

# DAMAGE ACCUMULATION IN WOOD STRUCTURAL MEMBERS UNDER STOCHASTIC LIVE LOADS

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## ABSTRACT

Damage accumulation in wood structural members was assessed using realistic stochastic modeling of live load. The model indicates that practically all damage occurs when the live load intensity is equal or nearly equal to the nominal live load,  $L_n$ , a quantity required by codes for design. Currently, basic allowable stresses for wood are calculated assuming a period of 10 years spent at the nominal live load. The model indicates that the time the live load is at or above the nominal is not 10 years but about 40 days in a reference period of 50 years, strongly suggesting that the 10-year period generally assumed for setting allowable stresses is much too long.

*Keywords:* Stochastic live loads, load modeling, floor loads, load history, damage accumulation, duration of load, creep rupture, structural engineering, wood.

## INTRODUCTION

Probabilistic load models for wood structural members are described and applied using data from surveys of sustained live loads plus scenarios of extraordinary live load events. Analysis of damage accumulation under sustained load follows. Then, by a combination of the two processes, insights are gained into features of the structural load process that are significant for damage accumulation.

The strength of wood is dependent on the rate and duration of applied stress

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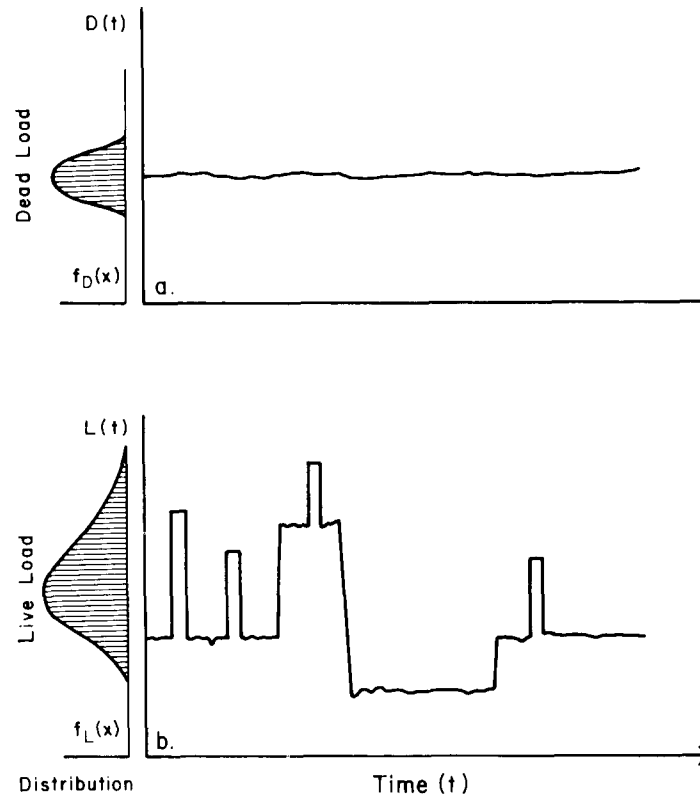
(Barrett and Foschi 1978; Gerhards and Link 1986; Madsen 1975; Wood 1951). As a result, the performance and reliability of wood structural members depend on their load-history. For over three decades, duration-of-load effects have been taken into account in structural design by adjusting the basic allowable stress,  $F_a$ , by factors that depend on the load combination considered. The normative load combination is dead plus live load. The value of  $F_a$  is based on the assumption of a 10-year duration of design load and is increased 15% for snow load, increased 33% for wind or earthquake loads, and decreased 10% for dead load alone. The basis for these adjustments is the "Madison" curve developed by Wood (1951), which was based on load duration tests of small, clear wood specimens. Corresponding to these adjustments in  $F_a$ , the implied load durations are 2 months for snow, 1 day for wind or earthquake, and 50 years or more for dead load. Why these durations were chosen is unknown. At the time, load modeling was too unsophisticated for construction of realistic load models or proper assessment of duration-of-load effects.

Structural loads, being random in time and space, are best modeled as stochastic (random) processes. During the past decade, considerable efforts have been made to develop such realistic load process models and to acquire supporting load data (Chalk and Corotis 1980; Corotis and Tsay 1983; Ellingwood and Culver 1977; Peir and Cornell 1973; Turkstra and Madsen 1980). Concurrently, recognition has grown among wood scientists that the Madison curve does not model appropriately the duration-of-load effect in members of structural size (e.g., Madsen 1975).

We are now working to develop procedures for assessing reliability of wood structures and ultimately to incorporate load duration in Load and Resistance Factor Design (LRFD) for wood (Hendrickson et al. 1987). These procedures take account of duration-of-load effects in wood members subjected to random loads. Because of the complexities of modeling load process and damage accumulation, this research has relied heavily on numerical analysis and simulation. However, we have developed approximate methods for evaluating damage accumulation under the effects of dead and live load without making unreasonable assumptions or simplifications. We use these approximate methods here to obtain insights into features of the structural load process that are significant for damage accumulation.

#### PROBABILISTIC LOAD MODELS

Because the strength of wood structural elements depends on load history, random variable characterizations of structural loads used for determining the reliability of steel and reinforced concrete structures (Ellingwood et al. 1982; Galambos et al. 1982) are not sufficient in themselves for use with wood structures. Instead, a stochastic characterization of the entire load process is required such as we are developing. Stochastic process models describe the variation of load in space and in time. Typical gravity load processes are illustrated in Figs. 1a and 1b for dead load and occupancy live load. These gravity loads are assumed to be uniformly distributed loads, statistically equivalent to the real loads. For the statistics of dead and live loads, which have been analyzed in detail, we use existing results (Chalk and Corotis 1980; Ellingwood and Culver 1977; Ellingwood et al. 1982; Galambos et al. 1982).



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FIG. 1. Typical sample functions for (a) dead load and (b) live load. (ML87 5300)

#### *Dead load*

We assume the dead load to be random in intensity but constant in time (Fig. 1a) and model dead load simply by a random variable with a normal probability distribution. The mean dead load,  $m_D$ , is approximately equal to the nominal value,  $D_n$  (American National Standards Institute (ANSI) 1982), and the dead load coefficient of variation is about 0.10 (Galambos et al. 1982).

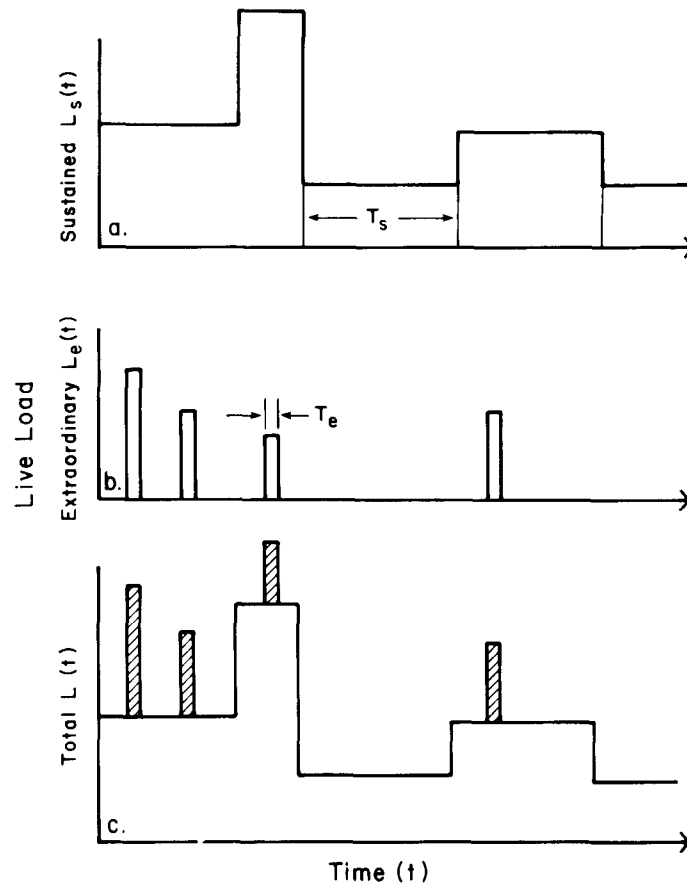
We found the variation in live load so much larger than the variation in dead load that, to a first approximation, the dead load can be assumed to be determinate and equal to  $D_n$ . This assumption enables the formulation of a stochastic load combination analysis simply in terms of the live load distribution.

#### *Live load*

The occupancy live load  $L(t)$  acting on a floor (Fig. 1b) is modeled as having two components,  $L_s(t)$  and  $L_c(t)$  as shown in Fig. 2a and 2b. Thus, the total load (Fig. 2c) at any time is

$$L(t) = L_s(t) + L_c(t) \quad (1)$$

The first component,  $L_s(t)$ , represents the sustained live load, which remains



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FIG. 2. Stochastic models of live load processes (a) sustained ( $L_s(t)$ ) (b) extraordinary ( $L_e(t)$ ), and (c) combined live load. (ML87 5301)

essentially constant during each tenancy period,  $T_s$ . This component of live load arises from the weight of people normally on the floor, their possessions, furniture, and movable equipment. We used data on  $L_s$  provided by recent surveys of live loads in buildings (Ellingwood and Culver 1977). The second live load component,  $L_e(t)$ , is termed an "extraordinary" or "transient" live load. It results from temporary crowding of floor areas because of parties, remodeling, or emergencies. Because it is rare and occurs for very short periods of time,  $L_e$  is seldom measured during a load survey. However, the extraordinary load is a significant contributor to  $L(t)$  and thus is important in evaluating the safety of floor structures (Chalk and Corotis 1980).

Turkstra and Madsen (1980) captured the essential features of the temporal variation of  $L_s$  and  $L_e$  by modeling the occurrences of the load pulses as Poisson point processes. For the sustained load component of a Poisson process (Fig. 2a), changes in the load events occur with a mean rate,  $\nu_s$ . Each load pulse has a duration,  $T_s$ , an exponential random variable with mean duration  $\tau_s = 1/\nu_s$ . The

TABLE 1. Typical live load statistics.

| Area of influence<br>$A_i$ | Sustained load        |                         | Extraordinary load    |                         |
|----------------------------|-----------------------|-------------------------|-----------------------|-------------------------|
|                            | Mean<br>$m_{L_s}/L_n$ | Variance<br>$\nu_{L_s}$ | Mean<br>$m_{L_e}/L_n$ | Variance<br>$\nu_{L_e}$ |
| $ft^2$                     |                       |                         |                       |                         |
| 400                        | 0.23                  | 0.68                    | 0.19                  | 0.67                    |
| 800                        | 0.30                  | 0.57                    | 0.19                  | 0.52                    |

$\nu_s = 0.125/\text{yr}$  (offices);  $\nu_e = 1-10/\text{yr}$ ,  $\tau_s = 8 \text{ yr}$ ,  $\tau_e = 0.01-0.02 \text{ yr}$ .  
 $L_n$  = specified nominal live load in ANSI Standard A58.1-1982 (ANSI 1982).

intensities of the load pulses are assumed to be statistically independent and identically distributed random variables, described by Peir and Cornell (1973) as a gamma probability density function

$$f(x) = \frac{\lambda(\lambda x)^{k-1} e^{-\lambda x}}{\Gamma(k)}; \quad 0 \leq x < \infty \quad (2)$$

The gamma distribution parameters  $\lambda$  and  $k$  are related to the mean,  $m$ , and standard deviation,  $\sigma$ , of the distribution by

$$\lambda = m/\sigma^2 \quad (3a)$$

$$k = (m/\sigma)^2 \quad (3b)$$

Statistics of the sustained load,  $L_s$  (the mean,  $m_{L_s}$ , coefficient of variation,  $\nu_{L_s}$ , and mean duration,  $\tau_s$ ) have been obtained by analysis of surveys of live loads in buildings (Chalk and Corotis 1980; Ellingwood and Culver 1977), and are summarized for influence areas of 400 and 800 square feet (37 and 74  $m^2$ ) in Table 1. Note that the mean value listed has been normalized by the specified nominal live load,  $L_n$ , computed according to ANSI Standard A58.1-1982 (ANSI 1982). In this form the statistics may be used for several common occupancies, including general offices and residences.  $L_n$  depends on the occupancy classification (ANSI 1982). Parameters  $\lambda_s$  and  $k_s$  of the gamma distribution describing  $L_s$  are obtained from Eqs. (3a) and (3b).

The occurrences of the extraordinary component of live load,  $L_e$ , are also modeled by a Poisson point process (Fig. 2b) with a mean rate of occurrence,  $\nu_e$ . The average duration of  $L_e$  is very short (2 weeks or less) in comparison with the mean time between extraordinary load events,  $1/\nu_e$  or  $\tau_e$ . Consequently, in earlier live load analyses (Chalk and Corotis 1980; Ellingwood et al. 1982; Galambos et al. 1982) when the focus of primary interest was the intensity of maximum total live load and not its duration, it was assumed that  $L_e$  could be modeled simply as an impulse.

In computing cumulative damage in wood, however, the random duration of  $L_e$  must be included. In the Poisson point process the duration,  $T_e$ , of the extraordinary load pulse is assumed to have an exponential distribution with mean value  $\tau_e$  (Turkstra and Madsen 1980). The probability is  $p = \nu_e \tau_e$  that the load process,  $L_e$ , is nonzero at any time,  $t$ . In contrast to the sustained live load statistics, which are obtained from load survey data, the statistics of  $L_e$  in Table 1 are based on analysis of extraordinary load event scenarios (Chalk and Corotis 1980; Ellingwood and Culver 1977). The distribution of  $L_e$  is modeled by a gamma distribution (Eq. (2)) with parameters  $\lambda_e$  and  $k_e$ .

TABLE 2. Combined live load levels for 50-year-lifetime design ( $A_1 = 800 \text{ ft}^2$ ).

| Load level, $\ell$ | Average total time above level, $\bar{t}$ | Average number of excursions, $(\bar{n})$ |
|--------------------|---|---|
| <i>psf</i>         | <i>yr</i>                                 |   |
| 5                  | 43.4                                      | 7.26                                      |
| 10                 | 26.9                                      | 20.8                                      |
| 15                 | 13.4                                      | 21.8                                      |
| 20                 | 5.90                                      | 15.8                                      |
| 25                 | 2.38                                      | 8.90                                      |
| 30                 | 0.908                                     | 4.39                                      |
| 35                 | 0.327                                     | 1.73                                      |
| 40                 | 0.113                                     | 0.616                                     |
| 45                 | 0.038                                     | 0.244                                     |
| 50                 | 0.013                                     | 0.121                                     |

The total time,  $t$ , that the combined load (sustained and extraordinary) process is at or above a given load,  $\ell$ , is the sum of the time,  $t_s$ , during a reference period of time,  $T$ , that the sustained load is above  $\ell$  plus the time,  $t_e$ , that the extraordinary loads cause the combined process to exceed  $\ell$ . The distribution of  $t$  may be approximated by a gamma distribution (Corotis and Tsay 1983), with an average value,  $\bar{t}$ , given by

$$\bar{t} = \bar{t}_s + \bar{t}_e \quad (4a)$$

where

$$\bar{t}_s = T[1 - F_{L_s}(\ell)] \quad (4b)$$

and

$$\bar{t}_e = \nu_e \tau_c T F_{L_s}(\ell) [1 - F_{L_c}^*] \quad (4c)$$

where  $F_{L_s}(\ell)$  is the cumulative distribution of  $L_s$  evaluated at  $\ell$ , and  $[1 - F_{L_c}^*]$  is the probability that the extraordinary load causes the combined load to exceed  $\ell$ . This probability is difficult to evaluate (Corotis and Tsay 1983); we computed it by simulation methods. Equation (4b) shows that  $\bar{t}_s$  is proportional to the probability that the sustained load is above  $\ell$ . Equation (4c) shows that  $\bar{t}_e$  is proportional to the probability that the extraordinary load is nonzero times the probability that the sustained load is below  $\ell$ , times  $[1 - F_{L_c}^*]$ .

The average number of excursions of the combined load process above load  $\ell$  in time period  $T$  is (Corotis and Tsay 1983)

$$\bar{n} = \nu_s T [1 - F_{L_s}(\ell)] F_{L_s}(\ell) + \nu_e T F_{L_s}(\ell) [1 - F_{L_c}^*] \quad (5)$$

The average time that the combined live load process spends above  $\ell$  and the average number of excursions above  $\ell$  in 50 years are tabulated in Table 2 for various load levels for  $A_1 = 800 \text{ ft}^2$ . The statistics in Table 2 are used in the analysis of damage accumulation.

#### CUMULATIVE DAMAGE ANALYSIS

Because the time to failure of a wood structural component depends on the level to which it is stressed, failure of the component appears to be governed by

a creep rupture phenomenon. Accordingly, we used a cumulative damage analysis to define the failure condition, similar to that used to analyze fatigue. In this approach, damage is measured by a state variable,  $\alpha(t)$ . In the undamaged (preload) condition,  $\alpha(t) = 0$  by definition. Failure is assumed to occur when  $\alpha(t) = 1$ .

Damage accumulates in a wood component subjected to the combined effects of random dead and live loads. The increment of damage occurring during time interval,  $\Delta t_i$ , subjected to stress,  $\sigma_i$ , is defined by

$$\Delta\alpha_i = \Delta t_i / T_f(\sigma_i) \quad (6)$$

in which  $T_f(\sigma_i)$  is the time to failure under a constant stress,  $\sigma_i$ . As the stress varies in time, damage accumulation is expressed by

$$\alpha(t) = \sum_i \Delta\alpha_i \quad (7)$$

This simple model assumes that damage accumulates linearly and is analogous to the Palmgren-Miner approach to analyzing cyclic fatigue (Miner 1945).

Relations describing time to failure,  $T_f$ , because they are necessary to evaluate damage increments, currently are the focus of intensive research (Barrett and Foschi 1978; Gerhards and Link 1986). One proposed relation has the general form

$$T_f(S) = \exp[A - (B)(S)] \quad (8)$$

in which  $S$  = stress ratio (applied stress/5-minute strength) and  $A$  and  $B$  are constants determined from duration-of-load tests of wood structural components. Typical values of  $A$  and  $B$  are 20–40 and 25–50, respectively, for  $T_f$  in days; thus, the predicted time to failure is extremely sensitive to small variations in applied stress.

#### WOOD DAMAGE UNDER LIVE LOAD

Having described the modeling of the random process of live loads and the damage accumulation of wood under constant load levels, we may combine the two processes to gain insight into the features of the structural load process that cause significant damage in the damage accumulation process. For illustrative purposes, we considered a floor system supporting an influence area of 800 ft<sup>2</sup> (74 m<sup>2</sup>) designed for a dead load,  $m_D$ , of 10 psf (0.48 kPa) and live load of 39 psf (1.9 kPa) (ANSI A58.1-1982). The supporting wood component of the floor was subjected to a random load process  $L(t) + m_D$ . The average time spent by the random load process above load level,  $\ell + m_D$  (referred to as a load exceedence curve), was determined using Eqs. (4a–4c). This average time is plotted as the dashed line in Fig. 3 up to the maximum, 60 psf, which is the unreduced live load (ANSI A58.1-1982)  $L_o = 50$  psf (2.4 kPa) plus the constant average 10 psf (0.48 kPa) dead load. According to the model we used (Table 2), the average time spent above this maximum load level in 50 years is 0.013 of a year or 4.75 days, and the average number of excursions above this level is 0.121 in 50 years or about once in 400 years.

For convenience, we divided the load exceedence curve into 11 discrete load levels shown in Fig. 3. The time,  $\Delta t_i$ , spent at each load level,  $i$ , is conservatively estimated as

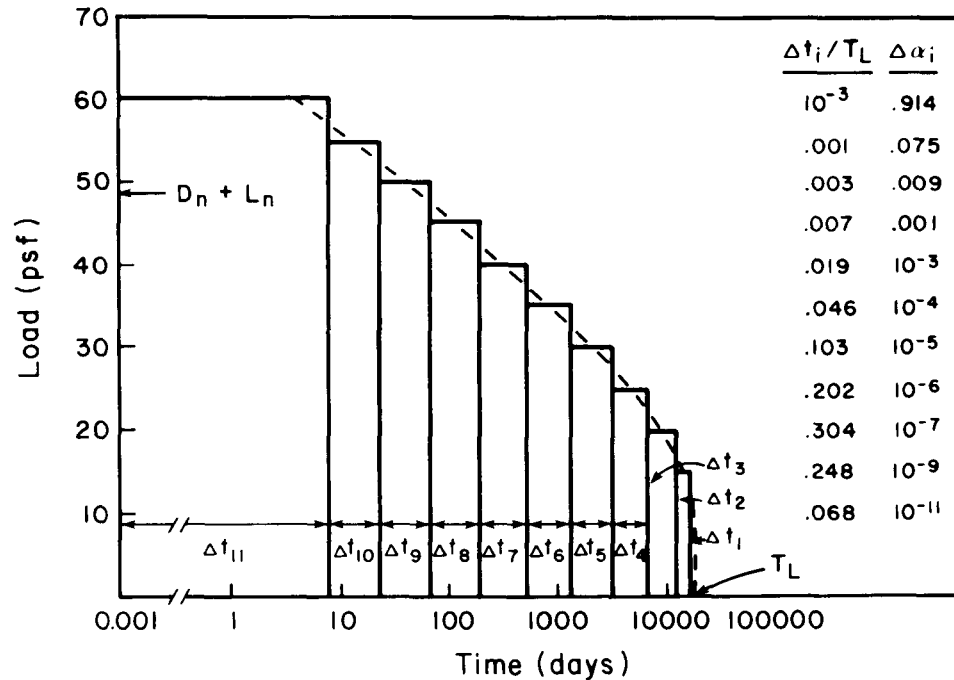
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FIG. 3. Time spent and damage incurred at each of 11 load levels. (ML87 5302)

$$\Delta t_i = T_i - T_{i+1} \quad (i = 1 \text{ to } 10) \quad (9a)$$

where (see Table 2)  $T_1 = T_L = 50$  years,  $T_2$  is the geometric mean of 50 and 43.4 years, . . . ,  $T_{11}$  is the geometric mean of 0.038 and 0.013 years, and  $\Delta t_{11} = T_{11}$  at the maximum load considered. Note that

$$\sum_{i=1}^{11} \Delta t_i = T_L \quad (9b)$$

Using Eq. (8) to describe load duration effects under constant stress, the average damage accumulated in  $T_L$  is approximately

$$\alpha(T_L) \approx \sum_{i=1}^{11} \frac{\Delta t_i}{e^{(A - B\sigma_i/R_0)}} \quad (10)$$

where  $\sigma_i$  is the stress associated with the average total time  $\Delta t_i$  and  $R_0$  is the 5-minute strength of the wood component, expressed in the same units as  $\sigma_i$ . The  $R_0$  can be determined that gives  $\alpha(T_L) = 1$  in Eq. (10). For illustrative purposes, we used the time-to-failure model (Eq. 8) of Gerhards and Link (1986) for select structural lumber from which  $A = 34.16$ ,  $B = 49.75$  when time is measured in years.

Figure 3 shows pairs of numbers for each load (stress) level. The first number,  $\Delta t_i/T_L$ , is the fraction of time in  $T_L = 50$  years spent at load level,  $\sigma_i$ , and the second,  $\Delta \alpha_i$ , is the average contribution to damage of the loads at that load level, such that  $\alpha(T_L) = 1$ . The figure shows:



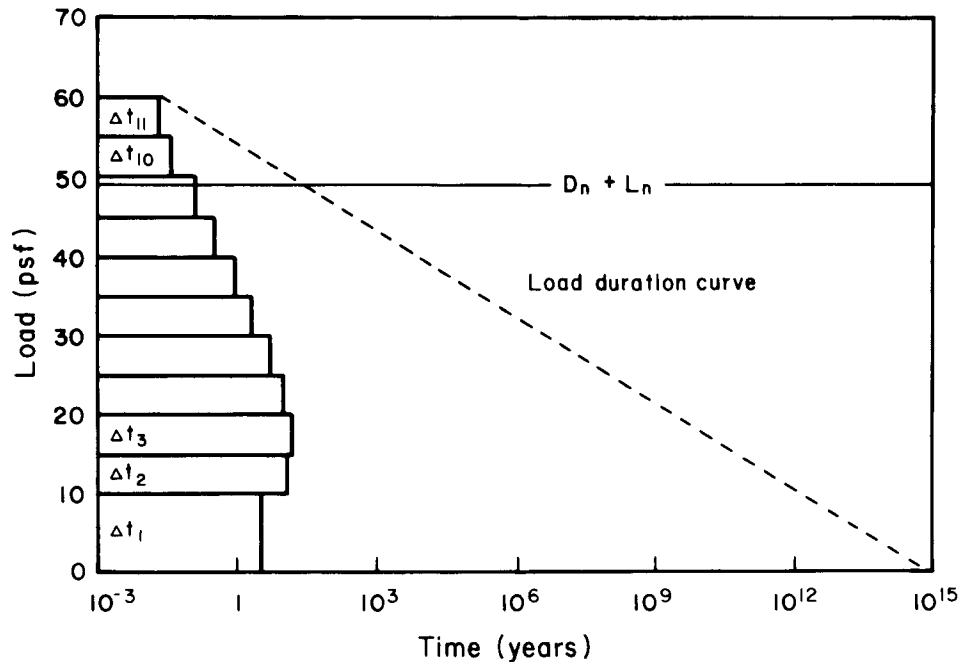
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FIG. 4. Relation between time increments of load levels and load duration curve. (ML87 5303)

1. Practically all the time is spent at the lower loads (stresses);
2. Most of the damage occurs at the near-maximum loads.

The  $R_o$  that gives  $\alpha(T_L) = 1$  is 78.95 psf. If all damage were ascribed only to the maximum load considered (i.e., 60 psf), then the  $R_o$  that gives  $\alpha(T_L) = 1$  would be 78.62 psf. Simulation studies (Hendrickson et al. 1987) also indicate that significant damage accumulates only during near-maximum load events.

Similar conclusions can also be drawn from Fig. 4 in which the 11 time increments are plotted, together with a (dashed) load duration curve. A change in the 5-minute strength  $R_o$  only changes the slope of the load duration curve, not the time intercept at zero load. As would be expected from Eq. (10), the damage at any load level is proportional to the ratio of the time at that level to that shown on the load duration curve. Virtually no damage accumulation appears during  $T_L$  at any but the largest loads (stresses).

Figures 4 and 5 also show the level of design load ( $D_n + L_n = 49$  psf or 2.3 kPa) that would be required by the A58 Standard (ANSI 1982) for an influence area,  $A_1 = 800$  ft<sup>2</sup>. It can be observed that the amount of time at or above the nominal design load is on the order of 40 days, which is a small fraction of the 10-year duration of live load that is used in current wood design specifications (Wood 1951) as a basis for determining the nominal allowable stress. Of course, the current procedure dates from a time when the stochastic nature of the live load was not well understood (Hendrickson et al. 1987). It seems likely that if the dead plus live load combination is to remain the basis of the allowable stress in working stress design, a significantly shorter duration of load should be used in

adjusting the basic wood strength data (ASTM D 245) for structural design purposes. Indeed, the short duration during which  $D_n + L_n$  actually occurs raises the distinct possibility that the basic allowable stress or strength could be determined directly from in-grade test results without significant adjustments for duration of load. However, when  $L_n$  becomes small relative to  $D_n$ , the duration of  $D_n + L_n$  is not short and will approach the duration of  $D_n$  alone. The allowable strength would then have to be based on this longer duration of maximum load.

#### SUMMARY

If failures of wood structures by damage accumulation and creep rupture are to be assessed realistically, sophisticated load modeling is required. We applied stochastic models of live load to the variation of live load in time, using statistical data from existing surveys of live loads in buildings.

Applying damage accumulation models to wood, we found that practically all damage occurs at times when the live load intensity,  $L(t)$ , is (nearly) equal to the nominal live load,  $L_n$ , and essentially none at live load intensities less than about  $0.8L_n$ . This finding is not surprising because of the highly nonlinear (exponential) nature of the relation between stress and time-to-failure shown in Fig. 4 (Eq. (8)). Such damage-causing live loads occur very infrequently. In 50 years, the time  $L(t)$  spent at or above  $0.8L_n$  is about 250 days, while that spent at or above  $L_n$  is about 40 days.

As a basis for setting allowable stresses for structural design, the assumed 10-year duration of design live load is excessive and a more realistic value should be used. Current adjustments in allowable stress for load combinations involving snow, wind, and earthquake effects also need reevaluation. For these purposes studies are in progress (Hendrickson et al. 1987).

#### ACKNOWLEDGMENT

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