A CUMULATIVE DAMAGE MODEL TO PREDICT LOAD DURATION CHARACTERISTICS OF LUMBER[1]

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ABSTRACT

The exponential damage model $d\alpha/dt = \exp[-a + b\sigma(t)/\sigma_s]$ is used in this paper to describe duration-of-load data on lumber tested in bending where $d\alpha/dt$ is rate of damage, σ_s is static strength, $\sigma(t)$ represents applied load history, and a and b are parameters. A specially selected set of Douglas-fir 2 by 4s was divided into six 49-specimen groups having similar distributions of edge knot size and modulus of elasticity. Each group was randomly assigned to one of three rates of ramp loading or one of three levels of constant loading.

The lognormal distribution $\sigma_s = \sigma_o\exp(wR)$ provided a reasonable description of static strength of the 2 by 4s where σ_o is the median static strength, w is a measure of variability, and R is a normal random variable. With $b' = b/\sigma_o$, the model used to fit the ramp and constant load experimental data by nonlinear least squares was $d\alpha/dt = \exp[-a + b'\sigma(t)/\exp(wR)]$; thus a, b′, and w were parameters that were estimated. The model fit some but not all of the ramp and constant load data reasonably well. The estimates of variability (w) were slightly greater under ramp loading than under constant loading. Residual strength of specimens surviving constant load was less than expected. A greater duration-of-load effect was observed for the edge knot 2 by 4 lumber than that previously indicated for small clear-wood specimens; however, the difference does not appear to be statistically significant.

Keywords: Duration of load, constant load, ramp load, static strength, bending, lumber, Douglas-fir, cumulative damage model, time, residual strength, load history.

INTRODUCTION

Current wood engineering design methods use adjustment factors for different end use conditions. These factors are not flexible enough to account for the effects of different load histories (duration of load) on the reliability of wood structures. Such flexibility can be provided by a cumulative damage model. This study was undertaken to evaluate a previously proposed cumulative damage model (Gerhards 1977b, 1979), using time-to-failure data from two different types of load histories (ramp loading and constant loading) for Douglas-fir 2 by 4 lumber containing an edge knot. A preliminary evaluation of this research was presented at the 1983 IUFRO Division 5 Conference (Gerhards and Link 1983).

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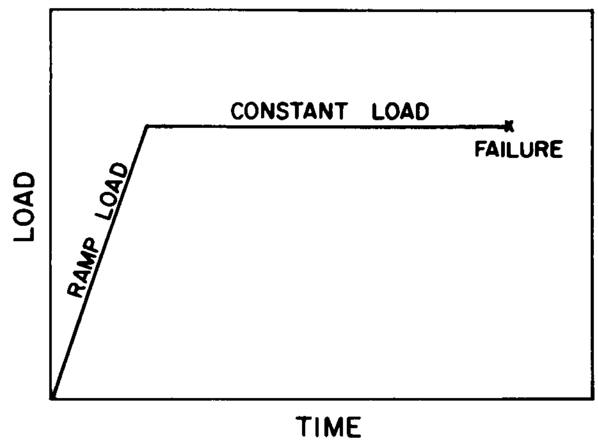

FIG. 1. Schematic load history for a constant load test. (ML83 5305)

BACKGROUND

Although wood structures have proven to be highly reliable, the reasons for their reliability are poorly understood. Actually, the reliability of wood structures could derive from several factors. Perhaps the most important is that design loads and their times of acting (load histories) are generally conservative. Also, much of the lumber in a grade is stronger than the allowable stresses for the grade would indicate, partly because of the natural distribution of strength of pieces that look alike and partly because pieces may be downgraded for characteristics other than strength. Another factor is that the greatest stress on a structural member might not fall at its weakest point.

The lack of precise design methods is partially caused by procedures used to adjust engineering design values for duration of load, a factor known to affect strength of wood (Gerhards 1977a; Wood 1951). In the United States, design values for structural lumber are based on "normal" loading, which implies a design load lasting for 10 years of either continuous or cumulative duration. Design values are adjusted upward for shorter durations of load (such as snow, wind, or earthquake) and downward for permanent loading. Recommended adjustment factors for loads of different duration are published by the National Forest Products Association (NFPA 1977).

These duration-of-load factors are based on a duration-of-load curve developed from rapid, pseudo ramp loading and constant loading tests of small clear-wood specimens in bending (Wood 1951). Wood's analysis of clear-wood data was based on the assumption that one continuous curve could be used in code applications to account for time to failure for both ramp loads and constant loads. That assumption is not strictly valid because the stress developed in a ramp load test theoretically should be higher than the stress that can be carried in a constant load test for equal time to failure. No such assumption is required when duration of load is accounted for by a cumulative damage model.

THEORY

The damage model (Gerhards 1977b, 1979) relates damage accumulation exponentially to load. The model may be written as:

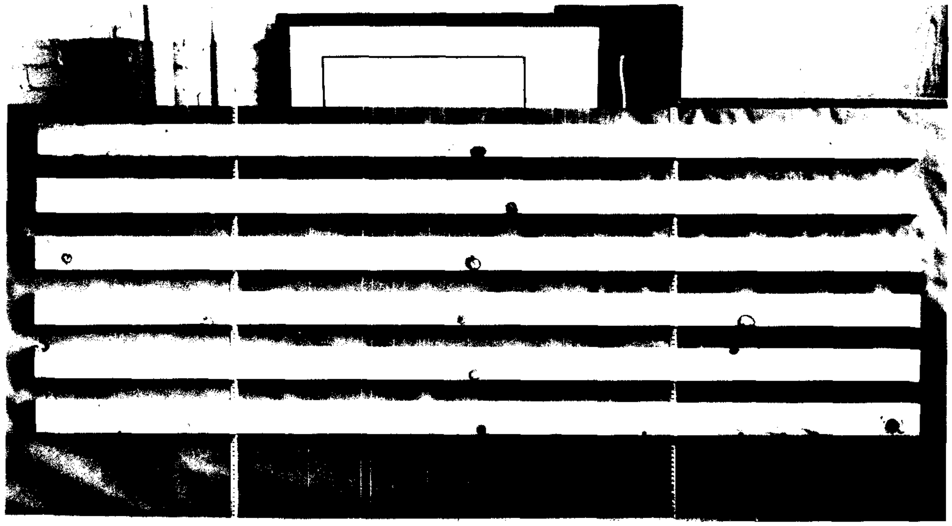

FIG. 2. Examples of edge knot 2 by 4s used in this study. (M 145 726-2)

$$d\alpha/dt = \exp[-a + b\sigma(t)/\sigma_s] \quad (1)$$

where α is the amount of damage (0 implies no damage, 1 implies failure), $d\alpha/dt$ is the rate of damage, $\sigma(t)$ is the applied load history, a and b are parameters, and σ_s is static strength. Equation (1) can be integrated for any stress history $\sigma(t)$, $t \geq 0$, to determine the accumulated damage α. The time to failure at $\alpha = 1$ will be designated as T.

In applying the damage model to lumber strength and duration of load, we recognize that the static strength, σ_s, varies from piece to piece for any population of lumber. For this study we believe the lognormal distribution:

$$\sigma_s = \sigma_o \exp(wR) \quad (2)$$

provides an adequate description of this variation in static strength, where σ_o is the median strength, w is a measure of variability, and R is a standard (mean 0 and variance 1) normal random variable. When Eq. (2) is substituted into Eq. (1), the two parameters, b and σ_o, appear as a fraction. Therefore, we introduce $b' = b/\sigma_o$ and Eq. (1) becomes:

$$d\alpha/dt = \exp[-a + b'\sigma(t)/\exp(wR)] \quad (3)$$

with a, b', and w as the parameters to be estimated.

Two traditional load histories are used for duration-of-load testing of wood: ramp loading and constant loading. In ramp loading $\sigma(t) = kt$ where k is a constant. In constant loading $\sigma(t) = \sigma_c$, a constant. Integration of Eq. (3) to failure yields:

$$T = \exp[a - b'\sigma_c/\exp(wR)] \quad (4)$$

for constant loading and:

$$T = [\exp(wR)/(b'k)]\ln\{[b'k/\exp(wR)]\exp(a) + 1\} \quad (5)$$

for ramp loading. If the load history is a combination of periods of ramp loading

TABLE 1. *Sorting variable statistics.*[1]

Type of loading	Mean	Standard deviation	Minimum	Maximum
	Edge knot size (in.)			
Fast ramp	1.23	0.139	1.02	1.54
Intermediate ramp	1.23	0.133	0.97	1.52
Slow ramp	1.23	0.136	0.99	1.54
High constant	1.24	0.144	0.97	1.51
Intermediate constant	1.24	0.136	0.99	1.52
Low constant	1.24	0.132	1.03	1.53
	Sorting modulus of elasticity (10^6 psi)			
Fast ramp	1.78	0.258	1.24	2.34
Intermediate ramp	1.78	0.245	1.28	2.31
Slow ramp	1.79	0.249	1.30	2.33
High constant	1.79	0.282	1.24	2.49
Intermediate constant	1.79	0.266	1.27	2.32
Low constant	1.78	0.256	1.28	2.33

[1] Differences between means for either sorting variable are not significantly different.

and constant loading, each segment of the load history may be integrated and damage accumulated until failure occurs. A typical experiment with lumber will involve a period of initial ramp loading until a desired level of constant load is attained, then a period of constant load to failure (Fig. 1). Integration of Eq. (3) for ramp loading followed by constant loading to failure yields:

$$T = \sigma_c/k - \exp(wR)/(b'k) + \{\exp[-b'\sigma_c/\exp(wR)]\}[\exp(wR)/(b'k) + \exp(a)] \tag{6}$$

Equation (4) is an approximation of Eq. (6) when constant loading time is long relative to the ramp loading time necessary to get to the constant load, as exp(wR)/(b'k) and σ_c/k are small relative to T.

From Eq. (4), the constant load that causes failure is linearly related to the logarithm of the median time to failure through:

$$\sigma_c = (a - \ln T)/b' \tag{7}$$

Similarly, from Eq. (5) the median strength in ramp loading to failure, σ_r, is approximately linearly related to logarithm of rate of loading:

$$\sigma_r \approx [a + \ln(b'k)]/b' \tag{8}$$

EXPERIMENTAL EVALUATION

Lumber specimens

Six hundred pieces of green, rough-sawn Douglas-fir lumber (2 by 4s, 2 by 6s, and 2 by 8s up to 16 ft long) were collected from a western Oregon mill over a period of several weeks. Each piece was selected on its suitability to furnish an 8-foot-long 2 by 4 test specimen with (1) a central 2-foot length with a nearly cylindrical knot of 1- to 1⅜-inch diameter (strength-controlling knot) at the edge of the wide face, (2) a central 4-foot length free of knots larger than ½ inch other than the strength-controlling knot, (3) slope of grain no steeper than 1 in 15, and (4) characteristics of the No. 1 lumber stress grade (Western Wood Products

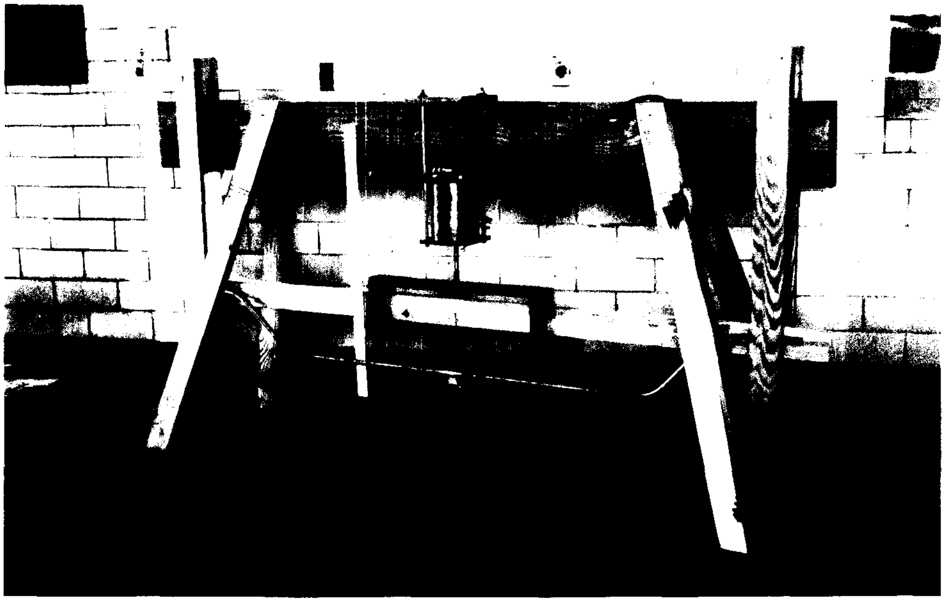

FIG. 3. Example of loading frame showing air cylinder mounted between strongback (upper horizontal member) and 2 by 4 specimen (lower horizontal member). (M 145 726-4)

Association 1979). The selected lumber was stickered to prevent stain or decay during sampling.

At the Forest Products Laboratory, a green 2 by 4 was cut from each selected piece so as to position the strength-controlling knot a small distance from an edge to allow for surfacing and to minimize grain slope. After kiln-drying with a conventional schedule, the 2 by 4s were stored for several months at 73 F and 50% RH. Then the 2 by 4s were surfaced four sides to 1.50 by 3.50 inches. The final edge knot size and its distance from the edge of the specimen were measured. Also, a modulus of elasticity for lumber sorting (E-sort) was determined from an edgewise, third-point bending test spanning 90 inches; the test load was kept low to minimize any possibility of damage.

Of the 600 prepared 2 by 4s, 294 were selected as having valid edge knots, with knot sizes ranging from 1 to 1½ inches. Figure 2 shows examples of selected specimens. The 294 specimens were assigned to six test groups (49 per group) in such a manner that all groups had nearly equal distributions of E-sort and edge knot size (Table 1). For each group, E-sort averaged about $1.78(10^6)$ psi, with a coefficient of variation (CV) of about 14%, and edge knot size averaged about 1.24 inches, with a CV of about 11%.

Loading apparatus

Forty-nine test frames were built to allow members of each group of specimens to be tested simultaneously. Each test frame consisted of a strongback, two pivoting supports, an air cylinder for applying load, a loading bar with two pivoting load points, a yoke with a potentiometer for measuring specimen deflection, a clock to time failure, and legs for support (Fig. 3). The air cylinder system was chosen

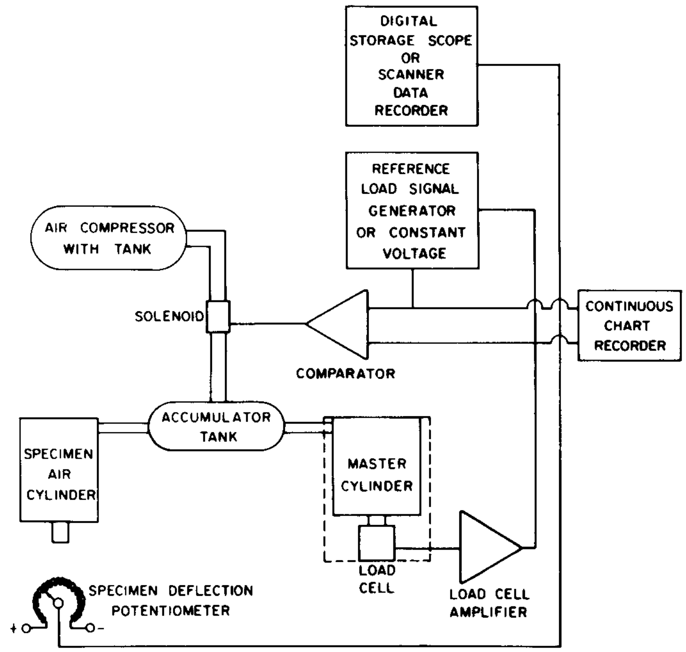

FIG. 4. Schematic of testing system (time clock not shown). (ML83 5310)

to provide flexibility in loading (ramp, constant, slow cyclic) and to simulate dead loading.

The test frames were sized for an edgewise bending span of 84 inches, with the two load points symmetrically located on the span and spaced 24 inches apart. Specimens were oriented with the edge knot on the tension side and between the load points.

A schematic of the testing apparatus is shown in Fig. 4. A master cylinder identical to the specimen air cylinders was equipped with a load cell in a rigid frame to monitor specimen loading. To maintain load, a comparator receiving signals from the load cell and a calibrated reference signal generator or a constant voltage source was used to control a solenoid valve to maintain proper air pressure. Whenever the load cell signal fell slightly below the reference signal, the comparator would switch the solenoid on until the load cell signal again matched the reference signal.

The schematic (Fig. 4) shows one air cylinder and one accumulator tank. In practice, there were 49 air cylinders and 4 accumulators interconnected in the system. The accumulators provided a large volume of air under the desired pressure so that system pressure would remain relatively stable anytime a specimen failed, which allowed sudden air cylinder piston movement. Three specimen air cylinders were equipped with load cells and one of the accumulator tanks was equipped with a calibrated pressure sensor to provide additional confirmation of loads during test. In addition, alarms and a shutoff device were built into the system to protect against overload.

The air cylinders used to apply load had negligible friction. Each air cylinder was equipped with a rubberized fabric that folded around the piston rather than

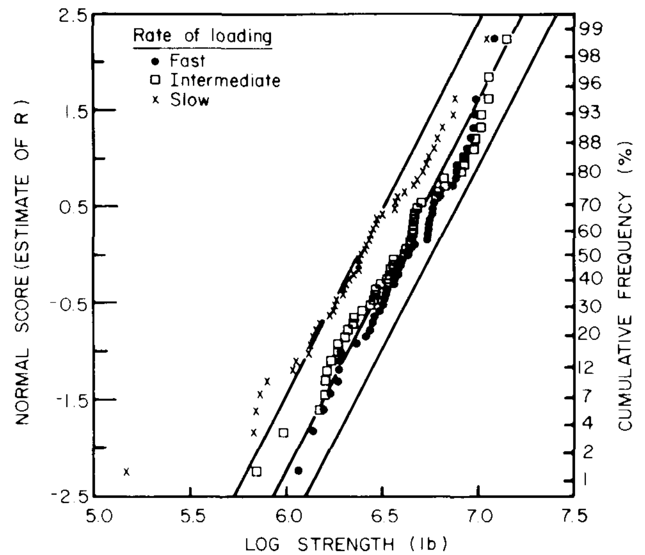

FIG. 5. Cumulative frequency of maximum loads (pounds) from ramp load tests. (ML86 5075)

a piston with ring seals. Also, each cylinder contained a linear ball bearing rather than a sleeve bearing for the piston rod, to further reduce friction.

Load histories

Three different rates of ramp loading and three levels of constant loading were used to test the specimens. All tests were done at about 75 F and 55% RH. Except for specimens tested at the rapid and intermediate ramp rates to failure, all specimens were given a brush coat of hot paraffin wax to retard changes in moisture content in case the conditioning equipment malfunctioned (which never occurred).

Ramp loading.—The three rates of ramp loading were 1,460 lb/min, 5.81 lb/min, and 0.0245 lb/min.[3] Median failing times were about 31 sec for the fastest rate, about 127 min for the intermediate rate, and about 16.9 days for the slow rate. The three rates were chosen to be approximately equally spaced on a logarithmic basis.

The air lines were too restrictive to allow loading of the whole 49-frame system at the fastest rate. Therefore, the 49 specimens of one group were tested one at a time to failure using only one of the loading frames and one accumulator tank. A digital storage oscilloscope was used to record specimen load and deflection versus time for the fast ramp test. For the two slower ramp tests and the three constant load tests, a chart recorder was used to monitor the master cylinder load cell continuously and a data scanner was used to record load-cell load and specimen deflections on a repetitive basis.

Constant loading.—The three levels of constant load were 702 lb, 631 lb, and 478 lb, to provide substantial differences in median times to failure. To reach the

[3] Although results of this study are discussed in pounds of specimen load, the load can be converted to bending stress in pounds per square inch by multiplying by 4.90.

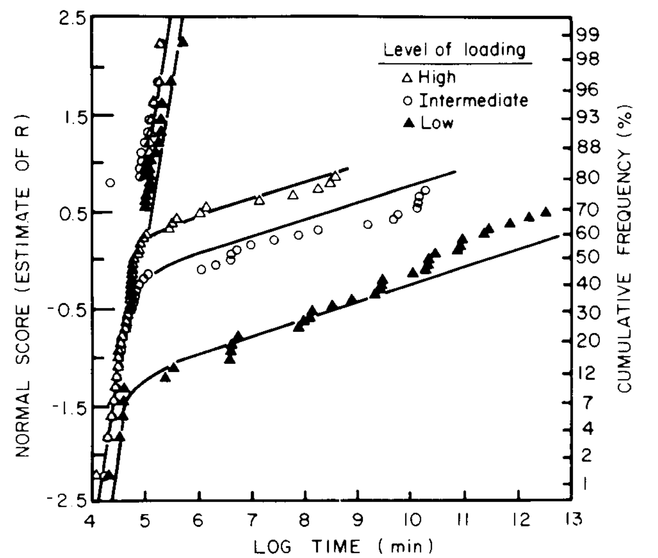

FIG. 6. Cumulative frequencies of times to failure (minutes) from constant load tests. Data on specimens failing during the initial ramp load and during the second ramp load of constant load survivors are included. (ML86 5076)

constant load level, specimens were ramp loaded at 6.0 lb/min to the two higher constant loads and at 4.8 lb/min to the lowest constant load.

The slower loading rate and the 478-lb load level for the low constant-load test were below those planned, because the master cylinder load-cell calibrating amplifier was inadvertently adjusted prior to the 478-lb load test. The change was discovered during test when the output from the master load cell differed from the output from the three specimen load cells and the air pressure sensor. For the low constant-load test, a precision pressure regulator was used to maintain constant load instead of the comparator, because electrical noise problems with the comparator system caused brief interruptions of load during tests at the higher constant loads. The air pressure (measured by a precision gauge just downstream from the pressure regulator) multiplied by the piston area of the air cylinders also confirmed the change in calibration of the master load cell amplifier.

Constant load tests were stopped before all specimens of a group had failed. Termination times were 4.65 days for the high constant load, 33.9 days for the intermediate constant load, and 220 days for the low constant load. Specimens surviving the constant load phase were unloaded for a day, then reloaded at their original ramp uploading rate to failure.

Determinations of time and load at failure

The original idea was to use the deflection-time record to establish when a specimen failed, with the time clock for backup. Because of the air cylinder load (simulated dead load), a specimen failing to support the attained load deflects very rapidly to a large value. The time to failure was taken as that time coinciding with the large sudden deflection.

For the specimens failing during ramp load, the maximum load attained was inferred from the load-time record for the master air cylinder load cell at the established failure time. This system worked well for establishing maximum loads for all specimens failing during uploading to a constant load level as well as those tested at the intermediate or slow ramp load to failure, as the backup clocks agreed closely with the inferred times of failure from the deflection-time records.

For the rapid ramp load, a stopwatch was used as a backup from the deflection-time record. However, the stopwatch time was missed for 3 of the 49 specimens. Also, the deflection-time record was incomplete on six other specimens, because the deflection apparatus became inoperable after a partial failure during uploading to the maximum load.

Because the rapid ramp load test setup consisted of one accumulator tank, two air cylinders, and the connecting short hoses, the sudden failure, and sometimes a partial failure, resulted in a rapid extension of the test specimen air cylinder. The rapid extension caused a small drop (blip) in the reference load cell output. Because the time of the blip agreed with the time of large deflection for all good deflection-time records, the blip in the load-time curve was used to establish maximum load. The time of the blip also agreed within 1 second of the time of failure recorded by the stopwatch.

A test modulus of elasticity (ET) was calculated from the slopes of the ramp load-time curve and the linear portion of the deflection-time curves.

ESTIMATING MODEL PARAMETERS

Equation (3) may be integrated for any load history, $\sigma(t)$. For ramp load tests to failure, $\sigma(t) = kt$, $0 \leq t \leq T$, where T is the failure time. For the constant load test used for this study, three load histories need to be considered.

1. If a specimen failed during the first ramp loading to the constant load level, $\sigma(t) = kt$, $0 \leq t \leq T$.
2. If a specimen failed during the constant load phase, a two-step load history must be accounted for: $\sigma(t) = kt$, $0 \leq t \leq T_1$, and $\sigma(t) = kT_1 = \sigma_c$, $T_1 \leq t \leq T$ where T_1 is uploading time to the constant load level.
3. For specimens surviving constant load but failing in the second ramp loading, a three-step load history is required: $\sigma(t) = kt$, $0 \leq t \leq T_1$, $\sigma(t) = kT_1 = \sigma_c$, $T_1 \leq t \leq T_2$, and $\sigma(t) = k(t - T_3)$, $T_3 \leq t \leq T$ where T_2 is the time when the constant load was removed from the surviving specimen and T_3 is the time when the second ramp loading was started.

In estimating model parameters, we found that a and b′ were very highly correlated with each other and to a lesser extent with w. Therefore, we substituted c for b′/a in estimating model parameters to decrease parameter correlations.

An iterative reweighted nonlinear least squares procedure was used to estimate the parameters: c, b′, and w. The dependent variable was ln(T) for all failures during ramp loading except for the constant load survivors for which it was $\ln(T - T_3)$. For specimens failing during constant load, the dependent variable could have two forms: $\ln(T - T_1)$ (total time on constant load) or ln(T) (total time of test). Both forms were used in separate analyses. The independent variable was an estimate of the underlying standard normal random variable, R, which is unknown. The estimates of R are the expected values of the order statistics of a sample of size n, 49 in this study, from a standard normal distribution.

When the dependent variable is not the logarithm of total time on test, some data points have to be deleted from the procedure for estimating the parameters. The deleted points result from inexactness in estimating R which can cause negative estimates in the argument of $\ln(T - T_1)$ or $\ln(T - T_3)$. The deleted data points occur at the beginning of constant load and at the beginning of final ramp loading. The logarithm of total time on test is a possible choice for the dependent variable for the constant load failures, but it is unrealistic for failures in the second ramp-load phase, as the values are nearly identical ($T - T_3$ is extremely small compared to T_3).

The iterative reweighting is necessary when ramp load and constant load data are used together because the variance of $\ln(T)$ is approximately equal to w^2 for ramp load data but on the order of $(b'\sigma_c w)^2$ for constant load data.[4] Therefore, the residuals for the constant load data must be weighted by $1/(b'\sigma_c)$ to get new estimates for c, b′, and w. Reweighting is continued until iteratively determined successive parameters converge.

The data

Table 2 lists descriptive statistics on physical properties of the specimens, other than the sorting variables, and the results (*P*-values) of one-way analyses of variance used to test equality of means. The groups were well matched in mean specific gravity (high *P*-value). The low *P*-values for moisture content and test modulus of elasticity imply significant differences, however. The lower value of modulus of elasticity for the slow ramp test was expected because of the viscoelastic behavior of wood. The 8.2% mean moisture content for the slow ramp is different from the means for the other groups for unknown reasons.

The ramp load and constant load data are shown in the form of lognormal probability plots (Figs. 5 and 6). The natural logarithms of maximum loads (ML) in the ramp load tests are plotted against the appropriate estimates of R, a function of cumulative frequency, in Fig. 5. While these plots are not totally linear, the Shapiro-Wilk test provides no evidence to reject the null hypothesis that the logarithm of static strength has a normal distribution (Shapiro and Wilk 1965).

Ramp loading	Standard deviation of ln(ML)	Shapiro-Wilk test, *P*-value to test H_0: normality
Fast	0.257	0.302
Intermediate	0.297	0.543
Slow	0.345	0.278

While the standard deviations of ln(ML)[5] show an increasing trend with a decrease in rate of loading, Bartlett's test (Snedecor and Cochran 1967) of the null hypothesis of equal variances could not be rejected (*P*-value = 0.130).

[4] Link, C. L., C. C. Gerhards, and J. F. Murphy. Estimation and confidence intervals for parameters of a cumulative damage model. Manuscript in preparation, Forest Products Laboratory, Madison, WI.

[5] Up to about 0.3, the standard deviation of ln(ML) is approximately equal to the coefficient of variation of ML.

TABLE 2. *Descriptive statistics on physical properties of the 2 by 4 specimens.*

Type of loading	Mean	Standard deviation	Minimum	Maximum	*P*-value for testing H_0: equal means
		Moisture content (pct)			
Fast ramp	9.1	0.43	8.2	10.0	
Intermediate ramp	9.8	0.37	8.9	10.4	
Slow ramp	8.2	0.25	7.7	8.7	<0.0001
High constant	9.1	0.35	8.3	9.7	
Intermediate constant	9.1	0.41	8.2	10.0	
Low constant	9.5	0.38	8.1	10.5	
		Specific gravity			
Fast ramp	0.49	0.042	0.39	0.58	
Intermediate ramp	0.47	0.040	0.39	0.55	
Slow ramp	0.47	0.050	0.37	0.64	0.6166
High constant	0.48	0.045	0.39	0.56	
Intermediate constant	0.48	0.046	0.38	0.62	
Low constant	0.48	0.037	0.40	0.56	
		Test modulus of elasticity (10^6 psi)			
Fast ramp	1.76	0.264	1.28	2.54	
Intermediate ramp	1.73	0.247	1.26	2.30	
Slow ramp	1.53	0.244	1.02	2.01	<0.0001
High constant	1.75	0.258	1.25	2.20	
Intermediate constant	1.77	0.279	1.28	2.33	
Low constant	1.78	0.288	1.25	2.37	

In Fig. 6, the natural logarithms of total time on test are plotted against the appropriate estimates of R for those specimens that failed either on first ramp loading to the constant load level (lower-steep portions of the distributions) or during the constant load phase (middle flatter portions of the distributions). For those specimens surviving the constant load phase (upper-steep portions of the distributions), the natural logarithms of only the time to failure under the second ramp loading are plotted, as the natural logarithm of total time would just show up as a vertical line on the graph.

Evaluation with the damage model

Results of fitting the damage model to various combinations of the data using T or $T - T_l$ for constant load failure times are given in Table 3. Regression models (RM) 1 and 3 were limited to constant load data only. RM 8 was limited to the three rates of ramp load data, while RM 5 was limited to the intermediate and slow ramp load data. The remaining models used all of the data including ramp load data for survivors of constant load, except the rapid ramp data were omitted from RM 2 and 4. RM 1, 4, and 7 used total time on test for all constant load failures; whereas RM 2, 3, and 6 used time on constant load only for those failures.

The suitability of the damage model may be evaluated, in part, by comparing fitted curves with actual data. This was done for all 8 models of Table 3, but only RM 3 is featured here (Figs. 5 and 6). The model assumes that the rate of loading effect on strength is approximately linearly related to the logarithm of rate (Eq. 8). While the model fits the median data for the intermediate and slow ramp

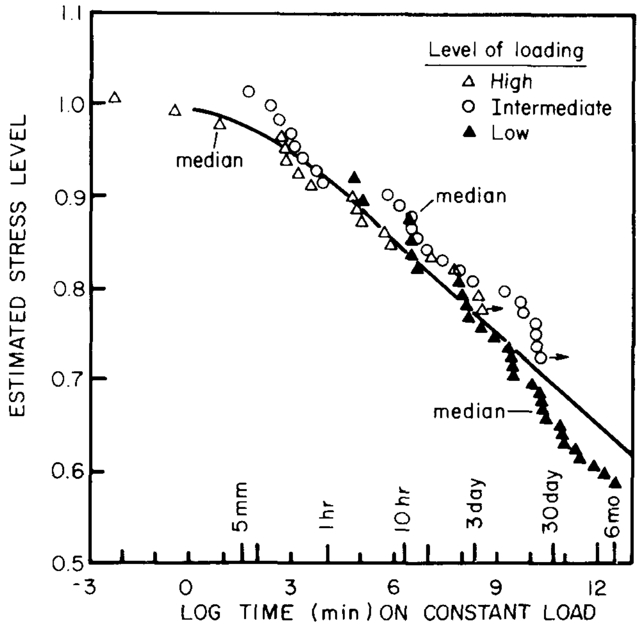

FIG. 7. Relationship between estimated constant load stress level and natural logarithm of time in minutes to failure under constant load. Curve is Eq. (6) with a = 31.199195 ln min, b′ = 0.0412522 lb, w = 0.268, and σ_o = 718 lb. (ML86 5074)

strengths, it does not fit the fast rate of loading strength data at all. The poor showing of the fast ramp data could be due to errors in determining maximum load or to experimental variation in groups. Otherwise, the lack of fit suggests that a more complicated model is needed to describe the rate of loading effect.

Another general observation pertains to models that include constant load data. Where the time variable is total time on test, T (RMs 1, 4, 7), the models fit longer time constant load data best. Where the time variable is total time on constant load, $T - T_1$ (RMs 2, 3, 6), the models fit shorter time constant load data best.

The assumption of common variability suggested by the damage model may also be questioned. The trend of the data in Fig. 5 suggests that variation in the logarithm of strength increases with a decrease in rate of loading. Moreover, the estimate of w is lower for RM 3 which is based on constant load data only, than for RM 8 which is based on the three rates of ramp load data. However, none of the differences between the estimates of w are statistically significant.

RM 3 fits the early constant load failures better than the later constant load failures (Fig. 6). In fact, it underpredicts some of the longer times at the high constant load and most of the times at the intermediate constant load but overpredicts many of the longer times at the low constant load.

The damage model predicts very little effect on strength of most specimens surviving constant load. This may be inferred from the upper portions of the distributions of time (Fig. 6, estimated R > about 1.0), which show a linear trend as an extension of the first ramp portions of the distributions, and the fact that strength is directly related to time under ramp load. The curving tails at the

TABLE 3. *Damage model regression statistics and some predictions.*

Regression model	Data[1]	Number of specimens included	Time used for constant load failures	Constant load failures excluded[2]	Degrees of freedom	Parameter estimates[3]				Predicted constant load time[4]	
						c	b′	a	w	100% of static strength	61.9% of static strength
						(10^{-3})	(10^{-2})			(min)	(yr)
1	C	71	T	0	68	1.19043	2.93005	24.6134	0.295	35.7	0.208
						(0.10598)	(0.70679)	(6.3205)	(0.062)	(2.5, 515)	(0.043, 1.020)
2	RCX[5]	245	$T - T_1$	12	230	1.33647	4.12022	30.8291	0.280	3.5	0.529
						(0.02120)	(0.24953)	(1.8629)	(0.014)	(1.5, 7.9)	(0.253, 1.104)
3	C	71	$T - T_1$	6	62	1.32222	4.12522	31.1992	0.268	4.9	0.748
						(0.02185)	(0.45117)	(3.4407)	(0.043)	(2.2, 10.5)	(0.096, 5.83)
4	RCX[5]	245	T	13	229	1.35247	4.25009	31.4247	0.283	2.5	0.539
						(0.03188)	(0.35911)	(2.7208)	(0.014)	(0.7, 9.2)	(0.221, 1.316)
5	R[5]	98	—	—	95	1.33554	4.42764	33.1524	0.333	3.9	1.379
						(0.06586)	(1.16431)	(8.8677)	(0.026)	(0.3, 52.0)	(1.019, 101.1)
6	RCX	294	$T - T_1$	19	272	1.40345	5.27875	37.6127	0.275	0.7	2.726
						(0.02188)	(0.53281)	(3.8333)	(0.012)	(0.2, 2.5)	(0.350, 21.2)
7	RCX	294	T	20	271	1.41447	5.30957	37.5375	0.276	0.6	2.205
						(0.02404)	(0.56852)	(4.0625)	(0.013)	(0.1, 2.1)	(0.246, 19.8)
8	R	147	—	—	144	1.42324	6.64345	46.6784	0.308	0.4	52.106
						(0.03068)	(1.27301)	(9.0001)	(0.020)	(0.04, 3.3)	(0.195, 15.610)

[1] C = constant load failures, R = ramp load failures, X = ramp load failures of specimens surviving constant load.
[2] Because the least squares estimates of $T - T_1$ or $T - T_3$ were negative.
[3] Based on time in minutes and stress in pounds. Numbers in parentheses are standard errors.
[4] Median static strength assumed to be 718 lb. Numbers in parentheses are 95% confidence intervals.
[5] Excludes rapid ramp data.

beginnings of the upper portions of the predicted distributions, however, represent the strength loss predicted by the model. The actual data on times to failure under the second ramp for constant load survivors suggest that there may have been more effect on strength than predicted by the model as the actual times tend to be shorter than predicted.

Although Fig. 6 suggests some lack of fit of the model to the data, another way to evaluate the model is to compare it against stress level-time data. To do so requires an estimate of static strength. Then the constant loads can be expressed as stress level ($SL = \sigma_c/\sigma_s$), which can be plotted against logarithm of time on constant load.

We assume that σ_s is lognormal (Eq. 2). Also, because the rapid ramp data may contain experimental errors, we assume the median static strength, σ_s, to be 718 lb based on the intermediate ramp data. The 718-lb value also results from Eq. (8) using k = 5 and parameter estimates for RM 3. Thus, σ_s is estimated from:

$$\sigma_s = 718 \exp(wR) \tag{9}$$

and SL is estimated from

$$SL = \sigma_c/[718 \exp(wR)] \tag{10}$$

where R is the estimate of the underlying random variable.

Figure 7 shows a plot of the estimated constant load stress levels determined from Eq. (10) versus the logarithm of actual time on constant load, that is, for $T - T_1$. The curve drawn through the data:

$$SL = -\{\ln[T - T_1 + 1/(b'k)] - \ln[\exp(a) + 1/(b'k)]\}/(b'\sigma_o) \tag{11}$$

is based on Eq. (2) and Eq. (6) expressed in terms of SL with R = 0 (median trend), k = 5, $\sigma_o = 718$, and a and b′ from RM 3. The curve, insensitive to the test values of k = 4.8 and 6 for $\ln(T - T_1) > 2$, appears to be a reasonable estimate of the trend of the constant load data.

Figure 7 has the same format used by Wood to show constant load durations for small clear-wood specimens (Fig. 1 in Wood 1951). It is worth noting that Wood's linear trend indicates a duration to failure of 3.8 years at 61.9% stress level (permanent load factor adjusted for 10-yr load duration). Figure 7 and Table 3 suggest a duration to failure of only ¾ year for RM 3 for the same relative stress level for the edge knot lumber; however, the confidence interval for the RM 3 prediction (Table 3) includes 3.8 years. Thus, although there appears to be a difference in duration-of-load behavior between the edge knot lumber of this study and Wood's small clear-wood specimen results, the difference does not appear to be statistically significant.

Although RM 3 was featured in the above discussion, the fit of the other models of Table 3 with the data was graphically evaluated, too. The need for brevity excludes the figures, but the evaluations can be summarized as follows:

1. If the parameter estimates are based on all of the data except that for the rapid ramp (RMs 2 and 4), then the model fit the data about equally with RM 3.
2. If the parameter estimates are based on including rapid ramp data (RMs 6,

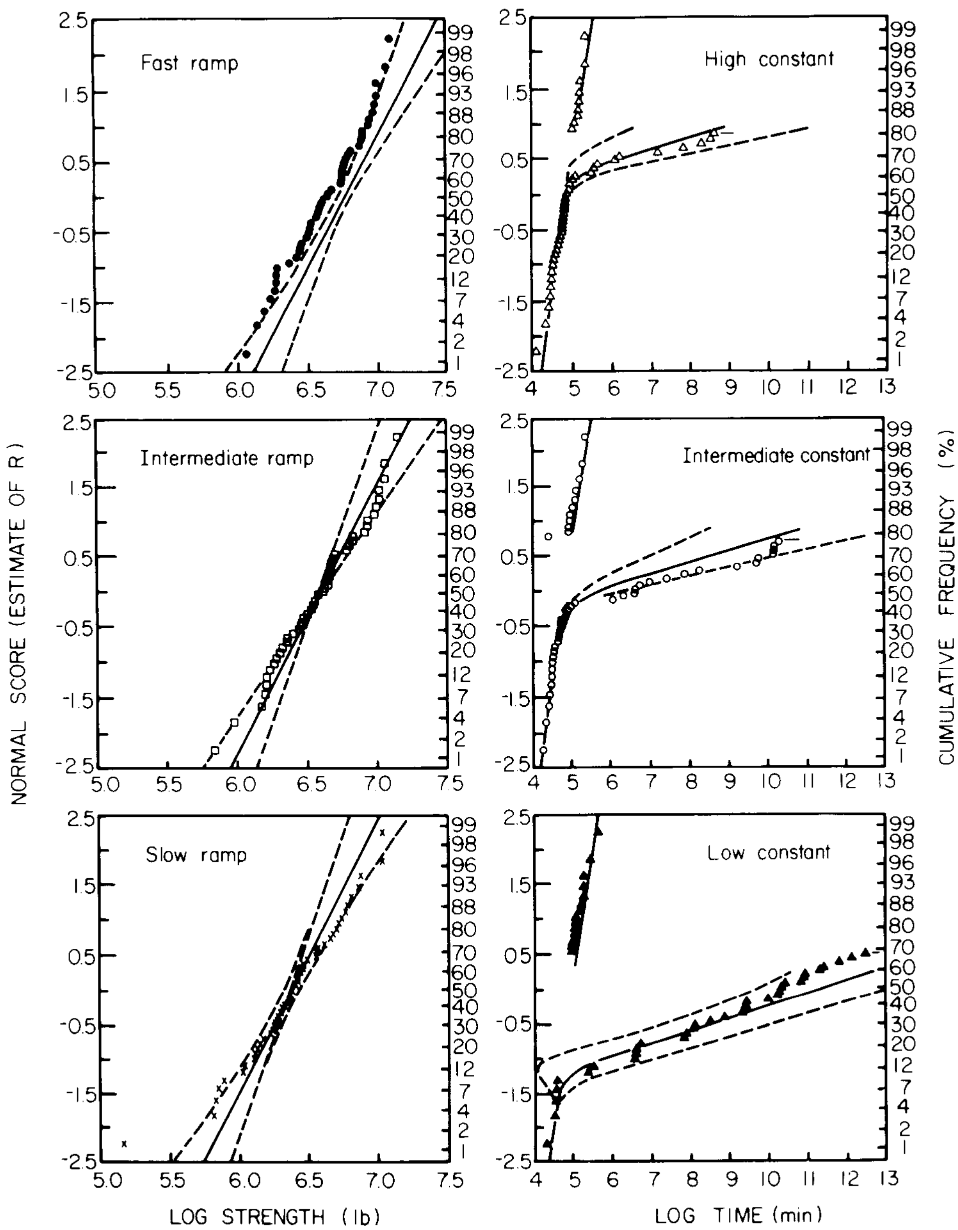

FIG. 8. Ninety-five percent confidence intervals on the RM 3 damage model for the six different test load histories. (ML86 5073)

7, and 8), then the model fits the constant load data poorly, especially notable for RM 8.

3. If the parameter estimates are based on ramp data only (RMs 5 and 8), then the model fits the constant load data poorly.
4. If the parameter estimates are based on total time on constant load only (RM 1), then the model fits most of the ramp load data poorly.

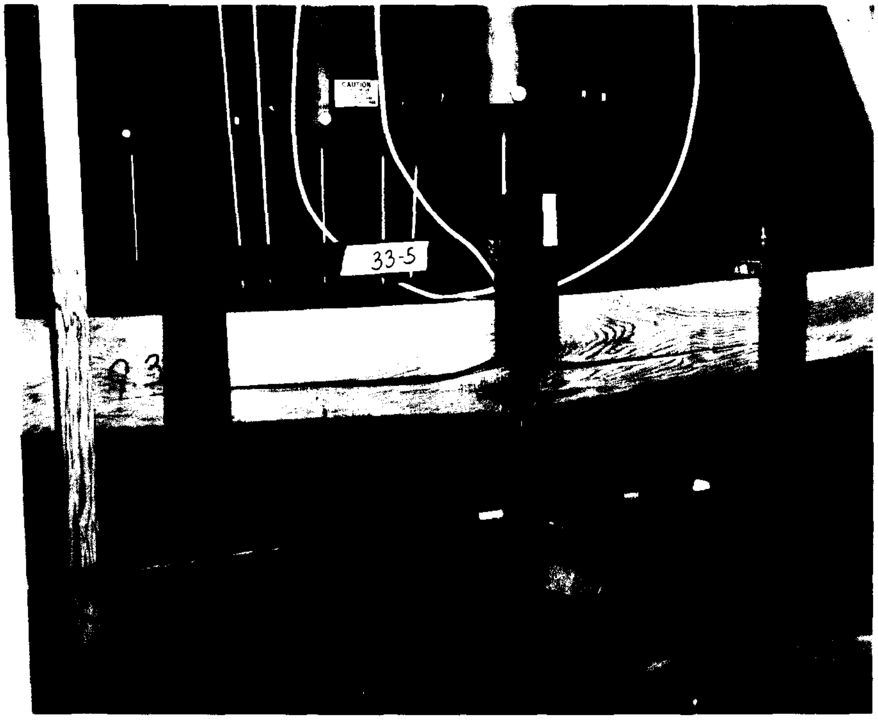

FIG. 9. Partial failure typical of many of the edge knot lumber specimens. (M 147 991-11)

Predictions of constant load times to failure offer another way to evaluate or compare the models of Table 3. Table 3 contains predictions of times to failure at 100% and at 61.9% of static strength (i.e., SL = 1.0 and 0.619) with 95% confidence intervals on those predictions. The predictions are based on a constant load immediately applied and a median static strength of 718 lb. Because the intermediate ramp rate was about 6 lb/min, time to median static strength was about 2 hours. Intuitively then, the constant load time of 36 min predicted by RM 1 in Table 3 for SL = 1.0 seems far too long. Also, the 52 years predicted by RM 8 for SL = 0.619 is too long based on reported data (Gerhards 1977a). Thus RMs 1 and 8 may be poor choices for the duration-of-load effect.

The confidence intervals on the predicted times at 100% and 61.9% of static strength are symmetrical in log time, thus are skewed in real time (Table 3). A few of the confidence intervals are very large (RMs 1 and 5 at 100% of static strength and RMs 5 and 8 at 61.9% of static strength).

Some idea of significant differences (although not exact) between models can be gleaned by comparing predicted values for a given model to the confidence intervals on the predicted values for other models. Thus, RMs 2, 3, and 4 are not different, RMs 1 and 8 predict differently from most other models, and RMs 5, 6, and 7 generally predict differently from RMs 2, 3, and 4. The comparisons for RMs 2, 3, and 4 support the conclusion based on graphical evaluations given above. It is also worth noting that the 3.8 years predicted for clear wood at 61.9% of static strength mentioned earlier falls outside several of the confidence intervals, suggesting that the edge knot lumber of this study seems to behave differently from small clear-wood specimens.

Confidence intervals and the model for each test load history data distribution

Figure 8 shows the model (solid lines) for each of the six test load histories along with dashed lines which represent the 95% confidence intervals on the expected values of the order statistics for RM 3. The meaning of these confidence intervals is that, if the experiment would be repeated many times, 95% of the confidence intervals for those repetitions would contain the true distribution of data. As the confidence intervals on the model are broad, one gets the impression that the damage model as represented by RM 3 is not unreasonable for the actual data except for the rapid ramp strength data (Fig. 8).

Specimen failure characterization

Specimen failures can be classed into three types. The most common failure started as a partial rupture through or directly around the knot with splits extending both ways just above the knot (Fig. 9); splits generally extended out to the load points. Then rupture would generally extend further into the 2 by 4 depth with more longitudinal splitting before finally breaking through the reduced cross section, generally somewhere other than above the knot. Final failure (specimen failing to support maximum load) for these most common types occurred at a higher ramp load or longer time under constant load than that causing partial failure. Also, the grain was relatively straight with shallow slope.

Specimens in the second class of failures, slightly less frequent than the most common type, contained moderate slope of grain (1 in 9 to 1 in 14) but not necessarily in the immediate vicinity of the knot. Failure in these slope-of-grain specimens also started as rupture through or around the knot with splits (as for the prominent type of failure), but one split would extend through the cross section out near a support. Most slope-of-grain edge knot specimens failed instantaneously once failure at the knot occurred.

The remaining type of failure consisted of a simple rupture through the depth of the specimen at the edge knot cross section, with only minor longitudinal splitting. The frequency of this type of specimen failure was small relative to the above types.

There are two related aspects concerning specimen failure characteristics: (1) Crack propagation rates vary within and between specimens, the latter substantially. (2) Structural lumber, particularly when used as a repetitive structural member, can have useful life even though cracking has occurred. This would allow time for taking remedial action to save a structure.

Specimens surviving constant load

Specimens surviving constant load all failed at ramp loads higher than the previously applied constant load, except for one of the intermediate constant load specimens. That specimen, having a long, moderate slope-of-grain split, persisted in carrying the imposed constant load. After removing the constant load, it failed at a ramp load of about ¾ of the sustained constant load.

CONCLUSIONS

Except for the rapid ramp results, the cumulative distribution model provides a reasonable fit to constant load and ramp load data for lumber containing an edge knot. However, the model seems to overpredict residual strengths of specimens surviving constant load, and the underlying distributional variance may be different for ramp than for constant loading.

A greater duration-of-loading effect was observed for the 2 by 4 edge knot lumber of this study than that indicated by Wood (1951) for small clear-wood specimens; however, the difference does not appear to be statistically significant.

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