THE FRACTAL NATURE OF WOOD REVEALED BY WATER ABSORPTION

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ABSTRACT

We use a simple experimental procedure of pressurized water absorption to show the fractal nature of the pore space in wood. Cubic blocks of wood were immersed in water at a given pressure for 14 days, and mass changes were measured. The plots of the mass changes versus block edge sizes are straight lines whose slopes correspond to the fractal dimensions of the void volume in wood. Results for different species and for distinct water pressures are shown, suggesting the fractal dimension as a new relevant parameter to characterize the porosity of wood.

Keywords: Water absorption, fractals, porosity, void volume.

INTRODUCTION

Nature provides us with a large variety of shapes and forms, from the simple ones to the more complex ones. In recent years, it has been recognized that many natural structures possess a rather special kind of geometric complexity: the statistical self-similarity upon changes in resolution power. The degree of geometric irregularity of such objects can be measured by the fractal dimension d_f (Mandelbrot 1977; Stanley and Ostrowsky; 1985; Vicsek 1992). The concept of fractal, usually an object with a noninteger dimension, has been applied to a vast range of different structures such as coastlines (Mandelbrot 1977), molecular surfaces (Avnir et al. 1984), aerogels (Fricke 1989), bacterial colonies (Vicsek et al. 1990), tumor cells (Vilela et al. 1995), axon terminals (Alves et al. 1996), and, in particular, porous materials (rocks) (Hansen and Skjeltorp 1988; Ruffet et al. 1991; Tsallis et al. 1992).

A typical property of fractals (central to this work) is related to their volume with respect to their linear size L:

$$V(L) \propto L^{d_f} \qquad (1)$$

where d_f, its fractal dimension, is in general a noninteger number between 0 and d, the Euclidean dimension of the space in which the fractal is embedded. This relation is familiar to us when dealing with the usual smooth (Euclidean) objects such as lines, discs, or spheres. For a line with length L, the volume and also the characteristic linear size are its own length L, and so for this simple example $V(L) = L^1$, and thus a line is an object with dimension 1. For a disc, the characteristic linear size is its radius R, and the volume is its area given by $V(L) = \pi L^2$; the disc is an object with dimension 2. Analogously, the characteristic length of a sphere is its radius R, and its volume is given by $V(L) = 4\pi L^3/3$; thus the sphere is a three-dimensional object. Note

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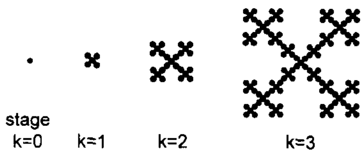

FIG. 1. The first four stages of construction of a simple deterministic mathematical fractal. Stage 0 (zero) is a disc of unit area. Each new stage is composed by the replication (five times) and aggregation of the figure of the anterior stage. The fractal dimension of this object is d_f = ln 5/ln 3.

that, in general, the dimension of a smooth Euclidean (as opposed to fractal) object is the smallest dimension d of the space the object can be embedded in. Note also that in the volume equation, all the information about the dimension of the object relies on the exponent of its characteristic linear size L, while the proportionality constant (π for a disc, 1 for a square or $4\pi/3$ for a sphere) is a geometric form factor characteristic of each object.

Fractal objects are in general 'thinner' than smooth objects, and their dimensions are smaller than the dimension of the space they are embedded in. To illustrate the emergence of noninteger dimensions, we use here one simple mathematical fractal. This fractal is embedded in two-dimensional space ($d = 2$), and its construction is based on the following replication and union of a simple motif (See Fig. 1). In stage 0 (zero) of construction, the object is one solid disc of area equal to 1. In stage 1, this disc is replaced by five other ones equal to that of stage 0 displaced in an aggregate or motif. In stage 2, the motif of stage 1 is replicated five times and these are aggregated in the same manner as in the first stage. This rule (the replication and aggregation) is repeated *ad infinitum.* As a result, at the end of the procedure, we obtain a deterministic mathematical fractal with fractal dimension $d_f = \ln 5/\ln 3 \simeq 1.46$. To obtain this fractal dimension, we can use the definition given in Eq. (1). Note that, at any stage k, the volume V (the area) of the object is simply equal to the number of discs aggregated in the structure, $V(k) = 5^k$. A characteristic length L of the object at stage k is the number of discs along the vertical direction, which is simply $L(k) = 3^k$. Thus, we have:

$$V(k) = 5^k = 3^{k\ln 5/\ln 3} = (L(k))^{\ln 5/\ln 3} \quad (2)$$

which gives the above-cited result. This simple example exhibits some common features of fractal objects such as: i) self-similarity (scale invariance), ii) a highly branching structure, and iii) dimension smaller than the embedding dimension. In contrast to the mathematical deterministic fractals, the fractal objects observed in nature are random fractals, their self-similarity is obeyed only in a statistical sense, and the anomalous scaling of the volume can be observed only in a restricted range of length.

The coastlines, for example, have been found to be fractals with $1 \leq d_f \leq 2$ (to a first approximation, $d_f = 3/2$). The higher the d_f value, the more wiggly and space-filling the line is. As another example, the surface area of the human lung is as large as a tennis court and is made up of self-similar branches with many lengths. The fractal dimension of the lung surface has been found to be 2.24 (Nonnenmatcher et al. 1994). The efficiency of gas exchange in the lung is optimized by this fractal property of space-filling, achieved by a design using branches with no characteristic length.

Concerning porous structures, for which wood is an example, various studies suggest that the pore spaces of rocks are fractals with fractal dimensions of order 2.69 (Hansen and Skjeltorp 1988 and references therein). It is also suggested that this fractal interpretation of pore space, combined with some other basic rock or fluid properties, may turn out to give a fundamental description and understanding of relative permeability, electric conductivity, and other physical quantities of great interest to the studies of reservoir engineering, underground storage of nuclear wastes, etc. (Hansen and Skjeltorp 1988; Ruffet et al, 1991; Tsallis et al. 1992).

A porous material is a solid with interconnected holes inside, which compose the void or pore space. Examples of porous materials are numerous: soils, rocks, ceramics, filter papers, and wood, to name just a few. The origin of the porosity in wood is in the lumens or voids and cell cavities present in its cellular structure composed of vessel members, parenchyma cells, etc. The importance of porosity for wood science and technology relies on its relation to wood density, hygroscopicity, thermal, acoustical and mechanical properties, as well as its susceptibility to preservative treatments, wettability by an adhesive, etc. (Tsoumis 1991).

Here we present a study of the void volume of different species of wood, suggesting that the fractal approach can give a useful method to characterize their porosity. Our objective here is to exhibit the anomalous scaling of the internal volume of wood with its linear size, a fingerprint of the fractal geometry.

MATERIALS

Four wood species were studied; three of them are of brazilian origin: jatoba (*Hymenaea stilbocarpa*) (which is used in railroad ties, furniture, etc.); cedro (*Cedrela* sp.) (used in furniture, naval construction, etc.); and cerejeira (*Amburana cearensis Fr. Allem.*) (used in furniture, barrels, etc.). The fourth species was the eucalipto (*Eucalyptus grandis*) (used in lamp-posts, etc.). All the species are of common use in Brazil and are obtained with no difficulty in small sawmills.

EXPERIMENTAL PROCEDURE

To measure the void volume in wood, we used a simple experiment of pressurized water absorption by cubic blocks of wood. As is known, when dry wood is immersed in water, there is active movement of water through cell cavities (free water) and cell walls. The free water fills the wood lumens and is limited by the fractional void space or wood porosity. Long immersion results in nearly complete saturation of the cavities, which allows us to use this mass of absorbed water to measure, or estimate, the internal void volume in wood.

The equipment used consisted of a closed cylinder 0.2 m in diameter and 1.8 m long, which was filled with liquid water and pressurized by the movement of a piston. The pressure P was indicated by a manometer and, in this work, we submitted the immersed wood to null manometric pressure (open cylinder), 1, 2, and 4×10^5 Pa.

To study a given species at a fixed pressure, we used a set of 10 cubic blocks of wood with edges varying from 5 mm to 50 mm (the first cube had an edge of 5 mm, the second cube of 10 mm, the third of 15 mm, etc.). The cubes of a given species were cut from a single block of wood to avoid the variation in wood structure between different trees and to minimize this variation within a tree. In all the experiments, three sample sets of each species were immersed in the pressurized liquid water at room temperature (25° C), and mass changes (ΔM) were measured for a period of 14 days of immersion. Since we did not use an oven to dry the wood before immersion, the moisture content of the wood was that dictated by the equilibrium of this wood with the humidity of the surrounding air (T = 25° C and relative humidity =60%). The moisture contents were 13.3% for the *Eucalyptus grandis,* 12.4% for the *Hymenaea stilbocarpa,* 13.4% for the *Cedrela* sp, and 10.4% for the *Amburana cearensis.* Immediately before mass measurements, the cubes were touched with paper on their surfaces to remove attached water.

As in Eq. 1, the mass absorbed by a cube, which was immersed in water at pressure P, is a function of its edge size L and of the water pressure ($\Delta M = \Delta M(L, P)$) and scales with L as

$$\Delta M(L, P) \propto L^{d_f(P)} \qquad (3)$$

where $d_f(P)$ is the fractal dimension of the wood pore space filled by this mass of water. The constant of proportionality is not important for us; only the scaling of the mass (or the volume) with the linear size is important to the fractal approach. Here we assume that the fractal dimension is a function of the water pressure P since we expect that for higher

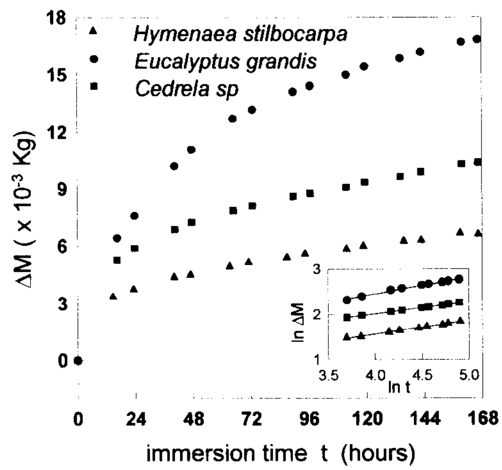

FIG. 2. Mass variations for blocks of some species, all of them with edge $L = 35$ mm, as a function of the immersion time at null manometric pressure. At the inset is shown the log-log plot of the same quantities for times at the beginning of the immersion.

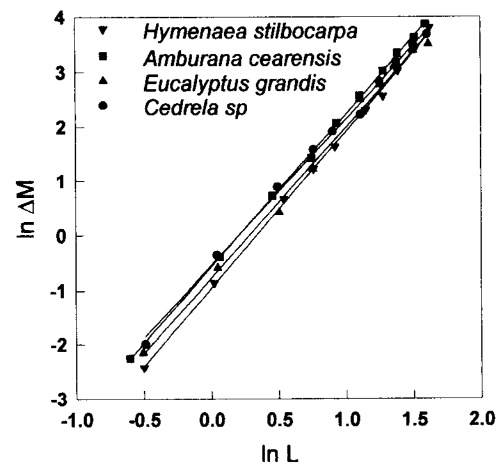

FIG. 3. Log-log plot of the wood cubes mass variations as a function of the cube edge for different species at manometric pressure $P = 2 \times 10^5$ Pa.

pressures, the water fills more space in the wood. It is clear that the fractal dimension must also be a function of the wood species. If the pore space were a smooth three-dimensional object, as the void of a bottle, we should have $d_f(P) = 3$ for any pressure and wood species.

RESULTS AND DISCUSSION

In Fig. 2 we show the absorption curve for some wood species at null manometric pressure (open cylinder). These mass variations were monitored two times per day for seven days to observe the saturation process. In the next seven days of immersion, our measures were less frequent, made only to certify the wood saturation. The mass variations used in our calculations were those after 14 days of immersion when all the species had ceased (under our experimental precision) to absorb water. The inset shows the log-log plot of the mass variations versus immersion time at the begining of the absorption process where the diffusion (due to the moisture content gradient) is the main process and Fick's law is obeyed (the slopes are approximately equal to 1/2).

In Fig. 3 we show, for various species, the log-log plot of the mass variations as a function of the block edges and fixed pressure $P = 2 \times 10^5$ Pa. These variations were measured after 14 days of immersion. The points represent average values taken for three immersed samples of each species. The straight lines are fitted lines whose slopes, according to Eq. 3, correspond to fractal dimensions of the pore space in wood as measured by the water at $P = 2 \times 10^5$ Pa. Similar curves are obtained for the other water pressures.

In Fig. 4 we show the log-log plot of the mass variations as a function of the block edges for the *Cedrela* sp. at the various water pressures after 14 days of immersion. The points are averaged over three samples of this species immersed at each pressure. Again, the straight lines are fitted lines whose slopes correspond to the fractal dimensions of the void of this wood species as measured by the water at different pressures. A larger pressure results in a larger slope for the fitted line and thus a larger fractal dimension. Similar curves are obtained for the other wood species.

All the results for the species studied at the chosen pressures are summarized in Table 1. ρ is the average mass density of the wood spe-

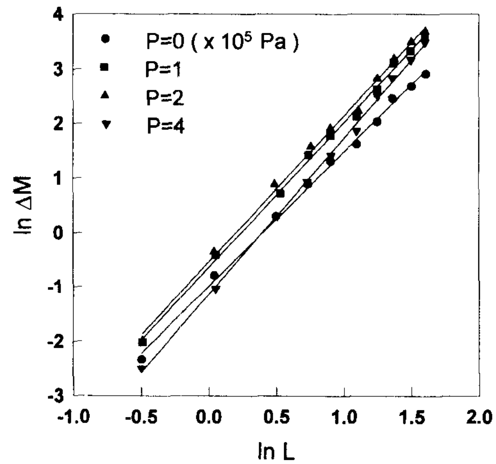

FIG. 4. Log-log plot of the wood cubes mass variations as a function of the cube edge for the same species, *Cedrela* sp. at different water pressures.

cies measured at the ambient conditions (T = 25°C and relative humidity = 60%).

In Fig. 5 we show the plot of the obtained fractal dimensions $d_f(P)$ as functions of the water pressure P for different species. The increase of the fractal dimension with the increase of the water pressure is a common behavior of all the species studied.

The values obtained here for the mass differences $\Delta M(L, P)$ are very sensitive to the nature of the absorbed fluid. This dependence, which is not investigated here, is clearly demonstrated by the distinct values for the wood specific volumes obtained by the displacement of water, helium, or benzene (Siau 1984). Therefore, the values of the fractal dimensions themselves must certainly depend on the nature of the fluid used in the experimental procedure. This is the reason we used the term fractal dimension 'measured by the water' at a given pressure. By this we mean that the geometric set composed of pores, voids, and microvoids in wood is revealed by an invasive fluid at a given pressure in a particular way. Other fluid, or the same fluid at an other pressure, reveals other fractal set, more 'fat' (larger d_f) or more 'thin' (smaller d_f); but for a fixed fluid and pressure, each species of wood

TABLE 1. *The fractal dimension values obtained for the studied wood species at different water pressures.*

	Wood species			
manometric pressure ($\times 10^5$ Pa)	*Eucalyptus grandis* $\rho = 0.94$ ($\times 10^3$ K g/m^3)	*Hymenaea stilbocarpa* $\rho = 0.88$ ($\times 10^3$ K g/m^3)	*Cedrela* sp. $\rho = 0.65$ ($\times 10^3$ K g/m^3)	*Amburana cearensis* $\rho = 0.63$ ($\times 10^3$ K g/m^3)
	d_f (P)			
0	2.52	2.59	2.46	2.53
1	2.67	2.81	2.66	2.74
2	2.77	2.87	2.68	2.80
4	2.97	2.92	2.87	2.89

has its own intrinsic fractal dimension, as can be seen in Table 1.

It is important to note that we do not take into account here the initial moisture content of the wood species since we are interested only in the scaling of the mass variations with the linear size of the wood blocks. A different moisture content corresponds to a different initial condition in the process of water absorption, which does not affect an intrinsic characteristic of the wood structure, namely, the noninteger exponent in the scaling of the volume of the void space with the size.

We also did not take into account the effect of shrinkage and/or swelling of the cube sides since the observed changes in the dimensions of the cube edges were very small (the vari-

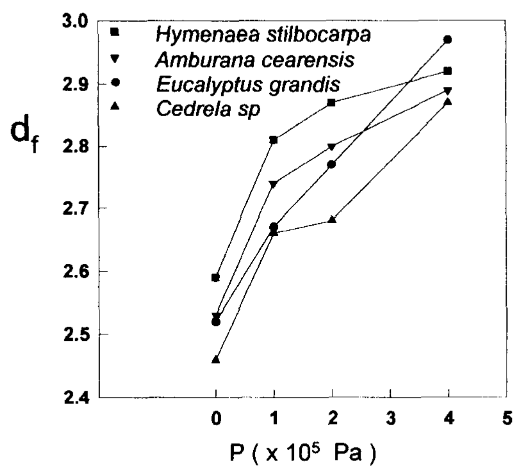

FIG. 5. Plot of the fractal dimension $d_f(P)$ as a function of the water pressure for different wood species. The lines are drawn by eye.

ation in ln L was of order 10^{-3}) and did not affect our results.

In wood science there are some quantities like the porosity or the specific gravity that give information about the void volume in wood. However, these quantities are densities, i.e., intensive quantities and so it is impossible to extract from them the information we are interested in here, namely, the scaling of the internal volume with the size of wood blocks. This information can be achieved by an experiment like the one we presented here where the internal volume is related to the linear size of wood blocks. An alternative approach might be a microscopic analysis of the wood structure. In fact, part of the motivation of this work came from some very interesting figures of the microscopic structure of wood presented in chapter 3 of Tsoumis 1991.

CONCLUSIONS

The fundamental purpose of this work was to investigate the hypothesis that the pore space in wood can be characterized by a fractal or a set of fractal dimensions. A very simple and reliable experiment was designed in order to answer this question using water absorption. Our results show, for the first time as far as we know, that each species of wood can be characterized by a set of fractal dimensions $d_f(P)$.

For a given pressure, each species has its own characteristic fractal dimension, which must be related to its intrinsic pore space geometry. Since the immersion time was sufficient for the saturation of the wood blocks, these results can not be attributed to an incomplete filling of the void volume by water. The fractal behavior revealed here is an intrinsic property of the set composed by the interconnected voids and microvoids of the wood structure and could turn out to be a new physical parameter in wood characterization. Apart from this, the fractal nature of wood can bring to wood science new research techniques developed for the study of fractal structures. In recent years, these techniques have been applied to a vast range of materials and biological systems, and it has become evident that fractal scaling represents an important characteristic of many growth structures.

The dependence of the fractal dimension with pressure shown here must be related to the microscopic pore-size distribution of wood. A specific pressure allows the water to penetrate in a subset of pores selected by the capillary tension and other mechanisms as the evaporation-condensation into the cells. Our results show, for all the studied wood species, a fractal dimension of the pore space which is a strictly increasing function of the water pressure. Since the fractal geometry is an intrinsic property of the wood void volume, we do not expect that for much larger pressures this fractal dimension converges to $d_f = 3$ (the dimension of the embedding space). If this was true, the set composed by the voids in wood would be, in fact, a smooth Euclidean object, which would be in contradiction to the fractal scaling shown here for the studied pressures. On the other hand, if the set of wood pores was a fat fractal (a fractal with dimension equal to that of the embedding space (Vicsek 1992)), in our belief, this fact would be revealed even for small water pressures. The dependence of the fractal dimensions with pressure would be best elucidated with the use of other invasive fluids such as nonpolar ones and gases, which could be subject of a future work.

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