INFLUENCE OF TRUSS PLATE CONNECTORS ON THE ANALYSIS OF LIGHT FRAME STRUCTURES

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ABSTRACT

A method of analysis for a plane framed structure with nonrigid connections is studied. The method encompasses the concept of matrix method. Structures are modeled using two types of elements, namely, a beam element and a joint element. Each two-dimensional joint element is composed of three linear springs having no physical dimensions, representing shear, axial, and moment resistance. The mechanical properties of the joint element are obtained experimentally. Two frames and several beams are tested, and results are compared to analytical results. Good correlations are obtained.

A solution of a truss with toothed metal plate connectors is presented and a general agreement with available solutions is obtained. A sensitivity study presenting the influence of joint stiffnesses is also presented. It is found that moment and axial spring properties have appreciable influence on members end forces, while the shear spring properties have little effect.

Keywords: Trusses, truss plates, analysis, connections, frames, stiffness, deflections.

INTRODUCTION

Mechanical connectors such as truss plates impart semirigid joints that have significant influence on internal forces and deflections. The analysis of such structures is complex and requires careful study of connectors' properties. Nonlinear behavior of the connections contributes to the complexity of the analysis. To circumvent this difficulty, linear-elastic assumptions have been made.

At the present time connections are assumed to be either rigid or pinned. In either of these two conditions, the forces obtained are unreliable and do not represent the actual structural behavior. Designs under either of the two assumptions are also inefficient and lead to an over- and/or under-designed member. The actuality of many connections is a partially rigid condition. Little research has been done in this area of analysis.

The objective of this study is the modeling of plane framed structures with nonrigid connections for predictions of the member forces and joint displacements. In addition, this investigation will set up an experimental method to measure nonrigid joint properties. The application of the method to a practical problem, such as trusses, will be presented. Little work has been reported to date to solve the problem of a truss with nonrigid joints. In 1967, Ngo and Scordelis

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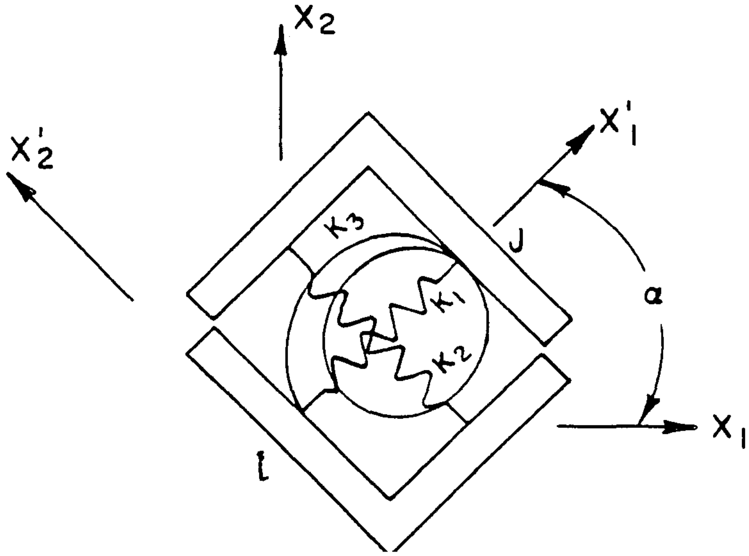

FIG. 1. Two-dimensional joint element.

developed an analytical method to determine the internal stress distribution in concrete and steel reinforcements of a member throughout its loading history. They modeled the concrete and steel slip by using two linear springs. In 1971, Reardon developed a method for analyzing plane framed structures having semi-rigid joints. In 1972, Suddarth of Purdue University developed a computer program, the Purdue Plane Structure Analyzer (PPSA), which performs rigid analysis of framed structures. The program is utilized in truss design and investigates the adequacy of truss elements under applicable code requirements.

Partial rigidity in the PPSA is handled by inserting fictitious, small elements of low stiffness. The difficulty in this approach is selecting proper properties of these fictitious elements to simulate actual conditions. The PPSA then uses rigid analysis.

In 1980, an analytical method for beam with joint slip was given by Soltis. In 1977, Foschi presented an analysis of truss-plate connection, considering the nonlinear load deformation relationship. This analysis accounts for the buckling capacity of the plate, and its yielding in tension and shear.

FORMULATION

Our method considers the structure as an assemblage of two elements, a beam element and a joint element. These two elements are combined using matrix methods of structures to formulate the stiffness matrix of a complete structure.

Beam element

A beam element is modeled as a line element defined by two nodes. Each node has three degrees of freedom. The stiffness matrix for the plane beam element is a 6 by 6 symmetric matrix (Eq. 1).

TABLE 1. *Properties of the lumber used in experimental work.*

Specimen no.	Width (in.)	Thickness (in.)	γ #/cf	Moisture content	E 10^5 psi
1	$3^{15}/_{32}$	$1^{15}/_{32}$	31.8	5.88	14
2	$3^{13}/_{32}$	$1^{15}/_{32}$	32.1	5.88	15
3	$3^1/_2$	$1^{15}/_{32}$	34.8	6.65	23
4	$3^{15}/_{32}$	$1^{15}/_{32}$	32.9	5.4	18
5	$3^1/_2$	$1^1/_2$	27.8	4.61	17

Joint element

There are several theoretical methods for modeling flexible connections. However, the attempt was made here to model the flexible connections so that their properties could be easily measured in a laboratory. Each connection was simplified as a set of linear elastic springs. Each joint element consisted of three springs representing axial, shear, and rotational stiffnesses. The plane joint element was conceptually modeled as a mechanism (Fig. 1). The joint element has no physical dimension; only its stiffness properties are of importance.

In Fig. 1, X_1 and X_2 are the global coordinate system, while X_1' and X_2' are the local coordinate system of the joint element. The stiffness matrix for a plane joint element in a local coordinate system (X_1', X_2') is represented in Eq 2.

$$[K] = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EL}{L} \end{bmatrix} \quad (1)$$

$$[K] = \begin{bmatrix} K_1 & 0 & 0 & -K_1 & 0 & 0 \\ 0 & K_2 & 0 & 0 & -K_2 & 0 \\ 0 & 0 & K_3 & 0 & 0 & -K_3 \\ -K_1 & 0 & 0 & K_1 & 0 & 0 \\ 0 & -K_2 & 0 & 0 & K_2 & 0 \\ 0 & 0 & -K_3 & 0 & 0 & K_3 \end{bmatrix} \quad (2)$$

In order to obtain the structural stiffness matrix, the element stiffness matrix must be transferred from the local coordinate system to the global coordinate system (X_1, X_2), by the product of $[T]^T[K][T]$, where [T] is the element transformation matrix,

$$[T] = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \quad (3)$$

where

$$[R] = \begin{bmatrix} \text{Cos } \alpha & \text{Sin } \alpha & 0 \\ -\text{Sin } \alpha & \text{Cos } \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

α is the angle between the local and global axis.

TESTING METHOD AND DATA COLLECTION

As mentioned previously, each nonrigid joint was modeled as a set of linear spring systems. The stiffness and transformation matrices were established for the joint elements. A computer program was developed capable of analyzing frames with nonrigid connections. The linear spring system characteristics for each joint were entered as input data.

To obtain the stiffness properties of the joint elements of the joint stiffness matrix, three sets of tests were performed, with five specimens used in each set. The properties of the lumber and connector plates used in this work are given in Tables 1 and 2.

The rotational spring constant

The resistance of the connection to flexural deformation was measured by the rotational stiffness of the joint. The stiffness of this spring was established by subjecting the specimen to pure bending and measuring the moment-rotational deformation property. Pieces of nominal 2 × 4 Douglas fir lumber, 64 inches long, were tested under pure bending as shown in Fig. 2. In order to determine the spring constant, K_3, each beam was subjected to load P, and the midspan deflection, Δ, was measured. Each beam was then cut in half (two 32-inch pieces) and rejoined with two light-gauge steel plates. The test was repeated to obtain the deflections, Δ', at the center of each beam (Fig. 3). Using deflection measurements made before and after cutting specimens, the rotation of one end of the joint with respect to the other end, θ, can be calculated. In Fig. 4 deflection of the beam at its center is Δ. The angle between the horizontal line and the cord, AB, is θ_1. When the lumber is cut in half and rejoined with steel plates (Fig. 5), the deflection at the center of the beam increases to Δ'. Cut and uncut beams exhibit the same

TABLE 2. *Dimension of the 18-gauge light plate.*

Length (in.)	Width (in.)	Thickness (in.)	Tooth size (in.)	No. of teeth per unit area
6	3	0.04	5⁄16	9

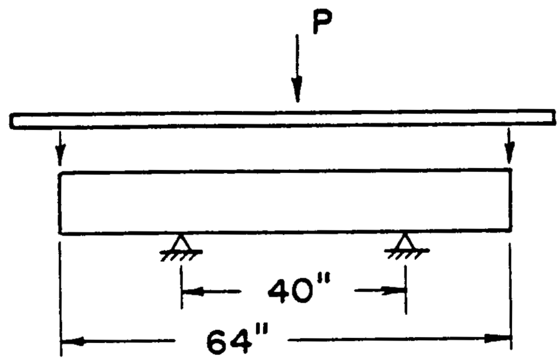

FIG. 2. Continuous beam, bending test.

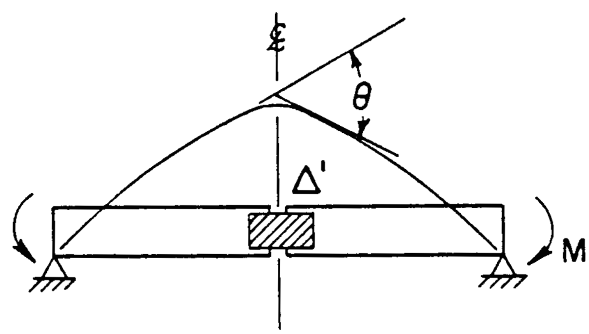

FIG. 3. Beam with steel plates.

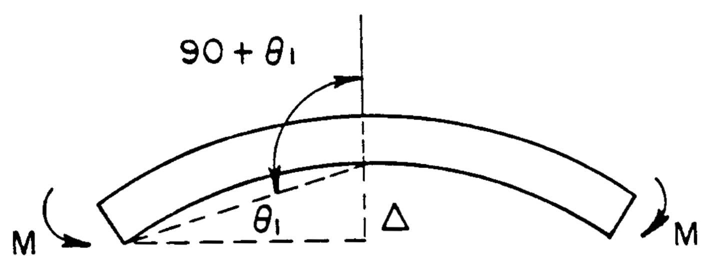

FIG. 4. Continuous, unjointed beam.

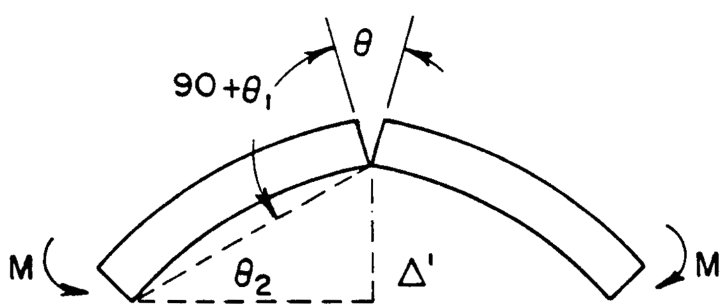

FIG. 5. Beam with metal plate connected joint.

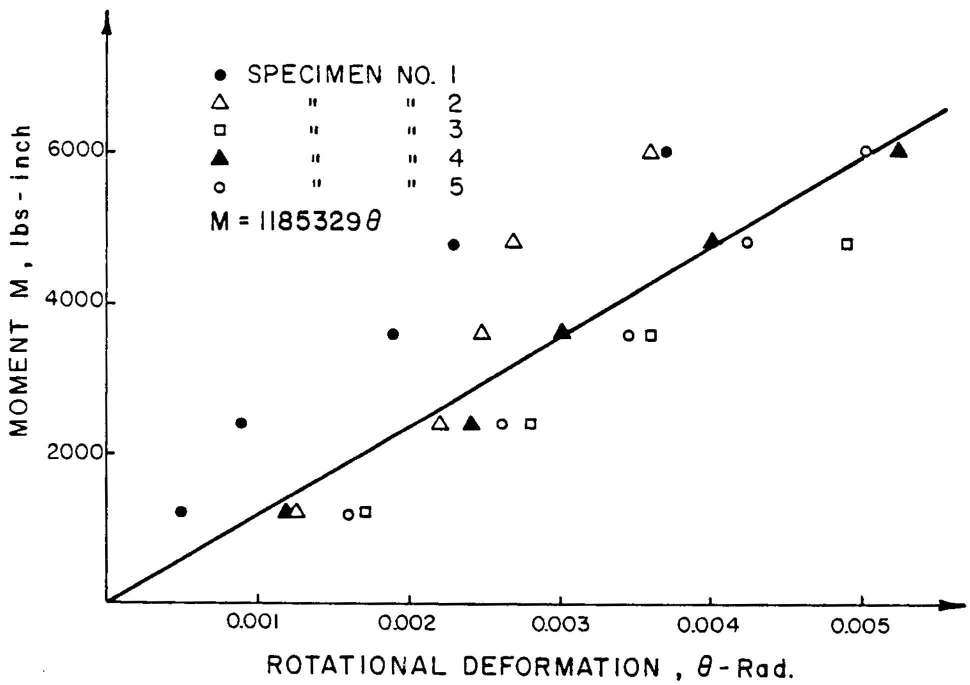

FIG. 6. Applied bending moment versus rotation.

curvature when the same moment is applied. Thus, each half of the cut beam rotates as a rigid body about the support. The rotation is $\theta/2$.

Thus:

$$\frac{\theta}{2} = \theta_2 - \theta_1 \tag{5}$$

For small deformations:

$$\text{Tan } \theta/2 = \text{Tan } 2(\Delta' - \Delta)/L \tag{6}$$

$$\theta = 4(\Delta' - \Delta)/L \tag{7}$$

Then,

$$M = 4\frac{\Delta' - \Delta}{L}K_3 \tag{8}$$

or

$$K_3 = 4\frac{ML}{\Delta' - \Delta} \tag{9}$$

Corresponding values of applied moment and measured rotation are plotted in Fig. 6. As can be seen from this graph, the relationship between bending moment

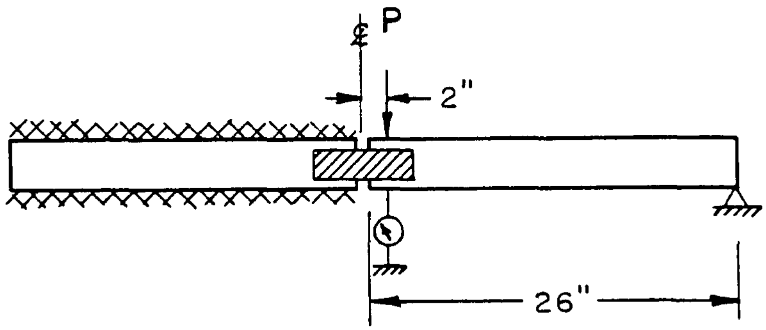

FIG. 7. Light toothed plates, shear test.

and rotation is nearly linear. A line was fitted to the pooled data for all five specimens using the method of least squares.
The following equation is obtained:

$$M = 1{,}185{,}329\theta \qquad (10)$$

A stiffness of 1,185,329 lb-in. per radian was used for this plate size. The coefficient of correlation, *r*, was 0.97, which is a very satisfactory value for wood properties variation.

The shear spring constant

The resistance of the joint to shear force is measured as a linear spring with stiffness of K_2. Two pieces of nominal 2 × 4 Douglas fir were connected by two toothed metal plates of the size and type used for the joint rotation test (Fig. 7). A total of five specimens were tested and six readings were taken for each specimen. The properties of steel plates and lumber are given in Tables 1 and 2.

Each member was subjected to vertical force P at a distance of 2 inches from the center of the joint. In this case the joint is very nearly under pure shear. The shear force which is carried by the joint can be calculated from the equation of equilibrium.

$$V = \frac{24}{26}P \qquad (11)$$

The shear force versus deflection is all plotted in Fig. 8. Because of nonlinearity of load slip curves, a wide scatter is obtained in points. For the purpose of linear analysis, a line was fitted to the pooled data for all five specimens using the method of least squares. The following equation is obtained:

$$V = 19{,}217\delta \qquad (12)$$

The coefficient of correlation, *r*, was 0.75, which could be regarded as typical for wood properties variation. A stiffness of 19,217 lb/in. was used for this plate size.

The method of analysis developed in this study has been tested by comparing computed and measured results for a structure loaded in such a manner that the shear loads are in the range where these points were measured.

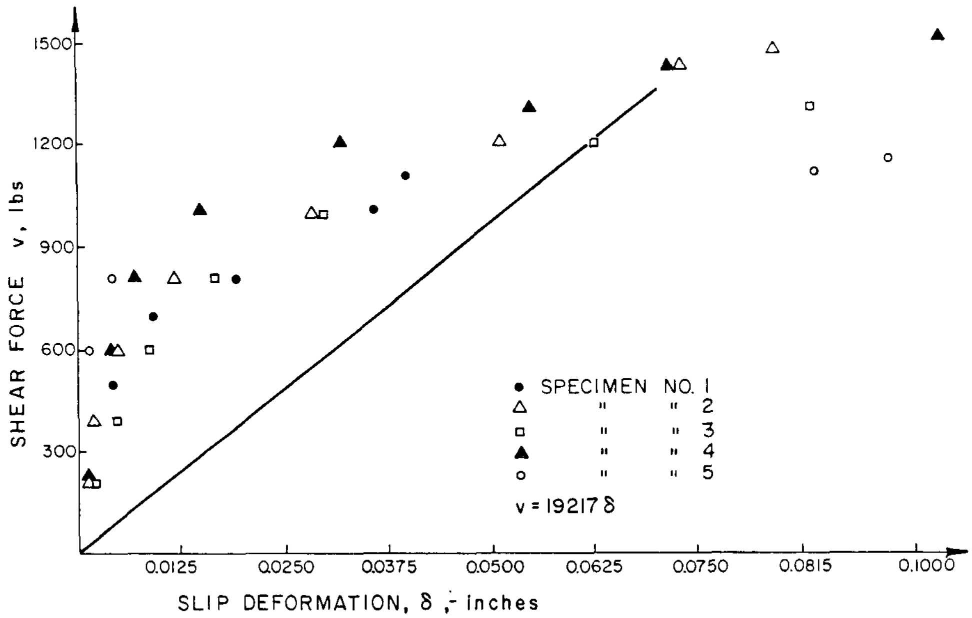

FIG. 8. Applied shear force versus slip, shear test.

The axial spring constant

To obtain the axial stiffness of the connection, K_1, two pieces of nominal 2 × 4 Douglas fir were connected by two toothed metal plates of the size and type used for the joint rotation test and shear test (Fig. 9). A total of five specimens

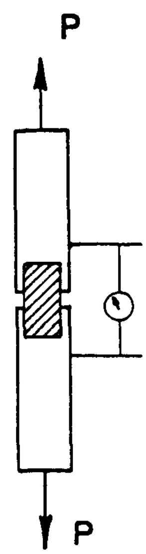

FIG. 9. Light-gauge toothed plates in tension test.

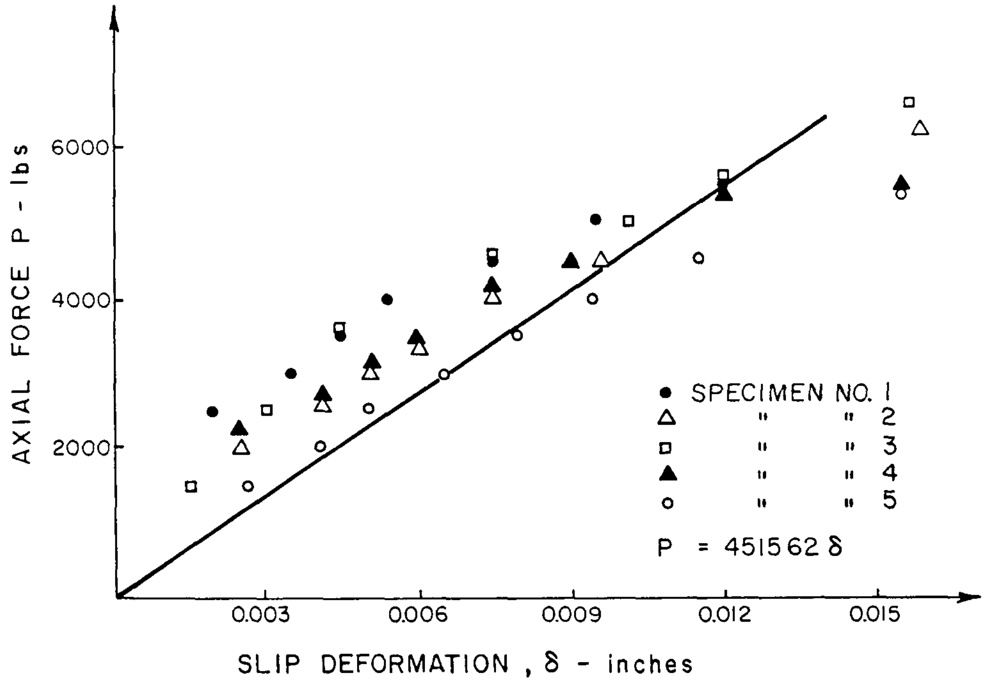

FIG. 10. Axial force versus slip.

were tested. The properties of steel plates and lumber are given in Tables 1 and 2. The applied force versus joint deformation was found to be nonlinear. For linear analysis a line was fitted to the pooled data for all five specimens using the method of least squares (Fig. 10). The following equation is obtained:

$$P = 451{,}562\delta \qquad (13)$$

A stiffness of 451,562 lb/in. was used for this plate size. The coefficient of correlation, r, was 0.96. This stiffness was used for axial tensile and compressive behavior of the joint, it being assumed that no lateral buckling of the metal plates would occur at the loads to be imposed and that the ends of the connected members would not come to bear on one another when load in compression.

THE BEAM AND FRAME TESTS

In order to examine the accuracy of the method of modeling, five beams and two frames containing metal plate connected joints were tested. The deflections were measured at certain points of the beams and frames. The properties of plates and lumber are given in Tables 1 and 2. The spring constants used in the analysis are those that were measured and described before. To obtain accurate analytical results, the stiffness matrix was modified to include the shear effects.

Beam test

A piece of nominal 2 × 4 Douglas fir lumber, 40 inches long, was cut and connected by two toothed metal plates (total of five specimens). These beams

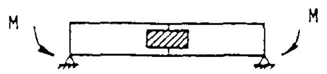

TABLE 3. *Deflection of the beam at the center due to bending.*

Moment (lb/in.)	Measured* deflection (in.)	Analytical deflection (in.)	Rigid deflection (in.)
1,200	0.0362	0.0381	0.0280
2,400	0.0741	0.0762	0.056
3,600	0.107	0.114	0.0840
4,800	0.141	0.152	0.1119

* Average of five beams.

were simply supported and subjected to two different load cases—case one, concentrated moment M at supports, and case two, a concentrated load P at its midspan. The average of midspan deflection of all five beams was compared with the analytical results (Tables 3 and 4). As Tables 3 and 4 indicate, there was acceptable agreement between analytical and experimental results. The measured and calculated deflections for nonrigid beams were compared with the continuous beams (rigid). The deflection at the center for the beam under applied moments increased 36% because of nonrigid joint; for the same beam this increase was 72% when the concentrated load was applied at midspan. The error in measured deflections at the center of the beams to calculated deflection at the same point for the nonrigid beams loaded with end moments M and the concentrated load P at the center ranged from 5 to 8% for the first case and 2 to 20% for the second case.

Frame tests

Metal plate connector behavior tests were carried out on two frames (Fig. 11). The properties of steel plates and lumber are given in Tables 1 and 2. The measured spring constants obtained previously were used in analytical solution. These two frames were tested, first using a concentrated load applied at the center of the beam, second, using a concentrated load applied at the distance of 24 inches from

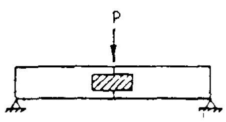

TABLE 4. *Deflection of the beam at the center under concentrated load.*

P (lb/in.)	Measured* deflection (in.)	Analytical deflection (in.)	Rigid deflection (in.)
120	0.04	0.032	0.0187
240	0.072	0.064	0.0373
360	0.103	0.096	0.0560
480	0.131	0.127	0.0746
600	0.163	0.160	0.0933

* Average of five beams.

TABLE 5. *Deflection of the frame at beam center.*

Load at center	Measured deflection at center (in.)	Analytical deflection at center (in.)	Rigid frame deflection at center (in.)
100	0.114	0.120	0.101
200	0.256	0.240	0.202
300	0.409	0.360	0.303
400	0.562	0.480	0.404
500	0.712	0.600	0.505

TABLE 6. *Deflection and side sway of the frame.*

Load at 0.25 lb	Measured deflec. at 0.25 lb (in.)	Analytical deflec. at 0.25 lb (in.)	Side sway (in.)		Rigid frame defl. (in.)	
			Measured	Analytical	Vertical	Side sway
100	0.082	0.07	0.042	0.05	0.057	0.05
200	0.171	0.14	0.085	0.101	0.115	0.10
300	0.216	0.21	0.127	0.15	0.172	0.151
400	0.340	0.28	0.180	0.20	0.229	0.20

the left column. The deflection of the beam under the applied load and the side sway were measured. The experimental and analytical results shown in Tables 5 and 6 indicate that the behavior of these two frames is nearly the same. Tables 5 and 6 show that the measured deflections and calculated ones are quite close. The error from measured deflections to calculated deflections ranged from 5 to 18%, depending on applied load. The comparison between the deflection of the nonrigid frame and the deflection of the rigid frame indicates 23% flexibility due to nonrigid connections.

CONTACT AREA AND STIFFNESS

Metal connectors are widely used in truss construction. At each joint, there are usually more than two structural members. It is common to use only one plate

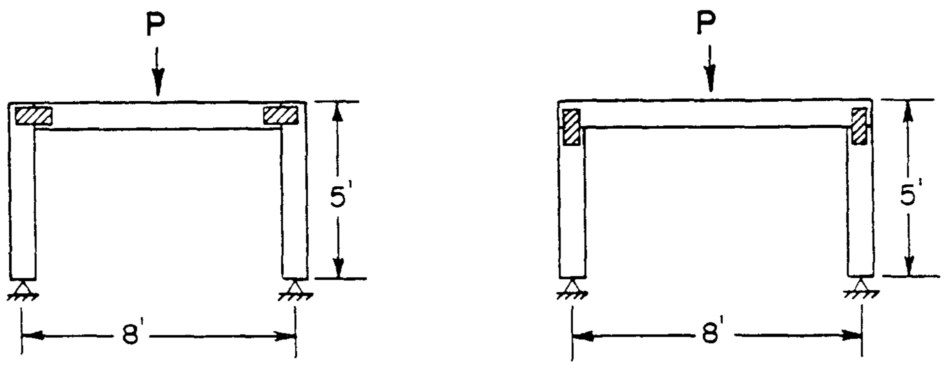

FIG. 11. Tested frames.

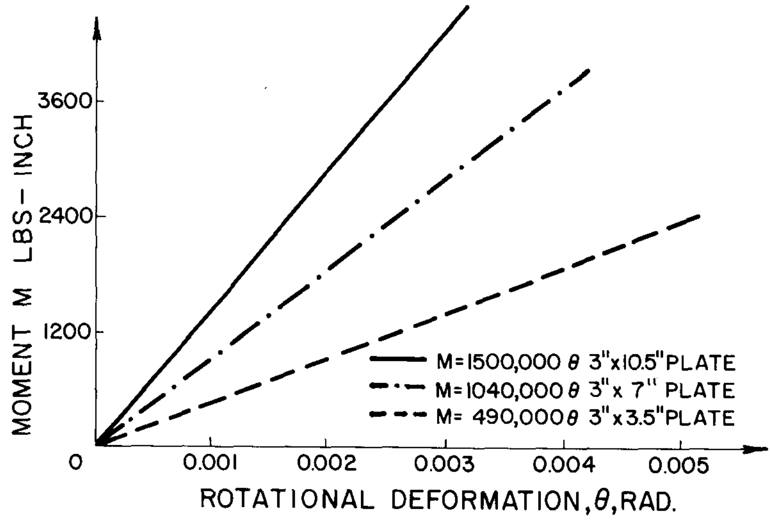

FIG. 12. Bending moment versus rotation for various plate sizes.

to cover all the members in the connection. Each contact area (defined as where the metal plate and corresponding members at a joint overlap) can be simulated as a set of three springs. Properties of these springs are essential for calculation of the system stiffness matrix. The relationship between stiffness and the contact area was studied experimentally by using three different plate sizes. A total of fifteen specimens were tested (five for each plate size). The rotational, axial, and shear stiffness for each plate size was calculated. The experimental procedure was

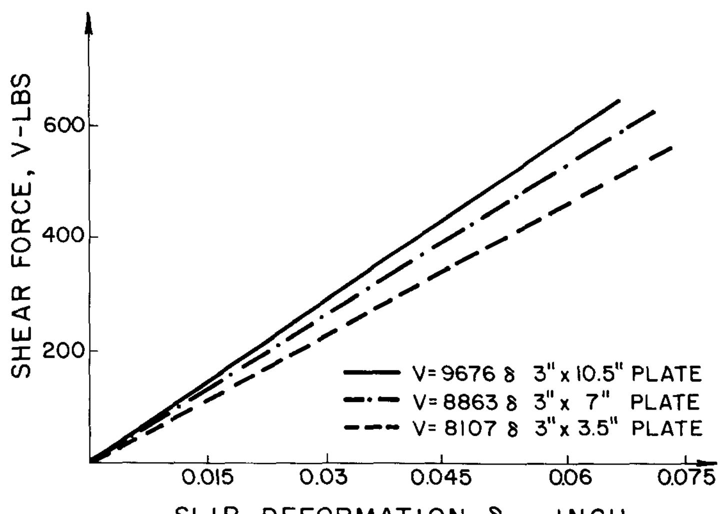

FIG. 13. Shear force versus slip for various plate sizes.

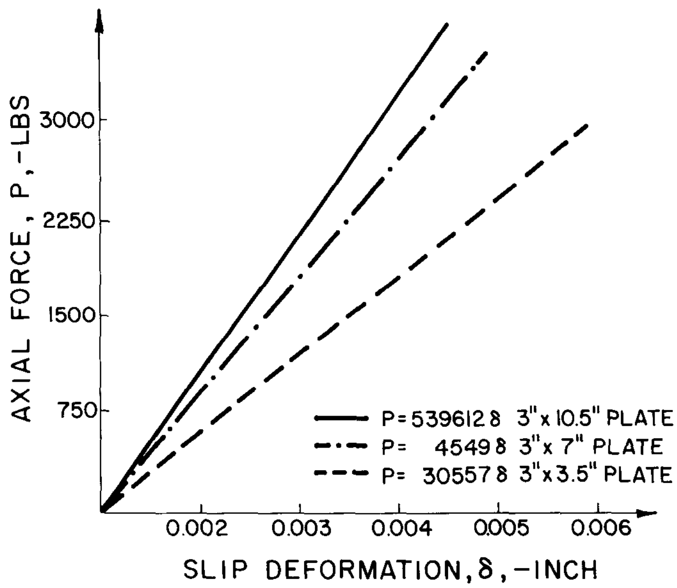

FIG. 14. Applied axial force versus slip for various plate sizes.

the same as previously detailed. It was found that the rotational stiffness is in direct proportion to the area of contact. No relationship was found for axial and shear stiffness and the area of contact. The measured deformation and applied load for each plate size are given in Figs. 12, 13 and 14.

PLANE TRUSS

In order to ascertain the validity of this method of analytical investigation, the following procedure was implemented. A plane truss was analyzed by two methods (Fig. 15). The first method is that of Reardon (1971); the second method is that developed in this study.

The analytical procedure undertaken by Reardon involved two different cases of semirigid joint behaviors.

Case 1. In this case all of the joints were semirigid in the axial direction and fully rigid in the other direction. The rotational rigidity was taken to be 100,000 in.-lb per radian.

Case 2. In this case all of the joints were semirigid in the rotational direction and fully rigid in the other directions. The rotational rigidity was taken to be 100,000 in.-lb per radian.

The truss analyzed by Reardon is studied by the method developed here. In Case 1 the axial stiffness of all the joints was taken to be 100,000 lb/in. The stiffnesses in shear and rotational direction were taken to be 10×10^{10} lb/in. and 10×10^{10} in.-lb per radian, respectively, in order to simulate the rigid connection in these directions. In Case 2 the rotational stiffness for all joints were assumed to be 100,000 in.-lb per radian. The stiffness in shear and axial directions was assumed to be 10×10^{10} lb/in.

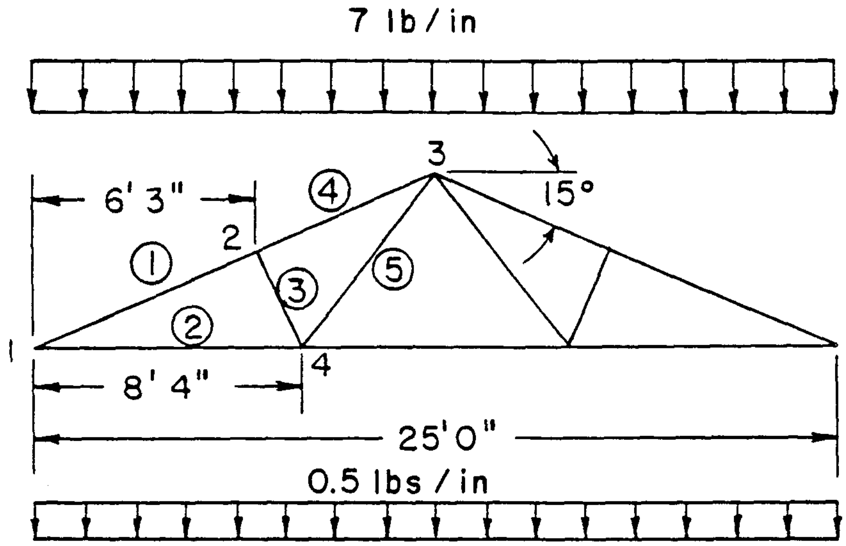

FIG. 15. Truss example.

The change in the end moments due to semirigidity was compared with Reardon's and tabulated in Table 7. Table 7 indicates the general validity of the method of modeling.

PLANE TRUSS WITH STEEL PLATE CONNECTIONS

This section is concerned with the application of the analytical method to a practical problem. A W-shaped truss with the dimensions as given in Fig. 15 was analyzed with the method presented in this study. The size of steel plate connectors was chosen arbitrarily (Fig. 16). It was assumed that the connector plates have the same stiffness per unit area as the first plate tested. Also it was assumed that the axial stiffness and the shear stiffness of the plates were proportional to the area of contact. In the following section of this study, it will be shown that the influence of the shear stiffness coefficients will have little bearing on forces and displacements. The properties of connections based on the contact area with wood are tabulated in Table 8. Results of rigid and nonrigid connections are compared

TABLE 7. *Comparison of member end moments for plane truss.*

		Percent of moment change			
		Reardon		Present study	
Member	Joint	Axial semirigidity	Rotational semirigidity	Axial semirigidity	Rotational semirigidity
1	1	7	−66	4	−79
2	1	7	66	4	−79
4	2	−7	15	−17	38
4	3	4	−70	2	−80

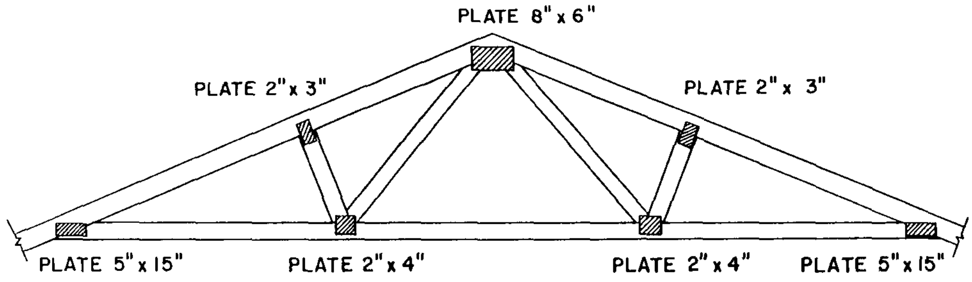

FIG. 16. Plane truss used in sensitivity study.

in Table 9 for joint displacements. The member forces are compared in Table 10.

As expected, Table 9 indicates that the displacements of joints are increased for nonrigid connections. However, the comparison of the members' forces indicates that the direction of the change in the end forces due to flexible joints cannot be predicted. The analysis shows that some members' end forces would increase (member 4) and some would decrease (member 2). For reliable design, an engineer must be aware of this condition.

SENSITIVITY ANALYSIS

The truss analyzed in previous sections was used in this study. The analysis consisted of three steps. First, the values of the axial spring stiffness for all of the

TABLE 8. *Joint elements properties for plane truss.*

Joint	Member	Contact area (in.²)	Stiffness per area for axial (lb/in./in.²)	Axial stiffness (lb/in.)	Stiffness per area for shear (lb/in.³)	Shear stiffness (lb/in.)	Stiffness per area for rotation (lb-in./Rad/in.²)	Rotation stiffness (lb-in./Rad)
1	1	37.5	25,087	940,754	5,214	195,527	65,852	24,694
2	1	37.5	25,087	940,754	5,214	195,527	65,852	24,694
3	2	6	25,087	150,521	5,214	31,284	65,852	39,511
3	4	4	25,087	100,347	5,214	20,856	65,852	263,407
4	3	12.14	25,087	304,553	5,214	69,389	65,852	799,439
5	3	4.28	25,087	107,371	5,214	24,464	65,852	281,845
5	4	4	25,087	100,347	5,214	20,856	65,852	263,407

TABLE 9. *Joints displacements of the plane truss.*

Node	Displacement	Rigid	Nonrigid
2	X	0.025347	0.041121
	Y	−0.216741	−0.289311
3	Y	−0.22394	−0.27324
4	X	0.004477	0.004658
	Y	−0.238523	−0.312295

TABLE 10. *Member forces for the plane truss.*

Member	Joint	Force*	Rigid	Pinned	Nonrigid
1	1	A	3,489	3,504	3,474
		S	225	255	244
		M	1,290	0	1,136
2	1	A	376	385	359
		S	5	19	9
		M	1,290	0	1,136
4	2	A	2,918	2,957	2,925
		S	249	261	265
		M	3,018	3,686	3,306
4	3	A	2,782	2,957	2,789
		S	259	261	242
		M	3,407	0	2,386

* A = axial, S = shear, M = moment.

joints were reduced by 75%, while the properties of the other springs were unchanged. Second, the values of the shear spring stiffness for all of the joints were reduced by 75%. In the third case, the values of the rotational spring stiffness for all of the joints were reduced by 75%. The member end forces for these three cases were compared to the member end forces for the truss without reduction in any spring properties. Results were tabulated in Table 11. This table indicates that the shear springs have little bearing on forces; however, the rotational and axial springs have considerable effect in members' end forces.

TABLE 11. *Percent change in member end forces due to a 75% change in joint properties.*

Member	Joint	Forces*	Change** in axial stiffness	Change** in shear stiffness	Change** in rotational stiffness
1	1	A	1.5	0.05	0.4
		S	5	0.25	4
		M	13	1.2	30
2	4	A	20	0	4
		S	5	0	12
		M	68	2	27
3	4	A	5	0	4
		S	40	2.5	90
		M	32	0	77
4	5	A	1	0	0
		S	6	0	8
		M	24	0	13
5	2	A	6	0	5
		S	170	2.5	63
		M	160	1	68

* A = axial, S = shear, M = moment.
** Change is applied to a stiffness coefficient while maintaining the other two at a rigid condition.

CONCLUSIONS

A method for analyzing semirigid light frame structures is introduced in this study. The method models the structures using beam and joint elements.

The joint element consists of a system of three linear springs having no physical dimensions. The properties of the springs were obtained experimentally. Frames and several beams were tested, and deflection was measured at the point where the load was applied. These deflections yielded good comparisons with results obtained by the method.

The relationship between each joint's stiffness and the contact area of the toothed-plates was investigated experimentally for three different sized plates. The test results indicate that rotational stiffness is in proportion to the area of contact. The shear and axial stiffness is not in proportion to the contact area.

The application of the method developed for nonrigid connections was applied to a truss. Results were compared with earlier published data and a reasonable comparison was obtained. A sensitivity analysis showed that the shear springs properties have little bearing on member end forces, while the rotational and axial spring properties have appreciable influence on the members end forces.

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