ROOF LOADS FOR RELIABILITY ANALYSIS OF LUMBER PROPERTIES DATA[1]

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ABSTRACT

With load information reported in previous studies, distributions of maximum lifetime roof loads are developed in a form suitable for use in reliability analyses of lumber properties data. A lognormal distribution is chosen as best representing normalized maximum lifetime roof snow load.

Examples are given in which contrasting lumber data sets are compared using the calculated load distributions and assuming that each set must provide equal reliability (equal safety) in the final design. A factor, k, resulting from this reliability analysis is shown to be a logical adjustment parameter for use in engineering design codes.

Keywords: Roof loads, reliability, lumber, strength behavior, adjustment factors.

PROBLEM STATEMENT

Structural performance is determined by the interaction of applied loads and the resistance provided by materials of construction. However, both the loads and the resistances exhibit inherent variability that must be considered if realistic estimates of structural performance are to be obtained. In timber design this inherent variability in material resistance is recognized by basing the design on a strength that would be exceeded by 95% of the pieces tested. Similar logic is used for specifying loads, except that attention is focused on the largest reasonable load to which a structure may be subjected during its lifetime. Lumber properties may also be affected by a number of "environmental" factors such as end-use moisture content, changes in temperature, fire-retardant and preservative treatment or even the rate at which the load is applied.

Ideally it would be desirable to determine actual structural performance for all possible combinations of load and material resistance. However, such evaluations are not always economically feasible. The alternative is to try to incorporate the effect of these factors into the design process by evaluating their effect on material resistance. Unfortunately, the effect of many of these factors on lumber strength is not constant but instead varies with position in the strength distribution. Because the current design process focuses attention on only one level of the strength distribution, safe and efficient utilization of the inherent strength of lumber that will be subjected to varying environmental conditions may not be possible.

An alternative approach to judging the effect of such environmental conditions on lumber performance is to use the concept of differential reliability (Suddarth et al. 1978) in which the probability of failure for one set of conditions is compared to that for a standard set of conditions. The objective of this paper is to identify distribution of dead load and maximum lifetime roof snow load that could be used in such a differential reliability analysis.

INTRODUCTION

In the United States, there are currently three possible procedures for assigning allowable design properties for native species: (1) The small, clear specimen procedure (ASTM 1981a, b), (2) full-size lumber testing (ASTM 1981c), and (3) machine stress rating (U.S. Dept. Comm. 1981). With all three procedures an estimate is first made of the strength of the population (grade, size, species, etc.) being considered, this estimate is modified to account for variability in strength estimates, and adjustments are made to account for various end-use considerations.

Throughout the design process, however, attention is focused on only two levels of the cumulative frequency distributions: the mean value for modulus of elasticity (MOE) and the fifth percentile for strength properties (ASTM 1981a, b, c; Bendtsen and Youngs 1981). The use of the fifth percentile represents an attempt to account for inherent variability by calculating a value that would be exceeded by 95% of the pieces tested.

Similar logic is used to develop nominal loads. The nominal value of a load is the value specified by the code authorities (Siu et al. 1975). Whereas loads acting on a structure during its design lifetime are highly variable, the basic concern is that a structure or structural component should be designed to withstand the largest reasonable load or load combination to which the structure will be subjected during its life. Therefore, nominal loads are developed to reflect a slight probability of occurrence of the maximum lifetime load (ANSI 1982).

By using the allowable design values for structural lumber given in the supplement to the National Design Specifications (NDS) (NFPA 1978) and a prescribed load combination, engineers have an established procedure to design light-frame lumber structures. These structures have an excellent record of service, but recent research indicates that this is not a unique result of the allowable strength values (Goodman et al. 1974; ICBO 1973; Polensek 1976). Also under close study are the factors used to account for end-use considerations (Gerhards et al. 1976; Green 1980; Madsen 1978). From the questions raised by these studies, it would seem that realistic estimates of structural performance may not be obtained if attention is restricted to only one point on the load and material property distribution

curves. It is also clear that basing estimates of the response of lumber properties to changing end-use environmental conditions on the response at only one level of the strength distribution may not efficiently utilize the inherent strengths of the material. How, for example, should one judge structural performance if subjecting lumber to a certain environmental factor causes one response from pieces at the extreme low end of the strength distribution and a completely opposite response from stronger pieces?

These effects can be judged by using the concept of differential reliability (Suddarth et al. 1978). To understand this concept, we must suppose that a resistance distribution and a load distribution are known, along with their relative positions on a coordinate axis. Then, a probability of failure associated with the two distributions can be determined. This probability of failure may have no significance by itself, but if a second, related resistance distribution is known, then comparative probabilities of failure can focus on differences between the two resistance distributions. This type of comparative evaluation has been termed differential reliability.

Suddarth et al. (1978) noted that, when studies of two or more distributions are made, the contrast between the probabilities of failure for these cases allows strong analytical focus on their differences. This strong analytic advantage occurs because all the assumptions used for one distribution can also be carried through for the others in a completely formal way. In other words, the effects of errors or biases induced by incomplete data describing load or resistance distributions should be minimized by the differential reliability analysis. However, to conduct a meaningful reliability analysis of lumber properties data and remain consistent with accepted design practice, load distributions that reflect actual maximum loads on a light-frame structure in service need to be identified. One load combination considered in design is the dead load plus snow load combination.

In this paper the distributions of dead load and maximum lifetime roof snow load that could be used in a differential reliability analysis are identified. A method for combining these distributions is discussed and several examples are given to illustrate the use of differential reliability in assessing the effect of treatments on lumber strength properties. Before the actual roof load distributions are considered, however, let us first review some basic concepts.

BACKGROUND

Probability of failure

Probability of failure is a measure of the underlying risk associated with a structure, member, or material due to the uncertainties of engineering design (Marin and Woeste 1981). A failure event occurs whenever the strength (resistance) of a structure or member (R) is less than the load (S). This can be described by the following equation.

$$P_f = Pr(failure) = Pr(R < S) \tag{1}$$

If the load and the strength are described by continuous probability distribution functions and are mutually independent, then the probability of failure can be shown to be

$$P_f = \int_0^\infty \left[\int_0^s f_R(r) \, dr \right] f_S(s) \, ds \qquad (2)$$

where

$f_R(r)$ is the probability density function of the resistance variable, strength
$f_S(s)$ is the probability density function of the load variable.

The probability of failure can be described as the sum of many small incremental probabilities of failure. An incremental probability of failure can be written as

$$d_{P_f} = \int_x^{x+dx} F_R(x) f_S(x) \, dx \qquad (3)$$

where

$F_R(x)$ is the cumulative distribution function of the resistance variable
$f_S(x)$ is the probability density function of the load variable.

Numerically, this incremental probability of failure can be approximated by two equations. The first equation is the upper limit and the second equation is the lower limit of the incremental probability of failure. They are

$$d_{P_{f_{upper}}} = F_R(x + dx)[F_S(x + dx) - F_S(x)] \qquad (4)$$

$$d_{P_{f_{lower}}} = F_R(x)[F_S(x + dx) - F_S(x)] \qquad (5)$$

where

F_R = the cumulative probability distribution of the resistance variable evaluated at x or x + dx
F_S = the cumulative probability distribution of the load variable evaluated at x or x + dx.

By choosing an appropriate increment, dx, and solving Eqs. (4) and (5) iteratively over a selected range of the resistance function, the probability of failure can be approximated. In other words, starting at a suitable point on the horizontal axis of the resistance distribution, such as the location parameter of a three-parameter Weibull distribution, and stepping from increment to successive increment, infinitesimal probabilities of failure can be calculated. The sum of these infinitesimal probabilities of failure is the probability of failure of the resistance variable under the influence of the load variable. If the increment dx is small enough, the lower and upper limits will approach one another. The range of calculation must be selected by trial and error. The range is the distance on the horizontal axis for which the calculation of incremental probabilities of failure is made. When the lower and upper limits of the probability of failure are no longer significantly changed by an increase in the range, the range is sufficient.

Differential reliability analysis

Differential reliability was first proposed by Suddarth et al. (1978) as a means of comparing contrasting sets of lumber data. With a simplified structural model to simulate a member from a more generalized structure such as a truss, an investigation was conducted on the effect of variability in MOE on truss reliability.

The concept of a "probability ratio" was also used as a means of comparing the probability of failure for a given load-material resistance combination to the probability of failure for an assumed standard, or benchmark, combination. Differential reliability was shown to be of value for code calibration purposes and for predicting future design-and-use payoffs for investments in material properties research.

Marin and Woeste (1981) used differential reliability to illustrate the potential use of proof loading in maintaining the allowable design value for a given grade of lumber in face of a shift in the strength distribution of the lumber being produced at a mill. Green (1980) suggested that a logical adjustment factor for evaluating the effect of moisture content on lumber properties might be obtained by requiring equal probabilities of failure for the strength distributions of two lumber samples conditioned to differing moisture contents. The "shift factor" resulting from this assumption was proposed as a more logical alternative to the moisture adjustment factor used in current design codes. A comparison between fifth percentile ratios for green and dry lumber strength and the comparable probability based ratios was given to illustrate the difference in the two approaches.

Differential reliability, as used in this paper, is based on the concept of equal reliability. That is, for the same end-use application, structural designs using lumber should exhibit the same degree of safety regardless of the size, grade, species, and moisture content of lumber, etc. Factors that produce similar probabilities of failure between contrasting data sets could be thought of as the adjustment factors relating the contrasting data sets on the basis of equal reliability. For example, assume that either green lumber or surfaced dry lumber could be used in a design in which a particular load combination acts. If this lumber is used as a roof rafter, then a dead load plus snow load combination is one design load that could be assumed to act on the roof. With the distribution of the load combination, the probabilities of failure of the two lumber data sets are calculated and compared. One of the data sets is chosen as the reference material. The other lumber data set is artificially altered by a factor, k, until a probability of failure similar to the probability of failure of the reference material results.[4] The k factor is therefore a measure of the strength of the contrasting lumber set related to the reference material.

Dead load

Dead load is the weight of the structure and all permanently affixed equipment, machines, and fixtures either installed initially or anticipated for future installation (ICBO 1973). Most researchers feel that the probability distribution of dead load is approximately normal (Ellingwood et al. 1980). Many researchers have assumed that the ratio of mean load to nominal load, $\bar{D}/D_n$, is unity and that the coefficient of variation, Ω_D, varies between 0.06 and 0.15 with a typical value of 0.10 (Allen 1976; Ellingwood 1978; Galambos and Ravindra 1973; Lind et al. 1978). Ellingwood et al. (1980) propose $\bar{D}/D_n = 1.05$ and $\Omega_D = 0.10$ because many design professionals feel that designers tend to underestimate the total dead load.

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[4] For different end-use applications, it may be desirable to set different risk levels (Galligan and Green 1980). In differential reliability, this could be accomplished by specifying the relationship between the load and the resistance distributions for the reference material.

All the above estimates of mean dead load to nominal dead load are applicable to heavy structures. Hence, the nominal dead load is the calculated weight of the structural members and permanent fixtures. For most light-frame structures, the nominal dead load is usually assumed to be 10 pounds per square foot (psf) (Hoyle 1978). Because the dead load is considered constant over the life of the structure, the mean dead load is assumed to be the calculated dead load for light-frame structures. Therefore, the ratio of mean dead load to nominal dead load should be calculated for each individual design case such as floor dead load or roof dead load.

Snow load

Snow loads are derived using climatological data and field studies that relate the roof snow load to the ground snow load and the roof exposure, geometry, and thermal characteristics (Ellingwood 1981). An estimate of the roof snow load can be expressed as

$$S = C_S q \tag{6}$$

where

S = maximum lifetime roof snow load
C_S = snow load coefficient relating roof snow load to ground snow load; C_S is determined by roof exposure, geometry, and thermal factors
q = maximum 50-year ground snow load.

All available data relate to the annual extreme ground snow load, but the maximum 50-year ground snow load can be derived from

$$F_{50}(q) = [F(q)]^{50} \tag{7}$$

where

$F_{50}(q)$ = the cumulative distribution function of the 50-year maximum ground snow load
$F(q)$ = the cumulative distribution function of the annual extreme ground snow load.

Ellingwood et al. (1980) developed a distribution of maximum roof snow load using Eqs. (6) and (7) as follows. Utilizing a recent analysis of annual extreme ground snow loads by the U.S. Army Cold Regions Research and Engineering Laboratory (CRREL) (Tobiasson and Redfield 1980), a more detailed analysis was conducted for a number of sites across the United States in which there was measurable snow accumulation in each of the years of record. The CRREL analysis indicates that the cumulative distribution function for annual extreme ground snow load is lognormal with parameters that vary from site to site. The best estimate of the probabilistic aspects of C_S is that it is symmetrical with a mean, $\bar{C}_S = 0.50$, and a coefficient of variation, $\Omega_{C_S} = 0.23$. The distribution of C_S is assumed to be normal. These estimates have been basically confirmed; however, the estimated coefficient of variation may be low.[5]

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[5] Ellingwood, B. 1981. Private communication. U.S. Dep. Commerce, Natl. Bur. Stand., Washington, DC.

Ellingwood et al. (1980) computed the maximum lifetime roof snow load by solving the convolution integral resulting from the combination of Eqs. (6) and (7). The integral describing maximum roof snow load at the different sites was solved for various cumulative values. The normal, extreme value type I, and extreme value type II distributions (Hahn and Shapiro 1967) were fitted to the calculated values. None of these distributions fitted over the entire range; but the extreme value type II distribution did provide an excellent fit in the upper percentiles for all sites. The type II distribution was chosen as the distribution of maximum roof snow load (Ellingwood et al. 1980). A single set of type II parameters was calculated by averaging the parameters for all sites. These parameters are

$$\mu = 0.72 \tag{8}$$

$$\alpha = 5.82 \tag{9}$$

These parameters correspond to a mean snow load to nominal snow load ratio, $\bar{S}/S_n$, and coefficient of variation, Ω_S, of

$$\bar{S}/S_n = 0.82 \tag{10}$$

$$\Omega_S = 0.26 \tag{11}$$

DEVELOPMENT OF THE LOAD DISTRIBUTIONS FOR A RELIABILITY ANALYSIS

Dead load distribution

In light-frame construction, floor joists, ceiling joists, and low-slope rafters not supporting a finished ceiling, the nominal dead load, D_n, is 10 psf (Hoyle 1978). Since the dead load is considered constant during the life of the structure, the mean dead load is assumed to be the calculated dead load for each particular structural application assumed in the differential reliability analysis. As an example, for a typical low-slope roof using rafter design, 2- by 8-inch No. 2 Douglas fir rafters, 16 inches on center with ½-inch plywood sheathing and asbestos shingles, a dead load of 5.7 psf can be calculated for the roof. The coefficient of variation of dead load, Ω_D, is taken to be 0.10 for this study. Therefore, the dead load parameters for this study case are

$$\bar{D}/D_n = 0.57 \tag{12}$$

$$\Omega_D = 0.10 \tag{13}$$

As most researchers feel the dead load is only approximately normally distributed, the distribution of dead load will be assumed to be lognormal. For small coefficients of variation, the differences between the normal distribution and the lognormal distribution are negligible (Fig. 1). Also, the lognormal distribution has the added advantage of being nonnegative. Based on these two observations, the lognormal distribution appears to be the best choice to model the dead load.

Maximum lifetime roof snow load distribution

In reliability analysis, the fit of the tail of a distribution to the data is very important; however, for a differential reliability analysis, a distribution should provide a good fit over the entire range of data. An analysis similar to that of
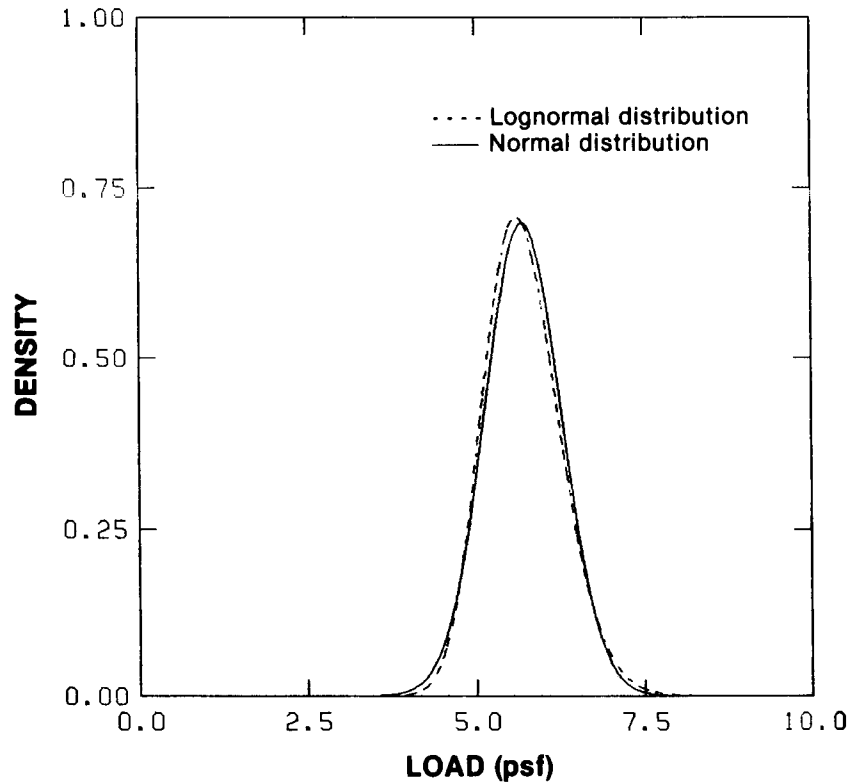
Fig. 1.   Comparison between a normal and a lognormal distribution of roof loads. Both distributions have a mean value of 5.7 psf and a coefficient of variation of 0.10.

Ellingwood et al. (1980) was conducted to determine a distribution that provided a good overall fit to the roof snow load data. A Monte Carlo simulation was employed in the analysis to describe the probability density function of the maximum lifetime roof snow load. New information concerning nominal snow loads from American National Standards Institute (ANSI) Standard A58.1-82 (1982) was incorporated into this analysis.

Because Ellingwood et al. (1980) had used ground snow load data from the latest available analysis by CRREL (Tobiasson and Redfield 1980), the same data were chosen for the Monte Carlo analysis. As mentioned, the annual extreme ground snow load is assumed to be lognormally distributed with parameters that vary from site to site (Tobiasson and Redfield 1980). The conversion factor, C_S, as previously noted, is normally distributed with a mean, $\bar{C}_S$, equal to 0.50 and a coefficient of variation, Ω_{C_S}, equal to 0.23. To be consistent, the nominal ground-to-roof conversion factor of $C_{S_n} = 0.7$ as taken from ANSI Standard A58.1-82 (1982) was used.

Combining Eqs. (6) and (7), we find the distribution of maximum lifetime roof snow load for each of the eight chosen sites, using the equation

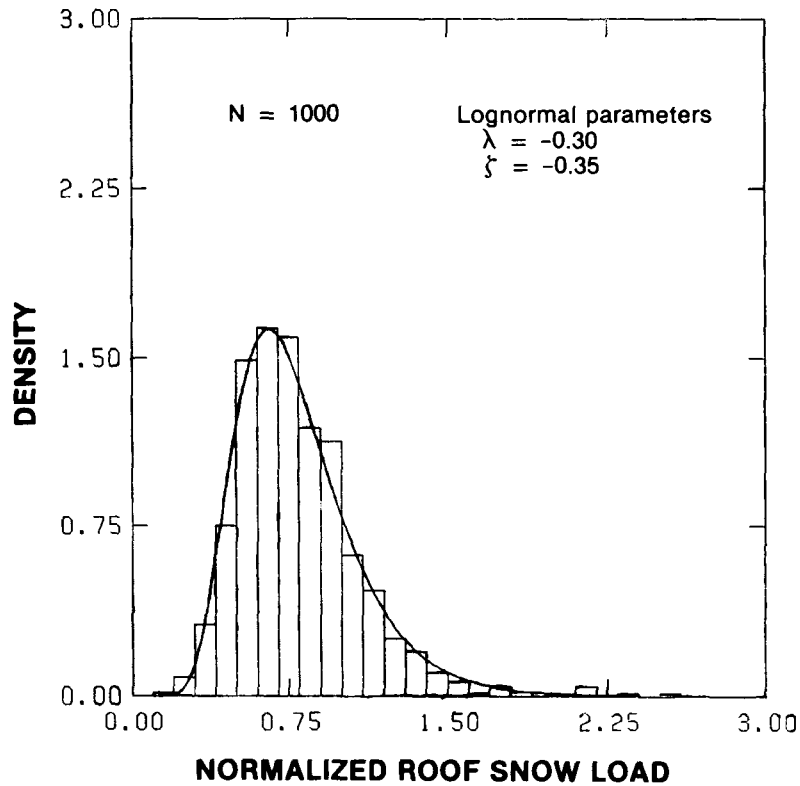$$S/S_n = (C_S/C_{S_n})(q_{50}/q_n) \tag{14}$$

Fig. 2. A histogram of the normalized maximum lifetime roof snow load for Rochester, NY, calculated from the results of a Monte Carlo simulation, with an overlay of the estimated lognormal density function.

where

S/S_n = the normalized maximum lifetime roof snow load

C_S = the normally distributed ground-to-roof snow load conversion factor

C_{S_n} = the nominal ground-to-roof snow load conversion factor specified by ANSI Standard A58.1-82 (1982)

q_{50} = the 50-year maximum ground snow load, which has the distribution of maximum lifetime ground snow load derived by Eq. (7)

q_n = the nominal ground snow load specified by ANSI Standard A58.1-82 (1982).

A histogram of the normalized maximum lifetime roof snow load for each site was constructed utilizing Eq. (14) in a Monte Carlo simulation (Thurmond 1982). An example histogram of S/S_n for Rochester, NY, is shown in Fig. 2. Inspection of the histograms suggested that the lognormal distribution provides a good fit. Accordingly, the lognormal distribution was overlaid on the normalized maximum lifetime roof snow load histograms and estimates of the lognormal parameters calculated.

Based on a visual inspection of the distribution fit and the results from Kolmogorov-Smirnoff (K-S) goodness of fit tests, it was judged that the lognormal
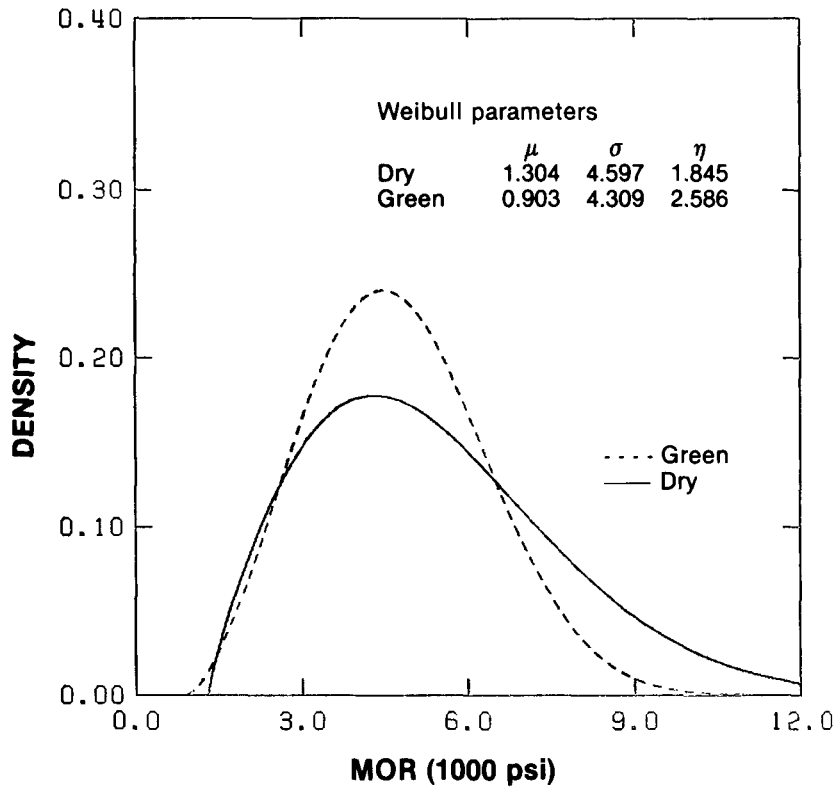
FIG. 3.   The three-parameter Weibull distributions of green and dry 2- by 8-inch No. 2 Douglas fir lumber utilized in the analysis of a moisture adjustment factor (Green 1980). The dry lumber was used to establish the benchmark safety level. The Weibull parameters, μ and σ, are given in 1,000 psi.

distribution adequately models the distribution of the roof snow load. Because none of the two-parameter distributions tested by Ellingwood et al. (1980) fit over the entire range, the lognormal distribution was chosen as the best distribution of normalized maximum lifetime roof snow load. The maximum lifetime roof load parameters, a combination of parameters from the selected sites, are

$$\bar{S}/S_n = 0.69 \tag{15}$$

$$\Omega_S = 0.44 \tag{16}$$

EXAMPLES OF THE APPLICATION OF THE DIFFERENTIAL RELIABILITY TECHNIQUE

Using the above-defined distributions, we can conduct many different analyses of lumber properties data. To demonstrate the analysis technique and its usefulness in diverse situations, two example reliability analyses are described. Although care was used in selecting material property distributions for these examples to ensure that they were not atypical of expected lumber performance, representativeness of the material property data set was not the major concern in its selection. Further, implementation of the differential reliability procedure in engineering design codes would probably be contingent upon standardization of certain aspects of the procedure (e.g., adoption of a standardized load distribution).

Determination of a moisture adjustment factor

Several recent studies have questioned the validity of the factors used in the American Society for Testing and Materials (ASTM) Standard D 245 (1981a) for adjusting the strength of lumber for changes in moisture content (Green 1980; Madsen et al. 1980). This concern is often initiated by calculating the dry-green ratio for fifth percentile strength and comparing this ratio to that given in the standard. However, when the density functions for green and dry lumber strengths cross each other (Fig. 3), there could be some question as to the proper procedure for comparing the results. If one were to look solely at the ratios of the fifth percentile strength estimates for the lumber (2,220 pounds per square inch (psi) for dry and 2,270 psi for green), one might conclude that the dry lumber is about 2% weaker than the green lumber. Yet this reduction may not be consistent with the expected performance of the two populations. Differential reliability offers an alternative approach for judging the effectiveness of drying in improving the structural performance of lumber.

To demonstrate the application of a dead load plus snow load combination in a differential reliability analysis, the contrasting lumber data set depicted in Fig. 3 is utilized. The data set, taken from Green (1980), consists of dry and green samples of 2- by 8-inch No. 2 Douglas fir lumber tested for modulus of rupture. The dry sample had a maximum moisture content of 19%.

In conducting a differential reliability analysis, the load and resistance distribution should reflect a specific design situation. Because our sample consisted of 2- by 8-inch lumber suitable for use as rafters, the design assumed is that of a low-slope roof utilizing rafter construction. The dead-load parameters are listed above as Eqs. (12) and (13).

The conventional design load for a dead plus snow load combination is nominal dead load plus nominal roof snow load if the resistance variable is appropriately adjusted by the correct load-duration factor. The nominal design dead load for a low slope roof is 10 psf (Hoyle 1978). In the moderate snow region of the United States, a typical nominal design roof snow load can be calculated by the method outlined in the proposed ANSI Standard A58.1-82 (1982) to be 20 psf. This design roof load is associated with a mean variance interval storm of 50 years. The total nominal design load is the sum of the nominal loads or 30 psf.

With the previously outlined method, we can now conduct the probability of failure analysis. In this example, the distribution of the sum of two lognormal variates cannot be derived in a form useful in the reliability analysis. However, noting that the coefficient of variation of the maximum lifetime roof snow load is large compared to the coefficient of variation of dead load, the parameters of the maximum lifetime total load may be approximated by adding the means and variances of the two independent lognormal distributions. Therefore, the parameters are calculated by

$$\mu_T = \mu_D + \mu_S \tag{17}$$

$$\sigma^2_T = \sigma^2_D + \sigma^2_S \tag{18}$$

where

μ_T = the average maximum lifetime load

μ_D = the average dead load

μ_S = the average maximum lifetime roof snow load

σ^2_T = the variance of the maximum lifetime total load

σ^2_D = the variance of the dead load

σ^2_S = the variance of the maximum lifetime roof snow load.

Using Eqs. (17) and (18), we can express the second moment parameters of the total load as

$$\mu_T = (\bar{D}/D_n)(D_n) + (\bar{S}/S_n)(S_n) \qquad (19)$$
$$= 0.57(10) + 0.69(20) = 19.5 \text{ psf}$$

$$\sigma^2_T = [\Omega_D(\bar{D}/D_n)(D_n)]^2 + [\Omega_S(\bar{S}/S_n)(S_n)]^2 \qquad (20)$$
$$= [0.10(0.57)(10)]^2 + [0.44(0.69)(20)]^2$$
$$= 37.19 \text{ (psf)}^2$$

The addition of the variances implies independence between roof snow load and dead load. This is believed to be a good assumption.

To test the above assumption, we conducted a simulation study. First, random lognormal deviates representing dead load and random lognormal deviates representing snow load were generated using their respective parameters as used in Eqs. (19) and (20). These deviates were summed and a relative frequency histogram was constructed. Next, a lognormal function described by the combined parameters calculated above was overlaid on the histogram (Fig. 4). The visual test indicated no obvious lack of fit. Also, a K-S test was performed at a 20% level of significance (recall that the 20% level is more likely to show a lack of fit than is the 5% level). On the basis of the test results, the lognormal distribution was accepted as a suitable distribution of dead plus snow load. Because the lognormal distribution was found to adequately model the load combination, the probability calculations can be conducted when the load and resistance distributions can be expressed in similar units.

For this reliability analysis, the units are pounds per square inch because resistance or strength data are usually expressed in these units. Also, it is easy to convert the units of the load distribution from psf to psi.

To provide compatible units for the load distribution, two basic steps are employed. First, the design load should be related to the design resistance. As mentioned previously, in conventional design situations, the design load is assumed equal to the design resistance. In reliability analysis, the normalized mean value, $\bar{X}/X_n$, is used to relate the load distribution to the resistance distribution. Because setting design load equal to design resistance is engineering practice, the nominal design load is set equal to the allowable design resistance. In other words, the nominal load, X_n, is interchangeable with the design strength, F_b. The load distribution is, in effect, positioned relative to the strength distribution by the normalized parameter, $\bar{X}/X_n$, because the mean value represents a more realistic load applied to a structure rather than the nominal load. The second step is to represent each type of load involved in the load combination in proportion to the total load. Each individual load is represented in the total load as a ratio of its nominal value to the total nominal load. The total load is the total nominal design load as adjustments for distribution have already been made by the previous factor.
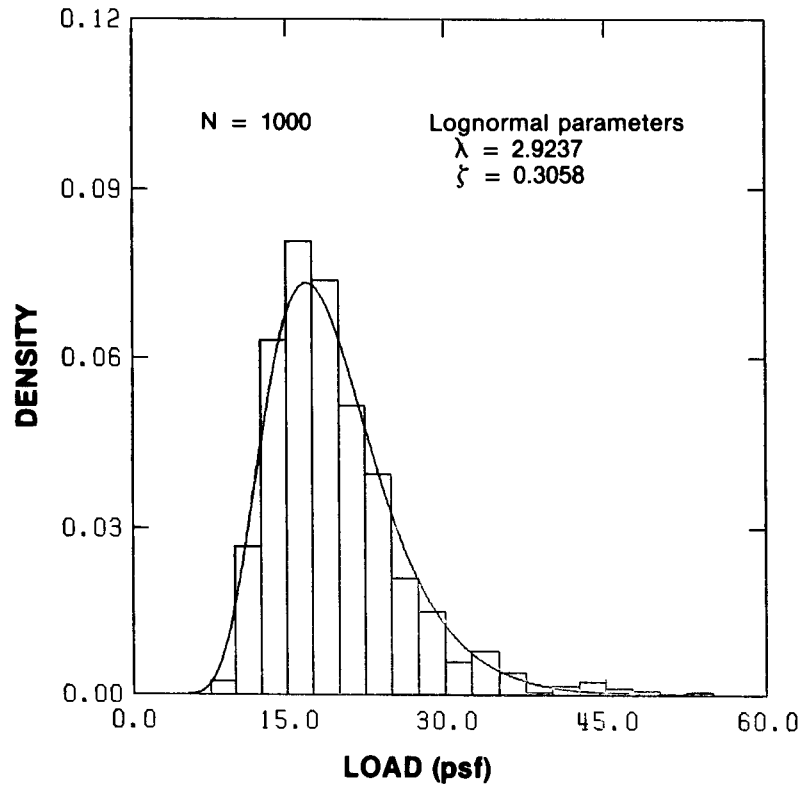
FIG. 4. The histogram of the sum of two random lognormal variables representing the dead load and the maximum lifetime roof snow load of a low-slope roof using rafter design is shown. The parameters of the overlaid lognormal distribution were calculated from the sum of the second moment parameters of the two lognormal distributions representing dead and snow load.

For the dead plus snow load combination, the equations for the parameters of total load are

$$\mu_T = \frac{D_n}{T_n}(\bar{D}/D_n)F_b + \frac{S_n}{T_n}(\bar{S}/S_n)F_b \tag{21}$$

$$\Omega_T = \frac{[(\mu_D\Omega_D)^2 + (\mu_S\Omega_S)^2]^{1/2}}{\mu_T} \tag{22}$$

where

μ_T = mean total lifetime load, psf

Ω_T = coefficient of variation of the total lifetime load

D_n = nominal dead load, psf

S_n = nominal snow load, psf

T_n = total nominal load ($D_n + S_n$), psf

$\bar{D}/D_n$ = normalized mean of the dead load distribution

$\bar{S}/S_n$ = normalized mean of the maximum lifetime roof snow load distribution

Ω_D = coefficient of variation of the dead load

Ω_S = coefficient of variation of the snow load

F_b = adjusted allowable design value.

The adjusted allowable design strength, F_b, is calculated from the contrasting data sets. The calculated fifth percentile from the reference data set is divided by the general adjustment factor of 2.1 and multiplied by 1.15 because a 15% increase in allowable stress is used for duration of load when a snow load is applied (Hoyle 1978). The adjusted fifth percentile historically has been the allowable design value. For this reason, the fifth percentile of the strength distribution is used to position the load distribution.

Because F_b is the factor in Eqs. (21) and (22) that positions the load distribution in relation to the resistance distribution, F_b determines the load parameters. Either the green or dry lumber can be chosen as the reference material, which then defines the data set from which F_b is calculated. In this example the dry distribution was chosen as the reference distribution because most dimension lumber will eventually equilibrate to a maximum moisture content of 19% or less in actual use. Under this assumption no adjustment would be made to the dry allowable values. In this case, an adjusted F_b was calculated from the dry lumber data set. The load parameters are then calculated.

On the basis of Eqs. (21) and (22), the parameters of the total maximum lifetime load are calculated as

$$\mu_T = \frac{10}{30}(0.57)(2.223)\left(\frac{1.15}{2.1}\right) + \frac{20}{30}(0.69)(2.223)\left(\frac{1.15}{2.1}\right)$$

$$= 0.23 + 0.56 = 0.79(1,000) \text{ psi} \tag{23}$$

$$\Omega_T = \frac{\{[(0.23)(0.1)]^2 + [(0.56)(0.44)]^2\}^{1/2}}{0.79} = 0.32 \tag{24}$$

By using a computer program developed for the probability of failure analysis (Thurmond 1982), we can calculate the probabilities of failure for the green and dry lumber. These are shown in the first two lines of Table 1; lower and upper limits are calculated to show that the step size of the numerical integration is adequate.

Using an iterative approach, the Weibull resistance parameters for the green lumber, μ and σ, are altered by a factor, k, until the probabilities of failure for green and dry lumber are similar. Altering the location and scale parameters of the three-parameter Weibull strength distribution by k is analogous to altering all of the individual green lumber strength values by k. The k factor is in essence a moisture conversion factor from green to dry lumber. For this example, k = 1.100, which is to say dry lumber is 1.100 times stronger than green. Table 1 values for green lumber adjusted by 1.100 show that the correct value of k was obtained. When the green lumber strength is increased by a factor of k, it yields a probability of failure almost identical to that of the dry lumber.

The lognormal function calculated from the combined second moment parameters utilizing Eqs. (19) and (20) fit the relative frequency histogram of the simulated data generated from the lognormal distributions of the dead load and snow
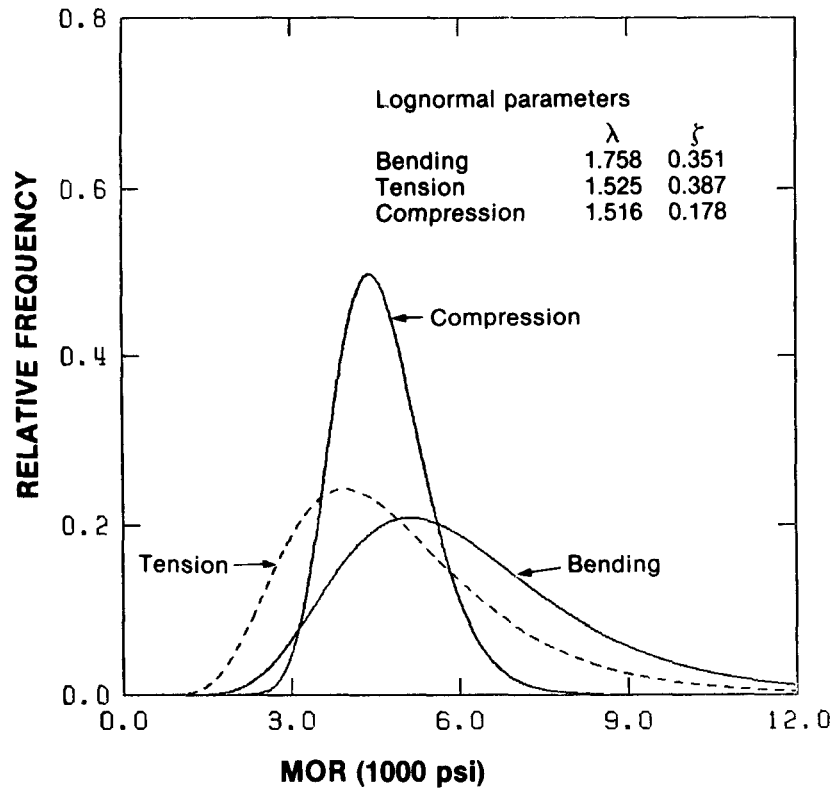
Fig. 5. The lognormal distributions of bending, tension, and compression strength of 2- by 4-inch 1650f-1.5E Hem-Fir lumber (Hoyle et al. 1979). Bending strength was chosen as the reference material. The lognormal parameters are the means and corresponding standard deviations of the logarithms of the data given in 1,000 psi.

load quite well. However, the probabilities of failure and the k factor could be different if a convolution integral describing the dead load plus snow load were employed to characterize the total load. This is supported by the known weakness of the K-S goodness of fit test for discriminating between data and a hypothesized distribution.

A computer routine was developed to numerically solve the convolution integral of the cumulative distribution function of the maximum lifetime total load. The total load is the sum of two independent lognormal variables describing the dead

TABLE 1. *Probabilities of failure calculated[1] for a 2- by 8-inch Douglas fir low-sloping rafter under a dead load plus snow load combination.*

Moisture condition	Lower limit	Upper limit
Dry	1.57×10^{-4}	1.58×10^{-4}
Green	3.28×10^{-4}	3.29×10^{-4}
Green, adjusted by k = 1.100	1.56×10^{-4}	1.57×10^{-4}

[1] The distribution of total load used in the probability of failure calculation was shown by the Kolmogorov-Smirnoff test to be adequately described by the lognormal distribution.

TABLE 2. *Probabilities of failure of rafter type structures using an exact integral approach.*[1]

Moisture condition	Lower limit	Upper limit
Dry	2.44×10^{-4}	2.50×10^{-4}
Green, adjusted by k = 1.080	2.39×10^{-4}	2.40×10^{-4}

[1] In this case, the load distribution is the sum of two independent lognormal variables—one representing dead load and the other representing snow load. These distributions were added or convoluted before the standard load and resistance reliability analysis was conducted.

and maximum lifetime roof snow loads. The green and dry probabilities of failure can now be calculated using the exact integral approach. Using the dry lumber to establish a benchmark reliability, the computer artificially alters the green lumber strength by a k factor until it has a probability of failure equal to the dry, k = 1.080. Table 2 shows that when the strength of the green lumber is artificially increased 8% it has the same probability of failure as the dry lumber.

Because the k value obtained by the simpler method summarized in Table 1 is close enough to be considered equal to the k = 1.080, the simpler approach should be used. This is based on the fact that the engineering results obtained by using the two factors would be the same. However, if the engineer feels that the dead and snow loads will not combine in a simple mathematical way, the more theoretical but complicated approach might be used. For example, if the coefficients of variation of dead and snow loads are approximately equal, the two lognormal distributions may not combine simply. These recommendations were supported by the results from several other reliability analyses of lumber data (Thurmond 1982).

Bending, tension, and compression allowable stresses

Lumber is assigned different allowable stresses in bending, tension, and compression parallel to grain. Using lumber tested in bending, tension, and compression, allowable stresses can also be calculated on the basis of the concept of equal reliability. If lumber tested in bending is chosen as the reference material, the allowable stresses for the lumber tested in tension and compression can be calculated on the basis of k factors and the allowable stress of the lumber tested in bending.

TABLE 3. *Probabilities of failure for 2- by 4-inch 1650f-1.5E Hem-Fir lumber tested in bending, tension and compression parallel to the grain (Hoyle et al. 1979). No design situation is specified.*

Test mode	Probability of failure	
	Lower limit	Upper limit
Bending	1.38×10^{-3}	1.39×10^{-3}
Tension	6.93×10^{-3}	6.94×10^{-3}
Tension, adjusted by k = 1.355	1.38×10^{-3}	1.39×10^{-3}
Compression	1.11×10^{-3}	1.11×10^{-3}
Compression, adjusted by k = 0.970	1.38×10^{-3}	1.38×10^{-3}

TABLE 4. *Differential reliability k factors describing the conversion from tensile or compression strength to bending strength.*

Modes for comparison	Differential reliability analysis (k)	Fifth percentile analysis[1]	
		Actual data (r)	NDS values (r)
Bending to tension	1.355	1.34	1.62
Bending to compression	0.970	0.96	1.25

[1] Ratio of fifth percentile for bending mode to fifth percentile for tension or compression modes.

The fifth percentiles of the lumber parameters in Fig. 5 are 3,250 psf for bending, 2,430 psf for tension, and 3,400 psf for compression. The allowable tensile strength as given by NDS (NFPA 1978) is 1,020 psf, the allowable compressive strength is 1,320 psf, and the allowable bending strength is 1,650 psf.

For an example of this analysis, the contrasting data sets consist of 1650f-1.5E Hem-Fir 2- by 4-inch lumber (Hoyle et al. 1979). The experimental strength distributions for the lumber tested in tension and compression are contrasted graphically to the reference bending distribution in Fig. 5.

Because a similar design situation is not apparent for all three cases based on a failure mode, the reliability analysis is conducted on the basis of load parameters calculated for a roof snow load without consideration of the dead load. The assumption of a design situation gives meaning to reliability comparisons; however, it is believed that the use of the load distribution reflecting only the roof snow load is sufficient for this comparison.

As in the previous examples for green versus dry lumber, the strength of lumber tested in tension and compression is artificially altered until a probability of failure results that is similar to the benchmark safety level calculated from the bending strength data. The failure probabilities and the k factors for the lumber tested by the various modes are given in Table 3. In this instance, the fifth percentile ratios obtained from the actual data are quite similar to the results of the differential reliability analysis.

Using the k factors given in Table 3, and given bending data, we can calculate the allowable design values for 2- by 4-inch lumber loaded in tension and compression, respectively, as

$$F_b = F_t k_{tb} \qquad (25)$$

$$F_b = F_c k_{cb} \qquad (26)$$

where

F_b = calculated allowable bending stress from the data, psi
F_t = calculated allowable tensile stress, psi
F_c = calculated allowable compressive stress, psi
k_{tb} = k factor for conversion from tensile allowable strength to bending allowable strength as determined by the reliability analysis
k_{cb} = k factor for conversion from compressive allowable strength to bending allowable strength as determined by the reliability analysis.

The conventional analytical technique for comparing strength properties is to calculate the fifth percentiles and compare the values in a ratio of bending strength to tensile or compressive strength. These ratios could be denoted as r_{tb} and r_{cb}, respectively (Table 4).

The calculated k factors are the adjustment from tensile or compressive stress allowables to bending stress allowable. Comparing the conventionally calculated factors, r, from the NDS analysis to the differential reliability factors, k, it is suggested that, if these data are representative of the properties of 1650f-1.5E hem-fir, then the allowable tensile stress can be increased to a level closer to the allowable bending stress and the allowable compressive stress can be increased to a level greater than the allowable bending stress. It is evidenced that the conventional analyses do not account for the stronger lumber in the data sets and therefore result in inefficient utilization of lumber. However, it must be remembered that the lumber used in this example was obtained from only one mill and may not be representative of lumber behavior in the general population.

SUMMARY AND CONCLUSIONS

The distribution of maximum lifetime roof snow load was developed for use in differential reliability analyses of lumber properties data. The models, assumptions, and data used in the development of the distributions reflect the state of the art of loads.

To give meaning to the reliability analyses, various design situations were assumed. These situations encompassed typical lumber design to reflect the physical state of the lumber when it was tested. On the basis of the design situation, the dead load parameters were calculated and then combined by a second moment approach with the roof snow load parameters to render the total load parameters for the assumed design. The distribution of the total load was found to be adequately described by the same distribution as the roof snow load. This simplification regarding the calculation of the total load had no effect on the results of the differential reliability analysis from an engineering viewpoint.

Examples were then given in which contrasting lumber data sets were compared on the basis of the concept of equal reliability using the calculated load distributions. A reference material was chosen from the lumber sets and this material was used to calculate the benchmark safety level. The other contrasting lumber data sets were artificially altered until a failure probability approximately equal to the benchmark safety level resulted. This technique assumes that all the lumber in the sample is fully stressed to the allowable design stress. This does not occur in actual design situations as the lumber strength is affected by cladding, non-structural components, and load sharing between members. However, any inaccuracies in the calculation of the failure probabilities are believed to be minimized by the comparative nature of the reliability technique.

The factor k resulting from this reliability analysis provides a logical comparison parameter between lumber sets because it compares the calculated probabilities of failure of the lumber in service. The probability of failure analysis can be carried out in a strictly formal and consistent way in every case. The end result is a number that can be compared to others on a logically uniform basis. While the failure probabilities are not absolute in the sense that they are the true probabilities of failure based on the design situation, they indicate the magnitude of difference between the contrasting lumber sets and the reference sample. This difference is therefore a measure of the effect of the study variable of the contrasting lumber data sets. This analysis is very powerful because the entire strength distribution is utilized.

In conclusion, the differential reliability technique is an integrated technique that is particularly well suited for the analysis of lumber properties data. The technique formally accounts for the inherent variability of lumber data. Lumber strength data can be analyzed within a framework of statistical principles while reflecting realistic design situations. It is believed that many factors that affect the strength of lumber can be rationally analyzed by differential reliability using the dead plus roof snow load combination.

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