POLYNOMIAL MODELS TO STUDY AND PRESENT WITHIN-TREE VARIATION OF WOOD PROPERTIES

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ABSTRACT

A method is presented for obtaining information on within-tree variation for wood properties. The polynomial models resulting from this study provide a technique for statistical analysis and for plotting within-tree variation patterns of fiber, vessel, and ray percentages as well as ring widths. With this method, it is possible to examine the oblique, horizontal, and vertical variation patterns of different wood properties, both statistically and graphically; hence, statistical inference can conveniently be made on within-tree patterns of variation. This can provide not only information of value in tree improvement work, but also a better understanding of the variability of wood.

INTRODUCTION

Information on within-tree variations in wood properties is necessary for successful evaluation of these properties and efficient conversion of wood into finished products. Quantitative data on variation of wood properties are also needed for guiding research aimed at improving wood quality and for providing the pulp technologist with basic information essential for making the best use of available wood. However, the existence of within-tree variation patterns of fiber properties, fiber length, and other wood properties forms a major obstacle to the evaluation of wood quality in terms of the wood's potential for paper making.

Within-tree Patterns of Variations

The conventional way to study within-tree variation patterns is to present the wood properties data in a special way in order to examine these patterns of variations graphically. Such a method has been used by several researchers to examine graphically wood properties variation within the tree (Duff and Nolan 1953; Walter and Soos 1962; Kandeel and Bensend 1969). This is done in terms of three sequences (Duff and Nolan 1953). In the first sequence (oblique), variations within each single ring are followed from the top of the tree downward (i.e. wood formed in the same year but from cambia of different ages). The second sequence (horizontal) is shown by plotting wood properties from the pith to the bark at intervals of height. In this sequence, the wood is formed by cambia of different ages during different seasons. In the third sequence (vertical), the wood properties are plotted for successive internodes down the tree but at constant ring number from the pith (i.e. wood formed by cambia of approximately the same age). Thus these three sequences (oblique, horizontal, and vertical) are used to project the within-tree trends of variation that are influenced by intrinsic as well as by extrinsic factors. These are factors such as cambial age, year of wood formation, and environmental factors.

However, in many wood properties studies, a knowledge of more than one independent variable is necessary to obtain better understanding or better prediction of wood properties responses. To accomplish this, multiple regression analyses that involve relations among more than two variables are used. This is done in situations where the wood property is influenced by several variables (Saucier and Hamilton 1967; Kandeel and Bensend 1969). To provide more information about the wood property being studied, vertical and hori-

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TABLE 1. *Regression equations (reduced models) for predicting wood properties in silver maple* (Acer saccharinum *L.*).

Property	Equation	Square root of multiple R square
Ring width (Y_1)	$Y_1 = 344.40 + 0.34X_1 - 61.04X_2 - 60.38X_3 - 0.04X_1^2 + 3.05X_2^2 + 3.06X_3^2 + 7.43X_2X_3$	0.80
Fiber % (Y_2)	$Y_2 = 62.22 - 0.59X_1 + 5.53X_2 + 0.54X_3 + 0.01X_1^2 - 0.38X_2^2 + 0.05X_1X_3 - 0.39X_2X_3$	0.70
Vessel % (Y_3)	$Y_3 = 75.16 + 0.59X_1 - 13.80X_2 - 14.10X_2 - 0.01X_1^2 + 0.75X_2^2 + 0.85X_3^2 + 1.50X_2X_3$	0.79
Ray % (Y_4)	$Y_4 = 27.23 - 0.86X_1 - 1.73X_2 - 1.56X_3 + 0.01X_1^2 + 0.07X_1X_2 + 0.06X_1X_3$	0.55

X_1 = height in feet
X_2 = coded year of formation
X_3 = coded cambial age

zontal patterns of variations are needed. This is done by plotting such patterns graphically. However, this approach provides two sets of results, not clearly related, from a given study. One set is the statistical regression analysis of the variables and the inferences about their relations. The other set of results is provided through the graphical presentation of variation patterns of the wood properties. This presentation provides no inferences regarding the significance of each variable influencing the given trend of the wood property. Since the sets of information are not clearly related, several researchers are inclined either to use regular statistical analysis or to present the graphically plotted variation trends within the tree stem. The technique presented here resolves this problem by presenting a method in which the patterns of variation are plotted in a way related to the regression analysis of wood properties data. This is done since the patterns of variations are controlled by known factors frequently included in the models used in multiple regression analysis of wood properties studies. Such variables are cambial age or year of wood formation.

When this approach was used, it resulted in the polynomial models in this study that provide a method of examining within-tree variation patterns graphically while analyzing the data by multiple regression techniques. This has the advantage of providing a way to test the statistical significance of these patterns of variation since the coefficients of the variables in the polynomial models are statistically tested for significance. Thus, a pattern of variation of wood fiber percentage or ring widths plotted by this method is known to account for a given amount of variation of that property in the tree. For instance, a graph plotted from the ring width equation (Table 1) will describe 63.51% (Table 4) of the total amount of variation in ring width within the tree. In such a way, both regression analysis of the studied properties and their patterns of variation within the stem will provide a coherent set of information to help in analyzing the variation of a given wood property.

MODEL CONSTRUCTION

In studying relationships of wood properties to specific independent variables, polynomial models describing these properties are constructed on the basis of hypothetical or reported trends. The variables chosen for these polynomial models are those most desirable for obtaining prediction equations for the wood properties in question. Fitting the terms in the polynomial models by the method of least squares will provide full regression models describing the wood properties (Draper and Smith 1967; Kandeel and Bensend 1969). This step is then followed by a type of backward elimination procedure, considering the biological known facts, to select the "best" regression equation for each wood property (Draper and Smith 1967). The deletion of the terms from the full model is based on testing the partial regression coefficients ("t" test) and considering the established importance of each of these terms.

The interrelations between these terms in the model are also taken into consideration. Thus, the reduced models obtained will have an advantage over the original full models since the reduced models contain fewer terms while still accounting for almost as much variability of each property as the full models. If the resulting prediction equation for each wood property is then used to present a pattern of variation of that property, then each pattern is controlled by the variation that is accounted for by its regression model. Hence, each plotted curve will represent a polynomial model in which the coefficients of the dependent variables are statistically tested for significance. From the multiple "R" of each model, the percentage of variability in the property that this model accounts for is known (Table 4). Hence, statistical inference can conveniently be made on each plotted graph.

PLOTTING METHOD AND RESULTS

In a study of a 48-foot silver maple tree, polynomial models were constructed to describe different wood properties. The above-described technique, considering the biologically known facts, was then used to select the "best" regression equation for each wood property. Table 1 presents polynomial models describing the ring width and proportions of wood elements. In order to cover the height range within the tree, five values of the coded cambial age covering the height range and matching the coded year of formation were used. There were two levels of coded year of formation used to study the oblique sequence. Each one of these two levels of the year of wood formation was matched by specific levels of coded cambial age. The two matching groups of cambial age levels were 9, 8, 7, 6, 5 and 7, 6, 5, 4, 3, respectively. This manipulation facilitated the calculation of five quadratic equations for each wood property studied as a function of height. Through algebraic substitution for the real values of the levels of cambial age, year of wood formation and height, these polynomial models were reduced to quadratic equations for each wood property.

TABLE 2. *General regression model (full model) used for predicting wood properties within the tree.*

Property	Equation
Y_i =	$B_0 + B_1X_{1i} + B_2X_{2i} + B_3X_{3i} + B_{11}X_{1i}^2 + B_{22}X_{2i}^2 + B_{33}X_{3i}^2 + B_{12}X_{1i}X_{2i} + B_{13}X_{1i}X_{3i} + B_{23}X_{2i}X_{3i} + e_i$

B's = partial regression coefficients
X_{1i} = height in feet from the ground level for the *ith* observation
X_{2i} = coded year of formation for the *ith* observation
X_{3i} = coded cambial age for the *ith* observation
Y_i = ring width, fibers percentage, vessel percentage, and ray percentage for the *ith* observation

This can be illustrated by considering the coded values of 1 and 9 as the levels of X_2 and X_3, respectively, in the regression equation for ring width (Table 1). When this polynomial model was reduced to a quadratic equation for ring width as a dependent variable and height as an independent variable, the terms in the equation could be recalculated. Since in the ring width polynomial equation (Table 1) there are no interaction terms that include X_1 (height), then the X_1 terms in the new quadratic equation will be the same as in the original polynomial model in Table 1. However, the change will occur in the value of B_0 in the ring-width equation (Table 2), i.e. the value 344.40 (Table 1) will change because of the substitution for the constant levels of X_2 and X_3 as 1, 9, respectively. Thus the equation for ring width will be:

$$
\begin{aligned}
Y_1 &= 344.40 + 0.34X_1 - 0.04X_1^2 - (61.04)(1) - (60.38)(9) + (3.05)(1)^2 + (3.06)(9)^2 + (7.43)(1)(9) \\
&= 662.18 - 604.46 + 0.34X_1 - 0.05X_1^2 \\
&= 57.72 + 0.34X_1 - 0.04X_1^2
\end{aligned}
$$

However, if the polynomial model includes interaction terms of X_1 and other variables, there will be a change in the values of B_1X_1 also. For example, in the equation for fiber percentage (Y_2) (Table 1), two terms occur that involve X_1, namely $B_{13}X_1X_3$ and B_1X_1. In the reduced equations, the coefficients of X_1 will be ($B_1 + B_{13}X_1X_3$). Thus, the first equation of the

TABLE 3a. *Calculated quadratic equations resulting from the regression models in Table 1.*

Wood property	Height range in feet (X_1)	Equation	Levels of X_2, X_3
Ring width (Y_1)	0.8–15.8	$Y_1 = 57.72 + 0.34X_1 - 0.04X_1^2$	1, 9
	17.8–21.8	$Y_1 = 58.58 + 0.34X_1 - 0.04X_1^2$	1, 8
	27.8–33.1	$Y_1 = 65.64 + 0.34X_1 - 0.04X_1^2$	1, 7
	35.3–38.4	$Y_1 = 78.82 + 0.34X_1 - 0.04X_1^2$	1, 6
	43.0–45.0	$Y_1 = 98.12 + 0.34X_1 - 0.04X_1^2$	1, 5
Fiber % (Y_2)	0.8–15.8	$Y_2 = 68.72 - 0.14X_1 + 0.01X_1^2$	1, 9
	17.8–21.8	$Y_2 = 68.57 - 0.19X_1 + 0.01X_1^2$	1, 8
	27.8–33.1	$Y_2 = 68.42 - 0.24X_1 + 0.01X_1^2$	1, 7
	35.3–38.4	$Y_2 = 68.27 - 0.29X_1 + 0.01X_1^2$	1, 6
	43.0–45.0	$Y_2 = 68.12 - 0.34X_1 + 0.01X_1^2$	1, 5
Vessel % (Y_3)	0.8–15.8	$Y_3 = 18.46 + 0.59X_1 - 0.01X_1^2$	1, 9
	17.8–21.8	$Y_3 = 16.51 + 0.59X_1 - 0.01X_1^2$	1, 8
	27.8–33.1	$Y_3 = 16.26 + 0.59X_1 - 0.01X_1^2$	1, 7
	35.3–38.4	$Y_3 = 17.71 + 0.59X_1 - 0.01X_1^2$	1, 6
	43.0–45.0	$Y_3 = 20.86 + 0.59X_1 - 0.01X_1^2$	1, 5
Ray % (Y_4)	0.8–15.8	$Y_4 = 11.46 - 0.25X_1 + 0.01X_1^2$	1, 9
	17.8–21.8	$Y_4 = 13.02 - 0.31X_1 + 0.01X_1^2$	1, 8
	27.8–33.1	$Y_4 = 14.58 - 0.37X_1 + 0.01X_1^2$	1, 7
	35.3–38.4	$Y_4 = 16.14 - 0.43X_1 + 0.01X_1^2$	1, 6
	43.0–45.0	$Y_4 = 17.70 - 0.49X_1 + 0.01X_1^2$	1, 5

fiber percentage Y_2 (Table 1) will be: (X_2 and X_3 at 1, 9 levels, respectively).

$$Y_2 = 62.22 - 0.59X_1 + (0.05)(9)X_1 + (5.33)(1) + (0.54)(9) - (0.38)(1)^2 - (0.39)(1)(9) = 68.72 - 0.14X_1 + 0.01X_1^2.$$

With this simple method, the polynomial models describing the wood properties (Table 1) were reduced to quadratic equations for each wood property (Table 3a and 3b). A simple computer program was written for each wood property group of

TABLE 3b. *Calculated quadratic equations resulting from the regression models in Table 1.*

Wood property	Height range	Equation	Levels of X_2, X_3
Ring width (Y_1)	0.8–15.8	$Y_1 = 71.86 + 0.34X_1 - 0.04X_1^2$	3, 7
	17.9–21.8	$Y_1 = 70.20 + 0.34X_1 - 0.04X_1^2$	3, 6
	27.9–33.1	$Y_1 = 74.66 + 0.34X_1 - 0.04X_1^2$	3, 5
	35.3–38.4	$Y_1 = 85.24 + 0.34X_1 - 0.04X_1^2$	3, 4
	43.0–45.0	$Y_1 = 101.94 + 0.34X_1 - 0.04X_1^2$	3, 3
Fiber % (Y_2)	0.8–15.8	$Y_2 = 70.98 - 0.24X_1 + 0.01X_1^2$	3, 7
	17.8–21.8	$Y_2 = 71.61 - 0.29X_1 + 0.01X_1^2$	3, 6
	27.8–33.1	$Y_2 = 72.24 - 0.34X_1 + 0.01X_1^2$	3, 5
	35.3–38.4	$Y_2 = 72.87 - 0.39X_1 + 0.01X_1^2$	3, 4
	43.0–45.0	$Y_2 = 73.50 - 0.44X_1 + 0.01X_1^2$	3, 3
Vessel % (Y_3)	0.8–15.8	$Y_3 = 11.86 + 0.59X_1 - 0.01X_1^2$	3, 7
	17.8–21.8	$Y_3 = 15.31 + 0.59X_1 - 0.01X_1^2$	3, 6
	27.8–33.1	$Y_3 = 16.16 + 0.59X_1 - 0.01X_1^2$	3, 5
	35.3–38.4	$Y_3 = 16.91 + 0.59X_1 - 0.01X_1^2$	3, 4
	43.0–45.0	$Y_3 = 20.26 + 0.59X_1 - 0.01X_1^2$	3, 3
Ray % (Y_4)	0.8–15.8	$Y_4 = 11.12 - 0.23X_1 + 0.01X_1^2$	3, 7
	17.8–21.8	$Y_4 = 12.68 - 0.29X_1 + 0.01X_1^2$	3, 6
	27.8–33.1	$Y_4 = 14.24 - 0.35X_1 + 0.01X_1^2$	3, 5
	35.3–38.4	$Y_4 = 15.80 - 0.41X_1 + 0.01X_1^2$	3, 4
	43.0–45.0	$Y_4 = 17.36 - 0.47X_1 + 0.01X_1^2$	3, 3

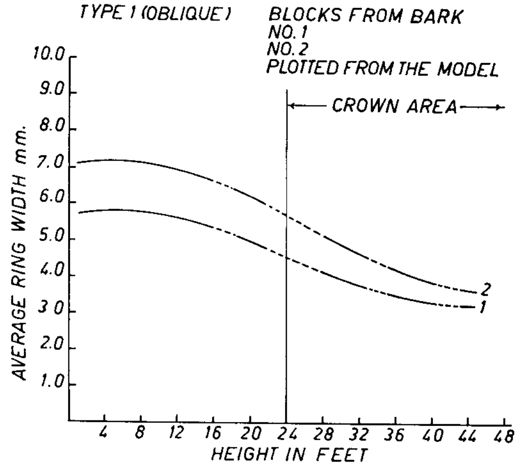

FIG. 1. Two curves of ring widths each one representing an average of three years of wood formation as plotted from the reduced prediction equation.

a. block No. 1 is the first block inside the bark (i.e. last three years of wood formation).
b. block No. 3 is the next block to block No. 1 inside the dark.

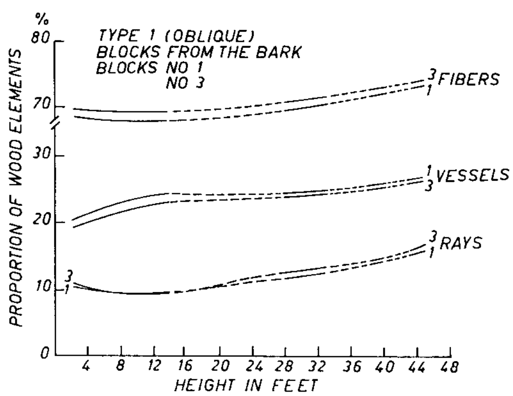

FIG. 2. A family of curves of percentages of wood elements each one representing an average of three years of wood formation as plotted from the reduced prediction equation.

a. block No. 1 is the first block inside the bark (i.e. last three years of wood formation).
b. block No. 3 represents the next sampled block inside the dark (i.e. formed in previous years to block 1).

quadratic equations. These equations were thus plotted as two-dimensional graphs (Figs. 1, 2). It should be remembered that each group of equations represents a specific wood property covering the whole

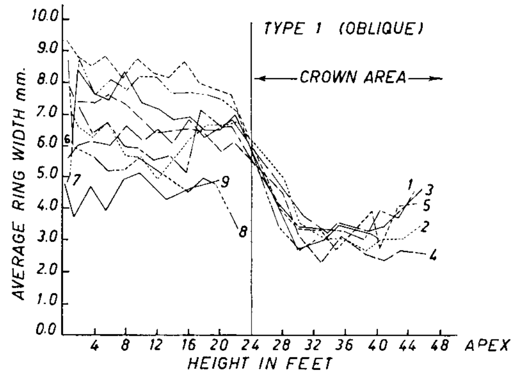

FIG. 3. A family of curves of ring widths, each one representing an average of three years of wood formation as plotted from the original data.

range of tree height (Table 3a, 3b). These specific wood properties were ring width, fiber percentage, vessel percentage, and ray percentage.

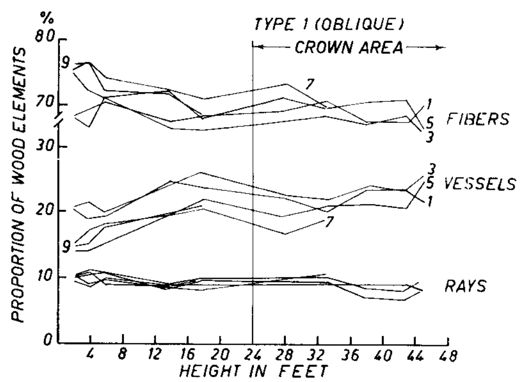

FIG. 4. A family of curves of percentages of wood elements, each one representing an average of three years of wood formation as plotted from the original data.

CONCLUSIONS

The predicted annual ring width, and fiber, vessel, and ray proportions were plotted as a function of height in Figs. 1 and 2. It is clear that Figs. 1 and 2 agree with the graphs in Figs. 3 and 4 from the original data. By this method, it is possible to investigate the other two patterns of variation (vertical and horizontal) if the proper constants and variables are used for

TABLE 4. *Percentages of variation accounted for by regression.*

Variable	Ring width	Fibers	Vessels	Rays
Percentage of variation	63.51	49.22	61.47	30.57

substitution in the polynomial model. The acceptance of the polynomial models for these properties is a prerequisite for using the method to plot the patterns of variation of wood properties within the tree stem. This depends on the percentage of variation in the property that is accounted for by each model (Table 4).

By this technique, it is possible to investigate the patterns of variation (oblique, horizontal, and vertical) both mathematically and graphically. However, in such cases the graphical presentation of the patterns of variation will be of known statistical significance, depending on the amount of variation that can be explained by each curve. Thus a graph plotted from the ring-width equation (Table 1) will present 63.51% of the total amount of variation in ring width within the tree (Table 4). This technique provides a way to study within-tree patterns of variation of a given wood property and tests the significance of the variables influencing its variation.

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