameter Weibull (model 9)[1] | | | | Three-parameter Weibull (model 8)[2] |
|---|---|---|---|---|---|
| | MOR[3] | | MOE[4] | | MOR[3] |
| Variable[1] | Shape, M | Scale, ω | Shape, M | Scale, ω | Shape, M |
| Constant | 0.95990E+01 | 0.54270E+01 | −0.11947E+02 | −0.45279E+00 | −0.54112E+01 |
| W | −0.45489E−01 | 0.38492E+01 | 0.62989E+01 | 0.58613E+00 | 0.50809E+01 |
| SR | −0.11587E+00 | 0.13688E+00 | 0.84498E+00 | 0.93137E−01 | 0.72533E+00 |
| $M_1$ | −0.11224E+01 | −0.10376E+01 | 0 | 0.63903E−01 | −0.10456E+01 |
| $(W)^2$ | 0 | −0.36469E+00 | −0.57900E+00 | −0.49959E−01 | −0.46670E+00 |
| $(SR)^2$ | 0.18427E−02 | 0 | −0.92431E−02 | −0.12810E−02 | −0.86848E−02 |
| $M_1^2$ | 0.75053E−01 | 0.27434E−01 | 0 | −0.52950E−02 | 0.69495E−01 |
| W(SR) | 0 | −0.30331E+00 | −0.32874E+00 | −0.35680E−01 | −0.29549E+00 |
| $W(SR)^2$ | 0 | 0.39487E−02 | 0.36436E−02 | 0.56389E−03 | 0.36732E−02 |
| $(W)^2(SR)$ | 0 | 0.26499E−01 | 0.29959E−01 | 0.30200E−02 | 0.26272E−01 |
| $(W)^2(SR)^2$ | 0 | −0.36134E−03 | −0.32636E−03 | 0.48760E−04 | −0.32546E−03 |
| $W(M_1)^2$ | 0 | 0 | 0.13638E−03 | 0 | 0 |
| $(W)(M_1)$ | 0 | 0 | 0 | 0 | 0.45144E−02 |
| $(SR)(M_1)$ | 0 | 0.70987E−01 | 0 | −0.13887E−03 | −0.80463E−03 |
| $(M_1)^3$ | −0.15075E−02 | 0 | 0 | 0.10986E−03 | −0.14128E−02 |
| $(SR)(M_1)^2$ | 0 | −0.20777E−02 | 0 | 0 | 0 |
| $W(SR)(M_1)$ | 0 | 0.43511E−03 | 0 | 0 | 0 |
| $(SR)^2(M_1)$ | 0 | −0.94275E−03 | 0 | 0 | 0 |
| $(W)^2(M_1)$ | 0 | 0 | 0 | 0 | 0 |
| $(SR)^2(M_1)^2$ | 0 | 0.25252E−04 | 0 | 0 | 0 |
| $(W)^2(M_1)^2$ | 0 | 0 | 0 | 0 | 0 |

TABLE A-3. *Continued.*

| Variable[1] | Three-parameter Weibull (model 8)[2] | Normal (model 7) | | | |
|---|---|---|---|---|---|
| | MOR[3] | MOR[3] | | MOE[4] | |
| | Scale, $\omega$ | Mean, $\bar{x}$ | Standard deviation, S | Mean, $\bar{x}$ | Standard deviation, S |
| Constant | −0.46134E+01 | 0.41601E+01 | 0.20322E+01 | −0.70262E+00 | −0.87410E−01 |
| W | 0.67475E+01 | 0.35300E+01 | −0.10199E+01 | 0.66508E+00 | 0.20859E+00 |
| SR | 0.63672E+00 | 0.13700E+00 | 0 | 0.10276E+00 | −0.81839E−02 |
| $M_1$ | −0.10768E−01 | −0.86352E+00 | 0.47008E+00 | 0.49413E−01 | 0.76781E−01 |
| $(W)^2$ | −0.57317E+00 | −0.33381E+00 | 0.88150E−01 | 0.58769E 01 | −0.19769E−01 |
| $(SR)^2$ | −0.68442E−02 | 0 | −0.3803E−03 | −0.13953E−02 | 0.38585E−04 |
| $M_1{}^2$ | 0.31177E−01 | 0.22914E−01 | −0.31081E−01 | −0.45713E−02 | −0.23567E−02 |
| W(SR) | −0.46079E+00 | −0.27410E+00 | 0.19693E−01 | 0.39874E−01 | 0.27204E−02 |
| $W(SR)^2$ | 0.59880E−02 | 0.35053E−02 | 0 | 0.60391E−03 | 0 |
| $(W)^2(SR)$ | 0.38465E−01 | 0.23919E−01 | −0.18626E−02 | 0.34526E−02 | −0.27128E−03 |
| $(W)^2(SR)^2$ | −0.52078E−03 | −0.32072E−03 | 0 | −0.52417E−04 | 0 |
| $W(M_1)^2$ | 0 | 0 | 0 | 0 | 0.10108E−02 |
| $(W)(M_1)$ | −0.16351E−01 | 0 | 0 | 0.96801E−03 | −0.35165E−01 |
| $(SR)(M_1)$ | 0.75538E−01 | 0.61020E−01 | −0.58373E−03 | 0 | 0 |
| $(M_1)^3$ | 0 | 0 | 0.58521E−03 | 0.95732E−04 | 0 |
| $(SR)(M_1)^2$ | −0.22569E−02 | −0.17886E−02 | 0 | 0 | 0 |
| $W(SR)(M_1)$ | 0.76219E−03 | 0.39894E−03 | 0 | −0.26215E−04 | 0 |
| $(SR)^2(M_1)$ | −0.10008E−02 | −0.80874E−03 | 0 | 0 | 0 |
| $(W)^2(M_1)$ | 0 | 0 | 0 | 0 | 0.33380E−02 |
| $(SR)^2(M_1)^2$ | 0.26900E−04 | 0.21579E−04 | 0 | 0 | 0 |
| $(W)^2(M_1)^2$ | 0 | 0 | 0 | 0 | −0.96428E−04 |

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[1] Equation (10) with location parameters ($\ell_1$ and $\ell_2$) = 0. Shape (M) and scale ($\omega$) parameters expressed as parameter = $a_0 + a_1(W) + a_2(SR) + \ldots + a_{20}(W)^2(SR)^2$. M = moisture content, %. W = standard dressed dry widths, 3.5, 5.5, 7.25, etc. SR = assumed minimum strength ratio for the grade, select structural = 65, No. 1 = 55, No. 2 = 45, No. 3 = 26, etc.

[2] Location parameters, $\ell$, are 2 × 4 SS = 3.200, 2 × 6 and 2 × 8 SS = 2.200, all No. 2 = 1,000 ksi and all No. 3 = 700 ksi.

[3] In $10^3$ psi.

[4] In $10^6$ psi.

TABLE A-4. *Coefficients to be used with normal and Weibull models for RS and EI.*

| | Two-parameter Weibull (model 9)[1] | | | |
|---|---|---|---|---|
| | RS[2] | | EI[3] | |
| Variable | Shape, M | Scale, $\omega$ | Shape, M | Scale, $\omega$ |
| Constant | 0.74936E+01 | 0.85539E+02 | −0.13768E+02 | 0.26771E+02 |
| W | 0.106475E+01 | −0.38568E+01 | 0.71750E+01 | −0.16962E+02 |
| SR | −0.11813E+00 | −0.62195E+01 | 0.95927E+00 | 0.63778E+00 |
| M | −0.99738E+00 | −0.69665E+01 | 0 | −0.37136E+00 |
| $(W)^2$ | −0.11052E+00 | 0.18675E+01 | −0.66776E+00 | 0.33342E+01 |
| $(SR)^2$ | 0.18847E−02 | 0.59482E−01 | −0.10198E−01 | −0.10703E−01 |
| $(M)^2$ | 0.77591E−01 | 0 | 0 | −0.14143E−01 |
| W(SR) | 0 | 0 | −0.37551E+00 | −0.20645E+00 |
| $W(SR)^2$ | 0 | 0.55665E−02 | 0.41275E−02 | 0.31682E−02 |
| $(W)^2(SR)$ | 0 | −0.18383E−01 | 0.34478E−01 | 0.75496E−02 |
| $(W)^2(SR)^2$ | 0 | 0 | −0.37318E−03 | 0 |
| $W(M)^2$ | 0 | 0 | 0 | 0.50787E−02 |
| (W)(M) | −0.17642E−01 | 0 | 0.50338E+02 | 0.19963E+00 |
| (SR)(M) | 0 | 0.78505E+00 | −0.87914E−03 | 0.10702E−02 |
| $(M)^3$ | −0.15523E−02 | 0.65916E−02 | 0 | 0 |
| $(SR)(M)^2$ | 0 | −0.21163E−01 | 0 | 0 |
| (W)(SR)(M) | 0 | −0.56632E−02 | 0 | 0 |
| $(SR)^2(M)$ | 0 | −0.88122E−02 | 0 | 0 |
| $(W)^2(M)$ | 0.74768E−02 | 0 | 0 | −0.43735E−01 |
| $(SR)^2(M)^2$ | 0 | 0.23945E−03 | 0 | 0 |
| $(W)^2(M)^2$ | 0 | 0 | 0 | 0 |

TABLE A-4. *Extended.*

| Normal (model 7) | | | |
|---|---|---|---|
| RS[2] | | EI[3] | |
| Mean, $\bar{x}$ | Standard deviation, S | Mean, $\bar{x}$ | Standard deviation, S |
| 0.43104E+02 | −0.42227E+02 | 0.12420E+02 | 0.10051E+02 |
| 0.90883E+01 | 0 | −0.14711E+02 | −0.55373E+01 |
| 0.10166E+01 | 0 | 0.11472E+01 | 0 |
| −0.87577E+01 | 0.76124E+01 | 0.10501E+01 | −0.23393E+00 |
| 0 | 0.64246E−00 | 0.29847E+01 | 0.90306E+00 |
| 0 | 0 | −0.15794E−01 | −0.16236E−02 |
| 0.23300E+00 | −0.45185E+00 | −0.52453E−01 | 0 |
| −0.10770E+01 | 0.54937E−01 | −0.21260E+00 | 0.18904E−01 |
| 0.18546E−01 | −0.57103E−03 | 0.30015E−02 | 0.43875E−03 |
| 0.85488E−01 | 0 | 0.92176E−02 | −0.52588E−02 |
| −0.12605E−02 | 0 | 0 | 0 |
| −0.16149E−01 | 0 | 0.55919E−02 | 0 |
| 0.57810E+00 | 0 | 0.14621E+00 | 0.97899E−01 |
| 0.44581E+00 | 0 | −0.638045E−01 | 0 |
| 0 | 0.85523E−02 | 0 | 0 |
| −0.11519E−01 | 0 | 0.18606E−02 | 0 |
| −0.52989E−02 | 0 | 0 | 0.21489E−03 |
| −0.51822E−02 | 0 | 0.72559E−03 | 0 |
| 0 | −0.17048E−01 | −0.39671E−01 | −0.11658E−01 |
| 0.13623E−03 | 0 | −0.20954E−04 | 0 |
| 0 | 0 | 0 | 0 |

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[1] Equation (10) with location parameters ($\ell_1$ and $\ell_2$) = 0. Shape (M) and scale ($\omega$) parameters expressed as parameter = $a_0 + a_1(W) + a_2(SR) + \ldots + a_{20}(W)^2(SR)^2$. M = moisture content, %. W = standard dressed dry widths, 3.5, 5.5, 7.25, etc. SR = assumed minimum strength ratio for the grade, select structural = 65, No. 1 = 55, No. 2 = 45, No. 3 = 26, etc.

[2] In $10^3$ psi.

[3] In $10^6$ psi.

TABLE A-5. *Regression coefficients of fixed quadratic surface models (model 6).*

| Coefficient | MOR[1] | MOE[2] | EI[2] | RS[1] |
|---|---|---|---|---|
| $D_0$ | −0.36161121E+03 | −0.7400681E−01 | −0.46983780E−01 | −0.37405354E+03 |
| $D_1$ | 0.29592626E+00 | 0.14654033E+00 | 0.10134717E−01 | 0.30897147E+00 |
| $D_2$ | −0.39783127E−04 | −0.12649923E+00 | −0.9818598E−01 | −0.40693328E−04 |
| $D_3$ | 0.12503366E−08 | 0.3051976E−01 | 0.2621110E−01 | 0.13410251E−08 |
| $E_0$ | 0.994437082E+01 | 0.195770E−02 | 0.150273E−02 | 0.10187048E+02 |
| $E_1$ | −0.815532E−02 | −0.414948E−02 | −0.342433E−02 | −0.82443200E−02 |
| $E_2$ | 0.95704266E−06 | 0.324488E−02 | 0.303597E−02 | 0.96345528E−06 |
| $E_3$ | −0.29208423E−10 | −0.78286E−03 | −0.76892E−03 | −0.30486442E−10 |

[1] In psi.

[2] In $10^6$ psi.

TABLE A-6. *Coefficients of linear strength ratio model (model 10).*

| Size | Grade | Coefficient symbol | Coefficient for indicated property[1] | |
|---|---|---|---|---|
| | | | MOR[2] | MOE[3] |
| | | | *psi* | |
| All | All | a | 11,486.116530 | 1.8667432 |
| | | b | −228.150890 | −0.0217115 |

[1] $F^* = (a + bM_2)/(a + bM_1)$ for lumber with SR = 100.

[2] In psi.

[3] In $10^6$ psi.

TABLE A-7. *Recommended property limits for the models presented in this report.*[1]

| Property | Size | Grade | Applicable range at MC = 15% | |
|---|---|---|---|---|
| | | | *psi* | |
| Modulus of rupture | All | Select structural | 1,000 | 12,000 |
| | | No. 2 | 1,000 | 10,000 |
| | | No. 3 | 1,000 | 8,000 |
| Modulus of elasticity | All | Select structural | 200,000 | 2,300,000 |
| | | No. 2 | 200,000 | 1,900,000 |
| | | No. 3 | 200,000 | 1,700,000 |

[1] The recommended limits on moisture content are 8% ≤ MC ≤ 24%.