STRENGTH AND DEFORMATION OF PITCHED-TAPERED DOUGLAS-FIR GLUED-LAMINATED BEAMS

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ABSTRACT

Strength tests were conducted on 12 pitched-tapered beams. Four of these beams failed in radial tension, the estimated maximum stress levels ranging from 176 to 286 psi. The other beams failed in bending at the extreme fiber. In all cases, the load-deflection and load-strain responses were linear to failure under short-term loading conditions. The radial strengths were marginally acceptable at the allowable unit stress level currently assigned in Canada for tension perpendicular-to-the-grain. Other studies on size effect suggest that working stress levels should be governed by the volume of wood subjected to perpendicular-to-glueline tensile stress.

Additional keywords: *Pseudotsuga menziesii*, *Larix occidentalis*, size effect.

INTRODUCTION

An early North American report on design and use of glued-laminated construction was that by Wilson (1939), in which he derived a simple formula for the maximum radial stress developed by bending of curved beams in the plane of curvature. The Wilson formula was confirmed by Norris (1963) as a good estimator of radial stresses for a pure-bending loading condition applied to curved beams. However, such beams had a constant cross section in contrast to the haunch or apex discontinuity exhibited by pitched-tapered beams (Fig. 1), for which there was no stress analysis available.

For pitched-tapered beams, which usually support roofs, several reports have indicated the success of a subsequent stress analysis (Foschi 1968, 1970a, 1971; Foschi and Fox 1970; Fox 1970a, 1970b, 1970c) and its application to structural design (Fox 1970d, 1971). The maximum radial tension stress formula for pitched-tapered beams (Foschi and Fox 1970) has been adopted by several code writing organizations (C.S.A. 1970; N.F.P.A. 1973; I.C.B.O. 1973).

Gopu et al. (1972) tested pitched-tapered beams of southern pine, whereas Kolb (1969) tested curved beams of an unidentified European softwood. Others have carried out related research in which the strength of Douglas-fir laminated blocks in tension perpendicular-to-gluelines has been studied (Thut 1970; Madsen 1972; Fox 1974). A relationship between such blocks and pitched-tapered beams has been proposed by Barrett (1974) based on an application of the size effect theory of Weibull (1939).

Since structural design requires a knowledge of material strength as well as stress distribution, 13 pitched-tapered beams of three configurations were tested to destruction at the Western Forest Products Laboratory. The test results of one beam were reported by Foschi (1971). This report presents an analysis of the test results of the other 12 beams and discusses the radial-tension in-service failure problem as reported by Hanrahan (1966). In this study, all beams were fabricated with Douglas-fir [*Pseudotsuga menziesii* (Mirb.) Franco] and/or western larch [*Larix occidentalis* Nutt.] which is accepted as an equivalent species (C.S.A. 1970).

METHOD

Material

Twelve beams were fabricated in two groups at four factories according to the

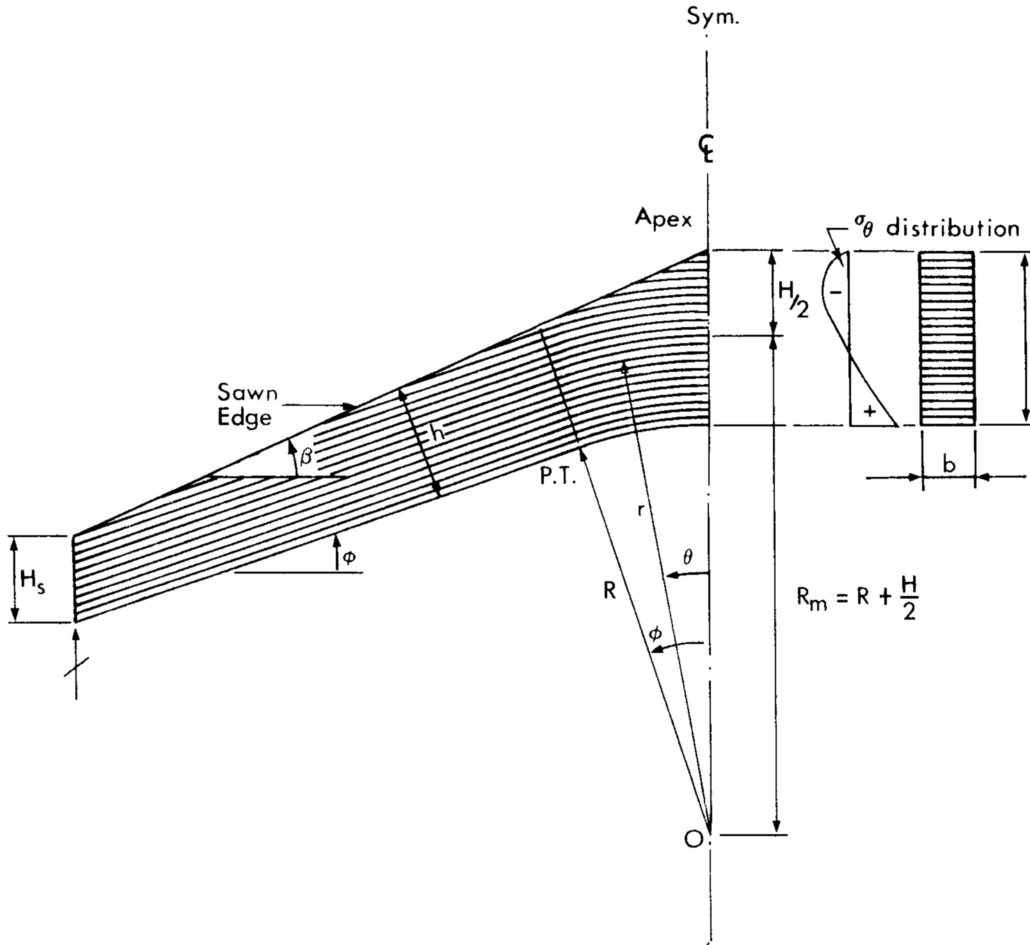

FIG. 1. Geometric and lamination layout.

dimensions of Table 1, which are defined in Fig. 1. All were made according to C.S.A. Standard 0122 (1969) with these group differences: Group 1 laminations were ½ inch thick and were carefully selected for straight edge grain and flat sawing; Group 2 laminations were about ¾ inch thick. Curved members are usually made with nominal 2-inch-thick laminations, but since radial stresses developed in a beam by applied loads are inversely proportional to the beam's radius of curvature, thinner laminations are necessary to promote radial tension mode of failure in relatively small beams. Any effect of kiln drying on inherent strength properties of nominal 2-inch versus 1-inch laminations has not been reported. The Group 1 beams were made for experimental strain analysis; hence a uniform orientation of annual rings was considered to be important. Both finger and hooked-scarf joints were supplied according to factory custom. Prior to destructive testing, Group 1 beams dried during storage time from about 12% moisture content (MC) to about 9% MC. Group 2 beams were tested within two months of their fabrication and were about 12% MC at the time of testing.

Beam histories

As reported by Fox (1970a, 1970c), one Group 1 beam was flexed many times to verify Foschi's analysis (1968, 1970a) of

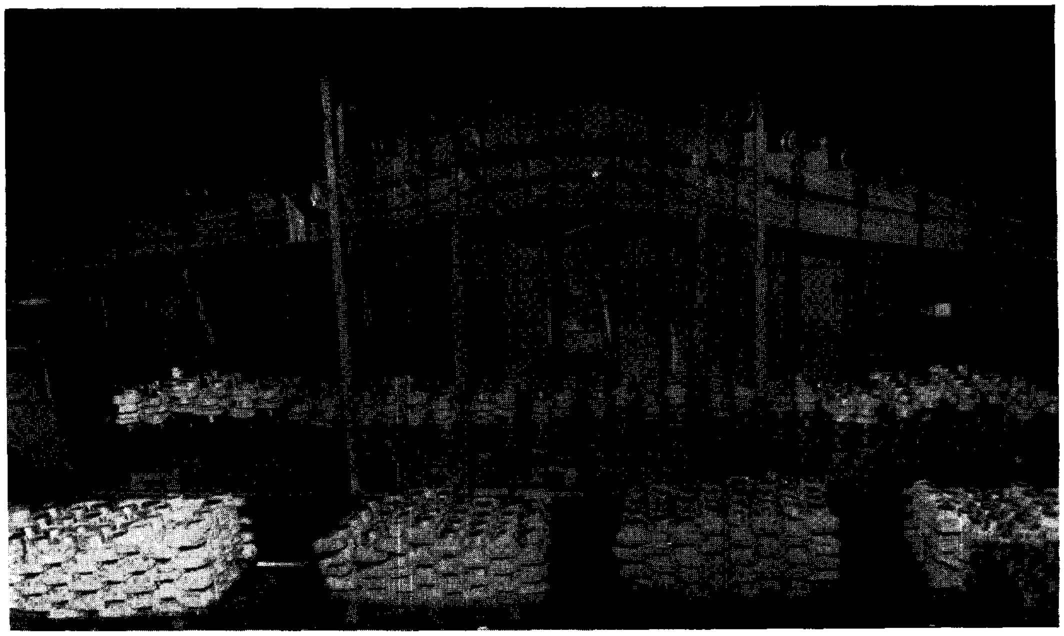

FIG. 2. Test setup and radial tension failure.

pitched-tapered beams. Using this analysis, the maximum radial tensile stress developed in this beam was 152 psi and the maximum tangential or extreme fiber stress was 3860 psi. For the other Group 1 beam, the maximum radial tensile and tangential stresses developed by a constant uniform load for one year were estimated as 140 psi and 3530 psi, respectively. Subsequently, this beam was loaded with lead ingots in an attempt to break it, but insufficient load was available. The maximum radial tensile and tangential stresses generated by this attempt were about 220 psi and 5600 psi, respectively.

The 10 beams of Group 2 had no loading history.

Testing

Each beam end was seated in plaster of paris on a steel rocker plate which was, in turn, supported by a load cell on roller bearings. Thus, each end of a beam was free to move horizontally.

To simulate snow loading, a uniformly distributed load was applied to the beams by means of 25-pound lead ingots placed on hanging platforms. The load was transferred to the top edge of a beam by tie rods and steel cylinders as shown in Fig. 2. Triangular blocks 12 inches long distributed the total load along the top edge. For Group 1 beams, the loading increment was 75 plf applied over 1 min with a 3-min resting period between increments. For

TABLE 1. *Pitched-tapered beam dimensions*

Group	n	Span in.	Roof slope	H in.	R in.	H_s in.	Width b in.	Central loaded length in.	H/R_m
1	2	218	4:12	14.3	69	7.6	4.0	192	0.188
2	10	240	3:12	12.0	112	6.0	3.0	120	0.102

See Fig. 1 for symbol definitions.

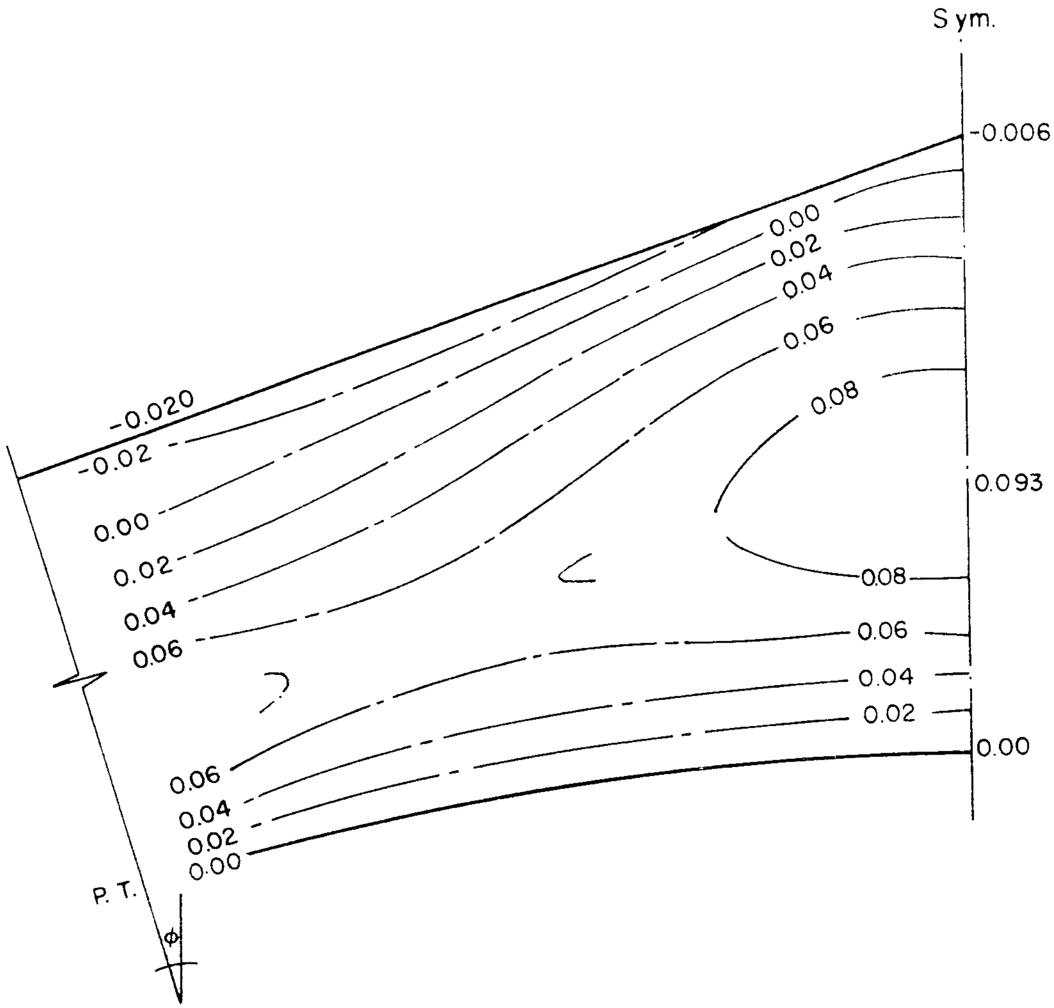

FIG. 3. K_r distribution for symmetric UDL.

Group 2 beams, a slower rate of 25 plf every 8 min was used in accordance with British Standard CP 112, clause 602 (B.S.I. 1967). Four horizontal restraints prevented overturning.

For Group 1 beams, the object of testing was to find the ultimate capacity and deformation of the beams when they were subjected to a uniform load distributed over the central 16 ft of the 18 ft 2 inch span. During the resting period between load increments, span increase and vertical deflection were read as well as surface strain measurements on two cross sections. A data acquisition system was used to scan five electrical-resistance strain gauges, while others were scanned and recorded manually.

Group 2 beams were tested according to British Standard CP 112, clause 602 (B.S.I. 1967) for built-up components of wood. This performance standard requires a *preload* test where "design long-term load" is applied and maintained for 30 min and then released; a *deflection* test where "maximum design load" is applied and maintained for 24 h and then released; and a *strength* test where twice maximum design load is applied for at least 15 min. In this latter test, the required time period

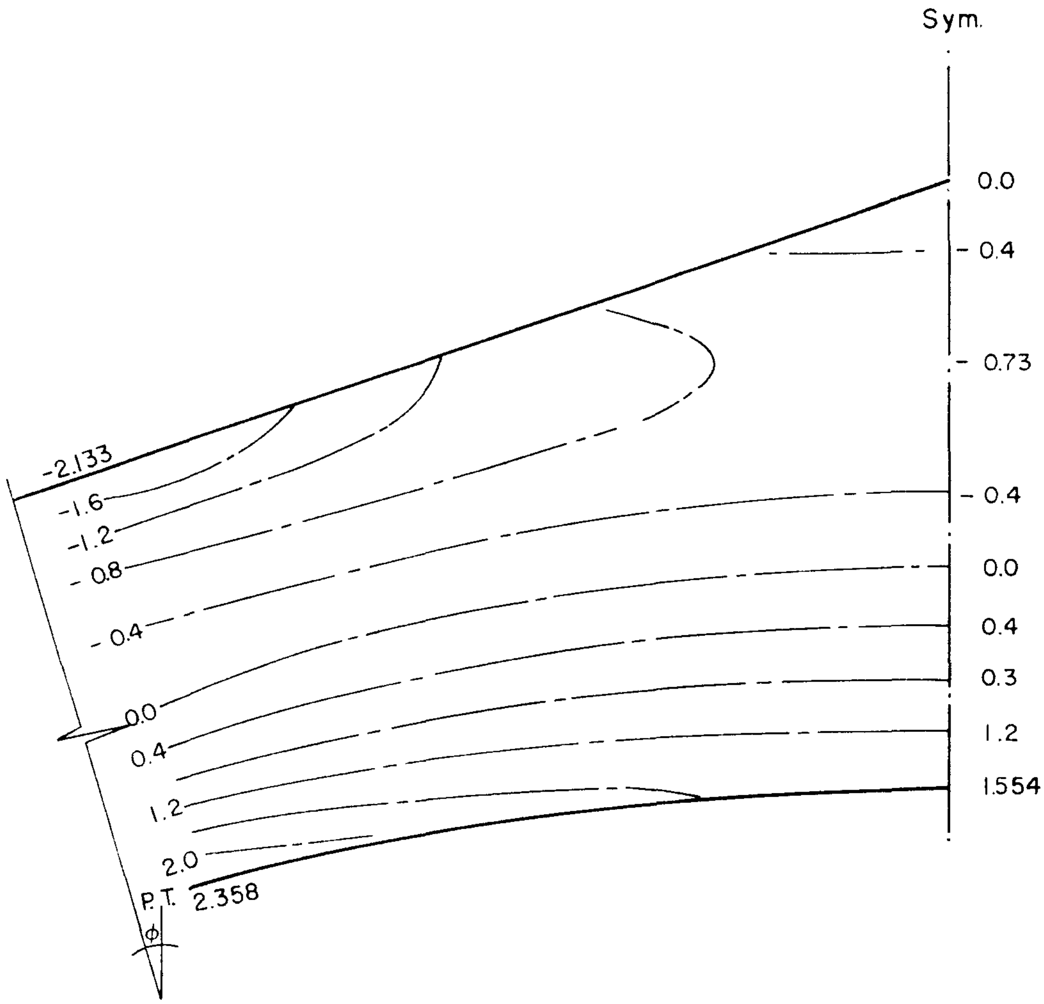

FIG. 4. K_θ distribution for symmetric UDL.

for satisfactory strength was increased to 24 h. The standard suggests that the structure may be loaded to destruction after the latter time period has been exceeded. This complete procedure was followed for one beam of each manufacture. Only the *strength* test and loading to destruction were applied to each of the remaining seven beams. A "maximum design load" of 155 plf was required to develop the Canadian allowable unit stress of 65 psi for radial tension (normal duration of load, dry service) as estimated by formulas of CSA 086 (1970). "Design long-term load" was therefore 10% less or 140 plf applied uniformly. The load was applied to the central 10-ft section of the 20-ft span and, during loading, readings were made of span increase and vertical deflection.

Stress analysis

While according to formulas of CSA 086 (1970), the maximum design load of 155 plf develops a maximum radial tensile stress of 65 psi, there was, in fact, a lesser stress developed in Group 2 beams by applying a uniformly distributed load (UDL). The CSA 086 formula overestimates maximum radial stresses developed

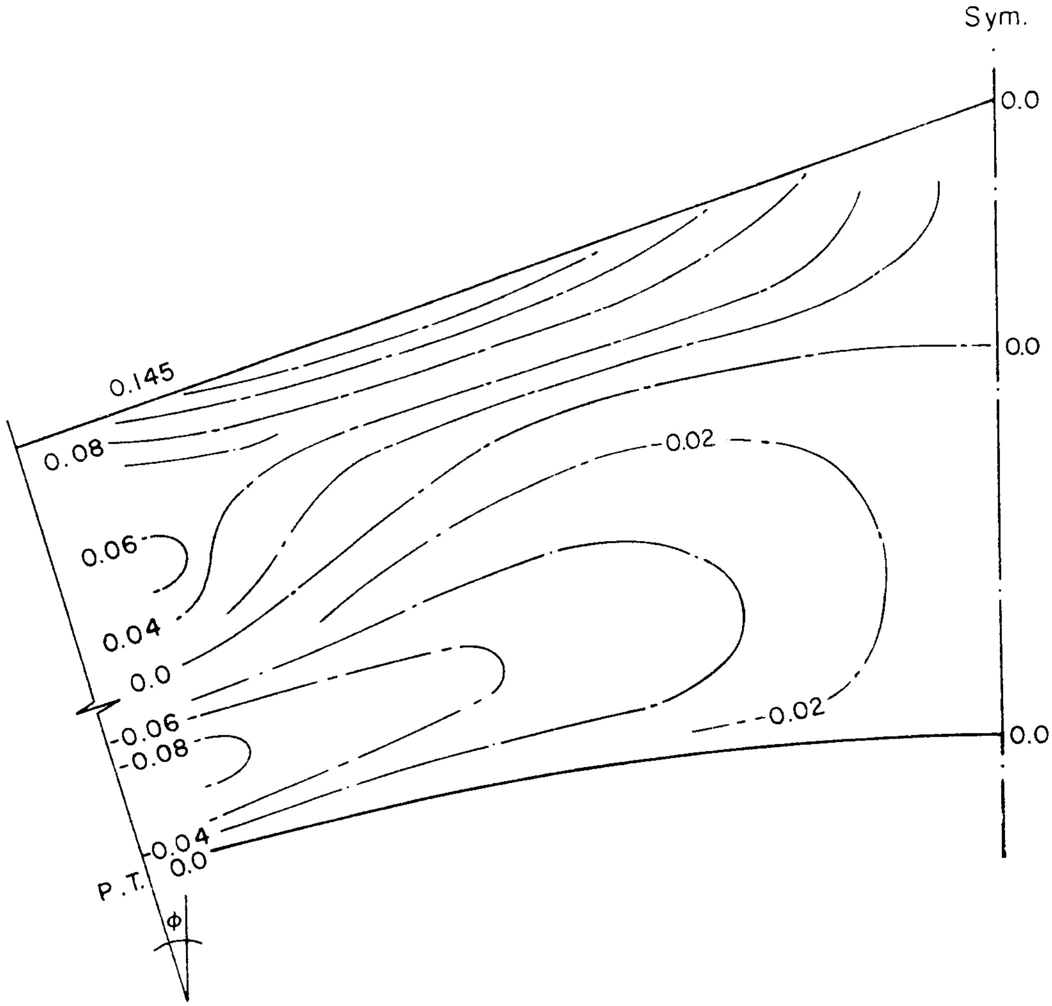

FIG. 5. $K_{r\theta}$ distribution for symmetric UDL.

by a UDL, since the formula was derived for a pure bending condition and was recommended as a conservative estimator for practical situations (Foschi and Fox 1970).

Assuming the load-deformation response of a beam to be linear to failure, the stress analysis program CGLB by Foschi (1970b) may be used to estimate stresses as well as an upper bound for maximum radial stresses developed by a failure load. This analysis may be summarized as

$$\sigma_r = K_r \frac{6M}{bH^2} \qquad (1)$$

where

K_r = radial stress amplification factor,
M = bending moment at apex cross section,
b = width of beam cross section, and
H = beam height at the apex cross section.

The amplification factor K_r is dependent on several variables including beam geometry, elastic constants, and the loading distribution applied to the top edge of a beam.

TABLE 2. *Pitched-tapered beam test results*

Beam	Failure load	Maximum stresses developed at apex cross section, psi		Failure mode	Notes (soffit lamination only)
	plf	σ_r	σ_θ		
Group 1					
1	535	177	2950	radial	Minor fracture
2	865	286	4770	radial	No fracture
Avg.	700	231	3860		
Group 2					
1	535	194	4820	bending	At collapse, joint failed at 7530 psi
2	735	266	6620	radial	" " " " " 7530 psi
3	635	230	5720	bending	" " " " " 8540 psi
4	460	167	4140	bending	Grain slope 1:10, " " 6120 psi
5	510	185	4590	bending	Prior to collapse, joint failed at 2690 psi
6	485	176	4365	radial	At collapse, joint failed at 4940 psi
7	560	203	5040	bending	" " " " " 7040 psi
8	410	149	3690	bending	" " " " " 4170 psi
9	485	176	4365	bending	Prior to collapse, joint failed at 5270 psi
10	610	221	5490	bending	At collapse, joint failed at 7370 psi
Avg.	543	197	4880		

For Group 2 beams, K_r is 0.066 from the CSA 086 formula, whereas the program CGLB (Foschi 1970b) computed a maximum K_r of 0.058 for a UDL when moduli of elasticity were assumed to be $E_\theta = 1.93$ million psi (C.S.A. 1970) and $E_r = G_{\theta r} =$ 0.1 million psi. Poisson ratios assumed were $\mu_{r\theta} = 0.02$ and $\mu_{\theta r} = 0.45$. Thus the maximum design load of 155 plf for Group 2 beams really developed an estimated maximum radial tensile stress of 56 psi, which is about 15% less than the CSA allowable unit stress of 65 psi. Similarly, two times maximum design load, or 310 plf, really developed 112 psi when 310 plf was applied as a UDL.

Both beams of Group 1 were made of the same lumber selection; hence their elastic properties have been assumed to be equal. Displays of K_r, K_θ, and $K_{r\theta}$, which are directly proportional to radial, tangential, and shear stresses, respectively, are reproduced from Fox (1970a, 1970c) as Figs. 3 to 5 to show how these stresses generated by a UDL are distributed between tangent points of Group 1 beams. From Fig. 3, at the apex cross section, $K_r = 0.093$ for maximum radial tensile stress calculation. This value was computed for a UDL by the program CGLB using the elastic constants determined experimentally (Fox 1970a, 1970c). If the CSA 086 formulas were used, a value of K_r greater than 0.093 would be found.

TEST RESULTS

In general, all beams appeared to be well made. Good bonding was apparent on the fracture surfaces exposed by failure. Beams of three of the four shipments showed noticeable kickback, i.e., the soffit face displayed a reverse curvature near the tangent points. Table 2 summarizes the test results found for all beams.

Group 1 beams

Both beams of Group 1 failed in radial tension (Fig. 2) at 21 and 40 min, respectively, after loading commenced. The dead weight of the beam, steel cylinders, tie rods, and triangular blocks was 85 plf. For the first beam, from 85 to 460 plf load-strain and load-deflection responses were very linear. Failure occurred immediately after a load increment had been placed, so that final strain and deflection measurements were impossible to obtain. An estimate of the vertical deflection of the apex cross section at failure is 1.60 inches with a corresponding increase in span of 1.33 inches.

For the second beam, from 85 to 835 plf load-strain and load-deflection responses were mostly linear—the strain measured by the data-acquisition system being very linear with load. Vertical deflection of the apex cross section at the moment before failure was estimated to be 3.13 inches with a corresponding span increase of 1.90 inches.

Radial and tangential stresses shown in Table 2 for Group 1 beams were determined from the output of program CGLB (indicated by Figs. 3, 4). A linear response has been assumed up to ultimate strength; hence the values of σ_r listed in Table 2 for Group 1 are estimates of an upper bound to the actual failure stresses.

The internal stress distributions of a structural member when summed over a cross section should balance the external forces acting at that cross section. Stresses were calculated through the use of Hooke's law written for plane stress and an orthotropic material. When the elastic constants found for the first beam were used with the strains measured for the second beam to assess the static equilibrium of its apex cross section, the neutral axis was located at a height of 5¼ inches above the soffit. Summation of estimated tangential compressive forces exceeded the summation of the estimated tangential tensile forces by about 7%. Similarly, when these forces were multiplied by lever arms measured from the apex point and were compared with the external moment, the difference was less than 10%.

When the radial stresses were estimated in a similar manner and compared with those predicted by the program CGLB, differences of the order of 50% were found. This error is attributed to poor estimation of the modulus of elasticity, E_r, for the laminations to which the strain gauges were attached.

Group 2 beams

All beams of Group 2 carried twice maximum design load, or 310 plf, for 24 h without failure. Every 8 min thereafter, 25 plf was added until collapse occurred. Various indicators, such as the relative location of fiber-breaking noise, small puffs of material from joints, and joint failure before collapse, were evidence of the failure modes noted in Table 2. Radial tension was attributed to two beams since this mode of failure was thought to have occurred before a subsequent bending failure occurred. Fracture surfaces emerged at a soffit joint in all beams except one where a cross-grain failure occurred in the soffit lamination.

Deflection data indicated a linear response during any loading sequence. The three beams, 2, 6, and 10, that were subjected to *preload* and *deflection* tests, exhibited creep during the 24 h they carried 155 plf (maximum σ_r = 56 psi) on the central half-span. This creep, expressed as a percentage of the vertical deflection due to 75 plf (the other 80 plf was due to apparatus weight), was 6.7, 2.7, and 7.0%, respectively. Creep recovery for these latter beams measured in the 15-min interval between the deflection and strength tests was unmeasurable.

In addition, all beams carried 310 plf (maximum σ_r = 112 psi) for 24 h before they were loaded to destruction. Expressed as a percentage of the vertical deflection due to 230 plf of lead ingots, the minimum creep was about 4.0%, the mean was 7.1%, and the maximum was 11.9%.

Most Group 2 beams broke at an extreme fiber, i.e., bending mode of failure. Thus, the maximum radial stress values, σ_r, shown in Table 2 for Group 2 are generally less than ultimate radial tensile strengths that could be expected for beams of the size and shape tested.

APPLICATION

Canadian and U.S. structural design practice differs on the subject of allowable unit stresses for Douglas-fir in tension perpendicular-to-grain. Hanrahan (1966) explained why an "interim precautionary measure" should be followed, utilizing a conservative working stress of 15 psi (normal duration of load, dry service) for all loads other than earthquake or wind. This recommendation is still published in the Uniform Building Code (I.C.B.O. 1973), even though it now includes a proper but conservative maximum radial stress formula recommended by Foschi and Fox (1970), which was adopted in the same year by Canadian code writers (C.S.A. 1970).

In Canada, a working stress of 65 psi is recommended pending data that would justify a reduction on a rational basis. During the 1960s as an interim measure, many manufacturers of pitched-tapered beams reduced this allowable unit stress by 50% until the maximum radial stress formula was adopted for CSA 086 (1970).

Canadian code writers have not reduced the allowable unit stress from 65 psi because Canadian building experience has been relatively good. A recent survey revealed that, of more than 1220 pitched-tapered beams erected in Canada between 1955 and 1973, only six have failed in radial tension and none catastrophically. Furthermore, the majority of these 1220 beams were designed by the Wilson formula, which is less conservative for pitched-tapered beams than the current maximum radial stress formula of Foschi and Fox.

Madsen (1972) reasoned that the relatively good Canadian experience might be due to higher live-load to dead-load ratios in Canada, as compared to those used in the U.S.A. He suggests that "the loss in strength under continuous loading (is) more critical."

Since there was little or no snow load applied to the six beams that failed in service, the actual causes of these radial tension failures are unexplainable. However, presence of ring shake in some laminations might be responsible. When long block specimens were tested by Fox (1974), two blocks failed at 12 and 21 psi. These low strengths were caused by the presence of ring shake.

Since laboratory tests produced catastrophic failures, while no in-service failures have been of that nature, support constraints under service conditions might have been influential. The laboratory tests used linear roller bearings to prevent development of horizontal thrust. Any thrust provided by beam supports reduces radial stresses generated by live load and, therefore, the probability of catastrophic failures.

No data on tension perpendicular-to-gluelines of Douglas-fir glued-laminated wood were available prior to those of Thut (1970). Although the averages obtained for laminated blocks by Thut (1970) and Fox (1974) are clearly below those determined by A.S.T.M.-D143 for clear wood (Kennedy 1965; U.S.D.A. 1955), they are also lower than those derived from this study. However, the stress distribution within a block is a poor approximation of the conditions in the apex cross section of a pitched-tapered beam carrying a symmetric load. In the apex cross section, shear stresses are negligible (Fig. 5), but there exists a nonlinear distribution of radial and tangential stresses (Figs. 3, 4). The presence of parallel-to-grain stresses and influence of combined stresses are discounted in an assumption that block tests represent beam strengths. Furthermore, in tests of blocks of commercially laminated Douglas-fir, Fox (1974) found that average tension perpendicular-to-glueline strength of blocks decreased with increasing specimen volume.

A relationship between inexpensive test-specimen blocks and structural-size beams has been proposed by Barrett (1974). His paper confirms that the shape and size of specimens are important factors affecting test results. As a consequence, since tests of blocks have demonstrated that low strength is associated with large volume, a size-effect formula might be developed to modify stresses in tension perpendicular-to-grain for pitched-tapered beams.

CONCLUSIONS

Despite loading histories, Group 1 beams developed upper-bound estimates of radial strength 2.7 and 4.4 times greater than the Canadian allowable unit stress of 65 psi for tension perpendicular-to-grain (normal duration of load, dry service). These values are based on linear response by the beams up to failure, and an analysis made by computer program CGLB for the uniformly distributed load applied. If the conservative CSA 086 (1970) formula based on pure bending is used, these values will be about 9% greater.

Assuming that the performance specifications of British Code of Practice CP 112:1967, clause 602(g), are applicable to the tests conducted, all Group 2 beams were satisfactory, since they deflected less than 0.8 times the maximum allowable during the *deflection* test and carried twice the maximum design load for more than 15 min. The strength of Group 2 beams was usually controlled by the extreme fiber in bending. A smaller radius of curvature and a greater top-edge slope are necessary to induce radial tension failures. The two Group 2 beams that failed in radial tension developed upper-bound estimates of 2.7 and 4.1 times 65 psi. If the CSA 086 formula is used, these values would be 14% higher.

Considering the limited results obtained for radial tension, i.e., four failures, the allowable unit stress of 65 psi for Canada would appear to be adequate. However, more recent studies on size effect (Fox 1974; Barrett 1974) indicate a need for confirmation that 65 psi provides an adequate safety margin for pitched-tapered beams of volumes larger than those reported here. Furthermore, the effect of time (several months to several years delay) in radial tension failures has not been estimated. The presence of occasional ring shake in laminated beams is known to exist but was not found in the subject tests. Establishment of an allowable working stress must depend on consideration of all factors influencing the strength of structural members.

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