MODELING MOISTURE GRADIENT EFFECTS ON BENDING PROPERTIES

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ABSTRACT

Moisture gradients are known to affect the bending properties of small clear beams, but no successful attempts have been made to quantify the influence of varying gradients. A three-dimensional, nonlinear finite element program was written for this purpose. Changes in compression and tension stress-strain relationships with varying equilibrated moisture contents were modeled from experimental results. These models were used with a paraboloid representation of moisture distribution to assign properties to individual elements in the finite element model. Comparisons with tests of equilibrated and non-equilibrated yellow poplar beams showed that the finite element model predicted expected trends, but moduli of rupture were consistently overpredicted.

Keywords: Finite elements, moisture gradients, stress-strain curves, mechanical properties, bimodularity, yellow-poplar.

INTRODUCTION

Because of the preponderance of tests on small, clear equilibrated specimens, slight notice has been taken historically of the potential effect that moisture gradients may have on mechanical properties of wood. However, some recent programs designed to test full-size lumber and pallet shook may have underestimated the significance of desorption moisture gradients on the test results. The objective of this research was to model the influence of moisture gradients in bending members and to make a quantitative assessment of gradient effects on bending properties.

LITERATURE

Tests of softwood lumber for the National In-Grade Testing Program (Galligan et al. 1980) have shown that the moisture content-strength property relationships established for small clear specimens by Tiemann (1906) and Wilson (1932) might not apply to all species and to all grades of lumber (Brynildsen 1977; Gerhards 1970; Green et al. 1986; Hoffmeyer 1978; Madsen 1975; Madsen et al. 1980). This is attributed to the fact that lumber often contains defects that tend to offset some or all of the strength gained during drying. High quality lumber generally has the greatest degree of stiffness and strength improvement with moisture loss, but significant variation has been reported, especially for lower grades. Hoffmeyer (1980) attributed this variability to the use of different seasoning and conditioning protocols in the testing laboratories and suggested that discrepancies may be due to desorption moisture gradients in some specimens. Clear beams with desorption moisture gradients have been shown to have increased strength properties compared to similar equilibrated beams at comparable average moisture contents (Tiemann 1906; Wilson 1932). However, tests by Madsen et al. (1980) showed only slight differences between equilibrated and gradient-containing lumber at identical average moisture contents. No data were published regarding the moisture distributions in the lumber tested.

Green, cut-to-size pallet parts were recently tested to assess their properties (McLain and Holland 1982; McLain et al. 1986). Many of these pieces are known to have had moisture gradients due to surface drying. It is not known whether these gradients significantly affected the bending properties.

Only one previous effort has been made to predict the influence of moisture gradients on wood beams. Wilson (1932) purposely induced one-dimensional desorption moisture gradients in beams with the intention of calculating the influence of moisture distribution on beam strength, but his attempt to predict the MOR of beams with gradients was unsuccessful. No attempts have been made to predict the MOE for nonequilibrated beams.

MATERIALS AND METHODS

To meet the research objectives, a finite element program was developed to model the bending actions of orthotropic wood beams. The properties of each of the elements in the model were specified according to the presumed moisture content of the element. Development of the model required several components:

a) A mathematical distribution of moisture content over wood cross sections given any combination of average wood moisture content (MC) and external equilibrium moisture content (EMC) conditions.

b) A model of how axial stress-strain relations in wood vary with changes in moisture content. This model may be different for tension and compression.

c) Relationships describing how orthotropic elastic constants vary with moisture content.

d) A general three-dimensional finite element program capable of solving nonlinear problems and a model of a beam with enough elements to adequately reflect MC variation.

With these components, the computer model could estimate the response of a wood beam subject to variable moisture conditions. Experimental testing was required in several instances:

a) Uniaxial data were collected to provide information for the development of uniaxial stress-strain models at several moisture levels.

b) Beam test data, together with associated moisture gradient data, were acquired to verify the finite element predictions of moisture gradient effects.

Moisture gradient model

Although both two- and three-dimensional moisture gradients are likely to be encountered in dry-out testing and in in-service conditions, this study was re-

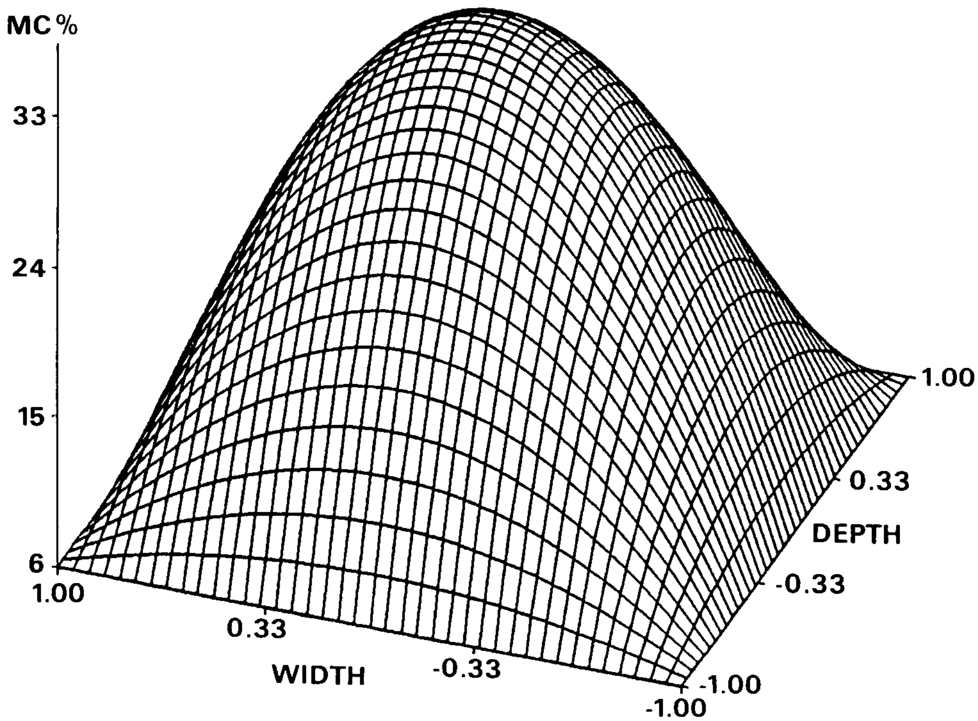

FIG. 1. Paraboloid moisture distribution over a square cross-section. Average MC = 18%, EMC = 6%.

stricted to the modeling of two-dimensional moisture gradients. The common one-dimensional parabolic model for moisture distribution in drying wood was extended to two dimensions, and moisture gradients were represented by a paraboloid. Since the diffusion coefficients for the radial and tangential directions are usually unequal, this model is probably not exactly correct. Nonetheless, discrepancies between the actual moisture content distribution and the simple paraboloid model were not expected to be significant enough to warrant a more complex model. The desorption moisture gradient model used in this study was represented for a rectangular domain (Fig. 1) by

$$MC = 2.25(MCBAR - EMC) \times (1.0 - k_1X^2) \times (1.0 - k_2Y^2) + EMC \quad (1)$$

where

MC = moisture content (%) at coordinate (X, Y)
X = horizontal distance from the centroid
Y = vertical distance from the centroid
MCBAR = average moisture content (%)
EMC = equilibrium moisture content (%)
k_1 = $1.0/(w/2)^2$
k^2 = $1.0/(d/2)^2$
w = width of the rectangular domain (cross-section)
d = depth of the rectangular domain (cross-section)

Stress-strain—MC models

An experimental program was designed to determine how axial stress-strain relations vary with changes in moisture content. Segmented models were developed to describe stress-strain curves, and other empirical relationships were determined to describe the stress-strain curve model parameters as functions of moisture content.

Test material.—Moisture gradients affect the mechanical properties of all species. However, yellow-poplar (*Liriodendron tulipifera*) was chosen for this study because freshly cut logs without existing moisture gradients were available, the anatomy is reasonably homogeneous, and some elastic constant data are available in the literature. Sapwood was used in this investigation because the diffusion coefficients for heartwood and sapwood are probably different (Choong and Fogg 1968; Choong and Skaar 1969) and because growth ring curvature could be minimized.

Several yellow-poplar logs were selected from freshly cut trees grown in southwest Virginia. These logs were chosen because of their straightness, freedom from defects, and wide sapwood. The sapwood was cut from the logs at a sawmill and resawn into shorter lengths. The pieces were then wrapped in heavy plastic and refrigerated to keep them green and free of fungi until the test specimens could be cut.

Uniaxial testing.—It is commonly assumed that the Young's moduli in tension and compression are equal (Sliker 1973), but some yellow-poplar data indicate that these moduli may be significantly different at around 9% moisture content (Stern 1944). Modulus inequality could not be confirmed or disproved at other moisture levels because of due to a lack of data, so both tension and compression specimens were tested to provide information for the finite element model. Four moisture levels were selected: 6, 12, 18%, and green.

The 1- × 1- × 4-inch compression specimen described in ASTM standard D 143 (1983) was used to take advantage of the available testing apparatus and to obtain the greatest number of specimens from the wood. The standard tension specimen was modified slightly; an 8-inch radius of curvature was used instead of the 17.5-inch standard curvature to permit the specimens to be cut by passing them at right angles to the arbor on a 16-inch table saw. This produced well-shaped specimens with minimum dryout. To accommodate these specimen dimensions, the refrigerated slabs were sawn along their lengths and planed into 1.125-inch and 2.25-inch squares; the larger pieces were reserved for beam specimens. Using a clear plastic template, each slab was marked on the end grain before cutting to ensure that the growth rings were aligned with the specimen axes as closely as possible. A scriber was used to orient the cuts so that they were parallel to the fiber direction. After each stick was cut, it was rewrapped and returned to the refrigerator.

Because of the size and amount of material available, only limited matching was practical. For tests at each moisture content category, sufficient 1.125-inch squares were randomly drawn such that approximately fifteen compression and fifteen tension specimens could be fabricated. This usually resulted in obtaining both a tension specimen and a compression specimen from the same stick, but sticks sometimes had to be cut differently to avoid minor defects. A wafer was

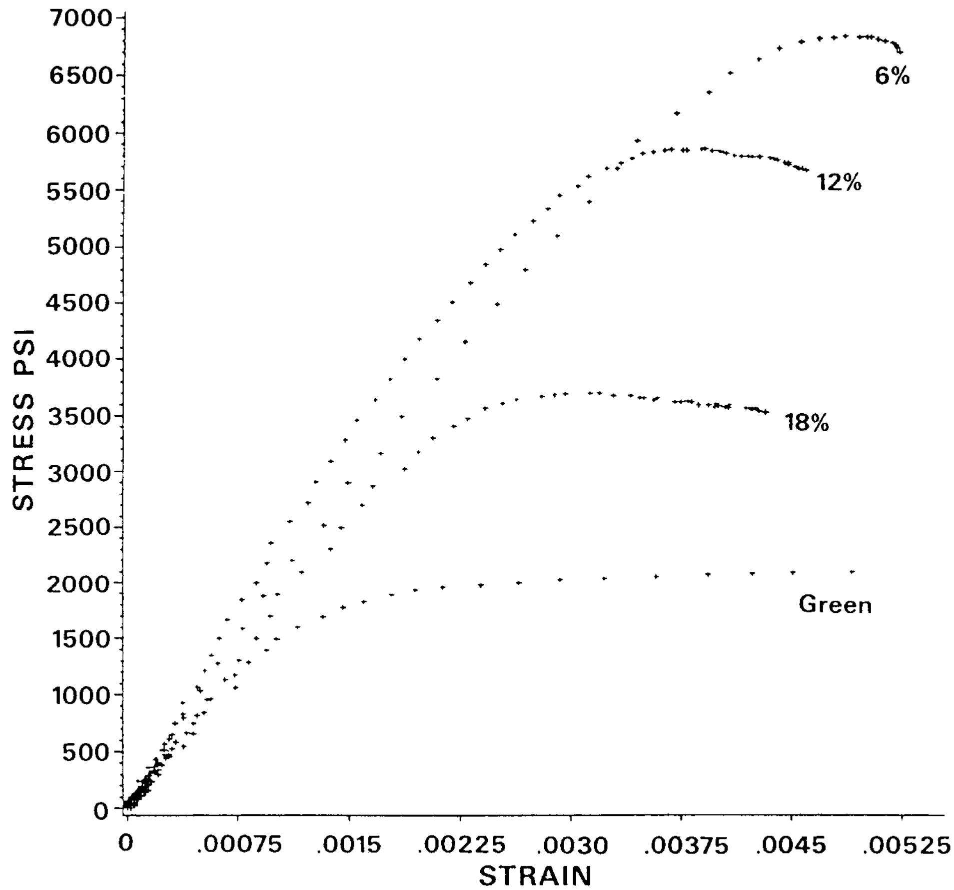

FIG. 2. Examples of digitized compression parallel-to-grain stress-strain relationships. The samples were equilibrated at 6, 12, 18% and green MC conditions.

removed from each stick for a specific gravity determination, and the sticks were then conditioned to the specified moisture content at 100 F, cut to size and shape, reconditioned, and tested. Wrapped green specimens were also equilibrated at 100 F before testing.

Data were acquired in digital form using a microcomputer interfaced with the load cells and strain-measuring devices used in the tests. Compression specimens were fitted with a reusable clip-type strain gauge harness to measure the longitudinal displacement over the central 2-inch gauge length. Longitudinal deformation in tension was measured over a 2-inch gauge length using a pair of miniature LVDTs mounted in a lightweight aluminum yoke. All uniaxial tests were conducted at a free-running crosshead speed of 0.012 inches per minute. The raw data were reviewed for accuracy and recalculated as stress-strain data; plots of representative compression and tension data for each moisture content category are shown in Figs. 2 and 3. Longitudinal Young's moduli were calculated by performing linear regressions on the appropriate data for each test. The tension and compression moduli were similar to each other at 6% and 18% MC, but at 12% MC the compression modulus was about 20% greater than the tension mod-

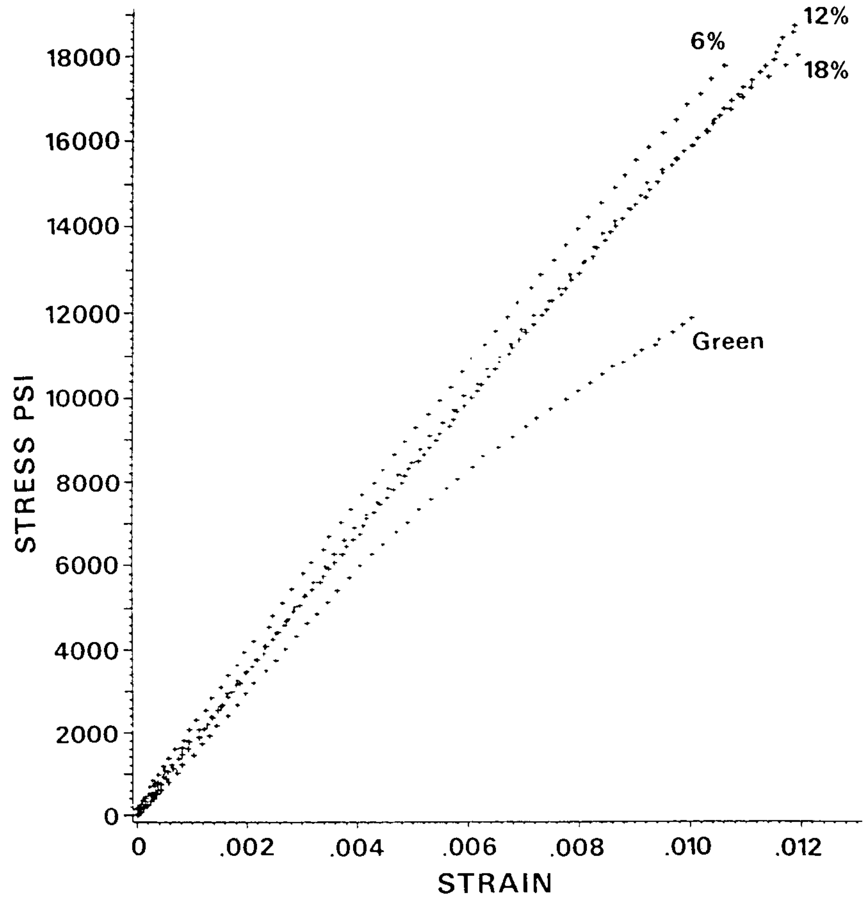

FIG. 3. Examples of digitized tension parallel-to-grain stress-strain relationships. The samples were equilibrated at 6, 12, 18% and green MC conditions.

ulus, and the green tension modulus was 17% greater than the corresponding green compression modulus. Maximum stresses and failure strains were also calculated from the digitized data and modeled by regression for use in the finite element program.

Modeling uniaxial stress-strain curves.—A mathematical model of the uniaxial stress-strain relation was developed for use in the finite element analysis. Previous stress-strain models (O'Halloran 1973; Ramberg and Osgood 1943; Urbanik 1982) have difficulty representing both the initial linear slope and the stress accompanying the strain at failure and beyond. Consequently, new, segmented models were devised for this study. The tension stress-strain model consisted of an initial linear segment followed by a smooth join to a quadratic function. The compression model was similar, but a second smooth transition was incorporated at maximum stress, followed by a linear segment with a zero slope to imitate plastic behavior. Three parameters were required to model each digitized stress-strain curve, one of which was the longitudinal Young's modulus; the remaining parameters were found using nonlinear, iterative regression techniques. Details of the model construction can be found in Conners (1985). The segmented models were able to fit

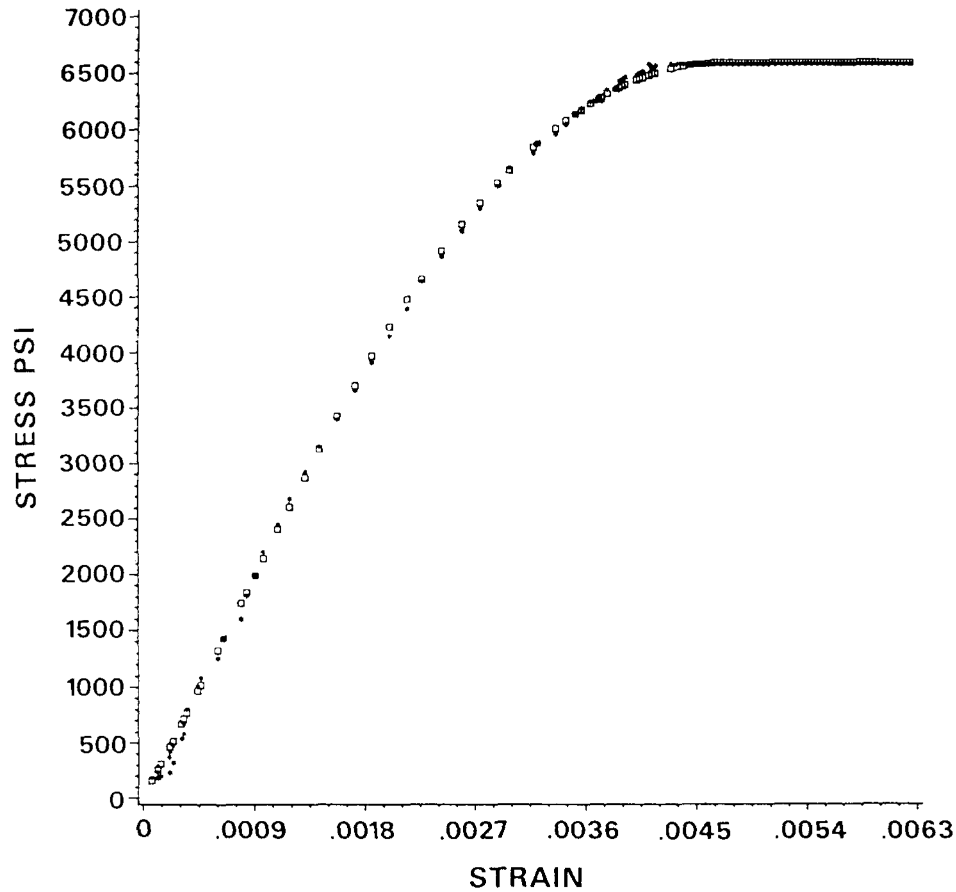

FIG. 4. Comparison of predicted to actual compression stress-strain relationship for one yellow-poplar specimen. Data = asterisks, predicted points = squares.

most of the individual tension and compression stress-strain curves with R^2 values in excess of 0.99 (Fig. 4).

The description of stress-strain diagrams at different moisture contents was accomplished by regressing the parameters for the segmented models as functions of MC. Typical tension and compression curves (similar to Figs. 2 and 3) could then be reproduced for any given moisture content.

Specification of other elastic constants

The remaining elastic constants and their variation with moisture content were specified next. Some of these values were available in the literature (Mark et al. 1970), but most had to be estimated. It was assumed that a linear relationship could be used to estimate each elastic constant between a maximum value at around four to six percent MC ("maximum value moisture content," or MVMC) and a minimum value at some intersection point MC. The term "intersection point" is used here to emphasize that this point may not be exactly the same as the fiber saturation point (Wilson 1932). The slopes of these linear relationships were estimated from data in the literature and used to adjust the tabulated values for yellow-poplar elastic constants at 11% MC (Drow and McBurney 1946) to the desired MC. The Poisson's ratios ν_{rl} and ν_{tl} were calculated using elasticity relationships. A sensitivity study showed that the finite element predictions were primarily dependent upon the longitudinal Young's moduli, and that inaccuracies in the relationships for the other elastic constants did not significantly affect the results.

Finite element modeling

The finite element program.—With finite element analysis, it is often desirable to take advantage of material uniformity to reduce the modeled structure to an assemblage of planar or even linear elements. In the present study, however, the presence of a two-dimensional moisture gradient necessitated the use of three-dimensional elements because of different material properties through both the beam thickness and the depth. Eight-node brick-type (linear) elements were selected after considering computer cost and storage constraints. Modeling of isotropic beams demonstrated that about 98% of the theoretical elastic deflection could be predicted using thirty-two linear elements between the support and the beam midpoint; these results were judged to be acceptable. Refinement of the cross-section mesh did not appreciably affect the results of these trials.

The models for the variation of uniaxial stress-strain curves and for the variation of the elastic constants with moisture content (Conners 1985) were incorporated into the finite element program, and a linear step-by-step method was chosen to model mechanical behavior beyond the elastic limit. In this method, the stiffness matrix for each element is updated once at the beginning of each step (i.e., load or deformation increment), and each elastic constant used to calculate this matrix is determined from the slope of its stress-strain diagram at the corresponding strain. The accuracy of the step-by-step method is dependent upon the successive imposition of small load or deformation increments. In spite of this restriction, it appeared to be the most practical choice for this study; more accurate methods suffer a significant penalty in computation time. Only the longitudinal Young's moduli and the Poisson's ratios ν_{lr} and ν_{lt} were programmed to vary with the elastic strain since Maghsood's (1970) computations indicated that the stresses for the other elastic constants do not exceed their respective proportional limits in beam bending beyond the elastic limit. "Failure" was defined whenever the slope of the calculated load-displacement curve became less than 100 pounds per inch of vertical displacement.

Small beam testing.—The Young's modulus data in compression indicated that E was greater at 12% moisture content than at 6%. Because the form of the Young's modulus-MC relationship is unknown between 6% and 12% MC, it was apparent that the MOE data from beam tests should be acquired at MCs between 12% and green to permit the best comparison to the finite element program predictions. However, because of various minor defects present in the original wood sample, too few 2.25-inch squares were available for an adequate number of replicates. Two additional green yellow-poplar logs similar in size and quality to the first were obtained to provide additional specimens. Twenty 2- × 2- × 40-inch pieces of sapwood were obtained from these logs; both these and the first group of 2.25-inch squares were sawn into 1- × 1- × 18-inch end- and side-matched beams. These specimens were then wrapped in plastic and refrigerated to await testing.

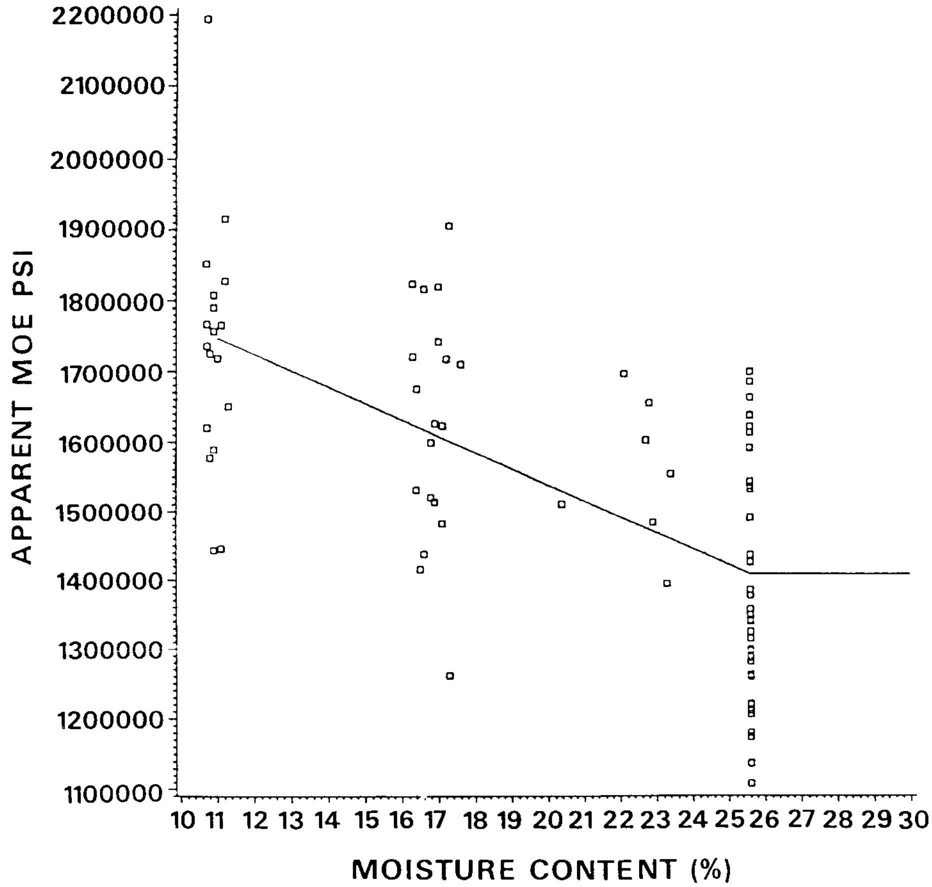

FIG. 5. Apparent moduli of elasticity of equilibrated yellow-poplar beams as a function of moisture content. The line represents the model criterion against which the finite element predictions were judged.

The second group of beams was tested first to provide the greatest number of tests and MC levels from a single source. Six of the eight matched specimens from each 2- × 2-inch beam were assigned the following moisture conditions at test: 12% average MC, no gradient; 18% average MC, no gradient; green, no gradient; 15% average MC, 12% EMC; 25% average MC, 12% EMC; 35% average MC, 12% EMC. Two specimens from each 2- × 2-inch beam were held in reserve. Twelve percent EMC was used to create the moisture gradients since the Young's moduli appeared to be greatest near this MC; material property differences were therefore maximized within each cross section. It was noted previously that beams with moisture content gradients can have strength properties greater than those of green beams even though the average moisture contents might be significantly greater than the fiber saturation point (Tiemann 1906; Wilson 1932). Since the compression data suggested that the intersection point for yellow-poplar was around 25% MC, average moisture contents of 15, 25, and 35% were chosen to bracket this value. All beam specimens were equilibrated at 100 F to be comparable to the uniaxial specimens.

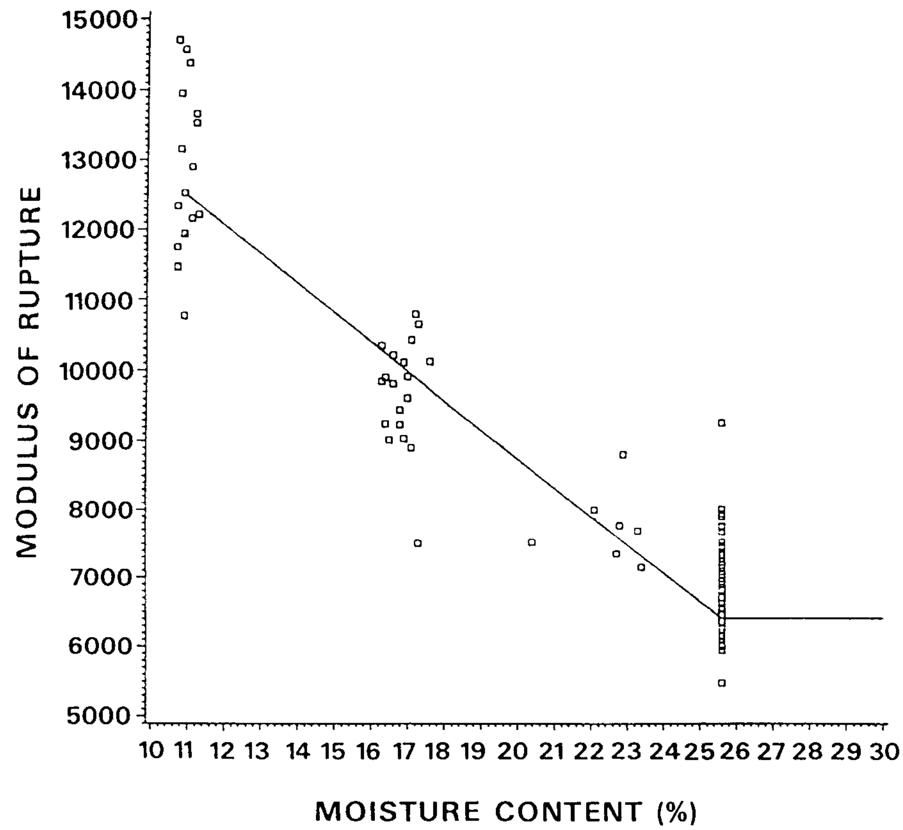

FIG. 6. MOR of equilibrated yellow-poplar beams as a function of moisture content. The line represents the model criterion against which the finite element predictions were judged.

Equilibrated beam specimens were conditioned at the same relative humidities as their uniaxial counterparts until constant weights were achieved. However, imposing the prescribed moisture gradients on nonequilibrated beams proved to be considerably more difficult. Preliminary experiments indicated that one approach was to remove a wafer from each beam end as a moisture content sample. Since the initial weight of the remainder of the specimen was also measured, the weight of each gradient-containing beam could be calculated at the target moisture content. Each end of these beams was coated with paraffin to prevent the formation of three-dimensional gradients in the beams while they dried to the target weights for testing. After testing in center-point bending, a 2-inch MC sample was immediately removed from the midpoint of every beam; an adjacent 1-inch wafer was also removed from twenty-five of the gradient-containing beams. These wafers were sliced into sixteen ¼-inch squares, weighed, and dried to determine the moisture distribution. The moisture contents for these sections agreed well with the values predicted by Eq. (1) when the predicted MC was less than the approximate fiber saturation point. Additional sampling of a few specimens verified that the gradients were essentially uniform along the entire beam length.

After the larger group of beams was tested, the original material (matched to the uniaxial specimens) was apportioned into four groups: green, no gradient;

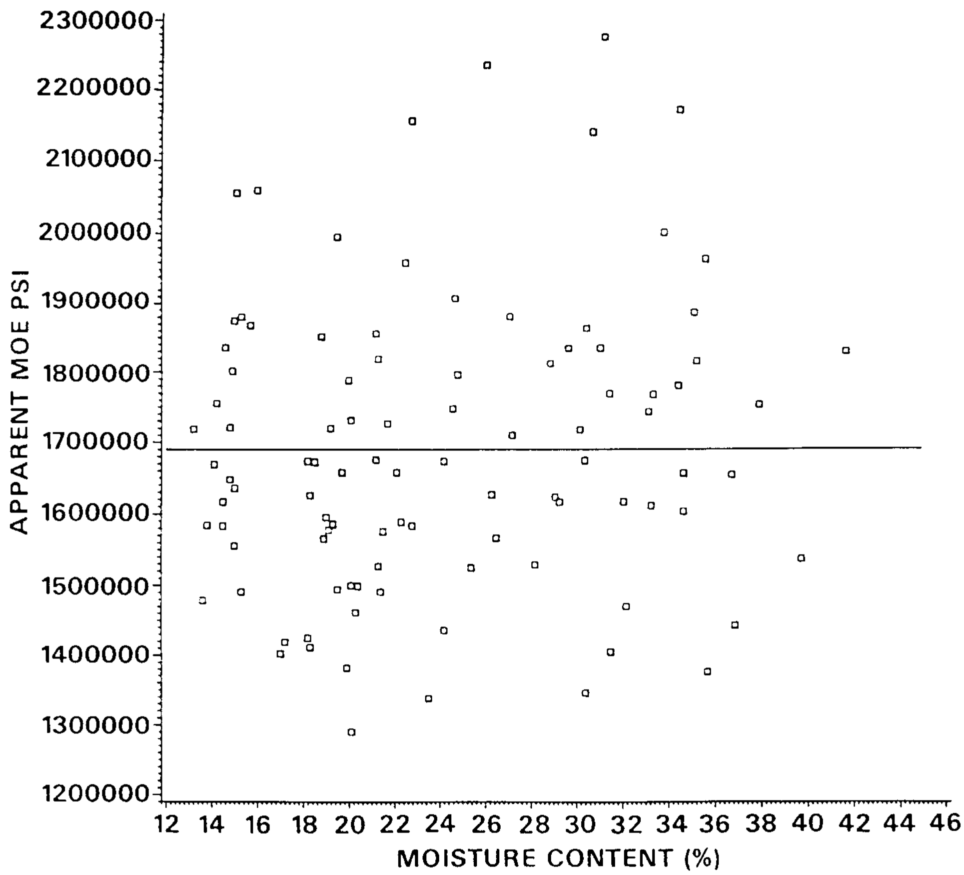

FIG. 7. Apparent MOE of nonequilibrated yellow-poplar beams shown as a function of moisture content. The line represents the average value for the collected data.

22% average MC, no gradient; 22% average MC, 12% EMC; 35% average MC, 12% EMC. The green condition was specified so that the properties of the two groups of wood could be compared, and the latter three groups were chosen to provide additional points at the indicated moisture contents. The beams were conditioned and tested in the same manner as the first group. Tests for differences in the mean properties of the two groups of green wood were not significant at the 95% level, so the data were pooled for further analysis.

In Figs. 5 and 6 MOE and MOR data from the equilibrated samples, overlaid by polynomial models, are presented. The green data were plotted at 25.6% MC since the compression tests indicated that this was the approximate intersection point. Figures 7 and 8 are comparable figures for nonequilibrated beams. It is evident that the target moisture contents were only approximated by the conditioning method chosen. It is also interesting to note that the nonequilibrated MOE data had no statistically significant relationship with moisture content. Since the thickness of the outer fiber region, which is close to the EMC, is inversely proportional to the severity of the gradient, application of the parallel axis theorem would suggest that lower MOE values should have been obtained for nonequili-

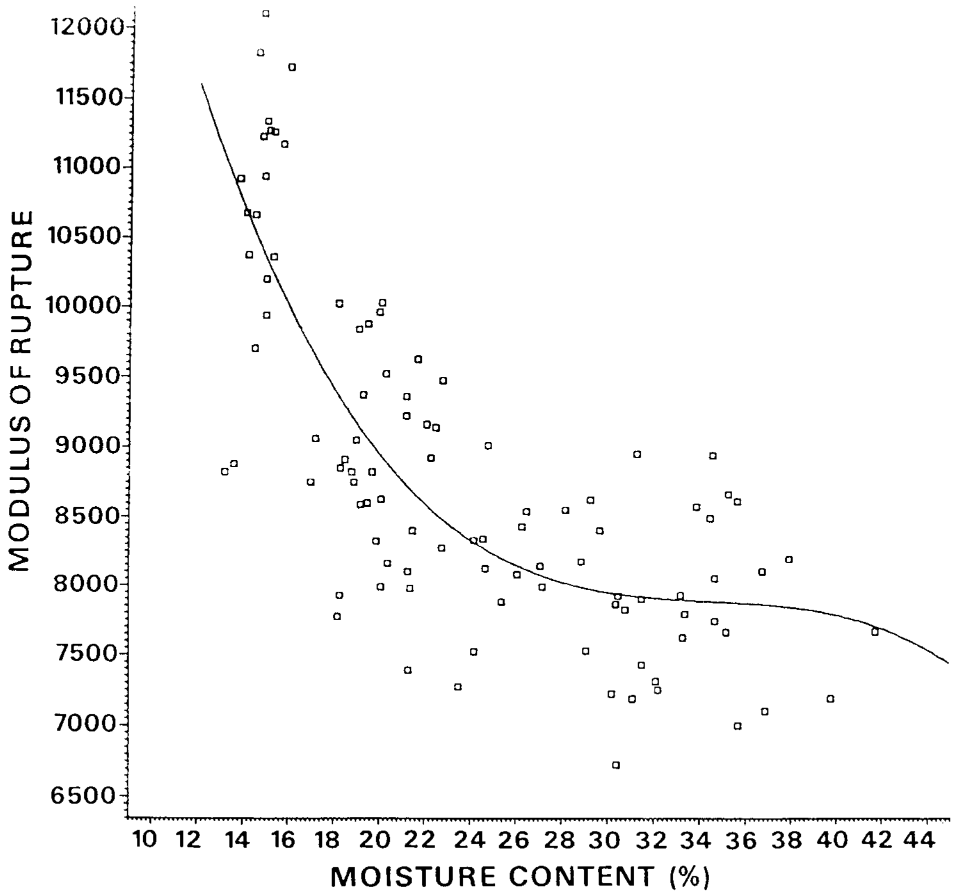

FIG. 8. MOR of nonequilibrated yellow-poplar beams shown as a function of average MC. A polynomial regression line is superimposed.

brated beams at higher average moisture contents. The model line should probably be viewed with suspicion, especially in light of the data variability.

Small beam modeling.—The equilibrated 1- × 1-inch beams were modeled as center-point loaded beams over a 14-inch span at 12%, 18%, 22% MC and green (greater than 25.6% MC) using the finest cross-section mesh consistent with the available computer storage. Specifically, the model was comprised of a uniform 32 × 6 × 3 mesh (l × h × w) imposed on a one-quarter beam domain (7 × 1 × 0.5 inches); the reduction to a one-quarter beam model was possible due to symmetry. The model beam was supported and loaded at the neutral axis (parallel to the tangential face). The apparent MOE values (uncorrected for shear) calculated by the program agreed well with the collected data; none of the predicted values differed from the data model by more than 5%. Modulus of rupture values were calculated by imposing successive 0.01-inch vertical displacements on the beam model to failure. A prior convergence study had shown that an even smaller incremental displacement would have been desirable, but this was not practical given the time and storage requirements of the program. The program calculated the correct trends for the MOR values, but the predictions were high because of

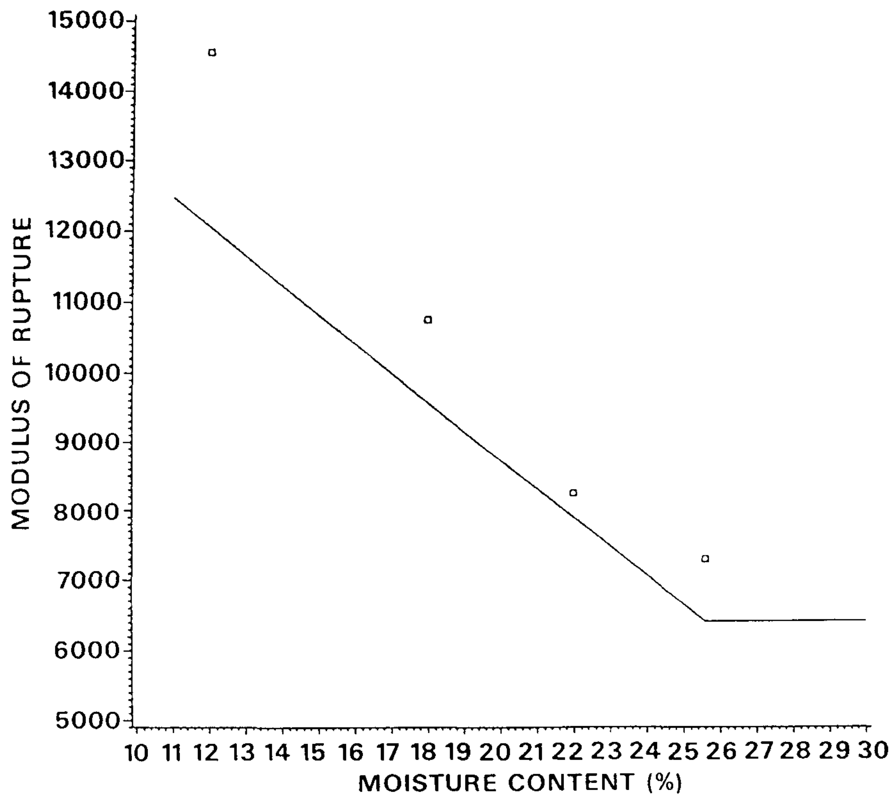

FIG. 9. Comparison of equilibrated MOR model to the MOR values predicted by the finite element program. The solid line is the fitted regression model shown in Fig. 6.

the shortcomings of the linear step-by-step procedure. Compared to the data regression, the errors ranged from about 4% to about 21% (Fig. 9).

Nonequilibrated beams were modeled at four representative average MCs (15%, 25%, 35%, and 45%) with a 12% EMC boundary condition. The 32 × 6 × 3 mesh used for these models was a compromise between those meshes necessary for modeling a beam in bending and modeling the two-dimensional moisture gradient. Preliminary trials on uniaxial models showed that a 6 × 3 cross-section mesh could model the effects of moisture gradients better if the elements were of unequal size, so the outer elements were made smaller than those near the model centroid (Fig. 10). Because of the coarseness of the mesh, it was expected that the apparent MOEs would be underpredicted, and the actual amount of the difference between predictions and observations ranged from about 6% at 15% average MC to about 12% at 45% average MC. The predicted trend of reduced MOE with increasing MC is consistent with the trend expected (but not seen) for the data.

Since the equilibrated beam models overpredicted MOR, similar results were expected for nonequilibrated beam models. The trend of the predicted values is similar to that of the experimental data, but mean MOR was consistently overestimated by approximately 20% to 27% depending on the moisture content (Fig. 11). The error is probably due to a combination of the step size used (0.01 inch) and the coarse cross-section mesh.

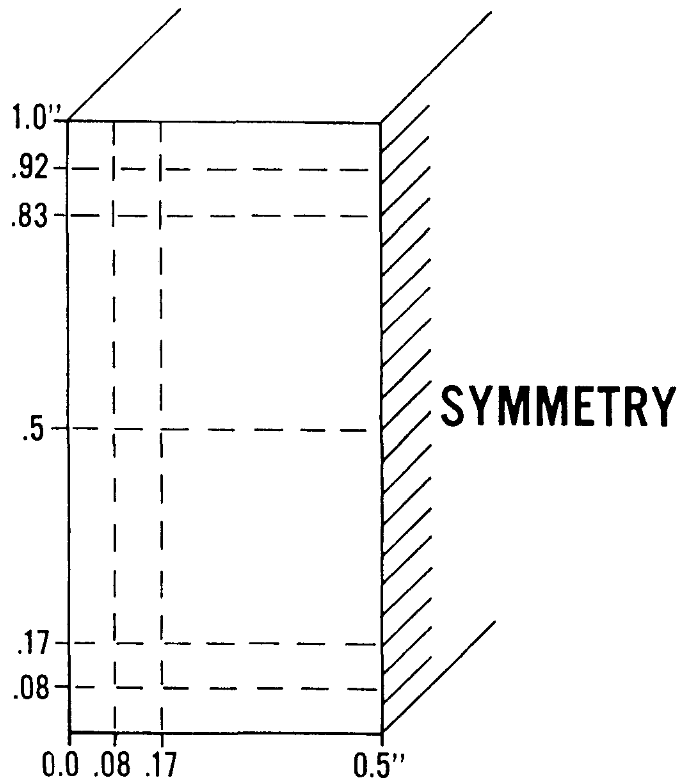

FIG. 10. Cross-section mesh used for the finite element model of nonequilbrated 1 × 1 beams.

Considering the number of subsidiary models incorporated into the program, the predicted values and noted trends were reasonable in most cases. The constraints imposed on this study can sensibly account for most of the discrepancies between the observed and predicted points.

2 × 6 beam modeling.—Lumber tested in full-size testing programs is often evaluated in the "green" state. Practically, however, green test lumber often sits at the test site for varying times after cutting, inevitably resulting in desorption moisture gradients. Structural-size lumber was not tested in this study, but nominal 2- × 6-inch beams were modeled with the finite element program in an effort to apply the methodology to a practical problem. It was recognized that the models incorporated into the program were valid only for small clear specimens, but it was felt that it might be instructive to apply the program to larger models to see what trends might be expected of clear material.

Modeling of these beams followed the same procedures as for small clear beam models, with the exception that a span to depth ratio of 17:1 was used. The actual dimensions of the model were 93.5 × 5.5 × 1.5 inches (l × h × w) and planes of symmetry were used to reduce the model to 46.75 × 5.5 × 0.75 inches (l × h × w). Previous efforts had shown that moisture gradients established at 12% EMC affected the MOR at average moisture contents above 20% MC, so beams with average moisture contents of 25%, 35%, and 45% were modeled with a 12% EMC boundary condition.

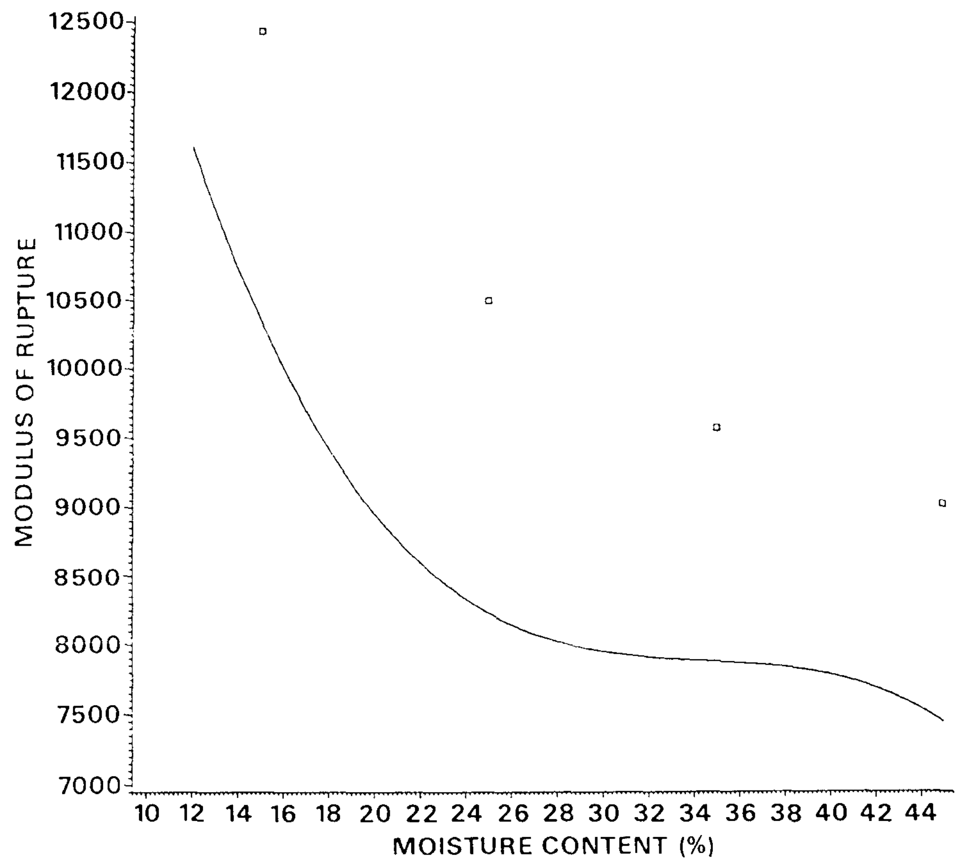

FIG. 11. Finite element predictions of MOR for nonequilibrated beam models compared to the polynomial regression model for nonequilibrated beam data (Fig. 8).

A convergence study was performed on uniaxial models using various meshes in an effort to find a cross-section mesh that could adequately represent the moisture gradients. The optimum meshes contained too many elements, so a 6 × 3 mesh similar to that used for the 1- × 1-inch beam models was finally selected. Predictions of Young's moduli using this mesh were within 1% of the predictions using the optimum meshes in the convergence study. A second convergence study and cost considerations directed us toward a compromise step size of 0.0625 inches.

The predicted MOEs for nonequilibrated "green" beams at 25% average moisture content were about 12% higher than equilibrated green beam data, and at 45% average MC the predicted MOEs were approximately 9% greater. MOR predictions for nonequilbrated (green) beams, inflated by the inadequate step size, were about 36% higher than equilibrated green beam data at 25% average MC; at 45% average MC the difference was still nearly 19%. Since the predicted values were not compared to actual beam data the degree of accuracy is unknown, but the trends are consistent with those predicted for the 1- × 1- × 14-inch beam models. These trends are also similar to data obtained by Tiemann (1906) and Wilson (1932). If the predictions are applicable to the grades of lumber tested by the In-Grade Testing Program, then nominally green beams with gradients can exhibit strength and stiffness properties that are representative of much drier wood. The amount of the potential difference is dependent upon the severity of the moisture gradient, but it can be significant; in this study the nonequilibrated "green" 2- × 6-inch beams (25% average MC) were predicted to have strength properties equivalent to wood equilibrated at 19% MC. Subsequent prescriptions for seasoning adjustments based on these "green" data may be excessive.

CONCLUSIONS

This study demonstrates the effect of moisture gradients on beams under controlled conditions. The results showed that desorption moisture gradients can significantly affect both stiffness and strength properties of small clear specimens, and that steep moisture gradients may result in observed properties that are more typical of drier wood.

The trends for MOR of equilibrated and nonequilibrated beams were predicted reasonably well, although the values were too high. MOE values were predicted with good accuracy for equilibrated beams, but it was difficult to evaluate the MOE predictions for nonequilibrated beams due to variation in the data. The predictions in the latter case were nevertheless consistent with expectations.

Finite element modeling of 2- × 6-inch beams indicated that structural-sized clear beams are also subject to moisture gradient effects. If the trends from these analyses are applicable to lumber with defects, then full-size test programs should exercise care to ensure that beams are as gradient-free as possible prior to test. Data resulting from tests of nominally green beams with moisture gradients may be more typical of data acquired at moisture contents below the fiber saturation point.

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