PREDICTING DAMPING OF SEMI-RIGID GLUED T-BEAMS

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(Received June 1985)

ABSTRACT

A theoretical model for predicting damping in composite wood T-beam is provided. The formulation is based on energy dissipation due to relative slip at the interface of the flange and joist. A loss factor parameter as obtained from stress-strain relationship of elastomeric adhesive has been introduced into the formulation. Damping computed from experimental tests is found to about 5% to 8% compared to theoretical values of 4.5% to 6.5%.

Keywords: Strain, energy, damping, beams, composite, nonrigid, floors.

NOTATIONS

The following symbols are used in this paper:

A_1, A_2	Cross-sectional areas of the subscripted layers
C_1, C_2	Constants
E_1, E_2	Young's moduli of the subscripted layers
F	Total shear force in the glue line at a section
G	Shear modulus of the adhesive
I_1, I_2	Moments of inertia of the subscripted layers
K_c	Effective stiffness of composite beam
L	Effective length of the beam
M	Total moment in the cross-section
M_1, M_2	Moments in the subscripted layers due to any load
P	Concentrated load
S	Slip modulus of adhesive $= \frac{Gb}{t}$
Y, Y_c	Deflections of the rigid and composite beams, respectively
Z, Z_c	Midspan deflections for rigid and composite beams, respectively
b	Width of the glue line
c_1, c_2	Extreme fiber distances for subscripted layers
t	Thickness of the glue line
x	Coordinate measured along the axis of the beam
z	Constant
Δ	Shear deformation (or slip) of the glue line
α	Loss factor of the adhesive
ζ	Damping ratio
ψ	Deflected shape of the beam

Wood and Fiber Science, 19(1), 1987, pp. 26–36

INTRODUCTION

Dynamic analysis is often conducted when a structural system is subjected to dynamic loading induced by machinery, earthquakes, and shock waves. In some cases, such an analysis is essential for the calculation of the resulting response such as deflections and stresses.

Among the parameters that are essential for dynamic analysis are the stiffness, damping characteristics, and mass of the structure. Much of the research that has been conducted concentrated on stiffness characterization. Little attention has been given to the damping properties of wood structures.

Damping is a phenomenon that describes the ability of a structure to dissipate energy. It is a decay of any response amplitude with time and is calculated from the logarithmic ratio of two peak amplitudes of vibration. Damping characteristics are needed for predicting such dynamic responses as deflections and stresses. The amplitude of these responses can be significantly reduced by increasing the damping characteristics of the structure.

Damping can be classified as material damping and system damping. Material damping is due to internal molecular friction and is commonly called hysteresis damping. The other type of damping is due to frictional energy loss at joints or connections at interface of components.

In most light-frame wood constructions, floors are relatively flexible. These floors are analyzed as composite wood T-beams with nails or glue acting as shear connectors between flange and joist. Based on past research (Yeh 1970; Polensek 1975), material damping for such systems is of less significance than that of slip damping. While there have been some studies on damping of mechanical connections, little information is available on slip damping in glued constructions. The purpose of this study is, therefore, to present a theoretical method for predicting damping characteristics of floors which are bonded with elastomeric adhesives.

LITERATURE REVIEW

A brief review of relevant research on damping is presented in the following paragraphs.

Pian and Hallowell (1951) investigated damping effect in a simple built-up beam. They found that joint slip had more influence on damping than internal friction of the material. Their conclusion showed that energy loss per cycle of vibration is dependent on the amplitude of vibration. They also concluded that the nonlinearity on the load deflection curve was a second order function.

Yeh (1970, 1971a) studied damping sources in wood structures and studied material damping in rigid frames and composite T-beams. He presented a theoretical model for slip damping in nailed T-beams using a mathematical expression that was based on the load-slip characteristics of a simple nailed joint in pure shear. His results showed good agreement for higher amplitudes of vibration.

Polensek (1975) determined experimentally the damping ratio on nailed wood-joint floors. He performed free vibration tests under different loading conditions. The average damping ratios were reported to be in the range of 7% to 11%.

Masuko et al. (1973) explained the mechanisms for energy dissipation in a jointed cantilever beam. They introduced slip ratio into the theoretical analysis from the experimental results of micro-slip between the jointed surfaces. They defined the damping ratio as energy loss to total energy introduced into the system and calculated strain energy from the first mode of vibration. A few years later he and Nishiwaki (1978) jointly studied the effect of frequency of excitation on slip ratio and damping capacity. They concluded that slip ratio increases with an increase in frequency and amplitudes of vibration.

FORMULATION

Typical floor systems can be modeled as T-beams in which the sheathing and web are joined by shear connectors such as nails and/or glue. In this paper the theoretical formulation for damping in composite beams is based on the evaluation of energy dissipation in the shear connectors of the T-beams.

Deflection and slip

The theory of partial composite action in beams has been applied to layered beams by several researchers. A discussion of this theory is provided by Goodman and Popov (1968), Goodman (1969), and McGee and Hoyle (1974). A brief summary of this theory is provided here only to enhance the readability of this paper and the subsequent formulations. For a detailed discussion of the partial composite action in beams, the reader is referred to the above references.

The model used for the partial composite action along a segmental length of a T-beam is shown in Fig. 1. The model simulates the resistance to external loads by moments in the layers and a couple consisting of an axial force F, applied along the centroidal axes of the two layers.

The differential equation relating to the axial force F, and the bending moment M, due to external loads is given by:

$$\frac{d^2F}{dx^2} - C_1F = -C_2M \tag{1}$$

while the relative slip Δ, between the flange and web, is given by:

$$\Delta = \frac{1}{S}\frac{dF}{dx} \tag{2}$$

Equation 1 can also be expressed as:

$$\frac{d^2Y_c}{dx^2} = \frac{d^2Y}{dx^2} + \frac{z}{C_1\Sigma EI}\frac{d^2F}{dx^2} \tag{3}$$

where Y_c and Y are the deflections of partial composite beams and a corresponding rigid beam, respectively.

From the differential Eqs. (1), (2) and (3), the expression for deflection Y_c and slip Δ can be obtained.

For a beam with simply supported ends and symmetric loading, the following boundary conditions apply,

at x = 0, $$Y = Y_c = F = 0 \tag{4a}$$

and at x = L/2, $$\frac{dY}{dx} = \frac{dY_c}{dx} = \frac{dF}{dx} = 0 \tag{4b}$$

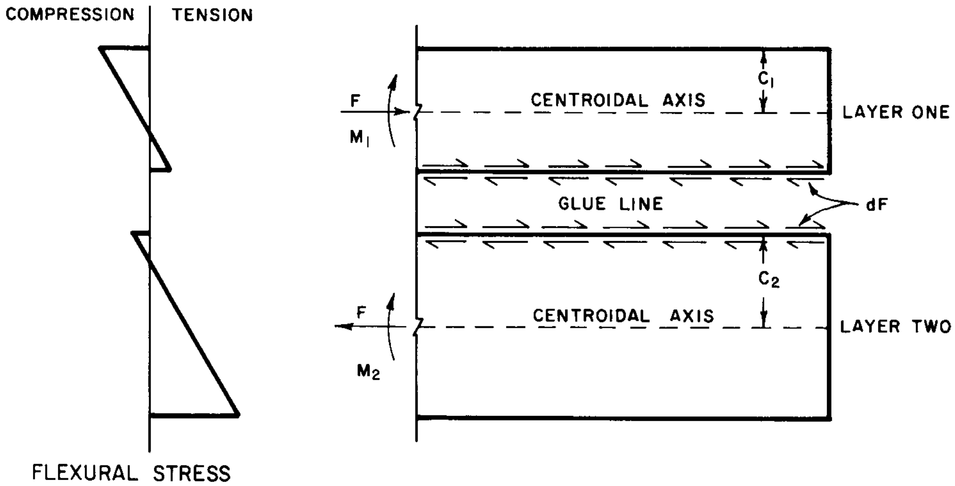

FIG. 1. Two-layer system with incomplete interaction.

The corresponding deflection Y_c and slip Δ can then be expressed as:

$$Y_c = Y + \frac{Fz}{C_1 \Sigma EI} \tag{5a}$$

and

$$\Delta = \frac{C_1 \Sigma EI}{Sz}\left(\frac{dY_c}{dx} - \frac{dY}{dx}\right) \tag{5b}$$

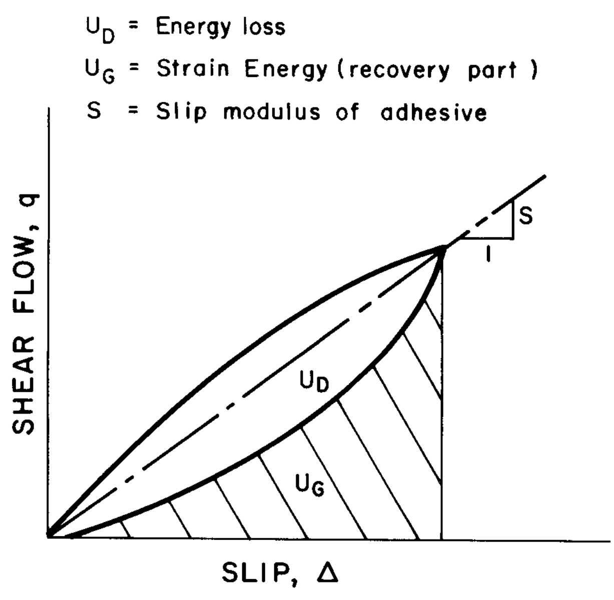

FIG. 2. Shear flow vs. slip in glue line.

TABLE 1. *Properties of plywood and lumber.*

Beam no.	Glue line thickness t	Cross-sectional area of plywood A_1	Cross-sectional area of lumber A_2	Moment of inertia of plywood I_1	Moment of inertia of lumber I_2	Modulus of elasticity of plywood E_1	Modulus of elasticity of lumber E_2	Effective stiffness of composite beam K_c
	in.	*in.*^{2}		*in.*^{4}		10^6 *psi*		*lb/in.*
A-1	0.067	1.929	4.125	0.101	2.60	1.80	1.98	493.82
A-2	0.036	1.929	4.125	0.101	2.60	1.80	1.98	550.58
B-1	0.125	3.904	8.250	0.204	20.80	1.72	1.89	2,773.10
B-2	0.094	3.904	8.250	0.204	20.80	1.72	1.84	2,823.28
B-3	0.061	3.904	8.250	0.204	20.80	1.72	1.94	3,151.49
B-4	0.042	3.904	8.250	0.204	20.80	1.72	2.01	3,447.97

Using Eq. (5) and assuming a deflected shape of the beam to be $\psi(x)$, a midspan deflection of the composite beam Z_c, and a midspan deflection of a rigid beam Z, the following expressions are obtained:

$$F = \frac{S\overline{EI}}{z\overline{EA}}[Z_c - Z]\psi(x) \tag{6}$$

and

$$\Delta = \frac{\overline{EI}}{z\overline{EA}}[Z_c - Z]\frac{d\psi}{dx} \tag{7}$$

As an example, using Eqs. (1) and (6) for a simply supported beam with a concentrated load P at midspan, the following deflections can be computed as:

$$Z = \frac{PL^2}{48\overline{EI}} \tag{8}$$

and

$$Z_c = \frac{P}{K_c} \tag{8}$$

where K_c is known as effective-stiffness of the composite beam and is given by:

$$K_c = \frac{PL^3}{48\overline{EI}} + \frac{PC_2z}{C_1{}^2\Sigma EI}\left[\frac{L}{4} - \frac{1}{2\sqrt{C_1}}\tanh(\sqrt{C_1}L/2)\right] \tag{9}$$

Strain energy

Total strain energy in a T-beam is the sum of the strain energy in the web, the strain energy in the flange and the strain energy in the glue line.

Strain energy, U_W, in the flange and the web is given by:

$$U_W = \int_0^L \frac{M_1{}^2}{2E_1I_1}dx + \int_0^L \frac{M_2{}^2}{2E_2I_2}dx + \int_0^L \frac{F^2}{2A_1E_1}dx + \int_0^L \frac{F^2\,dx}{2A_2E_2} \tag{10}$$

Substituting Eq. (6) into Eq. (10) and expressing M_1 and M_2 in terms of deflections, the following expression is obtained:

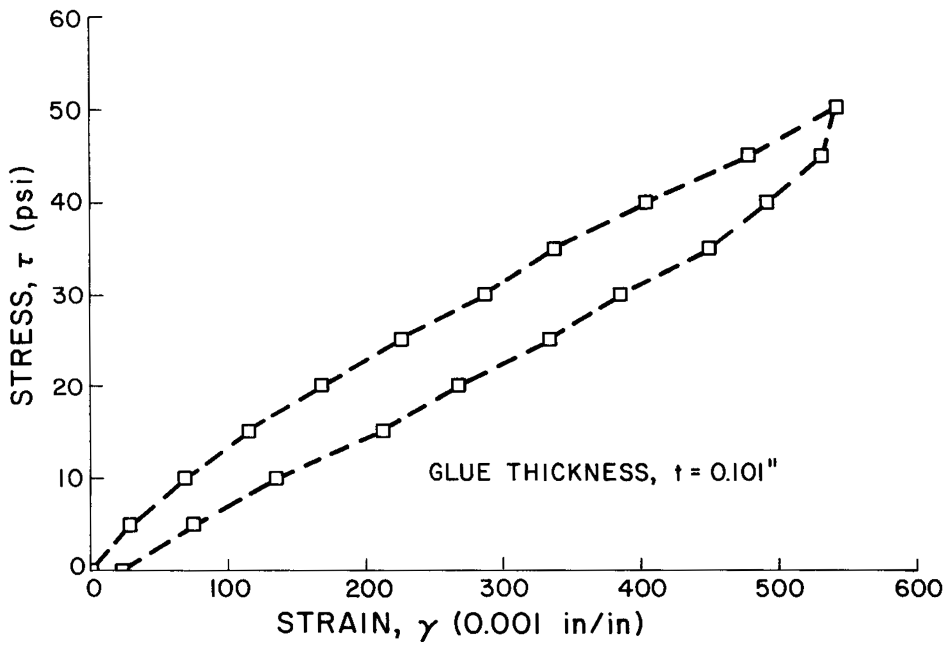

FIG. 3. Stress-strain relation for 3M adhesive.

$$U_W = (\Sigma EI)Z_c^2 \int_0^{L/2} \left(\frac{d^2\psi}{dx^2}\right)^2 dx + \frac{S^2(\overline{EI})^2}{(\overline{EA})^3 z^2}(Z_c - Z)^2 \int_0^{L/2} \psi^2 \, dx \qquad (11)$$

Strain energy, U_G, in the glue line is given by:

$$U_G = \int_0^{\Delta_{max}} F \, d\Delta \qquad (12)$$

Using Eqs. (6) and (7) into Eq. (12) yields,

$$U_G = \frac{S(\overline{EI})^2(Z_c - Z)^2}{(\overline{EA})^2} \int_{L/2}^{0} \psi \frac{d^2\psi}{dx^2} dx \qquad (13)$$

Equations (11) and (13) can be used in evaluating the term U_w and U_G.

Energy loss

The energy loss, U_D, per unit length of a glue line during vibration can be obtained by subtracting the elastic recovery part from the total energy of the glue line (Fig. 2). Therefore, the energy loss can be written as

$$U_D = \alpha(U_D + U_G) \qquad (14)$$

where α is known as a loss factor which varies with the thickness and shear modulus of adhesive.

Equation (14) is rewritten as:

$$U_D = \frac{\alpha U_G}{(1 - \alpha)} \qquad (15)$$

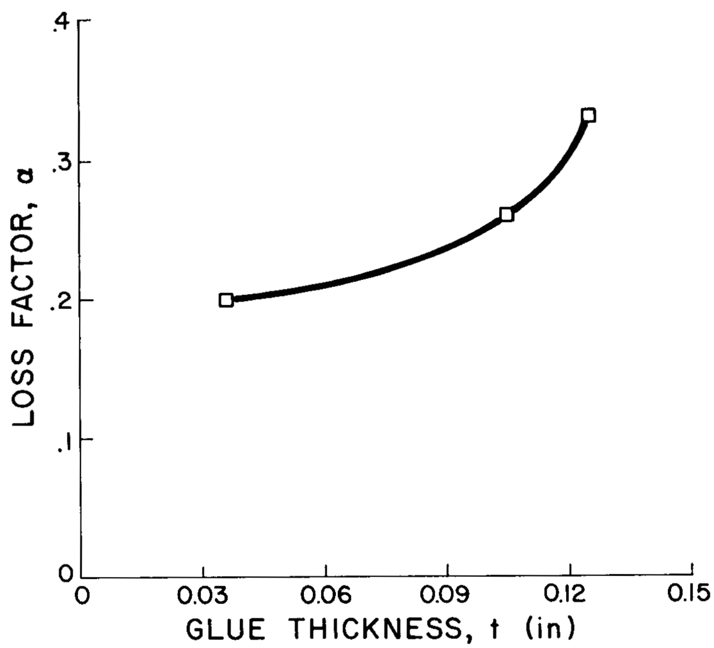

FIG. 4. Mean loss factor vs. glue line thickness.

Damping ratio

Damping ratio, ζ of the composite beam is defined by the ratio of energy loss to energy introduced into the system (Masuko et al. 1973).

Therefore, ζ can be expressed as:

$$\zeta = \frac{U_D}{U_W + U_G + U_D} = \frac{\alpha U_G}{(1 - \alpha)U_W + U_G} \tag{16}$$

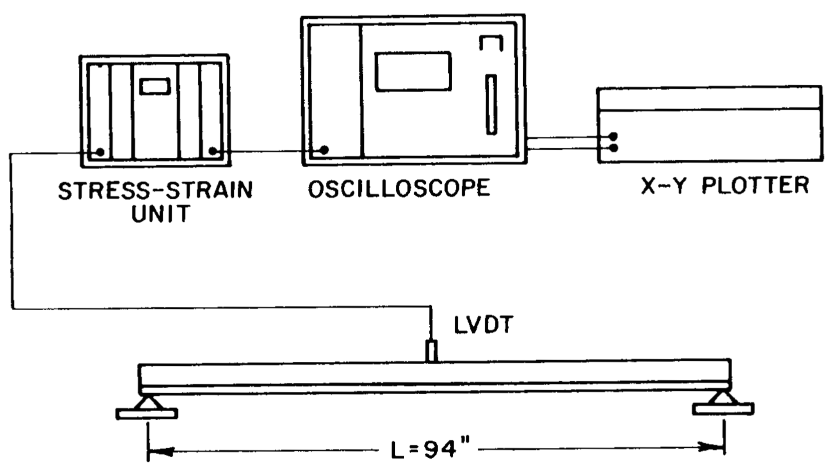

FIG. 5. Schematic diagram of test set-up.

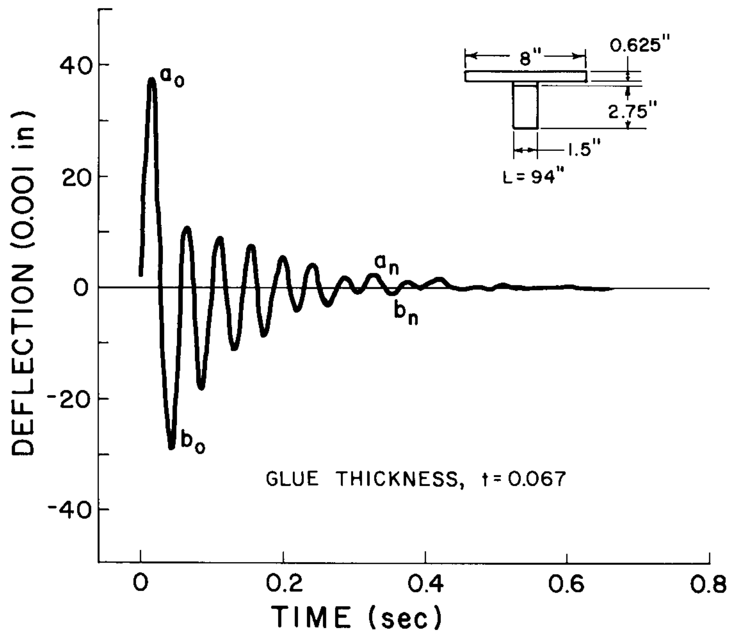

FIG. 6. Typical time-deflection trace obtained from free vibration test of beam no. A-1.

EXPERIMENTAL STUDY

Materials

Six T-beams, each 8 feet long, were made from the two different sizes of Douglas-fir lumber and from 4 ft × 8 ft × ⅝ in. AC-X plywood boards.

Beam "A": average size of web was 1.5 in. × 2.75 in. and average width of flange of plywood was 7.844 in.

Beam "B": Average size of web was 1.5 in. × 2.75 in. and average width of plywood was 15.875 in.

The effective moments of inertia and cross-sectional areas for the plywood were calculated from Hoyle (1978). Sectional properties of lumber and plywood are given in Table 1.

The elastomer used was 5230 Scotch-Grip Wood Adhesive. Nails were not used in the construction of the test beams.

Shear modulus and loss-factor of adhesive

The approach for determining shear modulus was similar to that used by Hoyle and Dong (1975). For a detailed description of the procedures, the reader is referred to that reference. Three double shear test specimens were made for each glue thickness of 0.036, 0.101, and 0.125 inch. The average value of shear modulus G, computed from these specimens, was 94.13 psi.

A typical shear stress-strain relationship for adhesive is shown in Fig. 3. Using the trapezoidal method of area calculation, the loss-factor, α, was calculated for each specimen by using Eq. (14). The difference between the maximum and

TABLE 2. *Overall damping and corresponding amplitude obtained from free vibration test of beams.*

Beam no.	Glue line thickness t (inch)	No. of traces	Damping ratio % Experimental Mean	Damping ratio % Experimental SD[1]	Damping ratio % Theoretical Mean	Theoretical vs. experimental differences (%)	Double amplitude ($a_0 + b_0$) (0.001 inch) Mean	Double amplitude ($a_0 + b_0$) (0.001 inch) SD
A-1	0.067	5	6.68	0.20	5.69	−15	70.2	10.7
A-2	0.036	5	5.12	0.77	4.45	−13	62.3	7.3
B-1	0.125	4	7.88	0.87	6.61	−16	56.3	12.3
B-2	0.094	4	6.42	1.28	5.13	−20	43.4	16.8
B-3	0.061	4	5.34	0.64	4.66	−13	54.7	13.7
B-4	0.042	4	5.02	0.90	4.57	−18	62.3	10.2

[1] SD, standard deviation among the traces.

minimum loss-factor values within a group of equal glue line thickness is about 0.02. The relationship between loss-factor and thickness of the glue line is plotted in Fig. 4.

Free vibration tests

A vertical free vibration method of testing was used in this investigation. The test set up consists of

i) a test specimen,
ii) a linear variable differential transformer (LVDT),
iii) a stress-strain unit,
iv) a digital oscilloscope and
v) an X-Y analog plotter.

A sketch of the experimental set-up is shown in Fig. 5.

The T-beam was placed in an inverted position on two simply supported edges and set to a span length of 94 inches. By giving an initial amplitude at midpoint of the beam, time deflection traces were observed on the oscilloscope. A typical time-deflection trace obtained from this test is shown in Fig. 6.

An overall damping ratio (Polensek 1975) is defined by:

$$\zeta = \frac{1}{2\pi n} \mathrm{Ln}\left(\frac{a_0 + b_0}{a_n + b_n}\right) \tag{20}$$

where,

a_0 = first initial positive peak amplitude
b_0 = first initial negative peak amplitude
a_n = positive peak amplitude after n cycle from a_0
b_n = negative peak amplitude after n cycle from b_0

RESULTS

To compute damping from the theoretical expression, the shape function, ψ, and loss factor, α, need to be assumed. In this paper, x is assumed to have the deflected shape of a simply supported beam with a mid-point loading. The loss factor is determined for a shearing stress level of 50 psi. It should be pointed out that, for a typical service load of 30 psf, the maximum shearing stress developed in the interfacing layer is less than 25 psi. In a single degree of freedom system, some forms of impulsive loading produce the maximum response to about twice the static response (Clough and Penzien 1975). Therefore, this value of stress was multiplied by a dynamic magnification factor of 2 to obtain the selected stress of 50 psi.

Experimental results are shown in Table 2 and compared with the theoretical analysis presented in this paper. It is seen that theoretical and experimental results are in good agreement. Table 2 shows that damping computed from experimental tests is about 5% to 8% compared with theoretical values of 4.5% to 6.5%. The theoretical results are 13% to 20% lower than experimental values. It is also shown that damping increases by about 45% as the thickness of adhesive layer is increased by about three times. The largest difference between the theoretical and experimental values is for a glue line thickness of $^3/_{32}$ inch.

One of the reasons for discrepancy between the theoretical and experimental results is due to the fact that the theory does not account for material damping of wood components. Material damping for wood in structural members is about 0.3% to 0.4% (Yeh 1971b).

Other sources of discrepancy are due to nonconformity of glue line thickness. The theoretical formulation assumes constant thickness, a condition difficult to achieve. In addition, the loss factor used in the formulation is based on one cycle of loading. Effects of repeated cycles of loading and unloading have not been considered in the theoretical analysis.

The effects of beam stiffness, K_c, on damping can also be seen from Tables 1 and 2. The beam stiffness is a function of glue line stiffness, S, which in turn depends on the glue line thickness. Table 2 shows a general decrease in the damping ratio corresponding to an increase in the beam stiffness. For Beam "B" the damping ratio obtained from experimental results shows a decrease of 34% corresponding to an increase in beam stiffness of 24%. The same comparison for theoretical values shows a reduction of 31%.

CONCLUSION

A theoretical formulation for predicting damping of composite T-beams has been presented. Theoretical results are compared from free vibration test of six beams considering the various parameters contributing to damping of beams: the theoretical results gave good comparison with experimental values. The following remarks can be drawn from this study:

1. Theoretical results gave lower values than those obtained experimentally.
2. Damping is influenced by glue line thickness.
3. Damping decreases by about 25% with six-fold increases in beam stiffness.

To make conclusive recommendations, an expanded testing program needs to be undertaken. Such a program is essential because of the limited data available on this topic of research.

ACKNOWLEDGMENT

The authors appreciate the help and assistance of Prof. Robert J. Hoyle, Jr. in the design of sample sizes and experimental set up.

REFERENCES

CLOUGH, R. W., AND J. PENZIEN. 1975. Response to impulsive loads. Dynamics of structures. McGraw-Hill Book Co.

GOODMAN, J. 1969. Layered wood systems with interlayer slip. Wood Sci. 1(3):148–158.

———, AND E. P. POPOV. 1968. Layered beam systems with interlayer slip. Proc. of the ASCE, Journal of the Structural Division, ST11. Pp. 2535–2547.

HOYLE, R. J. 1978. Effective section properties for plywood. Wood technology in the design of structures, 4th ed. Mountain Press Publishing Company, Missoula, MT. P. 62.

———, AND C. C. DONG. 1975. Elastomeric adhesive properties—Shear strength, shear modulus, creep and recover. Wood Fiber 8(12):96–106.

MASUKO, M., Y. ITO, AND K. YOSHIDA. 1973. Theoretical analysis for a damping ratio of a jointed cantibeam. Bulletin of the JSME 16(99):1421–1432.

MCGEE, W. D., AND R. J. HOYLE. 1974. Design method for elastomeric adhesive bonded wood joist-deck systems. Wood Fiber 6(2):144–145.

NEWMARK, N. M., C. P. SIESS, AND I. M. VIEST. 1951. Test and analysis of composite beams with incomplete interaction. Proc. of the Society for Experimental Stress Analysis 9(1):75–92.

NISHIWAKI, N., M. MASUKO, Y. ITO, AND I. OKUMURA. 1978. A study on damping capacity of a jointed cantilever beam. Bulletin of the JSME 21(153):524–531.

PIAN, T. H. H., AND F. C. HALLOWELL. 1951. Structural damping in a simple built-up beam. Proc. First U.S. National Congress of Applied Mechanics. Pp. 97–102.

POLENSEK, A. 1975. Damping capacity of nailed wood-joist floors. Wood Sci. 8(2).

YEH, C.-T. 1970. A study on the mechanisms of damping in wood structures. Ph.D. dissertation, University of Washington.

———. 1971a. Damping sources in wood structures. J. Sound Vibration 19(4):411–419.

———. 1971b. The prediction of damping in nailed and glued wood structures. J. Sound Vibration 19(4):421–435.