PREDICTION OF MODULUS OF RUPTURE FROM MODULUS OF ELASTICITY FOR SOME EGYPTIAN HARDWOODS[1]

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ABSTRACT

The MOE and MOR of 112 air-dry small, clear specimens (2 × 2 × 30 cm) of six species of hardwoods grown in Egypt were determined. Simple linear regression analysis revealed that MOR is highly correlated with MOE of *Eucalyptus camaldulensis, Khaya senegalensis, Tamarix articulata* and *Casuarina* spp. However, the "r" values for *Jacaranda ovalifolia* and *Melia azedarach* were not significant. In addition, covariance analysis showed that the six regression equations have different slopes and Y-intercepts and therefore cannot be grouped. The only grouping was that between the regression lines of the first three species mentioned above.

Introducing specific gravity in the regression equation did not result in improving the correlation coefficients, except in the case of *Casuarina* spp. Using specific gravity alone for predicting the MOR was found to be unreliable due to the relatively low "r" values obtained for the species except in the case of *Casuarina* spp.

Keywords: Eucalyptus camaldulensis, Khaya senegalensis, Tamarix articulata, Casuarina spp., *Jacaranda ovalifolia, Melia azedarach,* stress grading, bending strength, modulus of elasticity, specific gravity.

INTRODUCTION

During the past two decades, a considerable effort has been spent to develop accurate nondestructive tests for predicting the strength of wood. The majority of these efforts were directed towards a single concept: statistically correlating the modulus of rupture (MOR) with the modulus of elasticity (MOE). In spite of the inadequate theoretical basis for this correlation (Hearmon 1966), research laboratories in the United States, Australia, England, and Canada have developed this functional relationship into a useful, if not perfect, stress grading system.

Most of the research in this field has been conducted on softwoods (Miller 1963; Corder 1965; Johnson 1965; Hilbrand and Miller 1966; Hoyle 1968; Orosz 1968; Kennedy 1969; O'Halloran and Bodig 1972). A little information, however, is available on hardwoods (Badran et al. 1975; Senft and Della Lucia 1975; Walters and Reiss 1977). Badran et al. (1975) studied the relationship between MOR and MOE of *Casuarina stricta* Lab. They found that the best reduced regression model was one that included MOE only, with a correlation coefficient (r) of 0.63.

In a current research program at the Department of Timber Trees and Wood Technology, Alexandria University, an investigation of the strength and related properties of hardwood species grown in plantations in Egypt is underway. The objective of this study was to examine the correlation between the ultimate bending strength (MOR) and stiffness (MOE), for six hardwood species, namely, *Eu-*

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TABLE 1. *Average specific gravity (G), moisture content (MC), modulus of elasticity (MOE) and modulus of rupture (MOR) of small, clear specimens of six hardwood species.*

Species	Number of speci-mens	Specific gravity[a]			MC %	MOE, Kg/cm²			MOR, Kg/cm²		
		Average	Min.	Max.		Average	Min.	Max.	Average	Min.	Max.
Eucalyptus camaldulensis	48	0.658 ± 0.046[b]	0.575	0.748	14.6	63,900 ± 16,707	41,445	106,093	586 ± 177	354	1,069
Tamarix articulata	8	0.507 ± 0.040	0.453	0.563	13.7	43,065 ± 12,725	30,364	63,212	330 ± 131	125	542
Khaya senegalensis	19	0.551 ± 0.042	0.499	0.635	13.9	69,673 ± 6,113	56,391	81,006	641 ± 75	504	776
Casuarina spp.	18	0.561 ± 0.099	0.471	0.725	14.4	79,848 ± 29,754	30,713	143,591	717 ± 161	510	1,027
Jacaranda ovalifolia	10	0.454 ± 0.017	0.430	0.478	13.6	68,813 ± 9,076	57,042	86,756	570 ± 93	362	707
Melia azedarach	9	0.288 ± 0.013	0.273	0.309	13.6	44,286 ± 5,811	35,987	51,657	319 ± 33	274	381

[a] Based on volume at test and oven-dry weight.

[b] Standard deviation.

TABLE 2. *Linear regression equations for predicting modulus of rupture (MOR) from modulus of elasticity (MOE).*

Species	Regression equation	Number of measurements	SE_E[a]	Correlation coefficient
(A) *Eucalyptus camaldulensis*	MOR = −32.78 + 0.0097 MOE	48	76.97	0.905*
(B) *Tamarix articulata*	MOR = −76.07 + 0.0094 MOE	8	55.95	0.918*
(C) *Khaya senegalensis*	MOR = −118.35 + 0.0109 MOE	19	35.55	0.888*
Three species (A + B + C)	MOR = −69.39 + 0.0102 MOE	75	67.30	0.925*
(D) *Casuarina* spp.	MOR = 321.96 + 0.0049 MOE	18	66.84	0.915*
(E) *Jacaranda ovalifolia*	MOR = 403.25 + 0.0024 MOE	10	95.48	0.237
(F) *Melia azedarach*	MOR = 218.69 + 0.0023 MOE	9	31.85	0.403

[a] Standard error of estimate, Kg/cm^2.

* Significant at the 0.01 level of probability.

calyptus camaldulensis Dehn., *Tamarix articulata* Vahl., *Khaya senegalensis* Desr., *Casuarina* spp., *Jacaranda ovalifolia* R., and *Melia azedarach* Linn. and to examine the possibility of using this correlation for predicting the MOR.

MATERIALS AND METHODS

Wood specimens of *Eucalyptus camaldulensis* Dehn., *Casuarina* spp., *Jacaranda ovalifolia* R., *Tamarix articulata* Vahl., *Khaya senegalensis* Desr. and *Melia azedarach* Linn. were chosen from available stock in the Department of Timber Trees and Wood Technology. Eight air-dry pieces (nominal 8 cm × 4 cm × 80 cm) were randomly chosen for each species. Each piece was then machined down to a number of standard small clear test specimens (nominal 2 cm radially, 2 cm tangentially, and 30 cm longitudinally).

It was originally planned to use 50 test beams of each species, but because of the presence of natural defects such as large knots, tension wood, and grain irregularities, each species was represented by an unequal sample size (Table 1). However, the chosen species furnished satisfactory specific gravity and strength properties ranges for the purpose of this investigation (Badran and El-Osta 1967; El-Wakeel 1978).

Each beam was tested in bending according to British Standard Specification No. 373 (BSI 1957) using an Amsler 4-ton wood testing machine. Each small beam was loaded to failure flatwise at its midpoint over a 28-cm span. Then, MOR and MOE were calculated.

At the completion of each test, three sections were taken, one from each end and the third near the region of failure. The moisture content and specific gravity (based on oven-dry weight and volume at test) were determined for each section, the average of which was used in the statistical analyses. The volume at test was determined by mercury-immersion method using the Amsler Volume-Meter.

RESULTS AND DISCUSSION

Average values of MOE and MOR together with specific gravity (G) and moisture content are presented in Table 1. It can be seen from this table that the total number of specimens furnished a wide range of specific gravity (0.273 to 0.748) and hence a wide range in MOE and MOR. The simple linear regression analysis

TABLE 3. *Covariance analysis for testing the differences in slopes and intercepts of the regression equations for the six species.*

Group	Deviation from regression D.F.	S.S.	M.S.	F
Eucalyptus camaldulensis	46	272,521.9		
Tamarix articulata	6	18,785.7		
Khaya senegalensis	17	21,488.2		
Casuarina spp.	16	71,490.2		
Jacaranda ovalifolia	8	72,929.1		
Melia azedarach	7	7,101.2		
Total	100	464,316.3	4,643.2	
Difference for testing slopes	5	195,703.1	39,140.6	8.43*
Sums	105	660,019.4	6,285.9	
Differences for testing intercepts	5	189,380.8	37,876.2	6.03*
Combined regression	110	849,400.2		

* Significant at the 0.01 level of probability.

was conducted to develop a mathematical model that best represents the functional relationship between MOE and MOR for each species (Freese 1964). The regression equations are given in Table 2.

Examination of Table 2 indicates that in the case of *Eucalyptus camaldulensis*, *Tamarix articulata*, *Khaya senegalensis* and *Casuarina* spp. the "r" values are highly significant. This, in fact, reflects the reliability of the regression equations for predicting MOR from MOE. On the other hand, "r" values for *Jacaranda ovalifolia* and *Melia azedarach* are considerably lower (not significant at the 5% level). This indicates that the predictive equations for the above two species are unreliable. This result is not unexpected, since the species that gave high "r" values showed much wider ranges in specific gravity, MOE and MOR than did *Jacaranda ovalifolia* and *Melia azedarach* (Tables 1 and 2).

The above results raise the question of whether a separate prediction equation should be used for each species or whether these species should be represented by a single regression equation. Therefore, covariance analysis (Freese 1964) was carried out to test the heterogeneity of the six equations and the results of the analysis are presented in Table 3. Examination of Table 3 revealed that the differences for testing slopes and intercepts are significant at the 1% level. Consequently, the six linear regressions could not be pooled.

Covariance analysis was also carried out to determine whether the regression lines for the first four species in Table 2 could be combined; results are given in Table 4. It is clear from this table that the differences for testing slopes and intercepts are significant at the 1% level. Therefore, their regression lines cannot be pooled. The only group of species that can be pooled in one predictive equation is that of *Eucalyptus camaldulensis*, *Tamarix articulata*, and *Khaya senegalensis*, since their slopes and intercepts are not significantly different (Table 5).

In fact, the problem of grouping all species in one common relationship or

TABLE 4. *Covariance analysis for testing the differences in slopes and intercepts of the regression equations for* Eucalyptus camaldulensis *(A)*, Tamarix articulata *(B)*, Khaya senegalensis *(C) and* Casuarina *spp. (D)*.

Group	Deviation from regression			
	D.F.	S.S.	M.S.	F
(A) *Eucalyptus camaldulensis*	46	272,521.9		
(B) *Tamarix articulata*	6	18,785.7		
(C) *Khaya senegalensis*	17	21,488.2		
(D) *Casuarina* spp.	16	71,490.2		
Total	85	384,286.0	4,521.0	
Differences for testing slopes	3	171,759.8	57,253.3	12.66*
Sums	88	556,045.8	6,318.7	
Differences for testing intercepts	3	77,162.2	25,720.7	4.07*
Combined regression	91	633,208.0		

* Significant at the 0.01 level of probability.

separating them has been discussed previously (Hilbrand and Miller 1966; Hoyle 1968; Senft and Della Lucia 1975; Walters and Reiss 1977). Senft and Della Lucia (1975) in their investigation used three Brazilian hardwoods, namely banak (*Virola* spp.), tachi (*Tachigalia paniculatum*) and pequia (*Caryocar* spp.) and found that for any single species, the correlation between MOE and MOR was rather poor. However, combining all three species greatly improved the correlation. They attributed this improvement to the fact that the combined species represent a wide range of densities and strengths.

Some investigators prefer to use a separate equation for every species (Hilbrand and Miller 1966; Kennedy 1969; Walters and Reiss 1977). This seems to be undesirable because it will complicate structural design (Hoyle 1968). On the other

TABLE 5. *Covariance analysis for testing the differences in slopes and intercepts of the regression equations for* Eucalyptus camaldulensis *(A)*, Tamarix articulata *(B) and* Khaya senegalensis *(C)*.

Group	Deviation from regression			
	D.F.	S.S.	M.S.	F
(A) *Eucalyptus camaldulensis*	46	272,521.9		
(B) *Tamarix articulata*	6	18,785.7		
(C) *Khaya senegalensis*	17	21,488.2		
Total	69	312,795.8	4,533.3	
Differences for testing slopes	2	1,052.3	526.1	0.116 ns
Sums	71	313,848.1	4,420.4	
Differences for testing intercepts	2	16,802.5	8,401.3	1.90 ns
Combined regression	73	330,650.6		

ns, not significant.

hand, combining all species in one common relationship leads to some sacrifice of the efficiency of predicting bending strength of those species that gave higher correlations from their specific equations (Hilbrand and Miller 1966). It must be pointed out that combining *Eucalyptus camaldulensis, Tamarix articulata* and *Khaya senegalensis* in one regression equation did not result in any reduction in efficiency as is clear from Table 2.

The "r" value required for the purpose of the prediction in this respect has been previously discussed. Pellerin (1965) considered "r" values of 0.90 or higher to be essential for successful application. However, Hoyle (1968) provided a comprehensive summary of forty-four studies into the correlation between strength and stiffness in structural timber. He found that the "r" values were predominantly above 0.70 with the range being 0.57 to 0.88. In addition, using multiple regression analysis to predict MOR, Orosz (1968) improved the correlation coefficients but did not exceed 0.90.

In the light of the above information, the "r" values calculated in this study for each of the first four species in Table 2 and the combined equation for the first three species might be sufficient for prediction purposes and can be considered as a basis for further work using samples of larger dimensions and size.

In an attempt to improve the correlation coefficients given in Table 2, specific gravity (G) was introduced as a second independent variable in the regression equations. Multiple regression analysis was then carried out for each of the first four species in Table 1. The analysis revealed that the addtion of "G" did not significantly contribute to the variation in MOR, except in the case of *Casuarina* spp. whereby the following model was obtained;

$$\mathrm{MOR} = 60.51 + 0.0032\,\mathrm{MOE} + 716.59\,\mathrm{Ga}$$

$SE_E = 46.94$
$R = 0.9617$
$N = 18$

Where:

SE_E = Standard error of estimate, kg/cm^2,
R = Multiple correlation coefficient,
N = Number of measurements, and

A partial "F" value of 17.45 with 1,15 degrees of freedom was obtained which is significant at the 0.01 level of probability.

It is clear from the previous equation that the addition of "G" increased R^2 from 0.837 to 0.925. Accordingly, the addtion of G to the prediction equation of *Casuarina* spp. was justifiable.

Interest was next directed to the possibility of using "G" alone for predicting bending strength. Therefore, simple linear regression analysis technique was used to express MOR as a function of "G" and the fitted models are presented in Table 6. It is obvious from this table that the "r" values for *Tamarix articulata, Jacaranda ovalifolia* and *Melia azedarach* are not significant, whereas those of *Eucalyptus camaldulensis* and *Khaya senegalensis* are significant, but their values are not high enough to be used for prediction purposes. These results agree well with previous findings on softwoods (Miller 1963). The only good, reliable

TABLE 6. *Linear regression equations for predicting bending strength (MOR) from specific gravity (G).*

Species	Regression equations	Number of measurements	SE_E[a]	Correlation coefficient
Eucalyptus camaldulensis	MOR = −965.43 + 2,355.90 G	48	146.66	0.585*
Tamarix articulata	MOR = −762.15 + 2,155.01 G	8	106.06	0.659
Khaya senegalensis	MOR = −11.16 + 1,182.40 G	19	57.69	0.665*
Casuarina spp.	MOR = −83.88 + 1,428.05 G	18	79.95	0.876*
Jacaranda ovalifolia	MOR = −152.33 + 1,590.61 G	10	94.20	0.285
Melia azedarach	MOR = 214.50 + 360.17 G	9	34.42	0.145

[a] Standard error of estimate, Kg/cm^2.

* Significant at the 0.01 level of probability.

correlation between MOR and G is that of *Casuarina* spp. (r = 0.876). This might explain the significant contribution of "G" to the variation in MOR when it was included in addition to MOE in the multiple regression equation of *Casuarina* spp.

SUMMARY AND CONCLUSIONS

The MOE and MOR were determined for 112 small, clear specimens (2 × 2 × 30 cm) representing six hardwood species. From the results of this study the following conclusions are warranted:

1. Four of the species used, namely *Eucalyptus camaldulensis, Tamarix articulata, Khaya senegalensis,* and *Casuarina* spp., exhibited highly significant correlation coefficients btween MOE and MOR. On the other hand, *Jacaranda ovalifolia* and *Melia azedarach* gave a very poor correlation between MOE and MOR.
2. The differences for testing slopes and intercepts of the six regression lines were significant, and therefore they cannot be represented by one regression equation. The only possible grouping is that of *Eucalyptus camaldulensis, Tamarix articulata,* and *Khaya senegalensis.*
3. The addition of specific gravity as a second parameter in the regression equations did not improve the R^2 values, except in the case of *Casuarina* spp. whereby it became 0.925 instead of 0.837.
4. Specific gravity, as an independent variable, cannot be used for predicting MOR because of the low or insignificant correlation coefficients obtained, except in the case of *Casuarina* spp.
5. The high correlation obtained between bending strength and stiffness for the first four species indicated above is encouraging and can be considered as a basis for further work for obtaining higher "r" values.

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