COMPUTERIZED FINITE-DIFFERENCE METHOD TO CALCULATE TRANSIENT HEAT CONDUCTION WITH THAWING

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ABSTRACT

Transient temperature profiles of frozen logs subjected to axisymmetric thawing and heating were calculated by a temperature method and an enthalpy method. The present paper discusses only the temperature method, which uses the conventional (temperature) formulation of the nonlinear heat conduction equation. This approach required the specification of a thawing temperature interval over which the latent heat was incorporated in the specific heat. Thermal properties were varied with position and temperature, and changed discontinuously with the phase. The log surface temperature was specified. The computerized finite-difference program HEAT was used in conjunction with this method. Computed temperature profiles were in overall agreement with experimental data obtained from heating logs in agitated water.

Keywords: Heat transfer computer program, thawing model, phase change, nonlinear heat conduction.

INTRODUCTON

Computing transient heat conduction with thawing is important in a number of wood processing operations. As an example, northern veneer and flakeboard mill operators request estimates of log thawing times for wood frozen during the winter months. The additional time required for raising veneer block temperatures to the levels recommended for veneer cutting must also be computed, as temperature measurements inside the logs are not feasible from an economic standpoint.

Transient temperature profiles were calculated for frozen logs subjected to axisymmetric thawing and heating. The two computational methods found suitable were a temperature method after Bonacina et al. (1973) and an enthalpy method derived from Voller and Cross (1981). The temperature method uses the conventional (temperature) formulation of the nonlinear heat conduction equation, with temperature as the only dependent variable. In the enthalpy method, the dependent variables are enthalpy and temperature. This paper discusses the temperature method.

The method was used in conjunction with the computer program HEAT, which was prepared by Beckman (1972) in FORTRAN V language, for use on a UNIVAC 1108 computer. HEAT uses a finite-difference technique with predictor-corrector integrations (Hamming 1962) and can solve multidimensional heat conduction problems with various boundary and initial conditions. Thermal property data may vary with position and time or temperature. These data may also be discontinuous, which is important for phase change calculations.

Computations have been compared with data from experiments in which logs were thawed and heated in agitated water (Steinhagen 1977a). A more explicit

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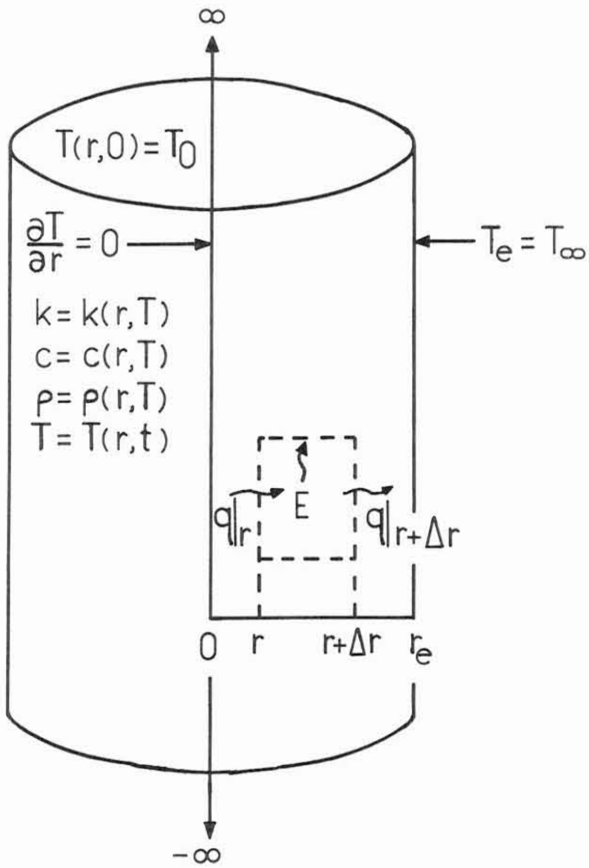

FIG. 1. Log transient with important energy terms.

description of the computational procedure is given in Steinhagen (1978), from which this paper was abstracted.

THEORY

Consider a long, cylindrical log with radius r_e (Fig. 1). The length of the log allows one to ignore any heating end effects; practically, this requires a length-to-diameter ratio in excess of four, according to data published by MacLean (1946). The log's initial temperature (T_0) is below 0 C, the freezing point of free water. At time zero, the log's surface is exposed to the heating bath temperature (T_∞) so that the surface temperature (T_e) is immediately raised to the level of the bath temperature, involving surface thawing. The question, then, is how much log heating time (t) is required to reach the temperature (T) at the radial depth (r) along the log's center cross section.

Note that the concept of two phases simultaneously existing during thawing is eliminated from consideration, given the assumption that the log's thermal conductivity (k), specific heat (c), and density (ρ) are functions of position and temperature and will discontinuously change with the phase. The latent heat is incorporated in the specific heat over a finite thawing temperature interval (concept

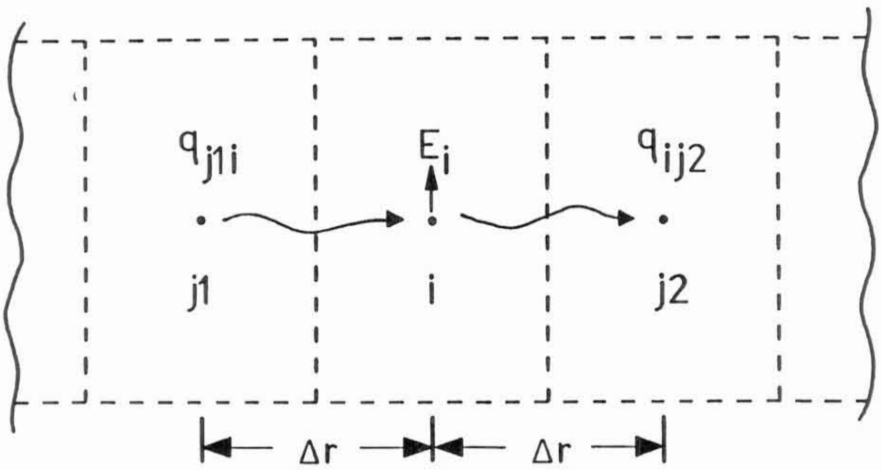

FIG. 2. Control systems with finite-difference terms.

of "equivalent specific heat," Bonacina et al. 1973). In this way, the thawing problem becomes equivalent to a problem of nonlinear heat conduction without phase change.

An energy balance on a control system (Fig. 1, broken lines) gives

$$q|_r = q|_{r+\Delta r} + E \tag{1}$$

where $q|_r$ is the rate of heat flow at position r, $q|_{r+\Delta r}$ is the rate of heat flow at position $r + \Delta r$, and E is the rate of energy storage. Substituting the conventional rate equations into the energy balance yields

$$-\left[kA\frac{\partial T}{\partial r}\right]_r = -\left[kA\frac{\partial T}{\partial r}\right]_{r+\Delta r} + \rho Vc\frac{\partial T}{\partial t} \tag{2}$$

where A is the area through which heat flows, and V is the volume. Further development will yield the nonlinear equation for axisymmetric heat conduction:

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)\right] = \rho c\frac{\partial T}{\partial t} \tag{3}$$

This equation cannot be solved because of the assumed temperature dependence of k, ρ, and c. However, an approximate solution is available via the finite-difference approach described below.

FINITE-DIFFERENCE APPROACH

A number of equidistant nodes are devised along the log radius (Fig. 2). Each node is successively considered the focal node i; nodes j1, j2 are the adjacent nodes. The heat flow from j into i is q_{ji} ($=-q_{ij}$), and the energy stored in the system surrounding node i is E_i. One may then write an energy balance on each system and substitute the rate equations into the energy balances. The negative expression of the derivative in the conduction rate equations, $-\partial T/\partial r$, is approximated by the difference notation $\Delta T/\Delta r$, for which one may write $(T_j - T_i)/d_{ji}$, where d_{ji} is the distance between j and i. This leads to a set of solvable equations analogous to Eq. (2):

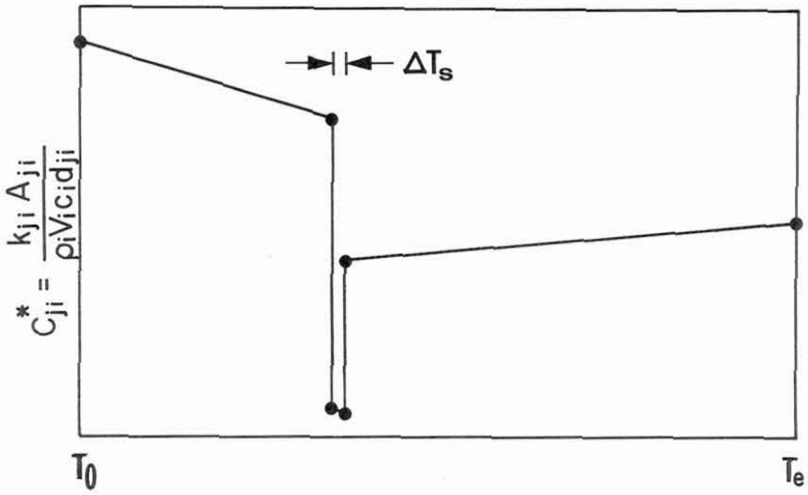

FIG. 3. Typical profile of C^*_{ji} over temperature.

$$\rho_i V_i c_i \frac{dT_i}{dt} = \frac{k_{jli} A_{jli}}{d_{jli}} (T_{jl} - T_i) + \frac{k_{j2i} A_{j2i}}{d_{j2i}} (T_{j2} - T_i) \tag{4}$$

Equation (4) may be rewritten as

$$\rho_i V_i c_i \frac{dT_i}{dt} = \sum_{j=jl}^{j=jn} C_{ji}(T_j - T_i) \tag{5}$$

where C_{ji} is the conductance $(k_{ji}A_{ji}/d_{ji})$, and n is the number of nodes immediately surrounding focal node i (i.e., 2 in our one-dimensional system, and 4, 6 in two- and three-dimensional systems, respectively). Equation (5) is of the type that HEAT (Beckman 1972) will solve simultaneously for all nodes in a multidimensional system.

As HEAT allows only the conductance (C_{ji}) but not the capacitance $(\rho_i V_i c_i)$ to be temperature-dependent, one must circumvent this shortcoming via dividing both sides of Eq. (5) by $\rho_i V_i c_i$, which yields

$$\frac{dT_i}{dt} = \sum_{j=jl}^{j=jn} C^*_{ji}(T_j - T_i) \tag{6}$$

where C^*_{ji} is the conductance per unit capacitance $(k_{ji}A_{ji}/d_{ji})/(\rho_i V_i c_i)$.

The quantity C^*_{ji}, in addition to the initial temperature (T_0) and surface temperature (T_e), is computer input data and must be evaluated for the temperature points shown in Fig. 3 by the dots. The thawing temperature interval (ΔT_s) shown in Fig. 3 is a computational parameter that must be adjusted so that agreement is obtained with experimental results. This thawing interval may or may not be fictitious. Within the thawing interval, the effective specific heat of a node (c_i) is

$$c_i = c_{i,s} + \frac{L_i}{\Delta T_s} \tag{7}$$

TABLE 1. *Thermal properties of log No. 10.*

Node i	r_i (m)	ρ_i (kg/m³)	$c_i \times 10^{-3}$ (J/kgK)						k_{ji} (W/mK)			
			−22 C	−1.5 (−) C	−1.5 (+) C[a]	1.5 (−) C[a]	1.5 (+) C	54 C	−22 C	−1.5 C	1.5 C	54 C
1[b]	0.2286	710	2.1	2.3	54	55	3.2	3.5	0.39	0.38	0.28	0.34
2	0.2159	650	2.1	2.3	49	50	3.1	3.4	0.35	0.34	0.26	0.32
3	0.2032	620	2.0	2.3	43	44	3.0	3.3	0.30	0.29	0.23	0.28
4	0.1905	570	2.0	2.3	35	36	2.9	3.2	0.27	0.26	0.22	0.26
5	0.1778	560	2.0	2.3	31	32	2.8	3.1	0.26	0.25	0.21	0.25
6	0.1651	540	2.0	2.3	26	27	2.7	3.0	0.27	0.26	0.22	0.26
7	0.1524	590	2.0	2.3	33	34	2.8	3.1	0.31	0.30	0.24	0.29
8	0.1397	620	2.0	2.3	40	41	3.0	3.3	0.33	0.32	0.25	0.30
9	0.1270	630	2.0	2.3	38	39	2.9	3.2	0.31	0.30	0.24	0.29
10	0.1143	610	2.0	2.3	35	36	2.9	3.2	0.30	0.29	0.23	0.28
11	0.1016	610	2.0	2.3	35	36	2.9	3.2	0.30	0.29	0.23	0.28
12	0.0889	590	2.0	2.3	33	34	2.8	3.1	0.29	0.28	0.23	0.27
13	0.0762	590	2.0	2.3	33	34	2.8	3.1	0.30	0.29	0.23	0.28
14	0.0635	610	2.0	2.3	33	34	2.8	3.1	0.27	0.26	0.22	0.26
15	0.0508	560	2.0	2.3	28	29	2.7	3.0	0.26	0.25	0.21	0.25
16	0.0381	530	2.0	2.3	26	27	2.7	3.0	0.31	0.30	0.24	0.29
17	0.0254	690	2.0	2.3	46	47	3.1	3.4	0.43	0.42	0.31	0.38
18	0.0127	850	2.1	2.3	59	60	3.3	3.6	0.57	0.56	0.39	0.48
19	0.0000	1,010	2.2	2.4	68	69	3.5	3.8				

[a] Includes latent heat.
[b] Surface node.

where $c_{i,s}$ is the true specific heat at the thawing temperature, and L_i is the latent heat. For wood, we have

$$L_i = L_w \left(\frac{MC_i - 30\%}{MC_i + 100\%} \right) \tag{8}$$

where L_w is the latent heat of water fusion (334,000 J/kg), and MC_i is the node's moisture content expressed in percent of the dry mass of wood. This concept considers only the free water ($MC_i > 30\%$) as frozen, the bound water phase change being negligible (Kubler 1962).

TABLE 2. *Computer input data C_{ji}^*, in seconds^{-1}, for log No. 10.*

j	i	−22 C	−1.5(−) C	−1.5(+) C[a]	1.5(−) C[a]	1.5(+) C	54 C
2	1[b]	—	—	—	—	—	—
1	2	0.0018	0.0016	0.000075	0.000054	0.00088	0.00098
3	2	0.0016	0.0014	0.000064	0.000048	0.00077	0.00086
2	3	0.0018	0.0015	0.000082	0.000062	0.00089	0.00099
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.
19	18	0.0010	0.0009	0.000034	0.000023	0.00043	0.00048
18	19	0.0064	0.0057	0.000200	0.000140	0.00270	0.00310

[a] Includes latent heat.
[b] Surface node i = 1 with specified temperature T_e = 54 C needs no C_{ji}^* value specification.

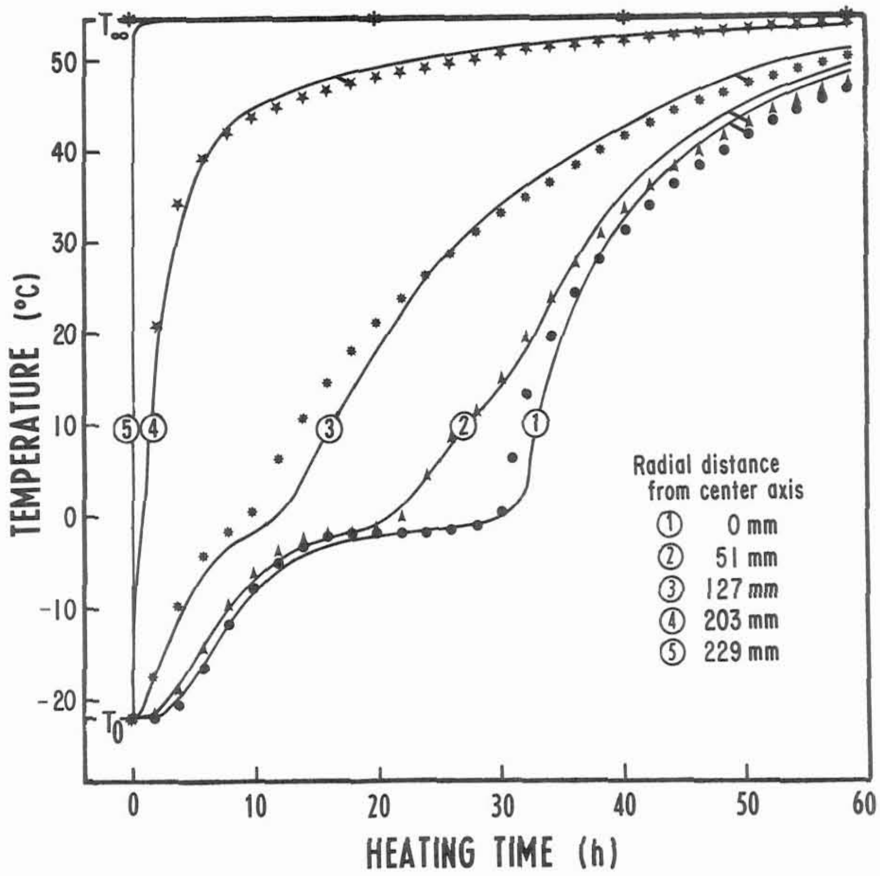

FIG. 4. Transient temperature profile of log No. 10.

COMPUTATIONS

Transient temperature profiles were computed for four logs identified in the above-mentioned experimental paper (Steinhagen 1977a) as Nos. 1 (aspen), 4 (black cherry), 7 (red oak), and 10 (eastern white pine). In accordance with the known log specifications, thermal conductive properties as a function of temperature, moisture content, and specific gravity were determined from the literature (Steinhagen 1977b). The thermal property values of log No. 10 are given in Table 1, for 19 nodes and an assumed thawing interval of 3 degrees (−1.5 to 1.5 C), with $T_0 = -22$ C and $T_e = 54$ C. These values were used to prepare the data of C^*_{ji} (Table 2). These data were submitted to the computer in tabulated form (the format is given in the user manual for program HEAT, Beckman 1972). HEAT performed linear interpolation of the table values. As the smallest value of the Euler stability limit criterion $(1/\sum_j C^*_{ji})$ was 1/0.0064 seconds (Table 2) which must not be exceeded by the integration time step, a value of 0.040 hours was chosen as a time step in the computations for log No. 10. Note that the computer input for capacitance had to be unity, due to the transformation of Eq. (5) into Eq. (6).

RESULTS AND DISCUSSION

The transient temperature profile of log No. 10 is given in Fig. 4. The solid lines were established in the earlier experiments (Steinhagen 1977a). The symbols represent the comparable computed data.

For a given position and temperature, the discrepancy between measured and computed time was usually within the 10% target, for all four logs studied in the project. The computational error seemed to increase when the latent heat entered the calculations. This is apparently due to the discontinuity and use of a fictitious thawing temperature interval to accommodate the latent heat effect. From a practical point of view, it appears, however, that the method has provided acceptable temperature-time solutions to the problem of axisymmetric thawing and heating of logs with position- and temperature-dependent thermal properties.

Increasing the number of nodes had no significant effect on the quality of results. Oscillations among the computed values were not observed in these computations.

Calculations with constant, though distinct, thermal properties in each phase were considerably less cumbersome to prepare than with variable properties. The resulting error was usually again within target.

Solutions have been generalized into charts (Steinhagen et al. 1980) covering a large range of values that the nondimensionalized parameters can assume.

CONCLUSIONS

Utilization of the temperature method in conjunction with computer program HEAT has provided temperature-time solutions to axisymmetric log thawing and heating cases.

The agreement between calculated and measured temperature profiles appears satisfactory.

Specification of a thawing temperature interval for wood appears to be a problem with this method.

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