MEASUREMENT OF THE SMALLER POISSON'S RATIOS AND RELATED COMPLIANCES FOR WOOD[1]

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ABSTRACT

Few direct measurements have been made of strains in the longitudinal L direction of wood when uniaxial loading takes place in the radial, R, or tangential, T, directions because of the difficulty of making the small measurements involved. Improved accuracy in measurements of this type were achieved by using low modulus strain gages having little or no sensitivity perpendicular to the gage axis, by having low noise signals in connecting wires, by making strain measurements with a resolution of 10^{-7} inches per inch, and by having a proper loading device. Compliances from loadings made on different specimens representing a number of species were found to be correlated in that the strain in the L direction per unit of stress in the R or T direction was proportional to the strain per unit stress in the load direction (R or T). The best estimates that could be found for Poisson's ratios ν_{TL} and ν_{RL} were 0.030 and 0.043, respectively. For some reason basswood data from a single board did not seem to fit with the rest of the data.

Keywords: Poisson's ratio, compliance, Young's modulus, stress, strain.

NOTATION

- a = slope of linear equation.
- b = y intercept of linear equation.
- i = subscript L, R, or T
- j = subscript L, R, or T
- L, R, T = longitudinal, radial, and tangential axes.
- E_i = Young's modulus in the i direction.
- S_{ij} = compliance: strain in the i direction per unit of stress in the j direction for loading in the j direction.
- ϵ_i = strain in the i direction.
- ν_{ji} = Poisson's ratio: ratio of strain in the i direction to that in the j direction for loading in the j direction; $i \neq j$.
- σ_i = stress in the i direction.

INTRODUCTION

This report is part of a continuing project to determine values for the less well-documented compliances and elastic constants for wood, which can be used in detailed mechanical analyses of structural components. In two previous papers (Sliker 1985, 1988) linear relationships were described between compliances measured at ninety degrees to each other along orthotropic axes from the loading of short columns in the L, R, and T directions. Two compliances not covered in the two references were for strains oriented in the L direction when column loading took place in either the R or T directions. Measurements of this type were left

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until later because of the great difficulty of measuring the small strains in the L direction when loading in the R and T directions. Special techniques for making the small measurements required, the resulting Poisson's ratios, and the relationships between the compliances resulting from the loading of a number of samples of different species in the R and T directions are the topics of this paper.

Only a few measurements have been made of strain in the L direction from the loading of wood in the R and T directions for computing Poisson's ratios and compliances. For uniaxial loading, the strain in the L direction per unit of stress in the R direction is equivalent to the compliance $-S_{LR}$. It is commonly seen in the literature as the result of dividing the Poisson's ratio ν_{RL} by Young's modulus E_R. Similarly, strain in the L direction per unit of stress in the T direction is equivalent to the compliance $-S_{LT}$ and the ratio of ν_{TL} to E_T. Hearmon (1948) lists measurements of ν_{RL}, ν_{TL}, E_R, and E_T made by a number of investigators prior to 1948. This listing included data from an extensive study made by Doyle et al. (1945–1946) at the U.S. Forest Products Laboratory. More recent data of this type were published by Bodig and Goodman (1973) and Goodman and Bodig (1970). They found little significant correlation between Poisson's ratios and specific gravity or Young's moduli. Their conclusion was that the best estimates of Poisson's ratios to use in engineering calculations were the averages of the values in their research and Hearmon's report. Values of ν_{TL} from the averaging of this data were 0.033 for softwoods and 0.027 for hardwoods, while average values of ν_{RL} were 0.041 for softwoods and 0.044 for hardwoods. Coefficients of variation for these numbers were between 41% and 70%.

In his experiments, Sliker (1985, 1988) experimentally determined Poisson's ratios other than ν_{TL} and ν_{RL} and also compliances for a number of different hardwood and softwood species. The average values for Poisson's ratios from his testing corresponded very closely to the values Bodig and Goodman found from averaging test results from various sources. Sliker found linear relationships between compliances from loadings performed in a given direction. The compliance equations are of the form:

$$\epsilon_j/\sigma_i = a\epsilon_i/\sigma_i + b \tag{1}$$

An ultimate goal of this project is to find reliable functions relating all the compliances and associated elastic constants to the quantity $1/E_L$, which equals S_{LL}. Combinations of the compliance equations from loadings made in the R, T, and L directions are the basis for achieving this goal. Successful completion of the modeling would provide engineering constants to be used in sophisticated computer programs for the accurate analyses of strains and stresses in wood structures and wood composites (Bodig and Jayne 1982).

EXPERIMENTAL PROCEDURE

Specimens for the testing were cut parallel to the grain from those used for a previous series of tests (Sliker 1988). These specimens were five layer laminates as shown in Fig. 1. Some are to be loaded in the R direction and others in the T direction. They were conditioned to between 9 and 12% moisture content. Species involved are listed in Tables 1 and 2. There was no specific attempt to match specimens for tangential loading with those for radial loading. However, radial

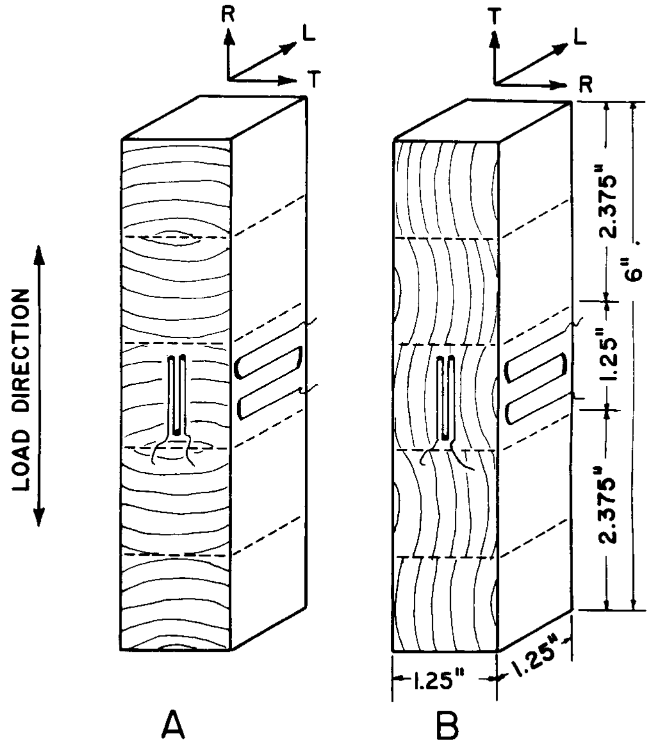

FIG. 1. Laminates for loading in the (A) radial and (B) tangential directions. Gages for measuring strain are shown oriented perpendicular to the specimen length (L direction) and parallel to the specimen length (R or T direction).

and tangential specimens for the following species were made either from the same board or the same log: the first sugar pine listed in each table, Douglas-fir, basswood, yellow-poplar, black cherry, and black locust. Each set of data in Tables 1 and 2 is the result of tests made on a single specimen. Details of specimen manufacture are described in a previous publication (Sliker 1988).

Of special concern when measuring strain in the L direction while loading in either the L or T directions is the possibility of picking up some of the large strains in the R and T directions with a gage oriented to measure the small strain in the L direction. This is a possibility with many strain gage designs as they have loops in their gage material that are perpendicular to their main strain axis. In order to overcome this problem, strain gages were made in which all the strain-sensitive wire was oriented in the L direction. This was accomplished by making strain gages with 12-mil constantan leads soldered to 1-inch lengths of 1-mil constantan strain gage wire having a resistance of 290 ohms per foot. Four of these gages were bonded parallel to each other at ¼-inch intervals perpendicular to the grain and with their long axes in the L direction on one side of the middle lamina of a specimen. These gages were connected in series and then were connected in series with a similar arrangement of 1-inch gages on the opposite side of the specimen. Nitrocellulose adhesive was used for bonding the gages to the substrate (Sliker 1967). See Fig. 2 for gage construction.

TABLE 1. *Slopes of stress and strain combinations from load and strain data for members loaded in compression in the T direction.*

Species	ϵ_T Versus σ_T (1/psi)	ϵ_L Versus σ_T (1/psi)	$E_T = \lvert\sigma_T/\epsilon_T\rvert$ (psi)	$\nu_{TL} = \lvert\epsilon_L/\epsilon_T\rvert$	EMC @ test %	Specific* gravity @ test
N. white cedar (*Thuja occidentalis* L.)	-24.10×10^{-6}	0.725×10^{-6}	41,500	0.0301	11.4	0.30
Sugar pine (*Pinus lambertiana* Dougl.)	−22.72	0.620	44,000	0.0274	11.0	0.31
Sugar pine (*Pinus lambertiana* Dougl.)	−15.10	0.404	66,200	0.0268	9.1	0.34
Douglas-fir (*Pseudotsuga menziesii* (mirb.) Franco)	−5.55	0.215	180,200	0.0388	10.4	0.58
Basswood (*Tilia americana* L.)	−25.00	0.300	40,100	0.0120	9.5	0.39
Yellow-poplar (*Liriodendron tulipifera* L.)	−10.59	0.278	94,400	0.0262	10.0	0.45
Black cherry (*Prunus serotina* Ehrh.)	−9.66	0.294	103,500	0.0304	9.7	0.53
Red oak (*Quercus* species)	−8.12	0.249	123,200	0.0307	10.2	0.62
Black locust (*Robinia pseudoacacia* L.).	−5.14	0.155	194,600	0.0301	11.7	0.64

* Based on oven-dry weight and volume at EMC (equilibrium moisture content) at time of test.

TABLE 2. *Slopes of stress and strain combinations from load and strain data for test members loaded in compression in the R direction.*

Species	ϵ_R Versus σ_R (1/psi)	ϵ_L Versus σ_R (1/psi)	$E_R = \lvert\sigma_R/\epsilon_R\rvert$ (psi)	$\nu_{RL} = \lvert\epsilon_L/\epsilon_R\rvert$	EMC @ test %	Specific* gravity @ test
N. white cedar (*Thuja occidentalis* L.)	-7.19×10^{-6}	0.497×10^{-6}	139,000	0.0691	11.2	0.28
Sugar pine (*Pinus lambertiana* Dougl.)	−8.62	0.389	116,000	0.0451	10.8	0.32
Sugar pine (*Pinus lambertiana* Dougl.)	−7.07	0.243	141,500	0.0344	9.5	0.33
White pine (*Pinus strobus* L.)	−8.24	0.285	121,300	0.0345	10.2	0.33
Douglas fir (*Pseudotsuga menziesii* (mirb.) Franco)	−4.11	0.109	243,100	0.0265	10.4	0.56
Basswood (*Tilia americana* L.)	−10.05	0.221	99,500	0.0220	10.8	0.40
Yellow-poplar (*Liriodendron tulipifera*)	−5.88	0.230	170,200	0.0391	9.6	0.45
Soft maple (*Acer* species)	−4.99	0.248	200,400	0.0497	10.9	0.51
Black cherry (*Prunus serotina* Ehrh.)	−4.48	0.197	223,300	0.0441	9.3	0.52
Red oak (*Quercus* species)	−4.30	0.182	232,600	0.0424	10.4	0.54
Black locust (*Robinia pseudoacacia* L.)	−3.18	0.130	314,400	0.0408	11.7	0.65

* Based on oven-dry weight and volume at EMC at time of test.

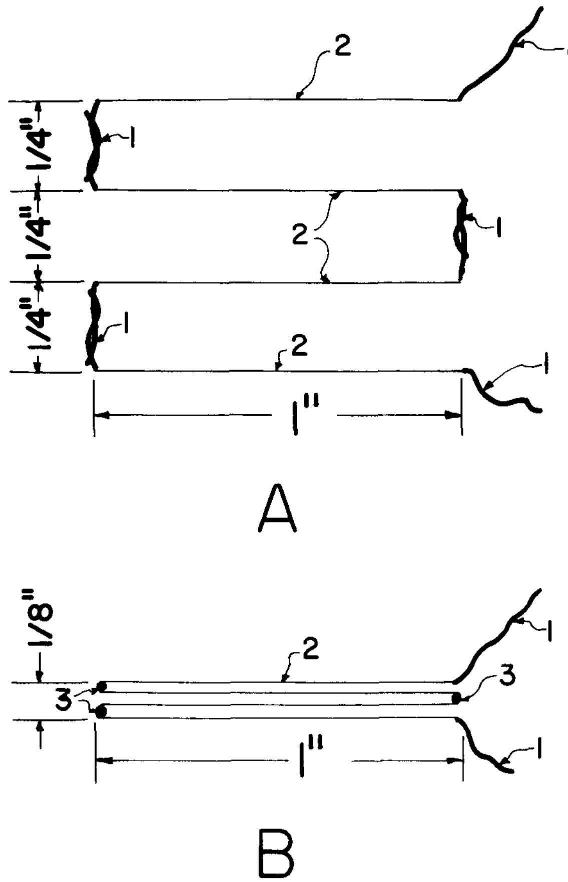

FIG. 2. Gage type A used to measure strain in the L direction; gage type B used to measure strain in the R and T directions. 12-mil diameter constantan lead wires are indicated by the number 1; 1-mil diameter constantan wires for measuring strain are indicated by the number 2; straight pins around which strain wire is looped are indicated by the number 3.

The gages used for measuring strain in the R and T directions were simpler to make: leads of 12-mil wire were soldered to 4-inch lengths of 1-mil constantan wire to form a gage of approximately 97 ohms. Gage length was kept at 1 inch by making three 360 degree bends in the 1-mil wire around steel straight pins when the gage was being bonded to the substrate. See Fig. 2. Although there might be a slight sensitivity to strain in the L direction in this design, the strain pickup in the L direction would be small compared to those in the R and T directions. Similar gages were placed on opposing faces of the middle section of the test specimen and were connected in series. Commercially manufactured strain gages with paper or plastic backing were not used because the backing material is stiffer than the substrate in the R and T directions and would reinforce it.

Another problem that had to be resolved was amplification of the low signal emanating from the gages in the L direction. Fortunately, a meter was available that could indicate strain to 10^{-7} inches per inch. It is Measurements Group's model 3800 Wide Range Strain Indicator. In order to keep the noise to signal

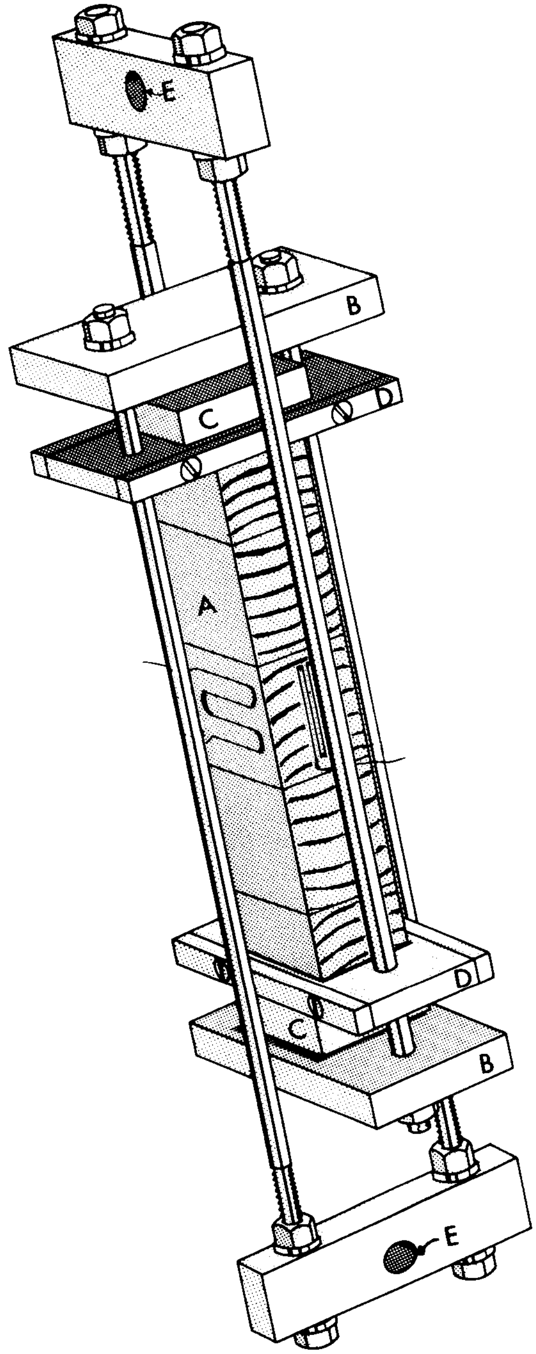

FIG. 3. Test specimen, A, in loading fixture. B is end block. C is end bearing block. D is centering guide. E is hole for metal dowel connection to universal joint. Ball bearing is centered between B and C at each end.

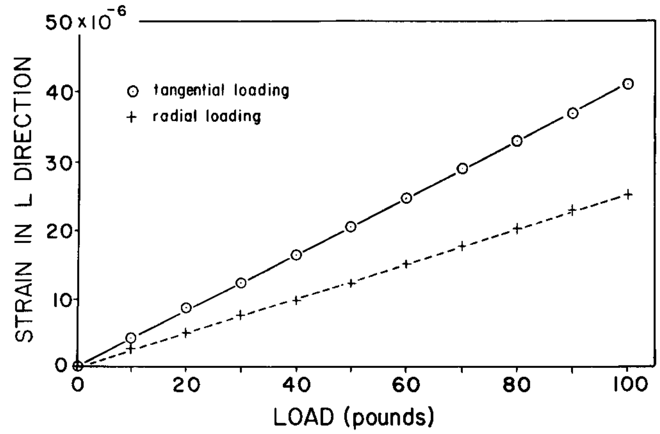

FIG. 4. Plots of strain in the L direction versus load applied in (1) the T direction and (2) the R direction for sugar pine specimens from the same board.

ratio low, shielded cable was used between the strain gage and the measuring instrument. A strain meter indicating strain to 10^{-6} inches per inch was used for gages oriented in the R and T directions.

Loads were applied to a specimen by hanging ten-pound weights from a compression cage containing it. Figure 3 is a drawing of a specimen in the compression cage, which is made of aluminum. Three-eighth-inch diameter ball bearings allow the blocks upon which the ends of a specimen bear to rotate (Bodig and Goodman 1969). Loosely fitting guides near the ends of the specimen keep it centered on the bearing blocks. The upper end of the compression cage is connected to a structural frame by a universal joint, while a load hanger is suspended from the lower end of the cage through another universal joint. Ten ten-pound weights were placed in rapid succession on the load hanger to produce a series of load and strain readings. A picture of a specimen under load is given in a publication by Sliker (1988). From these readings, slopes of strain versus stress or strain versus strain could be made for a given loading. Three replications of loading were made for each test specimen and then averaged. Time intervals between replications were at least 48 hours.

RESULTS

Plots of strain versus load and strain versus strain were linear for all specimens. The strains measured in the R and T directions were compressive or negative, while those measured in the L direction were tensive or positive. An example of a plot of strain in the L direction versus load in the T direction is given in Fig. 4 as is a plot from another specimen for the same board for strain in the L direction versus load in the R direction. Young's moduli, Poisson's ratios, and compliances resulting from the various tests are given in Tables 1 and 2. Each result is the average of three replications for one specimen. However, I would like to note that replications for a given specimen differed very little among themselves. Negative numbers indicate compressive strains.

The compliance $-S_{LT}$ $(=\epsilon_L/\sigma_T)$ could be plotted as a linear function of S_{TT} $(=+\epsilon_T/$

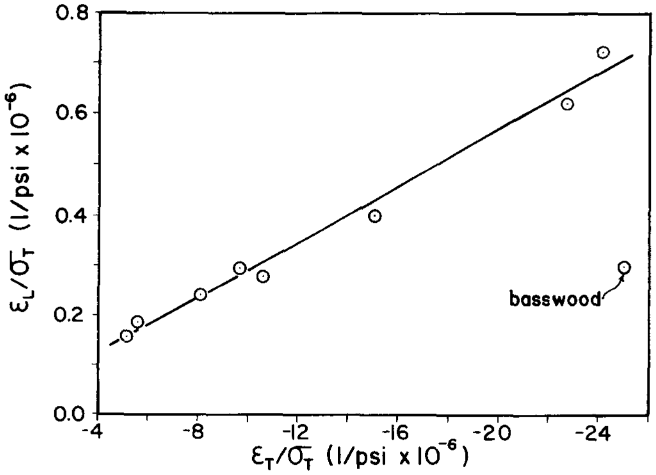

FIG. 5. Strain per unit stress perpendicular to the load axis as a function of strain per unit stress parallel to the load direction for specimens loaded in the T direction. Basswood was not included in the calculation of the regression line.

σ_T) for all the specimens loaded in the tangential direction if the data for basswood was omitted. This is shown in Fig. 5. The equation for the curve shown is:

$$\epsilon_L/\sigma_T = -0.0274 \quad \epsilon_T/\sigma_T + 0.0224 \times 10^{-6} \tag{2}$$

Units for ϵ_L/σ_T and ϵ_T/σ_T are strain (inches per inch) per psi. Since ϵ_T/σ_T is inserted as a negative number, indicating compressive strain per unit of stress, ϵ_L/σ_T will be positive. The correlation coefficient R for this equation is 0.99 and the standard error of estimate is 0.0333×10^{-6}.

A plot of the data for $-S_{LR}$ ($=\epsilon_L/\sigma_R$) versus $S_{RR} = (\epsilon_R/\sigma_R)$ shows greater variability around a regression line than exhibited by plots of other compliances (Sliker 1985, 1988). However, there is considerably less variability if basswood is again eliminated from the calculations. A linear regression line is shown in Fig. 6 based on all the data except for basswood. The equation for this line is:

$$\epsilon_L/\sigma_R = -0.0483 \quad \epsilon_R/\sigma_R - 0.0293 \times 10^{-6} \tag{3}$$

Again, ϵ_R/σ_R is to be inserted as a negative number to indicate compressive strain. The units for ϵ_L/σ_R and ϵ_R/σ_R are strain per psi. The correlation coefficient for this equation is 0.77 and the standard error of estimate is 0.0787. In two previous publications (Sliker 1985, 1988), equations with the tangential direction as a component had higher correlation coefficients than did those with the radial dimension as a component as is the case here. This may be related to the fact that strains in the tangential direction are averages of earlywood and latewood properties, while strains in the radial direction are summations of strains for earlywood and latewood acting independently of each other. Also, the radial orientation of rays in woods with different ray volumes may be a factor.

Why basswood data deviated to such a large extent from the regression lines for the other species is a mystery. Comparison of the averaged values for ϵ_R/σ_R and ϵ_T/σ_T from matched samples from this report and a previous report (Sliker

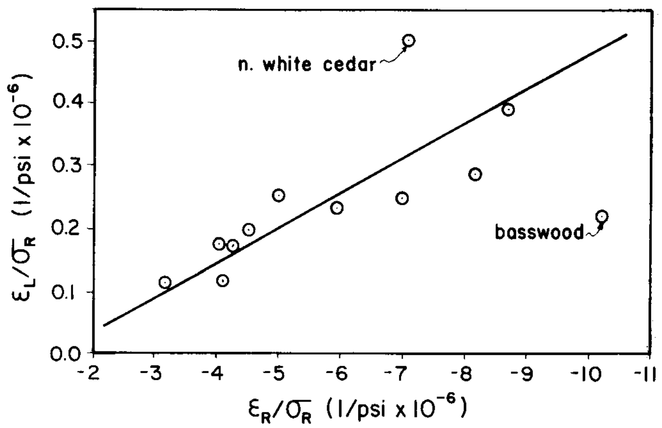

FIG. 6. Strain per unit stress perpendicular to the load axis as a function of strain per unit stress parallel to the load axis for specimens loaded in the R direction. Basswood was not included in the calculation of the regression.

1988) showed only slight differences in these numbers: 7% and 11% respectively. Looking at Figs. 5 and 6 suggests the values of ϵ_L/σ_T and ϵ_L/σ_R for basswood are about half of expected values. No anomalies with basswood appeared when strains were measured in the R and T directions for loadings made in the L direction (Sliker 1985). One clue that basswood may have some significant differences in properties from other woods is the extra large shrinkage it exhibits in the R and T directions: this shrinkage is much greater than that for any other United States species of lesser or equal specific gravity (U.S. Forest Products Lab 1987). Then, too, more pieces of basswood should be tested before reaching any conclusions. In addition to the basswood, the northern white cedar did not conform very well to the data for the other species for the loading in the R direction. No reason is offered for this.

Poisson's ratios in Tables 1 and 2 were obtained by plotting strain perpendicular to the load axis as a function of strain parallel to the load axis. They could also be determined by quotients of compliances:

$$\nu_{TL} = (\epsilon_L/\sigma_T)/(\epsilon_T/\sigma_T) \text{ and } \nu_{RL} = (\epsilon_L/\sigma_R)/(\epsilon_R/\sigma_R).$$

If the y intercepts in Eqs. 2 and 3 (0.0224×10^{-6} and 0.0293×10^{-6}) were statistically significant numbers, predictive equations for these Poisson's ratios could be obtained by dividing each term in Eq. 2 by ϵ_T/σ_T and each term in Eq. 3 by ϵ_R/σ_R. Unfortunately, the intercept terms in Eqs. 2 and 3 are not significant. At this point, the best estimate of these Poisson's ratios would be the averages of the test values. The average value and its standard deviation for ν_{TL} of all the test species except basswood is 0.030 ± 0.004, while the average value and its standard deviation for ν_{RL} for all the test species except basswood is 0.043 ± 0.011. These numbers compare very closely to Bodig and Goodman's calculated averages for ν_{TL} of 0.033 for softwoods and 0.027 for hardwoods and for ν_{RL} of 0.041 for softwoods and 0.044 for hardwoods.

Since the specimens used in this report were matched with specimens used in a previous report (Sliker 1988), the values for E_R and E_T can be compared.

Differences between paired values for E_R ranged from 0.7% to 15.5%, with the average being 5.0% Differences between paired values for E_T ranged from 0.9% to 12.6%, with the average being 5.9%. There are many possible sources for discrepancies when working with materials such as wood. Imperfect alignment of a gage with the assumed direction of measurement is one that creates large errors for small deviations. Another is the placement of the gages on different combinations of earlywood and latewood. The differences found between the two sets of values for E_R and E_T appear to be in the range that might be expected when possible error sources are considered (Perry 1984; Tuttle and Brinson 1984).

SUMMARY AND CONCLUSIONS

From stress and strain measurements made on a variety of hardwood and softwood species tested between 9 and 12% moisture content, the following conclusions were made:

1. The Poisson's ratios ν_{TL} and ν_{RL} and the compliances S_{LT} and S_{LR} can be measured for wood using low modulus strain gages with little or no sensitivity perpendicular to the gage axis, low noise signals in lead wires, strain instrumentation that can measure strain with a resolution of 10^{-7} inches per inch, and a proper loading system.
2. The compliance $-S_{LT}$ $(= \epsilon_L/\sigma_T)$ can be plotted as a linear function of S_{TT} $(= \epsilon_T/\sigma_T)$ for all test species with the exception of basswood. The correlation coefficient for the derived equation is 0.99.
3. The compliance $-S_{LR}$ $(= \epsilon_L/\sigma_R)$ can be plotted as a linear function of S_{RR} $(= \epsilon_R/\sigma_R)$ for all test species with the exception of basswood. The correlation coefficient for the derived equation is 0.77.
4. The larger than normal shrinkage of basswood in the R and T directions suggests that its stress-strain behavior may also be different in these directions than for most other species.
5. The best estimate that can be obtained for ν_{TL} for all the test species excluding basswood is 0.030 with a standard deviation of 0.004.
6. The best estimate that can be obtained for ν_{RL} for all the test species except basswood is 0.043 with a standard deviation of 0.011.

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