FLOOR LOADS FOR RELIABILITY ANALYSIS OF LUMBER PROPERTIES DATA[1]

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ABSTRACT

Utilizing load information reported in previous studies, we have developed distributions of maximum lifetime floor live loads in a form suitable for use in reliability analyses of lumber properties data. An extreme value type I distribution is chosen as best representing normalized maximum lifetime floor live loads.

Examples are given in which contrasting lumber data sets are compared using the calculated load distributions and assuming that each set must provide equal reliability, or equal safety, in the final design. A factor, k, resulting from the reliability analysis is shown to be a logical adjustment parameter for use in engineering design codes.

Combining these results with those of an earlier paper, the selection of load distributions for use in reliability analysis of lumber properties data is discussed.

Keywords: Floor loads, roof loads, reliability, lumber, strength, adjustment factors.

INTRODUCTION

To conduct a meaningful reliability analysis of lumber properties data, it is necessary to identify load distributions that reflect actual maximum in-service loads encountered by a light-frame structure. Although a significant body of literature exists on the distribution of various types of loads, few attempts have been made to assemble this information in a form that could be used in reliability studies. In a previous paper, the authors (Thurmond et al. 1984) summarized the available information on the distribution of roof loads in the United States and derived distribution parameters for use in reliability analyses.

The objectives of this paper are to identify the distribution of maximum lifetime floor live loads that could be used in a differential reliability analysis and to demonstrate their use in assessing lumber properties. The distribution of dead

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load and the method previously developed by the authors for combining two load distributions (Thurmond et al. 1984) are used to develop the distribution of the combined dead and live loads.

BACKGROUND

As noted by Zahn (1977), "the goal of probabilistic design methods is to provide unified procedure applicable to all materials, all loads, and all types of uncertainty" in the design process. Because probabilistic design offers a number of advantages over traditional deterministic methods, the use of probabilistic methods is of increasing importance in the use of wood as an engineering material. The advantages of probabilistic methods over traditional deterministic methods include:

1. Better relative safety of timber structures because of more consistency in the treatment of uncertainties.
2. The ability to assess the impact of changes in the specification of loads, stresses, and performance limits on the margin of safety for timber structures—an assessment not possible with deterministic procedures.
3. A more rational, realistic method that can be relatively free of semantic confusion.

Probabilistic methods have been used to quantify the potential impact of proposed lumber properties research (Suddarth et al. 1978) and to evaluate the effectiveness of improved quality control procedures (Suddarth et al. 1978; Marin and Woeste 1981). The use of these methods to determine resistance factors applicable to lumber at various moisture content levels has also been suggested (Green 1980).

In the sections to follow, a brief summary is given on the use of differential reliability. Recommendations from the previous paper concerning the use of a dead load distribution are reviewed and a synopsis of available state-of-the-art information on the maximum lifetime floor live load is presented.

Differential reliability analysis

Differential reliability is an analytical technique in which the probability of failure for one design situation is systematically compared to the probability of failure for a second design situation. Differential reliability is based on the concept of equal reliability—that is, structural designs using lumber should exhibit the same safety regardless of size, grade, species, moisture content, etc. The use of differential reliability was first proposed by Suddarth et al. (1978) as a means of comparing contrasting sets of lumber data. Using a simplified structural model to simulate a member from a structural assembly such as a truss, they conducted an investigation on the effect of variability in modulus of elasticity (MOE) on truss reliability. The concept of a "probability ratio" was used as a means of comparing the probability of failure for a given load-material resistance combination to the probability of failure for an assumed standard, or benchmark, combination. Differential reliability was shown to be of value for code calibration purposes and for predicting potential design-and-use payoffs of investments in material properties research.

Marin and Woeste (1981) used differential reliability to illustrate the potential use of proof loading as a quality control tool. Green (1980) suggested that a logical

TABLE 1. *Summary of available live load data for residential floors.*

Component of live load	Reference	Source of data	Live load Mean	Live load Standard deviation
			psf	*psf*
Sustained	Johnson 1953	Stockholm, Sweden 139 apartments	[1]5.2	2.1
	Karmen 1969	Budapest, Hungary 183 dwellings	11.4	4.0
	Sentler 1974	Sweden		
	Prior to 1940		6.0	2.5
	After 1940		4.7	1.9
	Combined		5.4	2.3
	Paloheimo and Ollila 1973	Helsinki, Finland	4.9	2.1
Transient	Johnson, 1953	Stockholm, Sweden	6.1	[1]3.1
	Paloheimo and Ollila 1973	Helsinki, Finland	5.7	1.4

[1] Includes weight of normal occupants assuming each adult weighs 154 pounds and each child 47 pounds.

adjustment factor for evaluating the effect of moisture content on lumber properties might be obtained by requiring equal probabilities of failure for the strength distributions of two lumber samples having different average moisture contents. An example comparing the fifth percentile ratios for green and dry lumber strength to comparable probability ratios was given.

Dead load

In light-frame applications such as floor joists, ceiling joists, and low slope rafters not supporting a finished ceiling, the nominal dead load, D_n, is assumed to be 10 psf (Hoyle 1978). Since the dead load is considered constant during the life of the structure, the mean dead load is assumed to be the calculated dead load for each particular structural application assumed in the differential reliability analysis. As an example, for a typical light-frame floor construction (2- by 8-inch, No. 2, Douglas-fir joists, 16 inches on center, with ½-inch plywood underlayment and pressed composition overlayment, pad and carpet), a dead load of 7.2 psf can be calculated. The coefficient of variation of dead load, Ω_D, is taken to be 0.10 for this study (Thurmond et al. 1984). Therefore, the dead load parameters used in this report are:

$$\bar{D}/D_n = 0.72 \quad (1)$$

$$\Omega_D = 0.10 \quad (2)$$

The distribution of dead load is assumed to be lognormal. The basis for this choice has been previously discussed (Thurmond et al. 1984).

Live load

The maximum lifetime floor live load is composed of two components, the sustained load and the transient (or extraordinary) load. The sustained load is the weight of all furnishings and movable fixtures, and includes the weight of all normal occupants. It is characterized by its long duration and essential uniformity over time. The transient load is of short duration, usually only several hours long. The transient load represents occasions when many people are gathered or furniture is placed together for some reason such as remodeling. This load is the most unpredictable since many transient loads are not measurable or even identified (Sentler 1975).

Sustained live loads.—A number of studies have been conducted on the sustained live load on residential floors (Table 1). Although considerable variation exists in estimates of the mean load, the coefficients of variation of the sustained live load in these studies are all near 40%.

Sentler (1974) fitted the gamma distribution and the lognormal distribution to his survey of flood loads. Although the gamma distribution appeared to best fit the data, he concluded that both distributions provided a good fit to residential load survey data. Johnson (1953) fit his survey data with the lognormal distribution.

Corotis and Doshi (1977) conducted an extensive analysis of five load surveys. After obtaining the histograms and basic statistics, Corotis and Doshi fit normal, lognormal, and gamma distributions to the data by the method of moments. They concluded that the gamma distribution provided the best overall fit to the distribution of sustained load.

Transient live loads.—The best method of surveying transient load is on a continuous time basis (Chalk and Corotis 1980). However, the expense and logistics of this sort of survey are prohibitive. Hence, none have been conducted in this manner. An alternative is a personal survey of occupants. This method introduces uncertainties, but is the only method that has produced results.

Only two surveys have attempted to quantify the actual transient load in residential buildings. Johnson (1953) and Paloheimo and Ollila (1973) both questioned families concerning the maximum number of people in the apartment at any particular time; however their results were considerably different (Table 1). Although neither of these studies attempted to fit a distribution to their data sets, several researchers feel that the gamma distribution is a good choice (Chalk and Corotis 1980; Sentler 1976).

Live load models.—Several live load models have been developed to quantify the stochastic nature of loads (Chalk and Corotis 1980; Sentler 1975; Wen 1977). Considering the total floor live load as a probabilistic combination of a maximum lifetime sustained load and a maximum lifetime transient load (Fig. 1), these models attempt to develop the maximum lifetime floor live load. Chalk and Corotis believe that the maximum lifetime floor live load can occur as decribed by one of three possible cases:

Case I—The sum of the maximum lifetime sustained load, S_{max}, and the largest transient load, T_s, occurring during the duration of the maximum sustained load denoted by subscript s.

Case II—The largest transient load, T_{max}, during the life of the structure plus the sustained load, S_t, acting at the time of this transient load denoted by the subscript t.

Case III—The sum of the maximum lifetime transient load, T_{max}, and the maximum lifetime sustained load, S_{max}. Case III is the largest possible floor load that can occur during the lifetime of the structure. However, the possibility of

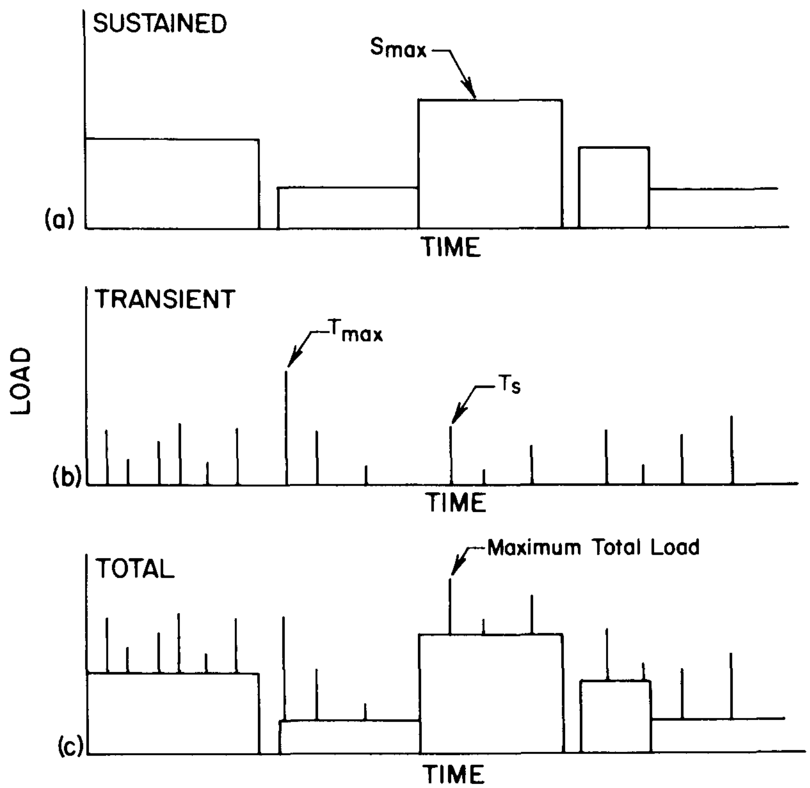

FIG. 1. A possible load realization for a light-frame structure is shown. (a) The sustained load history over the life of the structure. (b) The transient (extraordinary) load history over the life of the structure. (c) The combined load history.

two extreme events occurring simultaneously is slight; therefore, Case III has a reduced probability of occurrence.

The maximum lifetime total live load distribution can be calculated utilizing the above three cases relative to their respective probabilities of occurrence. Noting that S_{max}, T_{max}, and T_s are well modeled by the type I extreme value distribution, Chalk and Corotis (1980) have derived the parameters of the total maximum floor live load.

The moments of the three cases were found to be:

$$m_I = E[S_{max} + T_s] = m_{S_{max}} + m_{T_s} \tag{3}$$

$$\sigma^2_I = VAR[S_{max} + T_s] = \sigma^2_{S_{max}} + \sigma^2_{T_s} \tag{4}$$

$$m_{II} = m_{T_{max}} \tag{5}$$

$$\sigma^2_{II} = \sigma^2_{T_{max}} \tag{6}$$

$$m_{III} = E[S_{max} + T_{max}] = m_{S_{max}} + m_{T_{max}} \tag{7}$$

$$\sigma^2_{III} = VAR[S_{max} + T_{max}] = \sigma^2_{S_{max}} + \sigma^2_{T_{max}} \tag{8}$$

where

m = mean
subscript I, II, III = load case or type of load
σ^2 = variance

Equations 3, 4, 7, and 8 assume statistical independence between S_{max}, T_{max}, and T_s. Equations 5 and 6 were developed assuming that the instantaneous floor live load, S_t, acting during T_{max} can be approximated by m_{S_t}, the mean of S_t. This value is assumed to be deterministic (Chalk and Corotis 1980) and is taken to be the mean of a sustained load survey (Table 1).

Because all three cases are modeled by the type I extreme value distribution, the extreme value distribution parameters α and β can be calculated from the mean and variance of the load parameters (Hahn and Shapiro 1967). Following Chalk and Corotis (1980), the distribution of maximum lifetime floor live load can now be calculated as:

$$F_{L_t}(\ell) = \exp[-\exp(-w_1)]\exp[-\exp(-w_2)]\frac{T - E(\tau)}{T} + \exp[-\exp(-w_3)]\frac{E(\tau)}{T} \quad (9)$$

where

$F_{L_t}(\ell)$ = the cumulative distribution function of the maximum lifetime floor live load evaluated at ℓ
w_1 = $(\ell - \beta_1)/\alpha_1$, the reduced variate for Case I
w_2 = $(\ell - (\beta_2 + S_t))/\alpha_2$, the reduced variate for Case II
m_{S_t} = the mean of a sustained load survey (Table 1)
w_3 = $(\ell - \beta_3)/\alpha_3$, the reduced variate for Case III
T = the design life of the structure
$E(\tau)$ = the expected duration of the sustained load

Chalk and Corotis (1980) used their model to calculate maximum lifetime loads for several different types of building occupancies, including residential. For residential structures, two categories were selected based on occupancy: owner occupied and renter occupied. The two categories differ in average duration of occupancy. The renter-occupied residences averaged 2 years between occupant changes. For owner-occupied dwellings, the average occupancy duration was found to be about 10 years. Based on these facts, the calculated mean of the lifetime floor live load, m_{L_t}, was 35 psf for renter-occupied residences. The mean for owner-occupied residences was 39 psf. Both renter-occupied and owner-occupied residences had a calculated standard deviation, σ_{L_t}, equal to 7.7 psf.

Chalk and Corotis did not speculate on the distribution of maximum lifetime floor live load from this load model. Other researchers have ventured recommendations concerning the probability distribution. Ellingwood et al. (1980) advocated the use of the extreme value type I distribution. Sentler (1975) felt that the distribution of maximum lifetime floor load did not need to be chosen specifically as long as it was unimodal. The European Joint Committee on Structural

Safety (1976) recommended the extreme value type I distribution for describing the maximum lifetime floor live load.

Sentler (1975) also developed a stochastic model for floor live loads for application to offices, hotels, and apartments. He developed probability distributions based on live load surveys for sustained and transient loads. The data in a survey by Sentler (1974) were used for sustained loads, and the transient load survey by Paloheimo and Ollila (1973) was considered adequate for transient loads. The development of the maximum sustained and maximum transient floor live loads was based on equations that considered the temporal variation in live load. This rendered a closed-form development of the maximum lifetime floor live load distribution too complicated for practical use. Therefore, Sentler (1975) simplified the model by assuming no time dependence and calculated the maximum lifetime floor live load using both methods. He found no significant differences between the two methods and therefore recommended the simplified model.

The Joint Committee on Structural Safety (1976) based their recommendations of floor live load parameters on Sentler's model (1975) and survey (1974). For dwellings, a mean value of 20.89 psf and a standard deviation of 8.35 psf resulted for maximum lifetime floor live load.

Several Canadian researchers (Allen 1976; Siu et al. 1975) have advocated the ratio of the mean maximum lifetime floor live load to nominal floor live load be taken as 0.70, independent of tributary area. This value was developed based on a design live load of 50 psf for an office building in Canada. Allen (1976) stated that the expected maximum live load in 30 years would be 35 psf. Hence, the value of 0.7 was assumed. Based on the results of several live load surveys, a coefficient of variation of 0.30 was assumed for maximum floor load.

Ellingwood et al. (1980) indicated that the maximum floor live load varied significantly with respect to the tributary area; therefore, he recommended that the mean of the maximum floor load be calculated from an equation based on the ANSI Standard A58 (1982). The coefficient of variation of the maximum floor live load was taken to be 0.25. This result was comparable to the Canadian results based on the transformation from a 30-year return period to a 50-year return period (Ellingwood et al. 1980).

DEVELOPMENT OF THE MAXIMUM LIFETIME FLOOR LIVE LOAD DISTRIBUTION

The temporal and spatial aspects of the maximum lifetime floor live load render characterization of the load extremely difficult. Live load surveys are available that describe the sustained load with a good degree of accuracy. The transient load, however, has only been quantified twice in load surveys (Johnson 1953; Paloheimo and Ollila 1973). Because both of these surveys were based on the occupants' memory of the largest unusual personnel load event in the dwelling, they therefore reflect a large degree of uncertainty. Unfortunately, the sustained and transient load surveys cannot be directly combined and extrapolated to determine the maximum lifetime floor live load. Nevertheless, several models are available that express the maximum lifetime floor live load.

The models usually either assume the maximum lifetime live load distribution is extreme value type I or make no assumption regarding the distribution. Based on recommendations from current literature (Allen 1976; Corotis and Doshi 1977; Ellingwood 1978; Ellingwood et al. 1980; Galambos and Ravindra 1973), the maximum lifetime floor live load is assumed to follow an extreme value type I distribution. However, the parameters of the maximum floor live load distribution are not as apparent.

Next we must study the sensitivity of the differential reliability analysis to the parameters of the load distribution, and determine a recommended set or sets of floor live load parameters. The three sets of maximum lifetime floor live load parameters are utilized in the differential reliability analyses that follow. Also, a fourth set of floor live load parameters will be developed using the Chalk and Corotis model (1980). Existing live load data describing the sustained and transient loads will be utilized in the parameter analysis.

Live load parameters calculated by Chalk and Corotis

Chalk and Corotis (1980) calculated a mean maximum lifetime floor live load for owner- and renter-occupied residences. They note that the two occupancy types differ in their average lengths of duration of occupancy. Therefore, owner-occupied and renter-occupied live loads were developed separately. However, it was noted that such a division might be impractical due to the changing nature of housing use. Hence, for this analysis, the parameters for owner-occupied residences are combined with the parameters for renter-occupied residences. The combined mean and standard deviations are

$$\bar{L} = 37.5 \text{ psf} \quad (10)$$

$$s_L = 7.7 \text{ psf} \quad (11)$$

where

$\bar{L}$ = the mean maximum lifetime floor live load
s_L = the standard deviation of the lifetime floor live load.

Normalizing the mean by the nominal floor live load, L_n, equal to 40 psf (ANSI 1982) and calculating the coefficient of variation results in the parameters:

$$\bar{L}/L_n = 0.94 \quad (12)$$

$$\Omega_L = 0.21 \quad (13)$$

Live load parameters calculated by Sentler

As mentioned, the Joint Committee on Structural Safety (1976) recommended the use of Sentler's floor live load parameters for dwellings. The parameters calculated by Sentler (1975) are:

$$\bar{L} = 20.89 \text{ psf} \quad (14)$$

$$s_L = 8.35 \text{ psf} \quad (15)$$

and the normalized floor live load parameters with respect to the nominal floor live load are:

$$\bar{L}/L_n = 0.52 \quad (16)$$

$$\Omega_L = 0.40 \quad (17)$$

Live load parameters assumed by Canadian researchers

Canadian researchers (Allen 1976; Siu et al. 1975) have advocated the ratio of mean maximum lifetime floor live load to nominal floor live load be taken as 0.70. A coefficient of variation of 0.30 was assumed. In Canada, the lifetime of a structure is assumed to be 30 years.

These values are suspect because the mean maximum lifetime load value was not derived from an analysis that accounted for the effect of the transient load. Also, the coefficient of variation was assumed based only on sustained load survey results. However, the utilization of these parameters may provide useful knowledge concerning the sensitivity of the differential reliability analysis to the load distribution.

Since the lifetime for the above parameters is 30 years, a transformation is necessary to remain consistent with the accepted definition of structural life in the United States. Since the load distribution is extreme value type I, this transformation is a simple algebraic expression (Thurmond 1982). The resulting 50-year lifetime extreme value type I parameters are:

$$\alpha_{50} = 0.16 \tag{18}$$

$$\beta_{50} = 0.69 \tag{19}$$

and the resulting mean and coefficient of variation of the normalized floor live load ratios are:

$$\bar{L}/L_n = 0.78 \tag{20}$$

$$\Omega_L = 0.27 \tag{21}$$

Live load parameters developed utilizing the Chalk and Corotis model

Chalk and Corotis (1980) calculated live load parameters based on a pooled sustained load and a modeled transient load. An alternative set of parameters can be calculated by combining transient load survey information and the results from four individual sustained load surveys in the Chalk and Corotis model. In this manner, four sets of floor live load parameters could be calculated; then a grand average of these parameters could be obtained and utilized in the differential reliability analysis. The four sustained load surveys selected have been described in the Background. All four surveys were used by Chalk and Corotis in the pooled estimate of the sustained floor live load. A description of the input data essential for this analysis follows.

First, live load surveys describing the parameters of the sustained load are required. The four surveys that are utilized in the analysis are Johnson (1953), Karmen (1969), Paloheimo and Ollila (1973), and Sentler (1974). Second, estimates of the transient load parameters are required. Two surveys are available that describe the transient floor live load on a structure (Johnson 1953; Paloheimo and Ollila 1973). The transient load model developed by Pier and Cornell (1973) gives good results when compared to the mean of the above transient load surveys. Therefore, the transient load results calculated by Chalk and Corotis using the Pier and Cornell model will also be employed since the standard deviation was felt to be a better estimate than those of the load surveys.

The sustained load and the transient load are the two basic components required in the analysis. The moments of maximum lifetime sustained load, S_{max}, and the maximum lifetime transient load, T_{max}, can be calculated based on the sustained and transient loads. Also, the maximum transient load occurring during the maximum sustained load is required. Following Ott (1977) the moments describing T_s given by Chalk and Corotis were averaged. The resulting moments are:

$$m_{T_{s1}} = 16.4 \text{ psf} \quad (22)$$

$$\sigma_{T_{s1}} = 5.7 \text{ psf} \quad (23)$$

Wen (1977) derived an analytical solution for the mean and standard deviation of the maximum of a family of independent gamma variables. If L_{max} represents the maximum lifetime sustained or maximum lifetime transient load, then

$$E[L_{max}] = m[1 + \delta(C_1 + 0.5772C_2)] \quad (24)$$

$$\sigma_{L_{max}} = \frac{m\delta\pi C_2}{6} \quad (25)$$

where

E = expected value of L_{max}
m = the mean value of the sustained or transient load
δ = the coefficient of variation of the sustained or transient load
$C_1 = \frac{6}{\pi}\ln(N)$, where N = the mean number of independent repetitions of the load

$$C_2 = \frac{1 + C_1\delta}{2\delta + C_1}$$

As previously mentioned, the gamma distribution is favored as the distribution of the sustained load and transient load. Chalk and Corotis compared the above method with more exact solutions. They found that for Wen's method the means were within 10% and are always conservative. The standard deviations were also similar. Therefore, Chalk and Corotis adopted Wen's method in their analysis.

Utilizing Eqs. 24 and 25, the moments of maximum lifetime sustained load and maximum lifetime transient load are calculated. The results are shown in Table 2, referenced to the survey from which the original data were obtained.

Because not all sustained load surveys had an accompanying transient load survey, the moments of the maximum lifetime transient load were averaged for use in the analysis. The results are tabulated at the bottom of columns 4 and 5 in Table 2.

The moments of the three load cases are calculated using Eqs. 3 through 8 (Table 3). The average number of years between occupancy changes $E(\tau)$, is assumed to be 10 years as recommended by Chalk and Corotis (1980).

Now the parameters of the maximum lifetime floor live load can be calculated utilizing Eq. 9 and the moments listed in Table 3. Because the distribution of maximum lifetime floor live load is assumed to be the extreme value type I distribution, the mean of the distribution corresponds approximately to the 57th

TABLE 2. *Maximum lifetime load parameters calculated by the method derived by Wen (1977) for the surveys reported.*

Survey	Maximum lifetime sustained load		Maximum lifetime transient load	
	$m_{S_{max}}$	$\sigma_{S_{max}}$	$m_{T_{max}}$	$\sigma_{T_{max}}$
	psf			
Chalk and Carotis (1980)	—	—	29.4	7.0
Johnson (1953)	8.7	1.9	16.7	2.5
Karmen (1969)	16.5	4.4	—	—
Paloheimo and Ollila (1973)	8.4	1.9	10.4	0.9
Sentler (1974)	9.2	2.2	—	—
Average			18.8	4.3

cumulative percentile level. The corresponding standard deviation can be found by computing the load at a cumulative level of approximately 0.856 and subtracting the mean. The results and the combined parameters are listed in Table 4. Normalizing the combined mean by the nominal floor live load, L_n equal to 40 psf, results in the fourth set of live load parameters

$$\bar{L}/L_n = 0.73 \tag{26}$$

$$\Omega_L = 0.19 \tag{27}$$

APPLICATION OF THE DIFFERENTIAL RELIABILITY TECHNIQUE

The technique of differential reliability and its usefulness in diverse situations were demonstrated in the previous paper using a combination of a dead plus a roof snow load (Thurmond et al. 1984). Two example reliability analyses are now described to demonstrate the technique utilizing the four sets of floor live load parameters available. The results from these analyses are combined to determine the "best" available live load distribution. The results are also compared with results from the previous paper.

Determination of a moisture adjustment factor

To demonstrate the application of a dead load plus a floor live load combination in a differential reliability analysis, the contrasting lumber data set depicted in

TABLE 3. *Sustained load survey mean values (m) and moments (σ) of the three cases[1] for each survey.*

Survey	Sustained load m	Case I		Case II		Case III	
		m_I	σ_I	m_{II}	σ_{II}	m_{III}	σ_{III}
	psf						
Johnson (1953)	5.18	25.1	6.0	18.8	4.3	27.5	4.7
Karmen (1969)	11.35	32.9	7.2	18.8	4.3	35.3	6.2
Paloheimo and Ollila (1973)	4.93	24.8	6.0	18.8	4.3	27.2	4.7
Sentler (1974)	5.35	25.6	6.1	18.8	4.3	28.0	4.8

[1] Case I = sum of maximum lifetime sustained load and the maximum transient load that occurs during the duration of the sustained load.
Case II = sum of largest transient load during the life of the structure plus the sustained load acting at the time of the transient load.
Case III = sum of maximum lifetime transient and sustained loads.

TABLE 4. *Results of the floor live load analysis developed utilizing the model of Chalk and Corotis*[1] *(1980).*

Survey	Cumulative level, F		Standard deviation for $F_{0.856}$-$F_{0.57}$	Coefficient of variation
	0.57 mean	0.856		
	psf			
Johnson (1953)	27.4	32.5	5.1	0.186
Karmen (1969)	27.1	32.2	5.1	0.188
Paloheimo and Ollila (1973)	24.8	41.0	6.2	0.178
Sentler (1974)	27.7	33.0	5.3	0.191
Average	29.3		5.4	0.19

[1] Case D as discussed in the text.

Fig. 2 are utilized. The data set, taken from Green (1980), consists of two samples of 2- by 8-inch No. 2 Douglas-fir lumber tested in bending. One sample was green and the other had a maximum moisture content of 19%.

The distribution of the load combination must be known to perform the probability of failure analysis. The distribution of the sum of a lognormal variable (for dead load) and an extreme value type I variable (for maximum lifetime live load) cannot be easily derived in closed form. However, the coefficient of variation of dead load is relatively small compared to the coefficient of variation of the max-

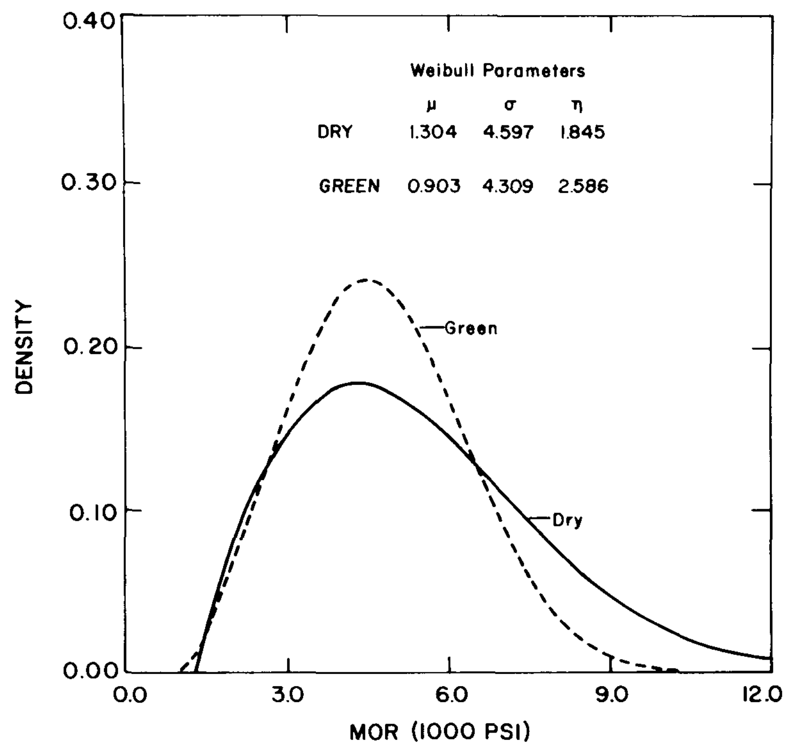

FIG. 2. The 3-parameter Weibull functions for green and dry 2- by 8-inch No. 2 Douglas-fir lumber. The parameters μ and σ are given in ksi (Green 1980).

TABLE 5. *Combined second moment parameters and resulting type I parameters tabulated for four different total loads.*

		Total load			
		Second moment parameters[2]		Type I parameters	
Load[1] case	Live load equations	μ_T	σ_T	α	β
		psf			
A	12, 13	44.7	7.7	6.0	41.2
B	16, 17	28.1	8.4	6.5	25.1
C	20, 21	38.6	8.4	6.6	34.8
D	26, 27	36.5	5.5	4.3	34.0

[1] A = dead load plus live load of Chalk and Corotis (1980); B = dead load plus live load of Sentler (1974); C = dead load plus live load from Canadian researchers (Allen 1976; Siu et al. 1975); D = dead load plus live load developed using Chalk and Corotis model.
[2] The load combination is dead plus floor live load combined according to Eqs. 28 and 29.

imum lifetime floor live load in all four cases. Therefore, the parameters of the maximum lifetime total load may be approximated by adding the means and variances of the lognormal and extreme value type I distributions. The resulting distribution of maximum total load is assumed to be extreme value type I. The moments of the total maximum load are calculated by

$$\mu_T = (\bar{D}/D_n)(D_n) + (\bar{L}/L_n)(L_n) \tag{28}$$

$$\sigma^2_T = [\Omega_D(\bar{D}/D_n)D_n]^2 + [\Omega_L(\bar{L}/L_n)L_n]^2 \tag{29}$$

where

μ_T = the mean of the total maximum load (psf)
σ^2_T = the variance of the total maximum load ((psf)^2)
$\bar{D}/D_n$ = the normalized mean of the dead load
$\bar{L}/L_n$ = the normalized mean of the maximum lifetime floor live load
Ω_D = the coefficient of variation of the dead load
Ω_L = the coefficient of variation of the maximum lifetime floor live load
D_n = the nominal floor dead load (40 psf (Hoyle 1978))
L_n = the nominal floor live load (10 psf (Hoyle 1978))

The above addition of variances implies independence between the dead load and the floor live load. The combined second moment parameters and the resulting type I parameters of the total load are shown in Table 5. The total load is identified in Table 5 in conjunction with the live load utilized in the calculation of the total load: load A is the total floor load comprised of the dead load and live load A, the average of Chalk and Corotis' results; load B corresponds to Sentler's work; and load C is from the results from Canada; and load D is from the previously described floor load analysis.

The applicability of the four extreme value type I functions described by the total load parameters shown in Table 5 was determined by overlaying the distributions on their respective histograms simulated from the individual load parameters. Visual inspection of the four figures indicated no obvious lack of fit. Based on this visual analysis, and a Kolmogorov-Smirnoff goodness of fit test (Thurmond 1982), the extreme value type I distribution was accepted as a suitable distribution of dead plus floor live load. Because the type I function adequately models the combined load, the probability of failure calculations can now be conducted once the parameters of the combined load are expressed in units of psi.

To convert the total load to units of psi, the procedure outlined in the previous paper (Thurmond et al. 1984) is again employed. First, the design load is set equal to the design resistance. In conventional design, this is achieved by calculating the load effect based on the nominal load. The calculated load effect is then compared to allowable design strength values to determine a suitably sized structural member. For the probability of failure analysis, the design resistance is the adjusted fifth percentile, F_b, of the strength distribution. The adjusted allowable design strength, F_b, is the fifth percentile of the dry lumber strength divided by the general adjustment factor of 2.1 (ASTM 1983). Since the design resistance is set equal to the design load, the adjusted fifth percentile is set equal to the nominal design load in the probability-of-failure analysis. When the normalized mean values of the dead load and floor live load are multiplied by the design resistance, the loads are expressed in units of psi, and they reflect the relative position of the dead load and the floor live load to the strength distribution.

Second, each load in the load combination should be represented in proportion to the total nominal load. For this load combination, the dead load is one-fifth of the total nominal load and the maximum lifetime floor live load is four-fifths of the total nominal load.

For the dead plus floor live load combination, the equations for the total load parameters are

$$\mu_T = \frac{D_n}{T_n}(\bar{D}/D_n)F_b + \frac{L_n}{T_n}(\bar{L}/L_n)F_b \tag{30}$$

$$\Omega_T = \frac{[(\mu_D\Omega_D)^2 + (\mu_L\Omega_L)^2]^{1/2}}{\mu_T} \tag{31}$$

where

μ_T = the mean of the maximum total load (psi)
T_n = $D_n + L_n$ = the total nominal load
F_b = the adjusted allowable design value
Ω_T = the coefficient of variation of the maximum total load

Only one set of parameters describing the load is calculated for use in the reliability analysis of lumber strength as affected by moisture content (Thurmond et al. 1984).

Probabilities of failure for the green and dry lumber were calculated assuming an extreme value type I distribution for load and a 3-parameter Weibull distribution for resistance. These are shown as lower and upper limits in the first two columns for each load case in Table 6.

Using an iterative approach, the green lumber Weibull strength parameters, μ and σ, are altered by a factor k until the probabilities of failure for the adjusted green and the dry lumber are the same. The k factor is shown in the third line for each load case in Table 6 along with the probabilities of failure. Since the adjusted green probabilities of failure are similar to the dry probabilities of failure, the correct k factors have been achieved.

To illustrate how the k factor could be used to adjust lumber data, suppose that the strength of a sample of dry lumber is known and its strength when green is

TABLE 6. *Probability of failure (P_f) of green and dry 2- × 8-inch Douglas-fir joists for the floor loads listed in Table 5.*

Moisture condition	Integral probability of failure[1]		Approximate probability of failure[2]	
	Lower limit	Upper limit	Lower limit	Upper limit
	$P_f \times 10^{-5}$			
	Load A[3]			
Dry	7.57	7.98	7.78	8.08
Green	29.60	3.07	29.70	30.90
Green, adjusted by k = 1.145	7.76	8.06	7.82	8.12
	Load B			
Dry	0.88	0.91	0.89	0.92
Green	2.95	3.06	2.98	3.09
Green, adjusted by k = 1.135	0.89	0.92	0.90	0.93
	Load C			
Dry	4.47	4.63	4.52	4.68
Green	14.60	15.20	14.70	15.20
Green, adjusted by k = 1.135	4.45	4.61	4.49	4.65
	Load D			
Dry	0.18	0.19	0.19	0.20
Green	2.10	2.22	2.14	2.26
Green, adjusted by k = 1.195	0.19	0.20	0.20	0.21

[1] Total load is the sum of two independent distributions: dead load = lognormal; maximum lifetime live load = extreme value type I.
[2] Total load approximated by extreme value type I distribution.
[3] A = dead load plus live load of Chalk and Corotis (1980); B = dead load plus live load of Sentler (1974); C = dead load plus live load from Canadian researchers (Allen 1976; Siu et al. 1975); D = dead load plus live load developed using Chalk and Corotis model.

desired. Recalling that k is the factor that a green strength distribution must be multiplied by in order to get the same probability of failure as a dry distribution, then

$$\mu_{\text{green}} = \frac{1}{k}\mu_{\text{dry}} \tag{32}$$

To test the assumption that the probabilities of failure and the k factor are not affected by simply combining the dead load and floor live load, an analysis was conducted using an exact integral approach (Thurmond et al. 1984). In the reliability analyses previously discussed (Thurmond et al. 1984) and in the present analysis, the k factor is incremented in the computer programs by $\Delta = 0.005$. The chosen k factor is therefore an estimate of the true k factor with an error of 0.005. Using a smaller increment of Δ to calculate probability of failure estimates closer to the benchmark reliability level is not justified considering the end use of the k factor. The resulting k values from the two methods presented in Table 6 were the same within an error of $\Delta = 0.005$. Also, the probabilities of failure calculated by the two methods are very close. Therefore the simpler approach is recommended. The researcher may wish to use the more exact integral approach if he feels that the dead and floor live load will not combine in a simple mathematical way.

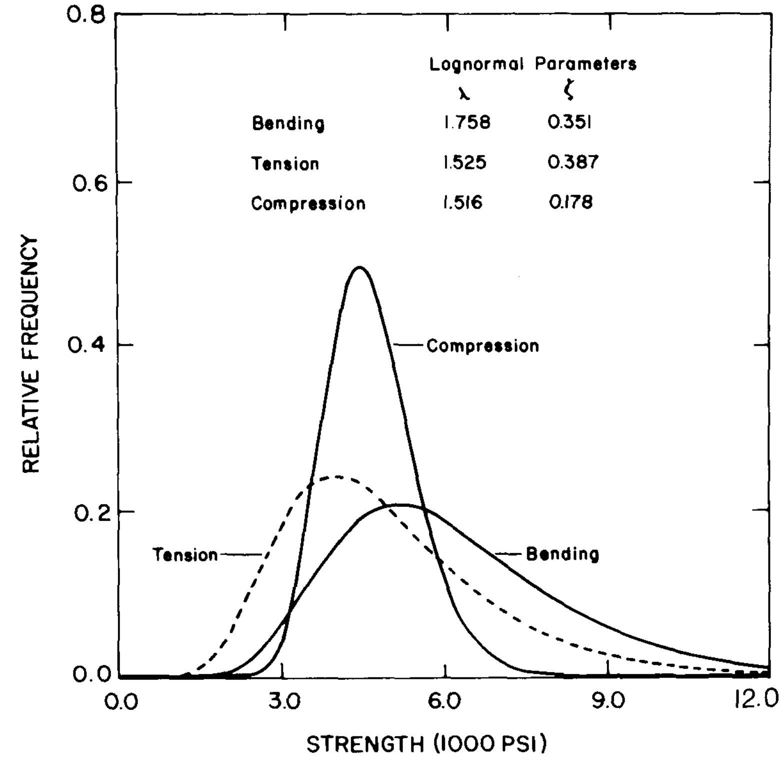

FIG. 3. Lognormal functions for 2- by 4-inch 1650-1.5E hem-fir lumber tested in bending, tension, and compression (Hoyle et al. 1979). The lognormal parameters are the means and corresponding standard deviations of the logarithms of the data, in ksi.

Bending, tension, and compression allowable stresses

Stresses in bending, tension, and compression parallel to the grain occur simultaneously in parallel chord floor trusses. Lumber is assigned different allowable stresses in each of these stress modes. Allowable stresses for these modes can also be calculated based on the concept of equal reliability. If lumber tested in bending is chosen as the reference material, the allowable stresses for the other two modes can be calculated based on k factors and the allowable bending stress.

As an example, the contrasting data sets for 1650f-1.5E hem-fir 2- by 4-inch machine-stress-rated lumber tested in bending, tension, and compression by Hoyle et al. (1979) are used. The experimental strength distributions for tension and compression are contrasted graphically to the reference bending distribution in Fig. 3.

Because no similar design situation is apparent for all three cases based on a failure mode, the reliability analysis is conducted based on load parameters calculated for a floor live load without consideration of the dead load. The assumption of a design situation gives meaning to reliability comparisons; however, it is believed that the use of the load distribution reflecting only the floor live load is sufficient for this comparison.

As in the previous examples, the strength of lumber tested in tension and compression is artificially altered until a probability of failure results that is similar

TABLE 7. *Probabilities of failure (P_f) calculated for 2- × 4-inch 1650-1.5E hem-fir lumber tested in bending, tension, and compression (Hoyle et al. 1979).*

Stress mode	Probability of failure	
	Lower limit	Upper limit
	$P_f \times 10^{-5}$	
	Load A[1]	
Bending	35.60	35.80
Compression	44.10	44.30
Compression, adjusted by k = 0.840	36.40	36.60
Tension	399.00	400.00
Tension, adjusted by k = 1.395	36.00	36.20
	Load B	
Bending	3.58	3.60
Compression	0.53	0.53
Compression, adjusted by k = 0.850	3.52	3.53
Tension	41.90	42.10
Tension, adjusted by k = 1.400	3.54	3.56
	Load C	
Bending	18.80	18.90
Compression	3.05	3.06
Compression, adjusted by k = 0.855	18.50	18.60
Tension	203.00	204.00
Tension, adjusted by k = 1.395	18.80	18.80
	Load D	
Bending	2.61	2.62
Compression	0.05	0.05
Compression, adjusted by k = 0.905	2.50	2.51
Tension	56.10	56.40
Tension, adjusted by k = 1.425	2.64	2.66

[1] A = dead load plus live load of Chalk and Corotis (1980); B = dead load plus live load of Sentler (1974); C = dead load plus live load from Canadian researchers (Allen 1976; Siu et al. 1975); D = dead load plus live load developed using Chalk and Corotis model.

to the benchmark safety level calculated from the bending strength data. The failure probabilities for the lumber tested by the various modes are given in Table 7, the k factors in Table 8.

A possible method to calculate allowable stresses for 2- by 4-inch lumber loaded in tension and compression—given bending data—is described as follows. Using the k factor listed in Table 8, the allowable design values for tension and compression, respectively, could be calculated by

$$F_b = F_t k_{tb} \tag{33}$$

$$F_b = F_c k_{cb} \tag{34}$$

where

TABLE 8. *Differential reliability k factors describing the conversion from tensile or compressive strength to bending strength.*

Stress to be converted	Ratio of 5th percentiles[1]	Ratio of allowable properties[2]	k factors for live load cases[3]			
			A	B	C	D
Tension	1.340	1.620	1.395	1.400	1.395	1.425
Compression	0.960	1.250	0.840	0.850	0.855	0.905

[1] The fifth percentiles of the lumber parameters are 3,250 psi for bending, 2,430 psi for tension and 3,400 psi for compression. These values taken from reference Hoyle et al. (1979).

[2] The allowable tensile strength as given by NFPA (1982) is 1,020 psi. The allowable compressive strength is 1,320 psi.

[3] A = dead load plus live load of Chalk and Corotis (1980); B = dead load plus live load of Sentler (1974); C = dead load plus live load from Canadian researchers (Allen 1976; Siu et al. 1975); D = dead load plus live load developed using Chalk and Corotis model.

F_b = the calculated allowable bending stress from the data (psi)
F_t = the calculated allowable tensile stress (psi)
F_c = the calculated allowable compressive stress (psi)
k_{tb} = the k factor for conversion from tensile allowable strength to bending allowable strength as determined by the reliability analysis (Table 8)
k_{cb} = the k factor for conversion from compressive allowable strength to bending allowable strength as determined by the reliability analysis (Table 8).

The conventional analytical technique is to calculate the ratio of the fifth percentiles of bending strength to tensile or compressive strength. These ratios could be denoted as r_{tb} and r_{cb}, respectively. These ratios are given in the second and third columns of Table 8 calculated from the actual data and tabulated National Design Specification (NDS) values (NFPA 1982). The calculated k factors are given in Table 8 for comparison.

Comparing the conventionally calculated factors (r) from the NDS analysis to the differential reliability factors (k) suggests that the allowable tensile stress can be increased to a level closer to the allowable bending stress, and the allowable compressive stress can be increased to a level greater than the allowable bending stress. This further suggests that conventional analyses do not account for the stronger lumber in the data sets. However, the lumber used in this example was obtained from only one mill at one time and may not be representative of lumber behavior in the general population.

SUGGESTED LOAD DISTRIBUTION FOR DIFFERENTIAL RELIABILITY ANALYSES

The results of the reliability analyses for the floor live load combination and the results from the previous paper in which a roof snow load combination was used (Table 9) were used, along with some results not summarized in this paper, to derive some conclusions regarding the "best" live load distribution.

Of the four floor live loads, loads A, B, and C produce k factors that are in excellent agreement, while load D produced k factors that generally are the most extreme of any load case. An exception is that for compression the k factor of load D is between the snow load k factor and the other k factors.

Ideally, one would recommend the snow load distribution (Thurmond et al. 1984) and the best available floor live load distribution as the load distributions to use in reliability analyses. A researcher could then choose a design situation that best describes the mode under which the data were obtained and then utilize

TABLE 9. *k factors used to gauge the sensitivity of the differential reliability analysis to changing load distributions.*

Load case [1,2]	Dry-green ratio	k factor if bending = 1.0	
		Tension	Compression
Snow[3]	1.100	1.355	0.970
Load A	1.145	1.395	0.840
Load B	1.135	1.400	0.850
Load C	1.135	1.395	0.855
Load D	1.195	1.425	0.905

[1] Dead load plus indicated snow load.
[2] A = dead load plus live load of Chalk and Corotis (1980); B = dead load plus live load of Sentler (1974); C = dead load plus live load from Canadian researchers (Allen 1976; Siu et al. 1975); D = dead load plus live load developed using Chalk and Corotis model.
[3] Thurmond et al. 1984.

the associated load distribution. However, choosing a "best" floor live load distribution is difficult because there are no clear criteria for such a choice.

The parameters of load A were determined by Chalk and Corotis (1980) using their model. Since this model is easy to use for determining load parameters and accounts for the stochastic nature of the floor live load, load A is recommended for use in reliability analyses along with the roof snow load. The parameters of load D were also calculated based on the Chalk and Corotis model. The statistical data base for load A and load D was essentially the same; however, the method of combining the load parameters was different. Also, actual transient load survey data were used to temper the results of the transient load model. Hence, the final parameters of the maximum lifetime floor live load are different for load A and load D. Nevertheless, there is no physical basis for distinguishing which of the two loads is the "best" load. Therefore, load D is also recommended for use in reliability analyses. The simultaneous use of load A and load D should effectively account for the uncertainties of load survey information. The parameters and distribution of the snow load and the floor live loads are given in Table 10.

If the reliability technique is used to determine a moisture content adjustment factor, then the chosen load distribution should be the one that results in a conservative k factor. The snow load produces a k equal to 1.100, load A gives a k equal to 1.145, and a k of 1.195 results from the use of load D in the reliability analysis. The conservative factor, k = 1.195, is thus the k factor that gives the lowest allowable bending stress for the green lumber.

A differential reliability analysis of allowable tensile or compressive stress in relation to allowable bending stress would reflect the same safety as the allowable bending stress. The assumption of equal safety between lumber grades, sizes, and species is presently implied for the tabulated design values in the Supplement to

TABLE 10. *Recommended snow and floor live load distributions utilized in reliability analyses of lumber properties data.*[1]

Load	Distribution	$\bar{X}/X_n$	Ω_X
Snow[2]	Lognormal	0.69	0.44
Load A	Type I	0.94	0.21
Load D	Type I	0.73	0.19

[1] These distributions should be combined with the lognormally distributed dead load. The normalized mean of the dead load should be calculated for each design situation. The coefficient of variation of the dead load Ω_D is 0.10.
[2] Thurmond et al. 1984.

the NDS (NFPA 1982). Since it is generally believed that tensile and compressive stresses are less than bending stresses, the conservative approach would be to select k_{tb} = 1.425 as the tensile-to-bending stress adjustment factor. This k_{tb} represents the lowest allowable tensile value that can be calculated from the adjustment factors shown in column 3 of Table 9.

The choice of a compression adjustment factor, k_{cb}, would also be made using a conservative approach.

SUMMARY AND CONCLUSIONS

The distribution of maximum lifetime floor live load was developed for use in differential reliability analyses of lumber properties data. The models, assumptions, and data utilized in the development of the distributions reflect the present state-of-the-art on loads.

Two floor load distributions are recommended for analyzing lumber properties data. Each total load distribution utilizes a lognormal dead load distribution with a coefficient of variation of 0.10.

The maximum lifetime floor live load distributions were modeled as extreme value type I. Two different extreme value type I distributions were selected due to the imperfect knowledge of floor live loads. The mean to nominal load ratios were 0.94 and 0.73, respectively. The coefficients of variation were 0.21 and 0.19. With both of these live load distributions, the recommended total load distributions are obtained by combining the lognormal dead load distribution with the extreme value type I distribution to yield another extreme value type I.

In implementing a reliability analysis of lumber properties data, we recommend that the data be analyzed under each of the three load cases to yield three conversion factors of interest. The conversion factor that is conservative from an engineering design standpoint should be chosen.

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