SHEAR MODULI DETERMINATION USING TORSIONAL STIFFNESS MEASUREMENTS

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(Received September 1991)

ABSTRACT

The orthotropic shear moduli were determined for three different reconstituted wood materials. Shear moduli determination was accomplished using the respective formulae that define torsional stiffness for a linear elastic orthotropic rectangular parallelepiped. Applied test procedures required the experimental evaluation of torsional stiffness constants for rectangular specimens of decreasing width to thickness slenderness ratio. Anticlastic plate bending tests were also conducted to derive in-plane shear modulus values using standard ASTM D3044 procedures. In-plane shear modulus values derived from applied torsional theory were found to be in reasonable agreement with the standard ASTM test procedure.

Keywords: Shear moduli, torsional stiffness, plate bending tests, reconstituted wood material.

INTRODUCTION

Orthotropic materials are characterized by nine independent elastic constants. Of these, three are elasticity moduli that define material resistance to shear distortion for each mutually orthogonal plane. Research to investigate shear distortion elasticity for wood and wood-composites has previously emphasized prescriptions of plate bending tests (Biblis and Lee 1976; Bodig and Goodman 1973; Gunnerson et al. 1973; and McMatt 1973). Two methods of plate bending test are prevalent for in-plane shear distortion elasticity determination. The more commonly implemented method has been the two-point or "anticlastic" square plate test. A second method is identified as either the modified anticlastic or three-point square plate test procedure. Anticlastic refers to the bent surface shape acquired by the plate element under applied flexural load and given support conditions. The actual term of anticlastic relates to any bent surface having two principal curvatures with opposite directional sense.

The established anticlastic plate test procedure for elasticity modulus measurement is ASTM D3044, "Standard Method for Shear Modulus of Plywood" (ASTM 1982). Unfortunately, reconstituted wood materials are manufactured in the form of thin sheets or panels which limits, if not prohibits, direct plate specimen preparation for two elastic orthogonal planes. This limitation seriously restricts plate test procedures to determine values for

Wood and Fiber Science, 24(4), 1992, pp. 392–400

shear moduli, G_{xz} and G_{yz}, for the transverse material planar directions where the major panel surface is defined by the x-y plane. Though most engineering design problems for sheathing applications may require only the in-plane shear modulus (G_{xy}) value, knowledge of remaining two moduli is still of importance. Transverse shear values are essential for experimental investigation of material mechanical response under extremely concentrated static loading conditions. Knowledge of all the shear elasticity constants is critical, not only for research to model localized deformation of rigidly restrained sheathing sections under concentrated static loads but more importantly the specific investigation of impact contact response behavior. Recent experimentation to predict the impact performance of several different reconstituted panel materials did indeed require the quantification of the shear moduli associated with each symmetric plane of orthogonal elasticity (Janowiak 1988). Greater understanding of transverse shear distortion elasticity may also be required if wood-based composites are to be used in more sophisticated structural unit geometries beyond current panel sheathing applications.

Review of the Annual Book of ASTM standards, Section 4 Construction, Volume 04.09 Wood, provides several additional test procedures established for shear strength and distortion elasticity determination of wood-based composites. These procedures include ASTM D1037.128-"Interlaminar Shear" along with ASTM D2718 and D2719. ASTM D2718, "Standard Test Method for Structural Panels in Planar Shear," is essentially the same test configuration described under D1037, which is devoted to nonstructural panel materials. The other options for shear modulus measurement include the three methods detailed within ASTM D2719 or "Standard Methods for Structural Panels Through-the-Thickness." Option methods within D2719 are: 1. Small Panel Shear Test; 2. Large Panel Shear Test; and 3. Two-Rail Test. These additional ASTM standards, along with plate test procedure, do not provide for a reasonable methodology for the simultaneous determination of orthotropic shear elasticity within a single test procedure scheme. Also, some doubt may be expressed for the identified ASTM procedures to induce pure shear strain for elasticity measurement. ASTM D1037 does in fact indicate the potential for secondary strains with respect to the test configuration.

The purpose of this article is to inform other researchers of an alternative shear modulus characterization test procedure based on torsional stiffness constants. This torsional procedure does not suffer from the general limitation as mentioned for the commonly implemented ASTM D3044 anticlastic plate bending test methodology. Rather, the torsional methodology provides for the simultaneous solution of both in-plane and transverse shear distortion moduli. Torsional shear moduli determination has the added advantage of elasticity measurement within a pure shear strain field in contrast to conditional secondary strains of the ASTM D1037 test configuration. Thus, torsional tests may permit a more efficient method for obtaining the orthotropic shear moduli values needed for investigative experimentation.

THEORY

Torsional loading is not a new procedural approach for shear moduli determination at least for solid wood material. Literature review indicates the application of torsion techniques for shear modulus measurement of wood sections used for aircraft production (Trayer and March 1930). More recently, torsion test procedures are reported for other material studies. A torsion test procedure and experimental results are presented on DSP-B resin-impregnated plywood (Semenov 1966). Shear modulus values were also obtained for beryllium sheet material (Dai 1966). Both shear strength and modulus of unidirectional carbon fiber reinforced composites were derived through the use of a torsion test apparatus (Hancox 1972). A torsion shear modulus test analysis has also been employed in the study of moisture-induced property changes for graphite-epoxy composites (Sumsion and Rajapakse 1978).

Shear moduli determination originates from the loading analysis solution, which defines the specific constant for torsional stiffness. From the rigorous torsional loading analysis for an orthotropic strip (Love 1944), the following torsional stiffness formula is observed:

$$K_x = (32/\pi^4)bh^3G_{xy}S(N)$$
$$= (32/\pi^4)b^3hG_{xy}S(1/N) \qquad (1)$$

where

K_x = torsional stiffness constant for the material x-axis

$$S(N) = \sum_{n=1,2,3}^{\infty} 1/n^4\left[1 - \frac{\tanh(\pi/2)n/N}{(\pi/2)n/N}\right]$$

$N = (G_{xy}/G_{xz})^{1/2}(h/b)$

G_{xy} = shear modulus of the x-y plane

G_{xz} = shear modulus of the x-z plane

b = strip thickness

h = strip width

Equation 1 represents the specific torsional stiffness formula for an orthotropic strip or parallelepiped in torsion about the longitudinal or x-axis. Noting that a second analogous formula for K_y, torsional stiffness about the y-axis, is obtained by the appropriate permutation of subscripts. Formulae for K_x and K_y are the explicit relationships that equate torsional stiffness to orthotropic shear modulus, and cross-sectional dimension. Initial examination of either torsional stiffness formula does not suggest an immediate computation method for solution of a single shear modulus value due to the inherent involvement of both unknown moduli. However, a computational method is possible in solution of the individual shear modulus values under conditions of either large or small width to thickness slenderness ratios. As indicated by Lempriere et al. (1969), computational difficulties are eradicated under the following slenderness ratio conditions:

$$b/h < (\pi/4)(G_{xy}/G_{xz})^{1/2}$$

or

$$h/b > (4\pi)(G_{xz}/G_{xy})^{1/2} \qquad [2]$$

For these expressed conditions, Eq. 1 reduces to the following approximate form:

$$(3/bh^3)K_x = G_{xy} - 0.63025G_{xy}(G_{xy}/G_{xz})^{1/2}(h/b) \qquad [3]$$

or, equivalently

$$(3/b^3h)K_x = G_{xz} - 0.63025G_{xz}(G_{xz}/G_{xy})^{1/2}\cdot(b/h) \qquad [4]$$

Equations 3 and 4 are linear functions that relate torsional stiffness capacity to the respective h/b and b/h slenderness ratios. The left-hand terms, $(3/bh^3)K_x$ and $(3/b^3h)K_x$, for these simplified expressions may be referred to as the reduced torsional stiffness constants. These linear relationships for the reduced torsional stiffness constants provide the basis for the simultaneous solution of both in-plane and transverse shear moduli values. Linear functionality associated with the independent h/b or b/h variables enables a relatively simple data reduction procedure in the computation of material shear moduli values. It should be emphasized that shear moduli computation using either of these reduced stiffness torsional mathematical expressions is valid only within the original conditional slenderness restrictions with respect to shear modulus anisotropy set forth by Eq. 2.

The actual test procedure for shear moduli determination requires the experimental data collection of several K_x values of varying slenderness ratio. Torsional stiffness, K_x, equals the applied torque times the torsional gage length divided by the observed twist angle. Using for example, Eq. 3, collected experimental data of the form $(3/bh^3)K_x$ are plotted versus the different corresponding h/b slenderness ratios. As suggested by the linearity of the reduced torsional stiffness function, G_{xy} is derived as h/b approaches zero. The in-plane modulus is uniquely defined through the linear plot with straight line extrapolation to the intersection point on the reduced torsional stiffness ordinate axis. With the explicit value for G_{xy}, substitution into the equation provides for solution of the transverse modulus value. The mathematical statements for G_{xy} at h/b = 0 and subsequent solution for G_{xz} computation are given as follows:

$$G_{xy} = (3/bh^3)K_x \quad [5]$$

and

$$G_{xz} = (0.3972\ G_{xy}^3)/k_s^2 \quad [6]$$

where

K_x = TL/Θ
T = applied torque, in-lb
L = torsional gage length, inches
Θ = measured torsional twist angle, radians
k_s = slope of the $(3/bh^3)K_x$ line

Equations 5 and 6 are developed within the postulate of an assumed system of free torsional deformation. However, a distinct limitation must be acknowledged and accounted for within any experimental torsional loading apparatus that prohibits free torsional deformation. An applied torsion couple requires rigid clamping through which rotational forces are imposed on the test specimen. Rigid clamping or grips affixed to specimen end cross sections generates torsional deformation restraint. This deviation from free or unrestrained deformation inherent to the torsional loading grip results in a potentially significant experimental source of error for moduli test computation.

Experimental error due to restrained deformation or grip effect may be avoided by using torsional angle measurements taken a sufficient distance from the application points of the torsional moment. Review of the literature reveals at least one recommended procedure for determination of the grip effect zone to isolate the experimental error associated with restrained deformation. Nikolaev and Novichkov (1968) proposed a relatively simple procedural approach to establish minimum length (z_{min}) beyond which grip effect becomes negligible. This procedural approach requires that the twist angle measurements be taken over a section with appropriate distance from the torsional loading points. The appropriate distance (z) is defined using the following empirical statement:

$$z > z_{min} \simeq c(G_\alpha/G_\beta)^{0.5} \quad [7]$$

where

c = the greater of dimensions b and h
G_α, G_β = respectively, the greater and smaller of the shear moduli G_{xy}, G_{xz}

Application of Eq. 7 to define the proper twist angle measurement position requires an initial assumption or estimation of shear modulus magnitudes. This procedural approach does not provide for restraint correction when twist angle measurements are obtained by monitoring grip rotation.

Direct grip rotation for torsional deformation analysis necessitates an alternative correction procedure. Experimental error correction in assessment of grip effect through analytical techniques is impossible. Alternatively, grip effect and the general suitability of varying grip configuration assemblies must be evaluated through experimentation. Nederveen and Tilstra (1971) introduced a viable experimental correction procedure based on the concept of the theoretical twist gage length (L) for free torsion being adjustable to real torsion through a virtual increase of elementary length. Elementary length (ΔL) serves as the correction term to compensate for grip effect. The correction method consists of a prismatic bar repeatedly loaded to constant torque magnitude for a decreasing gage length (L_i) with resultant twist angle ($\Delta\Phi_i$) measurement. Resulting ($\Delta\Phi_i$,L_i) × data extrapolation explicitly defines the real torsion correction term ΔL at Φ equals zero. Correction terms may assume either a positive or negative sense dependent on specimen torsional capacity and grip configuration enhances torsional stiffness. Conversely, a negative value is observed for reduced stiffness behavior. Reduction of torsional stiffness due to grip attachment is termed warping restraint behavior. Enhanced stiffness

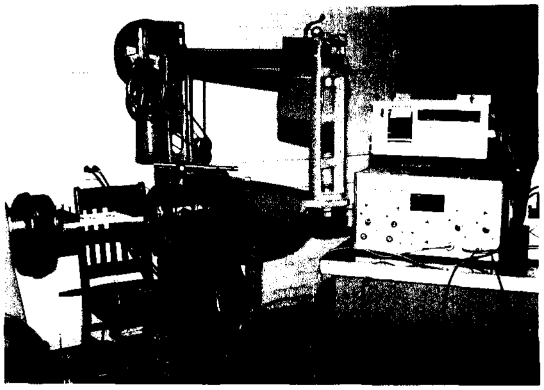

FIG. 1. Photograph of torsional stiffness apparatus.

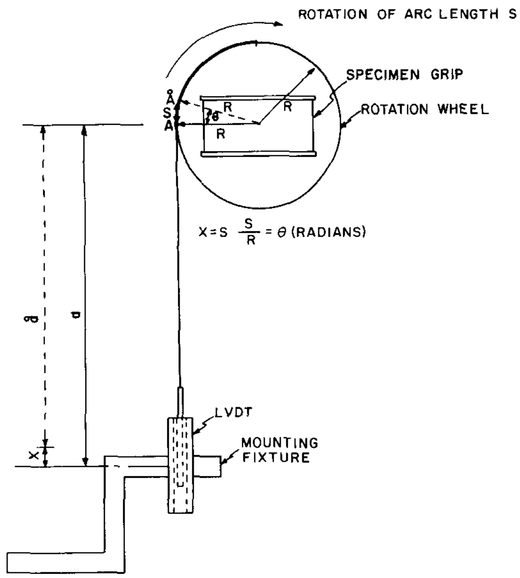

FIG. 2. Schematic illustration of the torsional angle measuring system.

inherent in the grip configuration is termed clamping restraint. Greater discussion on warping and clamping restraint phenomena is well-documented both in theory and investigation (Nederveen and Tilstra 1971; Tilstra 1962; and Timoshenko 1953, 1955).

EXPERIMENTAL METHODS

Torsional test apparatus

Experimental shear moduli characterization was conducted using a devised torsional stiffness test apparatus. The devised apparatus included five system components: a Tinius Olsen torsion machine, adjustable specimen grips, two displacement LVDT transducers, torque load cell, and support data acquisition instrumentation. Figure 1 shows the apparatus used for experimental evaluation of torsional stiffness constants. Adjustments within the grip assembly construction provided flexibility for variable specimen thicknesses while at the same time minimizing unintentional bending moment caused by improper axial elevation alignment. Grip mechanical connection to the torsion machine was provided through assembly shafts chucked to the machine carriage loading heads. Angular deformations were monitored using two Schaevitz DC-operated LVDTs (Linear Variable Displacement Transducers). As identifiable in Fig. 1, direct grip rotation was utilized to measure twist angle displacement between the fixed-reaction and torque-bearing grip assembly. Figure 2 provides further insight with a schematic depiction of the LVDT torsional angle measuring system coupled with the grip assembly. The second LVDT attachment to the reaction grip assembly was added to account for possible grip to loading head connection slippage and shaft deformation. Thus, the actual angular twist deformation was computed on the basis of absolute voltage output difference between the individual displacement transducers.

The torque load cell consisted of a wheatstone bridge installation on a reduced shaft section of the fixed grip assembly. Bridge construction utilized two Micro-Measurements CEA-06-250UR-350 three-element, 45-degree single-plane rosettes. The basic wheatstone bridge circuit consisted of the four active strain gage element arms paired with equal and opposite shear strain. The strain gage bridge installation provided maximum torque sensitivity while being insensitive to temperature, axial load, and bending effect. Bridge excitation for the torque load sensor was supplied by a TML model TDS-301 strain gage conditioner and multichannel data logger unit. Excitation voltage for LVDT operation was provided using a Metriguard model 821 signal conditioning unit. Maximum amplifier gain generated a calibrated output voltage of 99 mV (millivolts) per 0.001-inch displacement. Maximum LVDT output voltage in combination with the 3.49-inch radius grip assembly rotation wheel provided torsional deformation measurement resolution on the order of 2.87 $\times 10^{-6}$ radians (1.64×10^{-4} degrees). The calibrated load cell with TML signal conditioner was sensitive to one tenth in-lb in measurement of applied torque. Test measurement analog signals for both displacement transducers and the torque sensor were simultaneously recorded in the data logger buffer memory with subsequent printout to paper hardcopy. Data reduction included simple conversion of the recorded voltage to physical quantities of the torsional twist angle with respect to applied torque.

Torsional stiffness testing

Torsional stiffness specimens for determination of orthotropic shear moduli values were processed from each of three different reconstituted materials. The reconstituted wood materials for experimentation included ¾-inch underlayment grade particleboard (Composition Board Type A), ¾-inch structural waferboard (Composition Board Type B), and a ⅝-inch oriented strandboard (Composition Board Type C). Specimens dimensioned 3½ by 12 inches were processed from both the longitudinal (x-axis) and transverse (y-axis) in-plane panel directions. One specimen was obtained from each of fifteen different full-sized panel sheets with a total of fifteen test replications for both in-plane orientations. Supplemental specimens, three from each composition board type, were also processed for use in deriving the appropriate correction term associated with the experimental apparatus grip assembly configuration.

Individual test specimens were subjected to five independent torque loading cycles with determination of relative torsional stiffness. The first torsional stiffness term was determined for the original slenderness ratio with four other subsequent torsional stiffness terms derived for successively decreased slenderness ratio as the specimen width was reduced by approximately 0.50-inch increments. Adjustment of the specimen slenderness ratio was accomplished by removing ¼ inch of material from opposite specimen sides. Torque versus twist angle measurements were collected using the aforementioned data logger according to a programmed 3-second time interval scanning sequence. Measurements from individual loading cycles provided the required torque-twist angle data ($\Delta T/\Delta \Theta$) for computation of the reduced torsional stiffness constants respective to the decreasing slenderness ratio.

For experimentation, angular deformation within the different torsional loading cycles was maintained below a maximum 3-degree twist angle over a 7-inch torsion gage length. The applied torsional loading rate was 0.035 degrees/second. Test trials were also conducted, under the same experimental parametric conditions, to ascertain the appropriate correction term for adjustment to real torsion. Test trials for determination of ΔL to correct for grip effect included nine supplementary 3½- by 12-inch specimens. Specimens at their original and subsequently reduced gage length were subjected to a 400 in-lb torque with measurement of observed twist angular displacement. Angle of twist measurements were made for this constant torque level using free specimen gage lengths of 7, 6.5, 6.0, 5.5, 4.5, and 3.5 inches. All torsional specimens were equilibrated under the relatively constant ambient humidity conditions within the laboratory enclosure. Moisture contents of the specimens were found to average 5.6%. Torsional specimens were not conditioned to a standardized moisture content in an effort to avoid changing moisture content due to prolonged ambient exposure during the test sequence duration.

Anticlastic plate bending tests

Further experimentation for comparative purposes was conducted to characterize the in-plane shear modulus through the more standard testing practice based on anticlastic plate

TABLE 1. *Orthotropic shear moduli values derived from torsional loading.*

Property	Composition board type	Mean (psi)	Maximum value (psi)	Minimum vlaue (psi)	Standard deviation (psi)
G_{12}[1]	A	142,000	173,400	116,400	12,500
(G_{xy})	B	154,400	219,100	123,500	21,900
	C	217,700	258,600	188,500	19,300
G_{13}[2]	A	28,500	35,200	23,700	2,700
(G_{xz})	B	21,400	28,500	15,900	4,000
	C	30,800	37,500	25,400	3,500
G_{23}[2]	A	23,300	29,200	19,700	2,900
(G_{yz})	B	20,300	29,400	10,900	6,300
	C	34,200	41,600	29,500	3,500

[1] Based on 30 test specimens, with the exception of composition board type B which is based on 29 specimens obtained from 15 different panels.
[2] Based on 15 test observations, with the exception of composition board type B for which G_{yx} is based on 14 test specimens obtained from 15 different panels.

bending. As specified by ASTM D3044, inplane modulus of rigidity determination was pursued through plate deformation in formation of a hyperbolic paraboloid surface. Shear modulus was computed using the plate solution expression for an applied twisting moment. Plate specimens were not obtained from the same five full-sized panel sets used for preparation of the torsional stiffness specimens. However, both anticlastic and torsional panel sets for specimen preparation were randomly selected from a larger sample group population. Three specimens were processed from each of five full-sized panels with fifteen total test replications for each composition board type. Specimen materials were conditioned prior to test evaluation as prescribed under the ASTM standard. Average moisture content was found to equal 11.9%. Some clarification should be made to emphasize that the difference in moisture content between the two shear moduli determination test procedures was an unintentional experimental parameter variation. Irrespective of the moisture content variation, the assumption is made that general comparisons of in-plane shear modulus values derived from anticlastic versus torsional test methodology are valid.

RESULTS AND DISCUSSION

Torsional experimentation values for inplane (G_{xy}) and transverse (G_{xz} and G_{yz}) moduli have been summarized in Table 1. The results shown are based on linear regression analysis of the reduced torsional constants versus the corresponding slenderness ratio. Actual G_{xy} and G_{xz} computation follows from the use of Eqs. 5 and 6, respectively. Computation of shear moduli from torsional analysis data about the y-axis test orientation follows from the analogous expressions for K_y. Average in-plane shear modulus values with respect to x- or y-axis orientation torsional loading were found to be statistically equivalent. Thus, descriptive summary statistics in Table 1 for G_{xy} are a cumulative presentation from both test orientations. Figure 3 illustrates a typical $(3/bh^3)K_x$ versus h/b regression plot. Figure 3 indicates that the experimentally derived gage length correction term was incorporated for torsional stiffness calculation.

Experimental results with the nine supplementary test trial specimens yielded an average -0.75-inch correction term for grip effect compensation. Figure 4 illustrates a regression plot of $\Delta\Phi$ as a function of free specimen gage length. Elementary length for adjustment to free torsion conditions was not a constant value but ranged between -0.5 to -1.0 for the nine different test trials. During experimentation, significant sensitivity was observed for grip pressure on derived ΔL magnitude. This observation suggests correction terms for torsional stiffness measurements are optimized under conditions of similar grip clamping pressure.

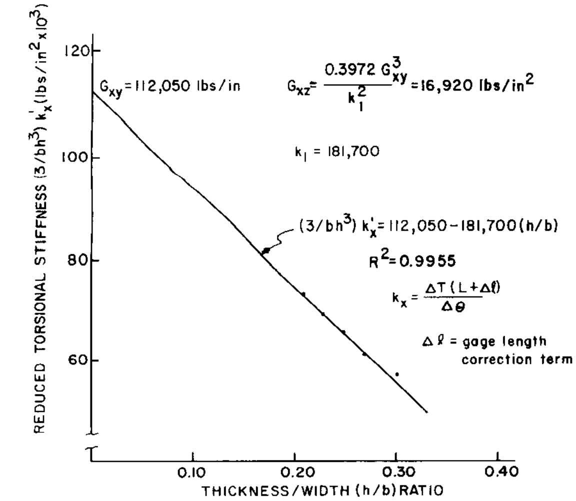

FIG. 3. Plot of reduced torsional stiffness versus slenderness ratio for an underlayment test specimen.

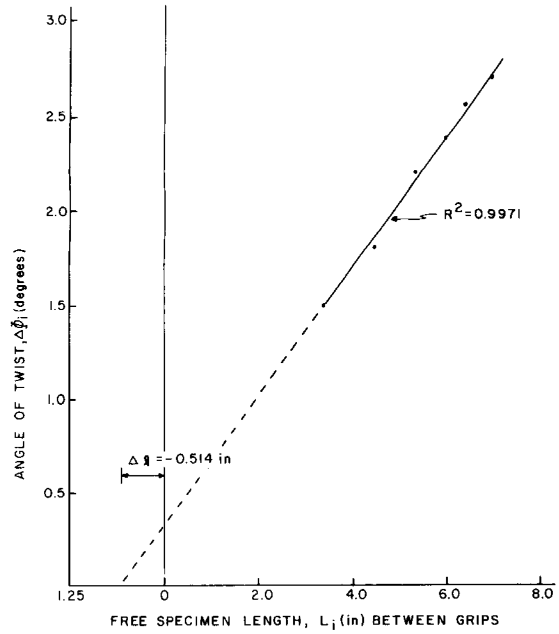

FIG. 4. Twist angle $\Delta\theta_i$ as a function of specimen gage length L_i.

Anticlastic plate bending test results are presented in Table 2. Examination showed that average in-plane shear modulus values obtained using standard D3044 procedures compare favorably in magnitude to those for torsional stiffness procedures. Torsional G_{xy} for composition board types A and C, respectively, averaged 11.1 and 10.9% higher than the values obtained using D3044. In contrast, composition board type B is 10.7% lower than corresponding anticlastic G_{xy} value. Higher shear moduli values associated with torsional analysis are reasonable within the context of lower material moisture content. The observed trend demarcation for composition board type B may be rationalized through assumption of inadequate sampling size. Further statistical contrast to analysis test methodology equivalence was not deemed appropriate. Statistically significant differences would be biased due to the disparity of specimen moisture content conditions.

SUMMARY

Assuming that plate bending test results accurately characterize in-plane distortion, elasticity suggests that effectiveness of the torsional stiffness methodology. However, transverse shear moduli determination is sensitive to experimental error. High accuracy requirements are essential in the establishment of the torsional stiffness versus slenderness ratio relationship. High accuracy requirements are self-evident with review of Eq. 6. Error in shear modulus G_{xz} increases as the square (or cube) of error in the determination of k_S (slope) or

TABLE 2. *In-plane shear modulus values calculated from anticlastic plate ending.*[1]

Property (psi)	Composition board type	Mean[2]	Maximum value	Minimum value	Standard deviation
G_{12}	A	128,300	141,700	102,900	11,800
(G_{xy})	B	172,800	229,400	122,300	36,600
	C	196,400	232,700	178,900	12,500

[1] Average moisture content of 11.9%.
[2] Based on 15 specimens obtained from 5 randomly selected panels, tested according to ASTM D 3044 (1982).

G_{xy} (intercept). Thus, a high order of test accuracy is critical for reliable shear modulus values. In addition, shear moduli calculations using Eqs. 5 and 6 must be valid within originally stated simplifying assumptions. To reiterate, restrictions on these equations are the mathematical conditions that the slenderness (thickness/width) ratio be small or large relative to anisotropy: $(b/h) < (\pi/4)(G_{xz}/G_{xy})^{1/2}$ or $(h/b) < (\pi/4)(G_{xy}/G_{xz})^{1/2}$. Experimentally derived shear values for each composition board material were found valid within these restrictions. Several supplemental recommendations are made as guidelines for computation of reliable shear moduli values:

1. Torsional stiffness constants should be evaluated for four or more slenderness ratios.
2. Correction for restrained deformation or grip effect needs to be considered.
3. Constant grip pressure should be maintained for all torsional testing.
4. Torsional stiffness data evaluation should be conducted through least squares regression fit.
5. Regression analyses of R^2 below approximately 0.9 may suggest excessive experimental testing variation.
6. Simplifying mathematical restrictions must remain valid for the shear moduli equations.

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