HEART SHAKES AND GROWTH STRESSES

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(Received 14 March 1972)

ABSTRACT

Heart shakes are often observed to accompany crosscutting of tree trunks and have been thought to be due to release of longitudinal strain energy in the trunk during cutting. Using a simple model of stresses in the trunk, the authors have attempted to link shake development quantitatively with release of axial strain. On the basis of the analysis, however, it appears unlikely that there is a direct relationship between the two. A more likely cause is the transverse stress concentration caused by the saw cut.

The residual or self-stresses in timber, usually called growth stresses, are technologically important because of their effects during cutting processes, especially in broad-leaved species. This paper attempts to show theoretically the relationship between growth stresses and the formation of heart shakes, which are radial splits in the log running for limited distances from the pith outward. An extensive treatment of this problem by Boyd (1950b) concludes that shakes in standing trees result from a pattern of transverse tensile stresses towards the centers of the stems. This view was supported by Kubler (1959a).

In the present paper we consider only heart shakes that occur upon crosscutting. According to Boyd, these are the most frequently encountered heart shakes and are attributed by him (1950a) to release of strain energy associated with longitudinal growth stresses. This hypothesis is assessed here in terms of redistribution of longitudinal growth stresses during crosscutting. On the basis of the assumptions made in this analysis, the authors feel that the direct effect of longitudinal stress redistributions is unlikely to cause transverse shakes. It is speculated that their actual cause is the transverse stress imbalance caused by the saw cut.

Analysis

Kubler (1959 a, b) proposed a theoretical model of growth stress development based upon constant initial stresses in each sheath of new cells added at the periphery of the growing stem. He assumes the elasticity of the stem to be transversely isotropic and derives in detail the expressions below for stresses in the radial, axial and circumferential directions, respectively.

$$\text{Radial: } \sigma_1 = \sigma_3^0 \ln(r/R) \qquad 1(a)$$

$$\text{Axial: } \sigma_2 = \sigma_2^0[1 + 2\ln(r/R)] \qquad 1(b)$$

$$\text{Circumferential: } \sigma_3 = \sigma_3^0[1 + \ln(r/R)] \qquad 1(c)$$

where σ_2^0 and σ_3^0 are the initial (new sheath) axial and circumferential stresses, r is a radial co-ordinate measured from the center of the stem, and R the outer radius of the stem. The principal stresses σ_1, σ_2, σ_3 from eq. (1) are plotted as functions of r/R in the form σ_i/σ_2 in Fig. 1 for $i = 1, 2, 3$.

Experimentally (Boyd 1950a), it is known that σ_2^0 is positive (tensile) while σ_3^0 is negative (compressive) and smaller in value.

In the figure σ_3^0/σ_2^0 is taken to be $-\frac{1}{4}$ as a typical value, obtained from measurements on a specimen of mountain ash (*Eucalyptus Regnans*) by Boyd (1950a). Note that quite large transverse tensions

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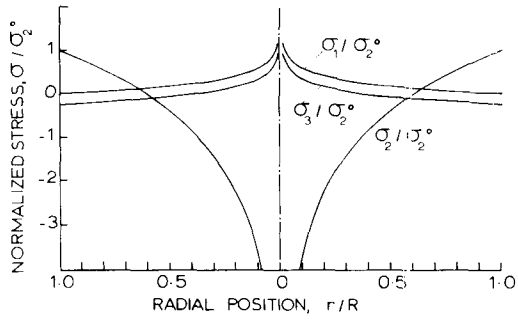

FIG. 1. Normalized principal stresses as functions of radial position.

are apparently supported by the central region of the standing stem.

The foregoing equations lead to complex three-dimensional elastic interactions during stress redistributions caused by cutting. However, Gillis (1972) showed that a good approximation of the solutions of such problems can be obtained by neglecting elastic interactions. Complete solutions show that stress changes in the axial direction and any of the two transverse directions are related through factors of the form $\sigma - \mu\sigma^*$, where σ is the axial stress change, σ^* is the transverse stress change, and μ is a Poisson's ratio. (Gillis 1972).

As transverse stresses and their changes are fairly small compared to axial, even for a relatively large Poisson's ratio, there is only a small fractional effect axially from changes of transverse stress. On the other hand, changes of axial stress do not have a substantial effect in the transverse direction because Poisson's ratio is typically only a few per cent in such cases.

On this basis, growth stress redistributions are treated here approximately as being uncoupled, which greatly simplifies the problem without sacrificing any essential features of the solution.

Let the stem be crosscut to some depth D (Fig. 2) and assume the inner edge of the cut to be straight. The remaining uncut cross section, A, responds to restore equilibrium by both stretching and bending. Assuming that plane cross sections remain

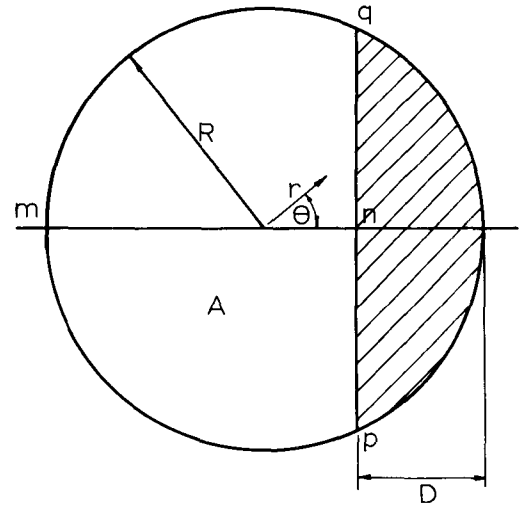

FIG. 2. Crosscut with segment removed.

plane, the change of axial strain at a point (r, θ), (see Fig. 2) can be written as

$$\epsilon = \epsilon_0 + \mathrm{krCos}\theta, \qquad (2)$$

where ϵ_0 is the strain at $r = 0$ and k is the curvature (reciprocal of the radius of curvature). This is equivalent to taking the strain change as a uniform extension ϵ_0 plus a position dependent contribution due to bending about the center of the disc (Gillis 1972).

The corresponding axial stress increment is obtained by multiplying ϵ, the strain change, in eq. (2) by Young's modulus for tension (E) along the longitudinal axis of the trunk. (It is assumed that Young's modulus for tension along the longitudinal axis is not significantly different from that for compression, i.e. that E does not vary greatly from the pith to the outer surface of the trunk.)

The total axial stress is then the original stress plus this increment

$$\sigma_2 = \sigma_2{}^0[1 + 2\ \ln(r/R)] + E\epsilon_0 + EkrCos\theta \qquad (3)$$

We normalize eq. (3) by defining

$$e = E\epsilon_0/\sigma_2{}^0 \text{ and } c = ERk/\sigma_2{}^0.$$

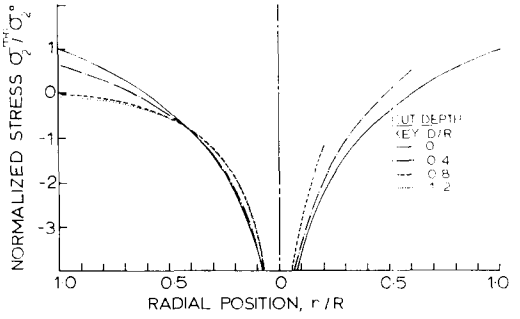

FIG. 3. Axial stress along truncated crosscut bisector versus distance from center.

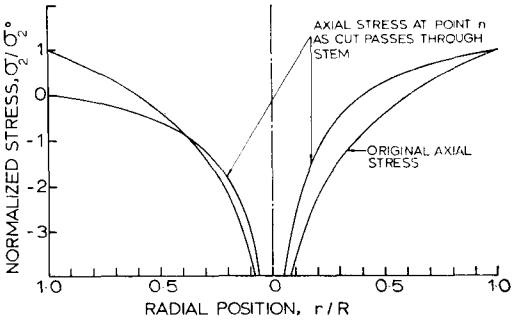

FIG. 4. Axial stress at center of cut face as cut passes through the stem.

Equation (3) then becomes

$$\sigma_2/\sigma_2^0 = 1 + 2\ln(r/R) + e + c(r/R)\cos\theta \quad (4)$$

The quantities e and c can be determined as functions of the cut depth D by using the two equilibrium conditions that the net force and bending moment on the cross section be zero. That is, where the integrations are carried out over the uncut portion, A, of the disc, we have

$$\int_A (\sigma_2/\sigma_2^0)\,dA = 0 \quad (5)$$

$$\int_A r\cos\theta(\sigma_2/\sigma_2^0)\,dA = 0 \quad (6)$$

substituting for σ_2/σ_2^0 from eq. (4) and integrating over A using polar co-ordinates gives us two linear simultaneous equations in e and c. (A numerical procedure is needed to integrate the logarithmic term in (6).)

Carrying out the integration for various values of D, we find the corresponding values for e and c after solving the linear equations, and can then substitute e and c into equation (4) to give the stress. In Fig. (3) axial stress profiles along the center line of the cross section (m–n in Fig. (2)) are shown for various values of D, (including $D = 0$ for the uncut disc). In Fig. (4), the axial stress at the center of the cut (point n in Fig. (2)) is plotted against D, and the original axial stress profile is shown for comparison.

Referring to Figs. 3 and 4, it can be seen that there are significant changes in σ_2 between the cut and the uncut discs. For instance in Fig. 3 at $r/R = 0.2$, the difference in σ_2 between the situations in which $D/R = 0$ (i.e. the uncut disc) and in which $D/R = 0.8$ is approximately 45%, both stresses being compressive, but with the compression less in the cut disc than the uncut one. Using figures from Boyd (1950a for *E. Regnans*), this corresponds to a drop in stress of about 6.9×10^6 N/m^2 (1000 lbs/inch^2). However, the Poisson's ratio that is operative is typically between 3% and 4% (Hearmon 1948), and when the changes in axial stress that occur are scaled down by this factor, the resultant changes in transverse stress are found to be of the order of 2.7×10^5 N/m^2 (40 lbs/inch^2). Since the existing transverse stresses are of the order of 2×10^6 N/m^2 (300 lbs/inch^2) (Boyd 1950a), we find it difficult to believe that this change of stress can be fractionally large enough to cause splitting.

Nevertheless heart shakes are often observed to accompany crosscutting, so it can be presumed that something substantially increases the transverse stress. We qualitatively attribute the increase to a transverse stresss concentration caused by the cut. Along the surface exposed by the saw teeth (p–q in Fig. 2), equilibrium requires that the normal stress components be zero. In terms of the radial and circumferential stress components at points on this surface, the requirement is that:

$$\sigma_1/\sigma_3 = \tan\theta. \quad (7)$$

Generally eq. (7) is not satisfied by the original stress distribution of eq. (1). Nevertheless, strain compatibility and the differential equations of stress equilibrium require a reasonably smooth transition between the stress state in the cut, eq. (7) at the cutting surface, and that within the adjacent material in the two portions of the stem being separated by the cut. As a result the adjacent stem material *tends* towards a transverse stress state satisfying (7) along p–q.

Readjustments of the growth stresses are required during the cutting, which could lead to increases in radial or circumferential tension at some points large enough to cause splitting.

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