POTENTIAL FAILURE OF A DECAYED TREE UNDER WIND LOADING

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ABSTRACT

Trees with decayed wood that are subject to moderate winds often collapse and cause property damage or injury and death to people. The purpose of this paper is to describe a decision-making aid to help identify a tree that may fail in the forest or be a potential hazard in the city. A tree may fail when the probability of radial shear cracks developing for a given wind load is sufficiently high.

Mathematical models are used to estimate the constant wind force on trees and to evaluate the cracking and collapse mechanisms under this loading. The physical dimensions are used to determine the wind force or drag on the tree, and the amount of decay in the tree is used to determine its ability to resist this load. Owing to uncertainties associated with accurately measuring and modeling a decayed tree, estimating the wind load, and specifying the wood strength of a tree species, reliability analysis is used to assess the potential risk of failure. Coupling this information with meteorological data for the largest wind speed value expected at the tree site and the topography of the tree site completes the analysis of potential failure. Case studies of balsam fir trees with the same exterior diameters but with different dimensions of decay columns, tree weights, tree heights, and wind speed conditions are analyzed and compared.

Keywords: Balsam fir, cracking, collapse, decay, failure, forecasting, reliability analysis, wind.

INTRODUCTION

Destructive field testing of balsam fir (*Abies balsamea*) with root and butt decay caused by *Armillaria mellea* showed that the amount of decay was the most significant factor in predicting cracking and collapse failure forces on trees subject to static forces. As a result, mathematical models were developed to simulate the cracking and collapse mechanisms of a balsam fir under constant horizontal loading. The mechanism consists of two sequential events. First, a pair of radial cracks were observed on either side of the decay column of the main stem. Second, tree collapse occurred when the bending stress equaled the modulus of rupture of the tree. The tree resistance to cracking and collapse was dependent upon the external dimensions of the tree and dimensions of the decay column. The drag or wind forces on a tree have been determined in wind tunnel tests. These tests have shown

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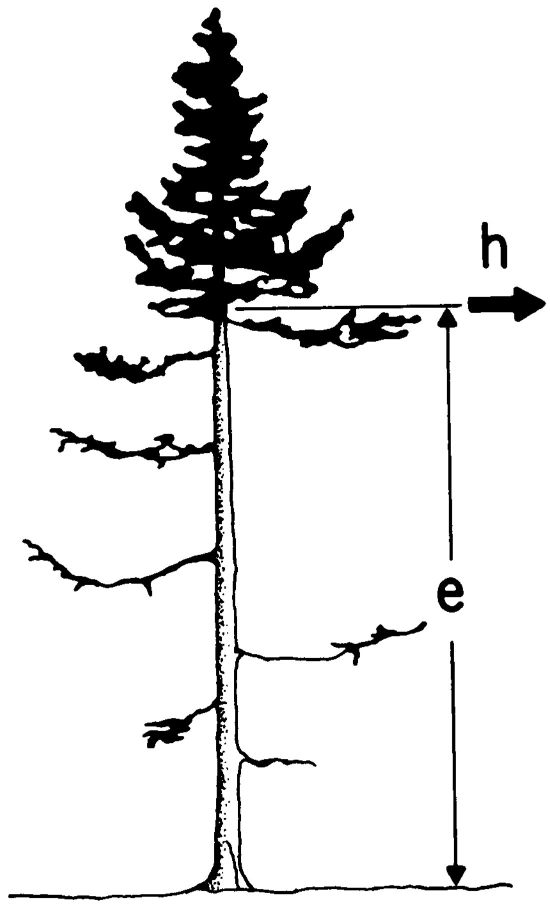

FIG. 1. The wind force diagram.

that the magnitude of the wind force is dependent upon the weight of the tree and the wind speed (Fraser 1962).

The mathematical models that are used to describe the cracking-collapse mechanism and to estimate the wind load are combined in an analysis procedure. Owing to the uncertainties associated with estimating model input variables of wind loading and tree strength and inherent variability of physical properties of wood, reliability analysis is used. The rationale behind the use of reliability analysis is described and its practical application is demonstrated with case studies.

CRACKING-COLLAPSE MECHANISMS

Analysis of destructive field test data of balsam fir trees leads to the development of a failure theory (Peters et al. 1984). The tree is assumed to be loaded with a wind pressure load that can be represented as a resultant force h acting at a distance e above ground (Fig. 1). Since a typical balsam fir tree is susceptible to decay at the base (root rot), cracking and bending failures are expected in this region.

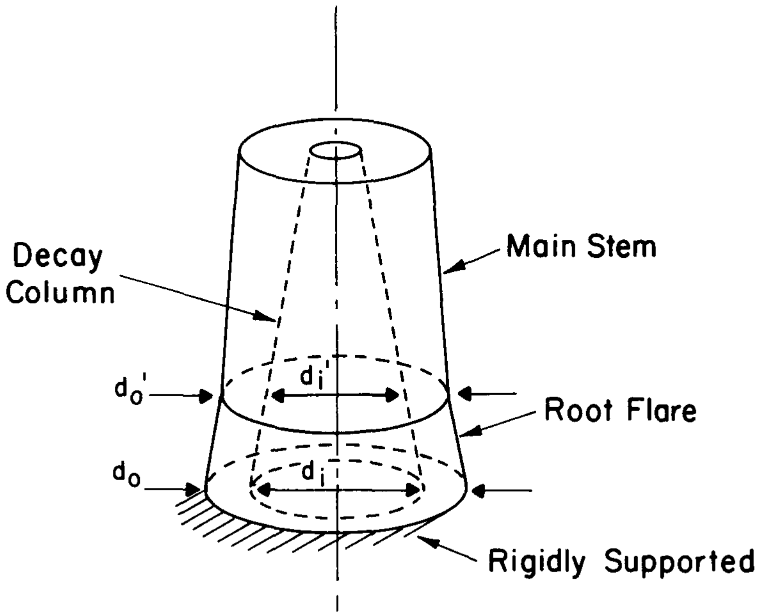

FIG. 2. The tree model.

The failure model

The tree is assumed to act as a tapered cantilever beam subject to a constant horizontal force h that is rigidly supported at the base (Fig. 1). Since cracking and bending failures are assumed to occur at the base, the tree is modeled as a tapered cylinder with a conical-shaped decay column (Fig. 2). The wood in the decay column is assumed to have zero strength. The wood in the main stem and root flare regions is assumed to be sound and of equal strength.

The wind force will cause the tree to bend. Since the tree is assumed to act as a cantilever beam, bending and shear stresses are developed. For simplicity, the result of bending will be represented as two internal forces, tension (T) and compression (C) forces (Fig. 3). Shear failure is assumed to be predominant in the phase that leads to radial cracking on either side of the decay column. The T and C forces act in opposite directions, tending to cause the tree segments on either side of the maximum shear plane to slide past one another. The maximum shear stress is assumed to occur on the plane of maximum shear stress (Fig. 3). The point of maximum shear stress on the shear plane is assumed to develop at point A, at the edge of the decay column at the base of the tree. From symmetry, the shear stresses at points A are equal. When the wind force h is sufficient, the shear stress at points A will equal the critical shear strength of the wood parallel to the grain. Cracks will develop at points A and propagate upward into the main stem and outward towards points B. If the wind load is sufficient to cause cracking to initiate at points A, it is assumed that cracking will proceed upward into the main stem and outward to the exterior edge of the tree. The net result is that the tree no longer reacts as a single cantilever beam, but as two cantilever beams. The cracking phase is assumed to be complete and the collapse phase begins.

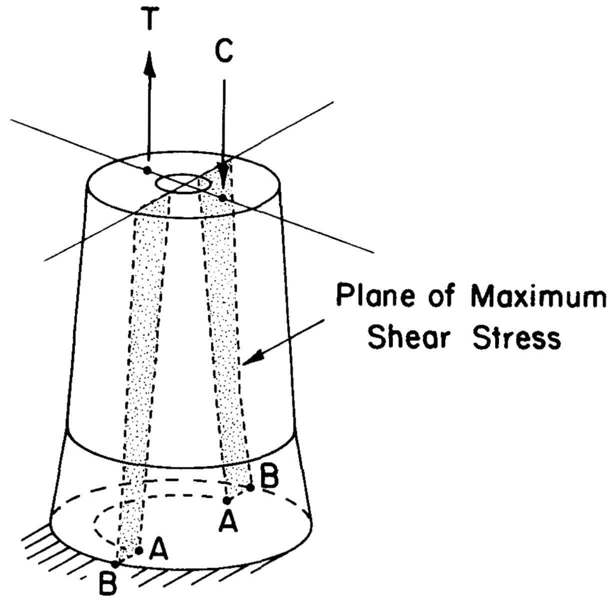

FIG. 3. The cracking phase.

Since the main stem is assumed to behave as two cantilever beam segments, the main stem of the tree behaves as two half hollow cylinders (Fig. 4). During the collapse phase, there is a redistribution of internal forces within each half hollow cylinder. The internal tension and compression forces are shown simply as T/2 and C/2 for a half tree segment because the two half cylinders are assumed to be equally strong. The bending strength of these segments is assumed to control during the collapse phase. When the wind force is sufficient, the tension and compression stresses caused by forces T/2 and C/2 equal the ultimate tension and compression strengths of the wood. Wood fibers will fail and the tree collapses. The collapse mechanism is expected to initiate above the root flare.

The hypothesis presented here is supported by visual observation during destructive field testing and analytical data gathered during these tests. The following is a mathematical model description of the failure theory.

Analytical models

The critical load to cause radial cracking h_c is estimated with the following mathematical model:

$$h_c = \frac{\tau}{2.089} \frac{(d_o^4 - d_i^4)}{(2.25d_o^2 + d_i^2)} \tag{1}$$

where d_o = inside bark tree diameter at the base, d_i = diameter of the decay column at the tree base, and τ = critical shear strength. The model assumes that the tree responds as a hollow cylindrical or tubular section under shear loading. Radial cracks that are present prior to loading are assumed not to affect the determination of h_c; therefore, they are not considered in this analysis. Further-

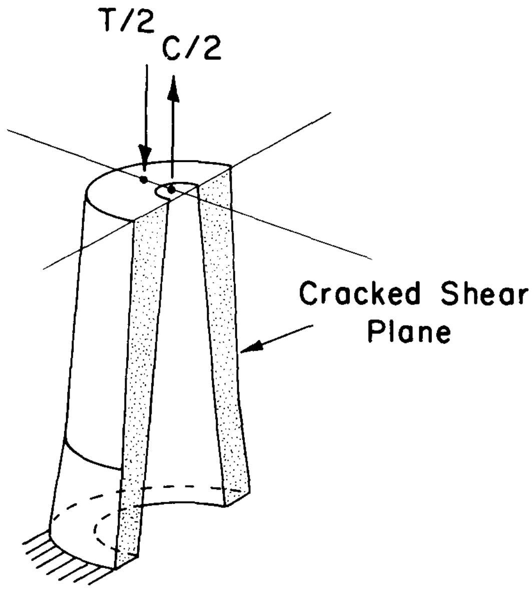

FIG. 4. The collapse phase.

more, it was found in our field and laboratory tests (Peters et al. 1984) that the average critical shear strength of 239 N/m^2 (39 psi) is significantly lower than the shear strength of 4,600 N/m^2 (668 psi) for sound wood samples. The coefficient of variation Δ_τ was estimated to be 0.125.

The critical load to cause tree collapse, the ultimate horizontal load h_u that may be placed on the tree, is

$$h_u = \frac{2\sigma I}{ec} \tag{2}$$

where I = moment of inertia of a half cylindrical section, c = critical distance between the neutral axis and extreme fiber to cause maximum bending stress in a half hollow cylindrical section, σ = modulus of rupture of green balsam fir, and e = moment arm or distance between resultant wind force and tree base. Since it is assumed that the radial cracking has occurred, a one half hollow cylindrical section is used for the collapse mechanism because the effective area for bending resistance is reduced to two half sections. In addition, it is assumed that the bending caused by the horizontal wind force is equally distributed between the two half hollow cylindrical sections. The equations of the moment of inertia I and critical distance c for a half hollow cylindrical section are respectively

$$I = \frac{\pi}{128}(d_o'^4 - d_i'^4) - \frac{1}{18\pi}\frac{(d_o'^3 - d_i'^3)^2}{(d_o'^2 - d_i'^2)} \tag{4}$$

$$c = \frac{d_o'}{2} - \frac{2}{3\pi}\frac{(d_o'^3 - d_i'^3)}{(d_o'^2 - d_i'^2)} \tag{5}$$

where d_o' and d_i' are the outside tree diameter and diameter of the decay column above the root flare.

WIND FORCES

Fraser (1962) conducted wind tunnel studies to determine the drag force of various tree specimens under constant wind speed. He found that wind speed v (knots) and tree weight w (pounds) are the significant factors in determining the resultant wind force. For coniferous trees, the regression equation is

$$h = 1.441v + 0.029vw - 0.328w + 7.426 \tag{6}$$

with a standard error estimate of 17.4 lb. The dry weight w_s in pounds of the above-ground components of a balsam fir tree (Tritton and Hornbeck 1982) is estimated with the regression equation

$$w_s = 1.81(\mathrm{dbh})^{2.4} \tag{7}$$

where dbh = tree diameter at breast height (1.37 m above ground) in inches. The total weight is equal to

$$w = w_s(1 + m) \tag{8}$$

where m is the moisture content (expressed as a decimal) of the tree. The average moment arm μ_e is estimated to be

$$\mu_e = 0.65L \tag{9}$$

where L = height of the tree. The moment arm estimates ranged from 0.52L to 0.84L (Peters et al. 1984).

RELIABILITY ANALYSIS

A tree is defined to be safe if the tree is sufficiently strong to resist a wind load without cracking. Otherwise, it is classified as hazardous and is considered to be potentially dangerous.

$$\text{Safe:} \quad h < h_c \tag{10a}$$

$$\text{Hazard:} \quad h \geq h_c \tag{10b}$$

The variables h and h_c will be treated as random variables H and H_c, respectively, because of the associated uncertainties found in nature and estimating the input variables of Eqs. (1) and (6). The difference in the magnitudes of wind load critical load is defined to be the margin of safety for cracking or

$$M_c = H - H_c \tag{11}$$

If a tree is safe $H < H_c$, then $M_c < 0$. Similarly, if a tree is hazard $H \geq H_c$, then $M_c \geq 0$. The probability that a tree is safe or hazard is expressed as

$$\text{Safe:} \quad P[M_c < 0] \tag{12a}$$

$$\text{Hazard:} \quad P[M_c \geq 0] \tag{12b}$$

Likewise, the following classification is used for tree collapse.

$$\text{No collapse:} \quad h < h_u \tag{13a}$$

$$\text{Collapse:} \quad h \geq h_u \tag{13b}$$

The variables h and h_u will be treated as random variables, H and H_u, respectively. The margin of safety for collapse is defined as

$$M_u = H - H_u \tag{14}$$

The collapse, $H_u \geq 0$, must be preceded by cracking, $M_c \geq 0$. Thus, the probability of no collapse and collapse are given by the conditional probabilities

$$\text{No collapse:} \quad P[M_u < 0 | M_c \geq 0] \tag{15a}$$

$$\text{Collapse:} \quad P[M_u \geq 0 | M_c \geq 0] \tag{15b}$$

The assumptions and derivations of the classification Eqs. (12a), (12b), (15a), and (15b) are given in the Appendix.

EXTREME WIND SPEEDS

In the evaluation of the sources of uncertainty, it was assumed that the wind speed is known. As a result, the probabilities of tree cracking or tree collapse for a given wind speed v may be expressed as conditional probabilities. Using the classification probabilities of Eqs. (14) and (15), they are rewritten as

$$\text{Hazard:} \quad P[M_c \geq 0 | V = v] \tag{16a}$$

$$\text{Collapse:} \quad P[M_u \geq 0 | (M_c \geq 0) \cap (V = v)] \tag{16b}$$

Statistical analyses of extreme wind speed data at 141 locations in the United States indicate that the Type I and Type II probability distributions of largest values most adequately describe the yearly occurrence of these extreme winds. The probability that a tree under evaluation will crack or collapse in any single year is estimated as

Hazard:

$$P[M_c \geq 0] = \int_0^\infty P[M_c \geq 0 | V = v] f_V(v)\, dv \tag{17a}$$

Collapse:

$$P[M_u \geq 0 | M_c \geq 0] = \int_0^\infty P[M_u \geq 0 | (M_c \geq 0) \cap (V = v)] f_V(v)\, dv \tag{17b}$$

where $f_V(v)$ is the probability density function of a Type I or Type II distribution of largest wind speed values.

Extreme wind speed distribution

The selection of $f_V(v)$ will be dependent upon the meteorological and terrain conditions at the tree site. For purposes of this evaluation, the extreme wind speeds recorded at airport locations will be used (Simiu et al. 1979). These wind speeds represent recordings taken in open terrain; thus they must be corrected to account for local surface roughness conditions. The wind-speed power law relationship for open country; wooded areas, small towns or suburbs; and central areas of large cities will be used to correct for local effects (Hart 1982; International Conference of Building Officials 1979).

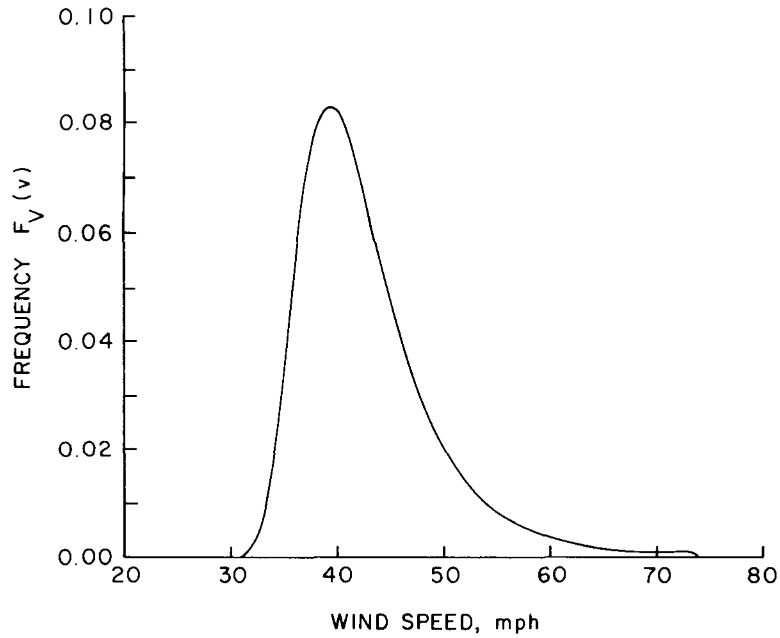

FIG. 5. Probability distribution of extreme wind speeds in Concord, New Hampshire.

The likelihood of tree collapse

Tree collapse occurs only if the applied load H is sufficient to exceed the tree cracking resistance load H_c and the ultimate load H_u. The probability $P[M_c \geq 0]$ is the likelihood that only radial cracking will occur in any year. Similarly, $P[M_u \geq 0 | M_c \geq 0]$ is the likelihood of tree collapse in any year, given that the tree is already cracked. The cracking may have occurred from an earlier extreme wind condition, or possibly, both cracking and collapse have occurred from the same extreme wind condition. The probability that a tree will collapse because of the same extreme wind is the joint probability that both cracking and collapse occur or

$$P[(M_u \geq 0) \cap (M_c \geq 0)] = P[M_u \geq 0 | M_c \geq 0] \cdot P[M_c \geq 0] \qquad (18)$$

It is possible that the critical cracking resistance load is equal to or greater than the ultimate load or $h_c \geq h_u$. Whether the critical cracking resistance h_c is less than, equal to, or greater than the ultimate load h_u will depend upon the configuration and extent of internal decay. When $h_c \geq h_u$, it is assumed that the wind load will be sufficient to cause cracking and collapse to occur simultaneously. There is no reserve strength in the tree after cracking occurs. In terms of probability, the conditional probability,

$$P[M_u \geq 0 | M_c \geq 0] = 1.0 \qquad (19)$$

is assumed to be equal to one.

DISCUSSION

In order to vividly describe the classification scheme, numerical results of the reliability analyses for several case studies are presented. In case 1, a tree with

TABLE 1. *Sources of measurement error.**

Model variable X	Range estimates $\epsilon = x_u - \mu_x = \lvert x_l - \mu_x \rvert$	Coefficient of variation δ_x
D_o	3.42 cm (0.125 in.)	0.012
D_o'	3.42 cm (0.125 in.)	0.012
D_i	3.42 cm (0.125 in.)	0.012
D_i'	3.42 cm (0.125 in.)	0.012
D_{bh}	3.42 cm (0.125 in.)	0.012
E	0.2L	0.154

* The uniform probability distribution is assumed for all model parameters except E
where

$$\delta_x = \frac{1}{\sqrt{3}}\left(\frac{x_u - x_\ell}{x_u + x_\ell}\right) = \frac{\epsilon}{\sqrt{3}\bar{x}}$$

with

x_u = upper range of x, and x_l = lower range of x, and μ_x = mean of x.

The model parameter E is assumed to be a normal probability distribution where

$$\mu_e = 0.65L$$

$$\delta_e = \frac{1}{2}\left(\frac{x_x - x_\ell}{x_u + x_\ell}\right)$$

with the upper and lower ranges of E assumed to be within two standard deviations of the mean μ_e of $x_u = 0.85L$ and $x_l = 0.45L$.

the following dimensions was analyzed: $d_o = 6$ in.; $d_i = 4.6$ in.; $d_o' = 5.7$ in.; $d_i' = 3.3$ in.; dbh = 4.8 in., and L = 360 in. The decay column for the tree samples used in the destruction sampling showed the following average relationships: $d_i' = 0.72d_i$; and $d_o' = 0.95d_o$. A balsam fir tree of these dimensions is representative of the average tree subject to destruction testing (Peters et al. 1984). It will be assumed that the tree will be subject to extreme wind speed recorded at the airport in Concord, New Hampshire. The mean extreme wind speed and coefficient of variation is 42.9 mph and 0.195, respectively. A Type II probability distribution for largest value with a tail length parameter of 9 will be used (Simiu et al. 1979) and is shown in Fig. 5. The sources of measurement error, inherent variability, and model uncertainty assumed in the analysis are recorded in Tables 1, 2, and 3.

The probability of tree cracking and collapse for a given wind speed v is presented as the conditional probabilities $P[M_c \geq 0 \mid V = v]$ and $P[M_u \geq 0 \mid (M_c \geq 0) \cap (V = v)]$, respectively. The relationships for case 1 are shown in Fig. 6. Owing to the greater certainty associated with model inputs for cracking mechanism as compared to ultimate load mechanism, a more sharply defined increase is observed for $P[M_c \geq 0 \mid V = v]$ than for $P[M_u \geq 0 \mid (M_c \geq 0) \cap (V = v)]$. The sources of uncertainty for H, H_c, and H_u as well as other model variables are given in Table 4. The major source of uncertainty in estimating H_u is the moment arm estimate of 3. The use of Eqs. (17a), (17b), and (18) gives the likelihoods of tree

TABLE 2. *Sources of inherent variability.**

Model variable X	Mean μ_x	Coefficient of variation Δ_x
Υ	239 N/m² (39 psi)	0.125
Σ	38.61 × 10^6 N/m² (5,600 psi)	0.125
M	0.70	0.082

* Υ and Σ are assumed to have a normal probability distribution. M is assumed to have a uniform probability distribution with a range of values from 0.60 to 0.80. See footnote of Table 1 for equations for estimating Δ_m.

TABLE 3. *Sources of model uncertainty.*

Predicted value Y	Bias factor ν_y	Coefficient of variation Δ_y
H_c	0.93	0.044
H_u	0.95	0.149
H	1.0	$17.4/\mu_H$
W	1.0	*

* Not reported in literature source. It is considered to be small value in relation to other model input variables; since it is considered negligible, $\Delta_w = 0$ is assumed.

cracking as $P[M_c \geq 0] = 0.659$ and tree collapse as $P[M_u \geq 0 | M_c \geq 0] = 0.505$ and $P[M_u > 0) \cap (M_c \geq 0] = 0.332$.

The effect of the extent of decay, tree weight, and tree height, and extreme wind speed have been studied in a sensitivity analysis. One or more model input variables are changed, and the results of the reliability analyses are compared to case 1. The results are presented in Tables 5 through 9 and in the following discussion.

Extent of decay

In cases 2 and 3 (Table 5), it is assumed that $d_i = 3.6''$ and $5.5''$, respectively. The conditional probabilities $P[M_c \geq 0 | V = v]$ and $P[M_u \geq 0 | V = v]$ for cases 2 and 3 are shown in Figs. 7 and 8. The analysis shows that the changes have a major effect upon estimates of the critical cracking loads, μ_{h_c} and collapse resistance loads μ_{h_u}. For case 2, since $\mu_{h_c} > \mu_{h_u}$ (i.e., 224 > 181), there is no reserve strength

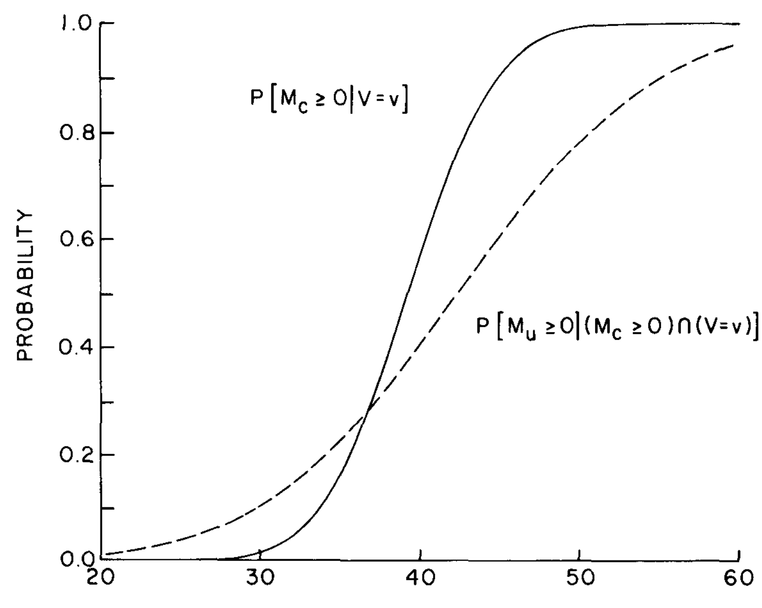

FIG. 6. Conditional probabilities for tree cracking and collapse, case 1.

TABLE 4. *Estimated coefficients of variation.*

Model output Y	Coefficient of variation Ω_y
H	0.046
H_c	0.058
H_u	0.249
w	0.044
c	0.016
I	0.002
e	0.154

in the tree; thus it assumed that cracking and collapse will occur simultaneously and $P[M_u \geq 0 | M_c \geq 0] = 1.0$. Thus $P[M_c \geq 0] = P[(M_u \geq 0) \cap (M_c \geq 0)] = 0.092$. For case 3, cracking is predicted to be a certain outcome, $P[M_c \geq 0] = 1.0$ because $\mu_{h_c} < \mu_{h_u}$ (i.e., 64 < 148). These results show that the extent of decay as measured by the single input estimate of μ_{d_i} has a dramatic impact upon tree behavior under extreme wind speed conditions.

In the destructive tests, the decay column was found to have different tapers. The values of d_i' ranged from $0.33d_i$ to $0.89d_i$. From Eq. (1), it is evident that the cracking load h_c is independent of d_i'. As a result, $\mu_{h_c} = 155$ lb and $P[M_c \geq 0] = 0.659$ for cases 1, 4, and 5 (Table 6) are the same. For case 4, the probability of tree collapse, $P[(M_u \geq 0) \cap (M_c \geq 0)]$ is less than for case 1. For case 5, since $\mu_{h_c} > \mu_{h_u}$ (i.e., 155 > 142) and cracking and collapse are expected to occur simultaneously, thus $P[M_c > 0] = P[M_u \geq 0 | M_c \geq 0] = 0.659$. The tree in case 5 has a very slight taper in the decay column as compared to cases 1 and 4. These case studies show that the extent of decay as measured by μ_{d_i} and $\mu_{d_i'}$ has a dramatic impact upon the ability of a tree to resist cracking and collapse.

Tree weight

The horizontal wind force h is assumed to be dependent upon wind speed v and tree weight w, Eq. (6) and, in turn, upon the solid tree weight w_s, Eq. (7), and moisture content m, Eq. (8). Since there have been several studies performed and equations derived to estimate w_s and the relationships give similar predictions (Tritton and Hornbeck 1982), it is assumed that the prediction of w_s can be estimated with reasonable confidence as compared to moisture content. For case 1 the moisture content of a tree is assumed to vary between 0.6 and 0.8 or $\delta_m = 0.082$. In case 6 (Table 7), the mean moisture is assumed to be equal to 1.4 with $\delta_m = 0.082$. The relationship of mean horizontal force μ_h with wind speed v is shown in Fig. 9 for m = 0.7 and 1.4, respectively. In case 7, it is assumed that the moisture content may lie between 0.5 and 1.5. Consequently $\mu_m = 1.0$ and $\delta_m = 0.289$. The results shown in Table 7 indicate that μ_{h_c} and μ_{h_u} are unaffected,

TABLE 5. *Effect of decay column diameter, d_i.*

Case	μ_{d_i}	μ_{h_c}	μ_{h_u}	$P[M_c \geq 0]$	$P[M_u \geq 0 \| M_c \geq 0]$	$P[(M_u \geq 0) \cap (M_c \geq 0)]$
1	4.6 in.	155 lb	167 lb	0.659	0.504	0.332
2	3.6 in.	224 lb	181 lb	0.092	1.000	0.092
3	5.5 in.	64 lb	148 lb	1.000	0.649	0.649

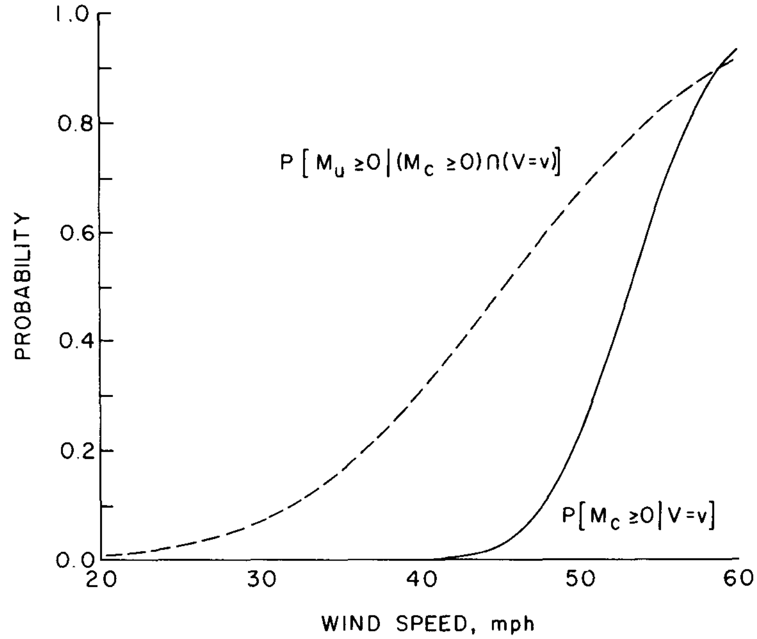

FIG. 7. Conditional probabilities for tree cracking and collapse, case 2.

but μ_h is significantly affected by moisture content as evidenced by the probabilities of cracking and collapse. These results indicate that a tree with a higher moisture content has a greater likelihood of cracking and collapse than a tree with the same dimensions and lower moisture content.

Tree height

The moment arm e is assumed to be related to the tree height, e = 0.65L and affects the ultimate load h_u. The taller the tree, the greater the risk of collapse as shown by comparing cases 1, 8, and 9 (Table 8).

Surface roughness

All estimated resistance loads, cracking and collapse probabilities given in Tables 5 through 8, are estimates for an unprotected tree in open country. The tree is assumed to be subject to an average extreme wind speed of 42.9 mph at 10 meters above ground in any given year. With use of the wind-speed power law, the estimated average extreme wind speeds for wooded and city areas are 26 mph and 15 mph, respectively. A tree stand and man-made structures increase surface roughness and tend to protect an individual tree from wind damage. The analysis (Table 9) shows that the cracking and collapse probabilities are significantly reduced. The increased surface roughness of the wooded and city environments offers protection against failure.

Reliability analysis gives quantitative measures of the potential risk of tree cracking and collapse under extreme wind speed for any given year. The probability measures are intended to be a decision-making aid. The actual assignment as to whether a tree is classified as potentially hazardous or safe will depend on

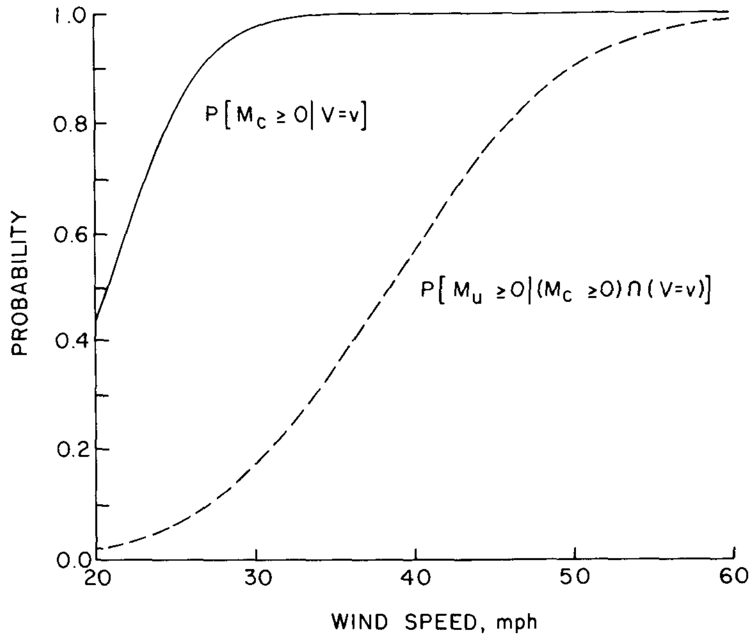

FIG. 8. Conditional probabilities for tree cracking and collapse, case 3.

the economic and physical risks associated with tree collapse. For example, if $P[M_c \geq 0] = 0.05$, there is a five in one hundred chance that the tree will crack under extreme wind loading in any given year. In a city or surburban environment, this risk may be considered significantly large to have the tree cut down and removed to avoid physical damage to people and property. The same tree situated in a remote rural or forest environment may also be removed for reasons of better forest management. The final decision will be based upon economics and the danger to humans.

CONCLUSION

Reliability analysis offers a method for evaluating the potential failure of a decayed tree under constant wind speed. The method is intended as a decision-

TABLE 6. *Effect of decay column taper, d_i'.*

Case	$\mu_{d_i'}$	μ_{h_c}	μ_{h_u}	$P[M_c \geq 0]$	$P[M_u \geq 0 \mid M_c \geq 0]$	$P[(M_u \geq 0) \cap (M_c \geq 0)]$
1	4.6 in.	155 lb	167 lb	0.659	0.504	0.332
4	1.5 in.	155 lb	199 lb	0.659	0.312	0.206
5	4.1 in.	155 lb	142 lb	0.659	1.000	0.659

TABLE 7. *Effect of moisture content, m.*

Case	δ_m	ν_m	μ_{h_c}	μ_{h_u}	$P[M_c \geq 0]$	$P[M_u \geq 0 \mid M_c \geq 0]$	$P[(M_u \geq 0) \cap (M_c \geq 0)]$
1	0.7	0.082	155 lb	167 lb	0.659	0.504	0.332
6	1.4	0.082	155 lb	167 lb	0.944	0.767	0.724
7	1.0	0.289	155 lb	167 lb	0.798	0.621	0.496

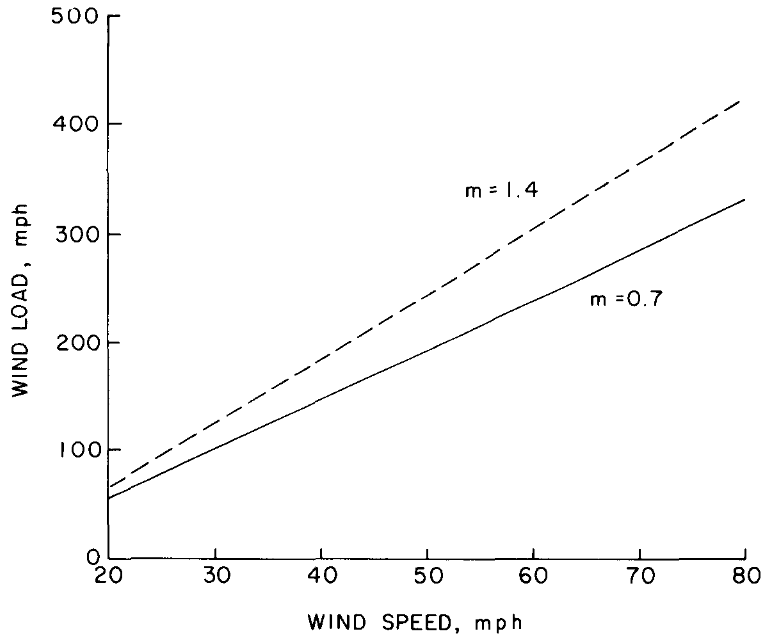

FIG. 9. The effect of moisture content on wind loading.

making tool for classifying a tree as hazard or safe. Sensitivity analyses show that care must be taken to accurately determine the input variables of the models. Analyses show that the most critical factor in our classification scheme is the exposure of the tree to wind forces. It was shown that rough terrain offers significant protection against cracking and collapse. In this paper, a case study of balsam fir with root and butt rot was undertaken. The mathematical modelling approach, presented here, is general and can be applied to other tree species if the failure mechanism of the species is known.

TABLE 8. *Effect of tree height, L.*

Case	L	μ_{h_c}	μ_{h_u}	$P[M_c \geq 0]$	$P[M_u \geq 0 \mid M_c \geq 0]$	$P[(M_u \geq 0) \cap (M_c \geq 0)]$
1	360 in.	155 lb	167 lb	0.659	0.504	0.332
8	240 in.	155 lb	250 lb	0.659	0.140	0.092
9	480 in.	155 lb	125 lb	0.659	1.000	0.659

TABLE 9. *Effect of surface roughness.*

Case	Terrain	V (mph)	μ_{h_c}	μ_{h_u}	$P[M_c \geq 0]$	$P[M_u \geq 0 \mid M_c \geq 0]$	$P[(M_u \geq 0) \cap (M_c \geq 0)]$
1	Open	42.9	155 lb	167	0.659	0.504	0.332
10	Wooded	26	155 lb	167	0.022	0.063	0.001
11	City	15	155 lb	167	0.000	0.004	~0.000

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APPENDIX

RELIABILITY ANALYSIS

A tree is defined to be safe if the tree is sufficiently strong to resist a wind load h without cracking. Otherwise, it is classified as hazardous and is considered to be potentially dangerous. The mathematical models, Eqs. (1) and (6), and the inequalities (10a) and (10b) are used to classify safe and potential hazard conditions. Likewise, Eqs. (2) and (6) and the inequalities (12a) and (12b) are used for predicting the no collapse and collapse condition of a tree. The essence of the classification method is summarized by evaluating these equations. This procedure is considered incomplete because of the uncertainties associated with estimating the model input variables of Eqs. (1) through (9); as a result, probabilistic methods will be used. The purpose of the Appendix is to explain in more detail the derivation of Eqs. (12a), (12b), (15a), and (15b).

The variables h, h_c, and h_u will be treated as random variables, H, H_c, and H_u. The classification scheme in terms of the random variable is as follows:

Safe:	$P[H < H_c]$
Hazard:	$P[H \geq H_c]$
No collapse:	$P[H < H_u \mid H \geq H_c]$
Collapse:	$P[H \geq H_u \mid H \geq H_c]$

where $P[H < H_c]$ is the probability that the wind force is less than the critical load H_c. $P[H \geq H_c]$ is the probability that the wind force is equal to or greater than the cracking load H_c, $P[H \leq H_u \mid H \geq H_c]$ is the probability that the wind force is less than the ultimate load given that radial cracking has occurred, and $P[H \geq H_u \mid H \geq H_c]$ is the probability that the wind forces is equal to or greater than the ultimate load given that radial cracking has occurred. The no collapse and collapse classifications explicitly incorporate the assumption that collapse is preceded by radial cracking.

Margin of safety

The concept of margin of safety, Eqs. (11) and (14), will be used to evaluate these probabilities. The margin of safety M, a random variable, is defined as the difference between the applied load H and resistance loads H_c or H_u. For cracking, $M_c = H - H_c$ and for collapse $M_u = H - H_u$. By rewriting the probability equations, the classification scheme equations in terms of M_c and M_u are

Safe:	$P[H - H_c < 0] = P[M_c < 0]$
Hazard:	$P[H - H_c \geq 0] = P[M_c \geq 0]$
No collapse:	$P[H - H_u < 0 \mid H - H_c \geq 0] = P[M_u < 0 \mid M_c \geq 0]$
Collapse:	$P[H - H_u \geq 0 \mid H - H_c \geq 0] = P[M_u \geq 0 \mid M_c \geq 0]$

By the total probability theorem, $P[M_c < 0] + P[M_c \geq 0] = 1.0$ and $P[M_u < 0 \mid M_c \geq 0] + P[M_u \geq 0 \mid M_c \geq 0] = 1.0$. Since the safe and hazard and the no collapse and collapse classes are simply related, the remaining discussion is restricted to the hazard and collapse classification equations.

In order to calculate these probabilities, the probability distributions $f_H(h)$, $f_{H_c}(h_c)$ and $f_{H_u}(h_u)$ must be known. Owing to insufficient statistical data, these distributions can only be estimated. The first order approximation (Ang and Tang 1984) will be used; the means μ_H, μ_{H_c} and μ_{H_u} and standard deviations, σ_H, σ_{H_c}, and σ_{H_u} of H, H_c, and H_u, respectively, will be estimated. Furthermore, it will be assumed that the random variables H, H_c, and H_u are adequately defined with the normal probability distribution. The probabilities will be determined with the use of the cumulative unit normal distribution.

The hazard event

The probability that a tree is a hazard is

$$P[M_c \geq 0] = F_U[u]$$

with

$$u = \frac{\mu_{M_c}}{\sigma_{M_c}}$$

$$\mu_{M_c} = \mu_H - \mu_{H_c}$$

and, for statistically independent random variables,

$$\sigma_{M_c} = (\sigma_H{}^2 + \sigma_{H_c}{}^2)^{1/2}$$

The collapse event

The probability of tree collapse is

$$P[M_u > 0 \mid M_c \geq 0] = F_U[u]$$

with

$$u = \frac{\mu_{M_u}}{\sigma_{M_u}}$$

$$\mu_{M_u} = \mu_H - \mu_{H_u}$$

$$\sigma_{M_u} = (\sigma_H{}^2 + \sigma_{H_u}{}^2)^{1/2}$$

Sources of uncertainty

Examination of Eqs. (1) through (9) shows that the mathematical predictions will be dependent upon the accuracy in measuring the input variables, d_o, d_o', d_i, d_i', and d_{bh}. Since these model variables are considered a source of measurement error, they are treated as uniformly distributed random variables, D_o, D_o', D_i, D_i', and D_{bh}. The moment arm estimate is assumed to be a normally distributed random variable E. It is expected that these variables may be accurately measured within the ranges shown in Table 1. The physical properties of τ, σ, and m are also sources of uncertainty, called sources of inherent variability error. As a result, they are treated as random variables, Υ, Σ, and M. The critical shear cracking and modulus of rupture are assumed to be normally distributed random variables with means and coefficients of variation shown in Table 2. It is assumed that the moisture content is adequately described with a uniform distribution with a range between 0.6 and 0.8.

The estimate of uncertainty is assumed to be associated with measurement error and inherit variability. Thus, the coefficient of variation of each input variable X, Ω_x, is estimated as

$$\Omega_x = (\delta_x{}^2 + \Delta_x{}^2)^{1/2}$$

where Δ_x = coefficient of variation of inherent variability and δ_x = coefficient of variation of measurement error.

The models of H, H_c, and H_u are functions of the input random variables

$$H = g(V = v, W, M, D_{bh})$$
$$H_c = g(\Upsilon, D_o, D_i)$$
$$H_u = g(\Sigma, D_o', D_i', I, C, E)$$

The wind speed v is assumed to be known for this part of the discussion; thus the random variable for wind speed V is equal to V = v. For simplicity in notation, these models will be represented in the generic model

$$\tilde{Y} = g(X_1, X_2, \ldots, X_n)$$

where $\tilde{Y}$ = model output, a random variable, and the model input random variables, $X_1, X_2, \ldots, X_n$. The relationship between the actual value of Y and $\tilde{Y}$, the model estimated value, is

$$Y = N_y \cdot \tilde{Y}$$

where N_y = model bias factor and a random variable. The predicted mean value of Y is

$$\mu_y = \nu_y \cdot g(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}) = \nu_y \cdot \mu_g \tag{20}$$

where ν_y = average bias factor, $\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}$ = average estimates of $X_1, X_2, \ldots, X_n$, μ_g = expected value of $g(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n})$ and μ_y = expected output. The average bias factors for H_c and H_u were calculated by Peters et al. (1984) and are shown in Table 3. It is assumed that the regression equation derived by Fraser is assumed to be a non-biased estimator. The coefficient of variation of Y, assuming no correlation between the input variables, is equal to

$$\Omega_y{}^2 = \Delta_y{}^2 + \frac{1}{\mu_y} \sum_i c_i{}^2 \sigma_{x_i}{}^2 \tag{21}$$

where

Δ_y = coefficient of variation of model error,

$c_i = \dfrac{\partial g}{\partial x_i}$ evaluated at $\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}$,

and

$\sigma_{x_i} = \Omega_{x_i} u_{x_i}$.

The means and coefficient of variations for H_c, H_u, and H are determined using Eqs. (20) and (21). The relationships for these random variables are determined as follows.

The critical cracking load H_c

The deterministic model for the critical cracking load is given by Eq. (1):

$$h_c = \frac{\tau}{2.089} \frac{(d_o{}^4 - d_i{}^4)}{(2.25d_o{}^2 + d_i{}^2)}$$

From Eq. (21), the coefficient of variation is

$$\Omega_{h_c}{}^2 = \Delta_{h_c}{}^2 + \frac{1}{\mu_{h_c}{}^2}[c_\tau{}^2(\Omega_\tau \mu_\tau)^2 + d_{d_o}{}^2(\Omega_{d_o}\mu_{d_o})^2 + c_{d_i}{}^2(\Omega_{d_i}\mu_{d_i})^2]$$

where the estimated mean for random variable H_c

$$\mu_{h_c} = \frac{\mu_\tau}{2.089} \frac{(\mu_{d_o}{}^4 - \mu_{d_i}{}^4)}{(2.25\mu_{d_o}{}^2 + \mu_{d_i}{}^2)}$$

with partial deviations:

$$c_\tau = \frac{\partial h_c}{\partial \tau} = \frac{1}{2.089} \frac{(d_o{}^4 - d_i{}^4)}{(2.25d_o{}^2 + d_i{}^2)}$$

$$c_{d_o} = \frac{\partial h_c}{\partial d_o} = \frac{\tau}{2.089}\left[\frac{4d_o{}^3}{(2.25d_o{}^2 + d_i{}^2)} - \frac{4.5d_o(d_o{}^4 - d_i{}^4)}{(2.25d_o{}^2 + d_i{}^2)^2}\right]$$

$$c_{d_i} = \frac{\partial h_c}{\partial d_i} = \frac{\tau}{2.089}\left[\frac{-4d_i{}^3}{(2.25d_o{}^2 + d_i{}^2)} + \frac{2d_i(d_o{}^4 - d_i{}^4)}{(2.25d_o{}^2 + d_i{}^2)^2}\right]$$

The ultimate load, H_u

The deterministic model for the ultimate load is given by Eq. (2):

$$h_u = \frac{2\sigma I}{ec}$$

From Eq. (21), the coefficient of variation is

$$\Omega_{h_u}^2 = \Delta_{h_u}^2 + \Omega_\sigma^2 + \Omega_I^2 + \Omega_e^2 + \Omega_c^2$$

The means and coefficients of variation for I and c are estimated with the following relationships. The deterministic model for I is given by Eq. (4):

$$I = \frac{\pi}{128}(d_o'^4 - d_i'^4) - \frac{1}{18\pi}\frac{(d_o'^3 - d_i'^3)^2}{(d_o'^2 - d_i^2)}$$

The coefficient of variation for I, is

$$\Omega_I^2 = \frac{1}{\mu_I^2}[c_{d_o'}^2(\Omega_{d_o'}\mu_{d_o'})^2 + c_{d_i'}^2(\Omega_{d_i'}\mu_{d_i'})^2]$$

with the estimated mean of random variable I is

$$\mu_I = \frac{\pi}{128}(\mu_{d_o'}^4 - \mu_{d_i'}^4) - \frac{1}{18\pi}\frac{(\mu_{d_o'}^3 - \mu_{d_i}^3)^2}{(\mu_{d_o'}^2 - \mu_{d_i'}^2)}$$

and the partial derivatives are

$$c_{d_o'} = \frac{\partial I}{\partial d_o'} = \frac{\pi}{32}d_o'^3 - \frac{1}{3\pi}\frac{(d_o'^3 - d_i'^3)d_o'^2}{(d_o'^2 - d_i'^2)} + \frac{1}{9\pi}\frac{(d_o'^3 - d_i'^3)^2 d_o'}{(d_o'^2 - d_i'^2)^2}$$

$$c_{d_i'} = \frac{\partial I}{\partial d_i'} = -\frac{\pi}{32}d_i'^3 + \frac{1}{3\pi}\frac{(d_o'^3 - d_i'^3)d_i'^2}{(d_o'^2 - d_i'^2)} - \frac{1}{9\pi}\frac{(d_o'^3 - d_i'^3)^2 d_i'}{(d_o'^2 - d_i'^2)^2}$$

The deterministic model for c is given by Eq. (5):

$$c = \frac{d_o'}{2} - \frac{2}{3\pi}\frac{(d_o'^3 - d_i'^3)}{(d_o'^2 - d_i'^2)}$$

The coefficient of variation, Eq. (21), for c, is

$$\Omega_c^2 = \frac{1}{\mu_c^2}[d_{d_o'}^2(\Omega_{d_o'}\mu_{d_o'})^2 + d_{d_i'}^2(\Omega_{d_i'}\mu_{d_i'})^2]$$

with

$$\mu_c = \frac{\mu_{d_o'}}{2} - \frac{2}{3\pi}\frac{(\mu_{d_o'}^3 - \mu_{d_i}^3)}{(\mu_{d_o'}^2 - \mu_{d_i'}^2)}$$

$$d_{d_o'} = \frac{\partial c}{\partial d_o'} = \frac{1}{2} - \frac{2d_o'^2}{\pi(d_o'^2 - d_i'^2)} + \frac{4}{3\pi}\frac{(d_o'^3 - d_i'^3)d_o'}{(d_o'^2 - d_i'^2)^2}$$

$$d_{d_i'} = \frac{\partial c}{\partial d_i'} = \frac{2d_i'^2}{\pi(d_o'^2 - d_i'^2)} - \frac{4}{3\pi}\frac{(d_o'^3 - d_i'^3)d_i'}{(d_o'^2 - d_i'^2)^2}$$

The wind load H

The deterministic model for wind loading is given by Eq. (6):

$$h = 1.441v + 0.029vw - 0.328w + 7.426$$

The coefficient of variation, Eq. (21), is

$$\Omega_h^2 = (c_w\Omega_w\mu_w/\mu_h)^2 + \Delta_h^2$$

with the estimated mean of the random variable H is

$$\mu_h = 1.441v + 0.029v\mu_w - 0.328\mu_w + 7.426$$

and the partial derivative of

$$c_w = \frac{\partial h}{\partial w} = 0.029v - 0.328$$

The mean and coefficient of variation of W is estimated with the following relationships:

$$w = 1.81(dbh)^{2.4}(1 + m)$$

The coefficient of variation, Eq. (21), for random variable W is

$$\mu_w = 1.81\mu_{dbh}{}^{2.4}(1 + \mu_m)$$

with the estimated mean of W equal to

$$\Omega_w{}^2 = \Delta_w{}^2 + \Omega_{dbh}{}^2 + \Omega_m{}^2$$

The coefficients of variation for H, H_c, H_u, w, c, F, and e are given in Table 4. These values are determined for a tree described as case 1 where $d_o = 6$ in., $d_i = 4.6$ in., $d_o' = 5.7$ in., $d_i' = 3.3$ in., dbh = 4.8 in., and l = 360 in. The sources of measurement error, tree strength and inherent variability, and model uncertainty used in these estimates are given in Tables 1, 2, and 3.

SYMBOLS

c = critical distance between neutral axis of bending and extreme fiber
d, d' = diameter
dbh = breast height diameter
e, E = moment arm
h, H = applied horizontal force or tree resistance load
I = moment of inertia of half hollow cylinder
L = tree height
M = margin of safety
m = moisture content
u = standard unit normal deviate
v = wind speed
w = weight of tree
μ = mean
δ, Δ, Ω = coefficient of variation of measurement error, inherit variability, and total
τ = shear strength
σ = modulus of rupture or standard deviation

Subscripts

c = radial cracking
i = decay column
o = outside of tree
s = dry weight
u = ultimate or collapse load
x = model input variable
y = model output variable